Towards an intelligent 6G: A Topos Perspective

Algo. Lab./Paris Research Center

www.huawei.com

Authors: J.-C. Belfiore, V. M. Nguyen and M. Debbah
Place and Date: 6G Summit - Levi - March, 26th 2019

HUAWEI TECHNOLOGIES CO., LTD.

Outline

A.I. for 6G : New problems arising

The key idea : Topology vs Logic

Grothendieck topos

Wireless AI challenges

- In every area, we will need to solve much more complex problems ranging from channel discovery (understanding the internal structures of the wireless channel) to intelligent Radio Resource Management, to intelligent networking.

Wireless AI challenges

- In every area, we will need to solve much more complex problems ranging from channel discovery (understanding the internal structures of the wireless channel) to intelligent Radio Resource Management, to intelligent networking.
- Perceptual AI, which only solves a **global** problem, will not be enough.

Wireless AI challenges

- In every area, we will need to solve much more complex problems ranging from channel discovery (understanding the internal structures of the wireless channel) to intelligent Radio Resource Management, to intelligent networking.
- Perceptual AI, which only solves a **global** problem, will not be enough.
- We want to explore how machines can acquire human-like communication and reasoning capabilities, with the ability to recognise **new situations and environments** and adapt to them.

AI: 3rd wave

- DARPA defined the first wave of AI as enabling **"reasoning over narrowly defined problems,"** but with a poor level of certainty.

AI: 3rd wave

- DARPA defined the first wave of AI as enabling **"reasoning over narrowly defined problems,"** but with a poor level of certainty.
- The second wave, it claims, enables **"creating statistical models and training them on big data,"** albeit with minimal reasoning.

AI: 3rd wave

- DARPA defined the first wave of AI as enabling **"reasoning over narrowly defined problems,"** but with a poor level of certainty.
- The second wave, it claims, enables **"creating statistical models and training them on big data,"** albeit with minimal reasoning.
- We are at that point. To go further, we need a third wave of AI, implying to define new research directions.

What is currently missing in ML

What is missing with Deep Learning?

- Deep Understanding
- Multiple time scales to handle very long-term dependencies.
  - Better unsupervised learning.
  - Discovering the underlying causal factors.

What is currently missing in ML

What is missing with Deep Learning?

- Deep Understanding
- Multiple time scales to handle very long-term dependencies.
  - Better unsupervised learning.
  - Discovering the underlying causal factors.
- Today, machines lack contextual reasoning capabilities and their training must cover every eventuality, which is not only costly, but ultimately impossible.

Learning from the physical world

- Our machine learning models, now, only rely on superficial statistical regularities. These models are vulnerable to out-of-distribution examples.
  - Humans generalize better than other animals thanks to a more accurate internal model of the underlying causal relationships.
  - To predict future situations far from anything seen before, we need an essential component of reasoning.

Without ...

- ... intuitive physics, intuitive psychology, compositionality, causality.

Figure: A woman riding a horse on a dirty road

Without ...

- ... intuitive physics, intuitive psychology, compositionality, causality.

Figure: An airplane is parked on the tarmac at an airport

Without ...

- ... intuitive physics, intuitive psychology, compositionality, causality.

Figure: A group of people standing on top of a beach

Brain and senses

Figure: AI first wave: Brain alone (symbolic Logic)

Brain and senses

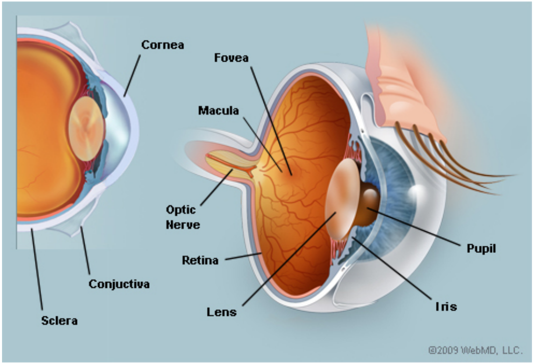


Figure: AI second wave: Senses+preprocessing (learning from data)

Brain and senses

Figure: AI third wave: Senses+preprocessing+reasoning (learning from data and ...)

Outline

A.I. for 6G : New problems arising

The key idea : Topology vs Logic

Grothendieck topos

Causality

- **Programming:** if A then B

Causality

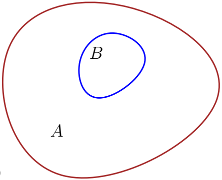
- **Programming:** if A then B
- **Probability:** Conditions where a realization A of random variable X determines the outcome B for random variable Y.

Causality

- **Programming:** if A then B
- **Probability:** Conditions where a realization A of random variable X determines the outcome B for random variable Y.
- **Logic:** Implication $A \rightarrow B$

Causality

- **Programming:** if A then B
- **Probability:** Conditions where a realization A of random variable X determines the outcome B for random variable Y.
- **Logic:** Implication $A \to B$
- **Topology:** $A \supseteq B$ or more generally $A \succeq B$ (partial order)



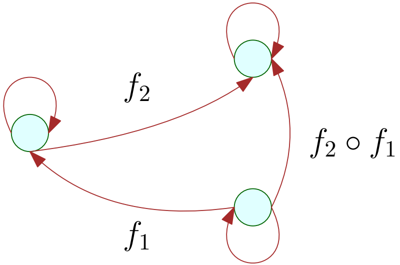
From global to modular: Compositionality

- What does deep learning do nowadays?
  - Solving a global learning problem at once.
- What it should do
  - Not always start from scratch

- Learn small subtasks
- Reuse them by composition
- **Categorical** framework to package everything.

A (very) short primer in category theory

- A category is a collection of "objects" and "arrows" satisfying some rules.



- The composition of morphisms (or arrows) is associative.
- There is an identity morphism.

- **Example 1:** The category of Sets
  - Objects are sets
  - Arrows are functions
- **Example 2:** The category Top of topological spaces
  - Objects are topological spaces
  - Arrows are continuous functions
- **Example 3:** Any oriented graph is a small category
  - Objects are vertices
  - Arrows are edges

Analogies

- "Every sufficiently good analogy is yearning to become a functor." -- **John Baez**

Analogies

- "Every sufficiently good analogy is yearning to become a functor." -- **John Baez**

- **Functor:** A functor F is a map between two categories C and D satisfying the constraints,
  - $F(\mathrm{id}_X) = \mathrm{id}_{F(X)}$ for any object $X \in C$.
  - $F(g \circ f) = F(g) \circ F(f)$

- A pair of adjoint functors (left and right) gives the best "approximation".
- A way to approximate objects in D relative to the image of F by objects in C.

From perception to reasoning

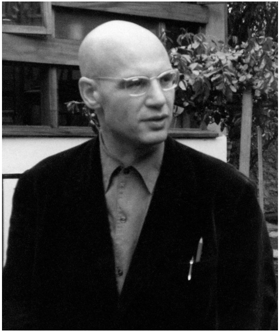
- We need to make perception and reasoning interact in a systematic way.

- Perception is related to the notion of space.
- Reasoning is related to the notion of logic.

- We also need theories.
- One important kind of theory is that which describes the fundamental processes underlying natural phenomena: once validated on sufficient data, it becomes a scientific theory

Outline

Then came Grothendieck



"C'est le thème du **topos** *qui est ce "lit", ou cette "rivière profonde" où viennent s'épouser la géométrie et l'algèbre, la topologie et l'arithmétique, la logique mathématique et la théorie des catégories, le monde du continu et celui des structures "discontinues" ou "discrètes". Il est ce que j'ai conçu de plus vaste, pour saisir avec finesse, par un même langage riche en résonances géométriques, une "essence" commune à des situations des plus éloignées les unes des autres provenant de telle région ou de telle autre du vaste univers des choses mathématiques".*
A. Grothendieck

Definition and intuition behind

Definition

A **Grothendieck topos** is the category of sheaves of Sets on a site.

- A (Grothendieck) site is a generalisation of topological space (for instance, a graph, a manifold, $\mathbb{R}^3$,...)
- The main idea is that, instead of studying the space, we may study what kind of things happens on spaces.
- These “things” are the **sheaves**.
- Sheaves of Sets are defined as **functors** between the site and the category of Sets.
- A topos is a too huge category to be apprehended. But we can do computations using representations or invariants of a topos.
  - The defining site
  - The internal logic

Definition and intuition behind

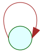
Definition

A **Grothendieck topos** is the category of sheaves of Sets on a site.

- A (Grothendieck) site is a generalisation of topological space (for instance, a graph, a manifold, $\mathbb{R}^3$,...)
- The main idea is that, instead of studying the space, we may study what kind of things happens on spaces.
- These "things" are the **sheaves**.
- Sheaves of Sets are defined as **functors** between the site and the category of Sets.
- A topos is a too huge category to be apprehended. But we can do computations using representations or invariants of a topos.
  - The defining site
  - The internal logic
    - Logic is the syntax. Topos is the semantics.

The topos of Sets vs the topological topos

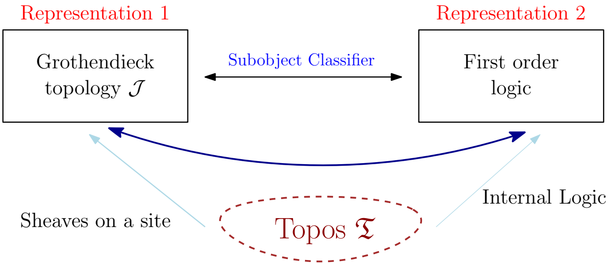
- The Grothendieck site



- The corresponding topos is the category of Sets
- The internal logic is the Boolean logic with one point of view.
- A proposition is True or False: One point of view.

- The site is the category of topological spaces.
- This topos is much huger.
- The truth values are open sets of the site.
- A proposition can be True on some open sets and False on some other ones.

Perception and reasoning



- Theories are classified by toposes (classifying topos)
- The important fact about a theory—a justification for taking the trouble to create and use it—is that it is a way of packaging propositions that work together logically to describe a world

Some thoughts

- The logic embedded in a neural network may be contained in its structure.
- The notion of topos gives a framework to learn using perception and some master/disciple training (logic).
- The notion of topos contains everything that we need to incorporate thinking and perception.

Some thoughts

- The logic embedded in a neural network may be contained in its structure.
- The notion of topos gives a framework to learn using perception and some master/disciple training (logic).
- The notion of topos contains everything that we need to incorporate thinking and perception.
- **If we want the revolution of thinking to happen, we have to learn topos theory.**

Thank you

www.huawei.com