Name:____ **Date:**______________________

Worksheet: Double Angle Identities

- $\sin(2x) = 2\sin x\cos x$
- $\cos(2x) = \cos^2 x - \sin^2 x$
  $= 1 - 2\sin^2 x$
  $= 2\cos^2 x - 1$
- $\tan(2x) = \dfrac{2\tan x}{1-\tan^2 x}$

I. Write as a single, simplified trig expression.

1. $2\sin A\cos A$
2. $\dfrac{2\tan B}{1-\tan^2 B}$
3. $1-2\sin^2 C$
4. $2\sin 105°\cos 105°$
5. $\cos^2 15° - \sin^2 15°$
6. $\dfrac{2\tan 67.5°}{1-\tan^2 67.5°}$
7. $2\cos^2 112.5° - 1$
8. $\sin 75°\cos 15° + \sin 15°\cos 75°$
9. $\dfrac{\tan 52.5° + \tan 7.5°}{1-\tan 52.5°\tan 7.5°}$
10. $\cos 127.5°\cos 7.5° + \sin 127.5°\sin 7.5°$
11. $\sin\left(\dfrac{\pi}{12}\right)\cos\left(\dfrac{\pi}{12}\right)$

II. Find the *exact* values of sin2*x*, cos2*x* and tan2*x* without using a calculator.

12. $\sin x = \dfrac{5}{13}, x$ in QII $\boxed{\left\{\dfrac{-120}{169}, \dfrac{119}{169}, \dfrac{-120}{119}\right\}}$
13. $\cos x = \dfrac{-2}{3}, x$ in QII $\boxed{\left\{\dfrac{-4\sqrt{5}}{9}, \dfrac{-1}{9}, 4\sqrt{5}\right\}}$
14. $\tan x = \dfrac{1}{2}, x$ in QIII $\boxed{\left\{\dfrac{4}{5}, \dfrac{3}{5}, \dfrac{4}{3}\right\}}$
15. $\cot x = -4, x$ in QIV $\boxed{\left\{\dfrac{-8}{17}, \dfrac{15}{17}, \dfrac{-8}{15}\right\}}$

III. Solving (#3 requires use of the quadratic formula)

16. $\sin 2x = \sin x;\ [0°, 360°)$ $\boxed{\{0°, 180°, 60°, 300°\}}$
17. $\sin 2x - \cos x = 0;\ [0°, 360°)$ $\boxed{\{90°, 270°, 30°, 150°\}}$
18. $\cos 2x + 2\sin x = 0;\ [0°, 360°)$ $\boxed{\{201.47°, 338.53°\}}$

HW: {5.4} pp. 475-476 #s 5-10all; 23-25all

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Precalculus: More Identities - From Double Angle to Power Reducing to Half-Angle

Power Reducing Identities: $\begin{cases} \sin^2 u = \dfrac{1-\cos 2u}{2} \\ \cos^2 u = \dfrac{1+\cos 2u}{2} \\ \tan^2 u = \dfrac{1-\cos 2u}{1+\cos 2u} \end{cases}$

1. Solve for $\sin^2 u$: $\cos(2u) = 1 - 2\sin^2 u$

2. Solve for $\cos^2 u$: $\cos(2u) = 2\cos^2 u - 1$

3. Determine $\tan^2 u$ using the quotient identity.

Half-Angle Identities: $\begin{cases} \sin\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{2}}, \quad \cos\dfrac{A}{2} = \pm\sqrt{\dfrac{1+\cos A}{2}} \\ \tan\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{1+\cos A}} = \dfrac{1-\cos A}{\sin A} = \dfrac{\sin A}{1+\cos A} \end{cases}$

4. Find $\sin u$: $\sin^2 u = \dfrac{1-\cos 2u}{2}$

5. Find $\cos u$: $\cos^2 u = \dfrac{1+\cos 2u}{2}$

6. Find $\tan u$: $\tan^2 u = \dfrac{1-\cos 2u}{1+\cos 2u}$

Let $2u = A$, $u = \dfrac{A}{2}$

Problems: {5.4} pp. 475-476 #s 31-36all, 50, 52

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Trigonometry: Review for Identities Quiz

ANSWERS

1. Use a **sum/difference formula** to find the <u>exact</u> value of $\cos\frac{11\pi}{12}$. Show all work.

$\boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$

2. Use a **half-angle identity** to determine the <u>exact</u> value of $\tan 67.5°$. Show all work.

$\boxed{1+\sqrt{2}}$

3. Solve over the interval **[0, 2π)**: $\sin 2x + 2\cos x = 0$

$\boxed{\left\{\frac{\pi}{2}, \frac{3\pi}{2}\right\}}$

4. Solve over the interval **[0°, 360°)**: $\cos 2x + 2 = -4\cos x - 2\cos^2 x$

$\boxed{\{120°, 240°\}}$

5. Find the *acute* angle formed by the lines $4x+3y+1=0$ and $2x-4y+1=0$

$\boxed{\theta = 79.70°}$

6. Find the *acute* angle formed by the lines $x=10$ and $y=\frac{-2}{9}x+3$.

$\boxed{\theta = 77.47°}$

7. Use the **double-angle identities** to find the <u>exact</u> values of $\sin 2x$, $\cos 2x$, and $\cot 2x$ if $\sec x = -\sqrt{5}$ and x lies in quadrant III. Draw a diagram.

$\boxed{\left\{\frac{4}{5}, \frac{-3}{5}, \frac{-3}{4}\right\}}$

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8. Matching (Some of the expressions on the right may be used more than once or not at all.)

a. $\sin(\alpha - \beta)$	i. $\sin\beta$
b. $\cos(\alpha+\beta)$	ii. $\sin\alpha\cos\beta+\cos\alpha\sin\beta$
c. $\sin(180°+\beta)$	iii. $-\cos\beta$
d. $\sin(180°-\beta)$	iv. $\cos\alpha\cos\beta+\sin\alpha\sin\beta$
e. $\cos(180°+\beta)$	v. $\sin\alpha\cos\beta-\cos\alpha\sin\beta$
f. $\sin(\alpha+\beta)$	vi. $\cos\alpha\cos\beta-\sin\alpha\sin\beta$
g. $\cos(90°-\beta)$	vii. $-\sin\beta$
h. $\cos(\alpha-\beta)$	viii. $\cos\beta$

9. Which expressions are equal to $\sin 15°$**?** (There may be more than one correct choice.)

A. $\sin 45°\cos 30°+\cos 45°\sin 30°$

B. $\sin 45°\cos 30°-\cos 45°\sin 30°$

C. $\sin 60°\cos 45°-\cos 60°\sin 45°$

D. $\cos 60°\cos 45°-\sin 60°\sin 45°$

10. Matching

a. $\sin\frac{\beta}{2}$	i. $2\sin\beta\cos\beta$
b. $\cos 2\beta$	ii. $1-2\sin^2\beta$
c. $\cos\frac{\beta}{2}$	iii. $\cos^2\beta-\sin^2\beta$
d. $\sin 2\beta$	iv. $\pm\sqrt{\frac{1+\cos\beta}{2}}$
	v. $\pm\sqrt{\frac{1-\cos\beta}{2}}$

11. Thinking Graphically – which equations have no solution?

A. $\sin x = 1$

B. $\tan x = 0.001$

C. $\sec x = \frac{1}{2}$

D. $\csc x = -3$

E. $\cos x = 1.01$

F. $\cot x = -1000$

G. $\cos x + 2 = -1$

H. $\sec x - 1.5 = 0$

I. $\sin x - 0.009 = 0.99$