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A DSmT-based Fusion Machine for Robot's Map Reconstruction

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Abstract: *Characteristics of uncertainty and imprecision, even imperfection is presented from knowledge acquisition in map reconstruction of autonomous mobile robots. Especially in the course of building grid map using sonar, this characteristic of uncertainty is especially servere. Jean Dezert and Florentin Smarandache have recently proposed a new information fusion theory (DSmT), whose greatest merit is to deal with uncertainty and conflict of information, and also proposed a series of proportional conflict redistribution rules (PRC1∼PCR5), therein, presently PCR5 is the most precise rule to deal with conflict factor according to its authors, though the complexity of computation might be increased correspondingly. In this chapter, according to the fusion machine based on the theory of DSmT coupled with PCR5, we not only can fuse information of the same reliable degree from homogeneous or heterogeneous sensors, but also the different reliable degree of evidential sources with the discounting theory. Then we established the belief model for sonar grid map, and constructed the generalized basic belief assignment function (gbbaf). Pioneer II virtual mobile robot with 16 sonar range finders on itself served as the experiment platform, which evolves in a virtual environment with some obstacles (discernable objects) and 3D Map was rebuilt online with our self-developing software platform. At the same time, we also compare it from other methods (i.e. Probability theory, Fuzzy theory and Dempster-Shafer Theory (DST)). The results of the comparison shows the new tool to have a better performance in map reconstruction of mobile robot. It also supplied with a foundation to study the Self-Localization And Mapping (SLAM) problem with the new tool further.*

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14.1 Introduction

The study on exploration of entirely unknown environment for intelligent mobile robots has been a popular and difficult subject for experts in the robotic field for a long time. Robots do not know the environment around themselves, that is, they have no experienced knowledge about the environment such as size, shape, layout of the environment, and also no signs such as beacons, landmarks, allowing them to determine their location about robot within the environment. Thus, the relation between self-localization and map building for mobile robot is like the chicken and egg problem [3, 16]. This is because if the mobile robot builds the map of the environment, then it must know the real position of itself within the environment; at the same time, if the robot wants to know its own position, then it must have a referenced map of the environment. Though it is hard to answer this question, some intelligent sensors such as odometer, electronic compass, sonar detector, laser range finder and vision sensor are installed on the mobile robot as if a person has perceptive organs.

How to manage and utilize this perceptive information acquired by organs, it's a new subject in information fusion, which will play an important role herein. As far as we know, experts have not yet given a unified expression. Just aiming to the practical field or system, proposed architecture of control such as hierarchical, concentrative, distributive and composite, and then according to the different integrated hierarchy, we compared the validity of all kinds of classical (Probability) and intelligent (Fuzzy, Neural-Networks (NN), Rough Set theory, Dempster-Shafer theory (DST), etc.) arithmetic. As far as the mobile robot is concerned, the popular arithmetic of self-localization in an unknown environment relying on interoceptive sensors (odometer, electronic compass) and exteroceptive sensors (sonar detector, laser range finder and visual sensor) is Markov location [10] or Monte Carlo location [28]. The map of the environment is built by applying some arithmetic such as Probability theory, Fuzzy Set theory and DST. The information of environment can be expressed as grid map, geometrical feature or topological map, etc., where the grid map is the most popular arithmetic expression [8, 9]. In this chapter, a new tool of the Fusion Machine based on DSmT [5, 6, 22] coupling with PCR5 is introduced to apply to the map reconstruction of mobile robots. DSmT mentioned here that has been proposed by Jean Dezert and Florentin Smarandache based on Bayesian theory and Dempster-Shafer theory [21] recently is a general, flexible and valid arithmetic of fusion. Its largest advantage is that it can deal with uncertain and imprecise information effectively, which supplies with a powerful tool to deal with uncertain information acquired by sonar detector in the course of building the grid map. Moreover, through the rule of PCR5, which is also proposed by Jean Dezert and Florentin Smarandache [23–25], we can refine and redistribute the conflict mass to improve the precision and correctness of fusion. The comparison of the new tool from other methods is done to testify it to have a better performance to solve the puzzle.

14.2 The fusion machine

14.2.1 General principle

At first, here the fusion machine is referred to a theory tool to combine and integrate the imperfect information without preprocessing it (i.e. filter the information) according to the different combination rules (i.e. DST, DSmT, etc.). It even redistributes the conflict masses to other basic belief masses according to the constraints of system using the different redistribution rules (i.e. PCR1$\sim$PCR5, minC [2], WAO [11], etc.). Of course, how to adopt the fusion rule must be considered according to the different application. Here we consider the application, and give a special fusion machine (shown in Fig. 14.1). In Fig. 14.1, k sources of evidences (i.e. the inputs) provide basic belief assignments over a propositional space generated by elements of a frame of discernment and set operators endowed with eventually a given set of integrity constraints, which depend on the nature of elements of the frame. The set of belief assignments need then to be combined with a fusion operator. Since in general the combination of uncertain information yields a degree of conflict, say K, between sources, this conflict must be managed by the fusion operator/machine. The way the conflict is managed is the key of the fusion step and makes the difference between the fusion machines. The fusion can be performed globally/optimally (when combining the sources in one derivation step all together) or sequentially (one source after another as in Fig. 14.1). The sequential fusion processing (well adapted for temporal fusion) is natural and more simple than the global fusion but in general remains only suboptimal if the fusion rule chosen is not associative, which is the case for most of fusion rules, but Dempster's rule. In this chapter, the sequential fusion based on the PCR5 rule is chosen because PCR5 has shown good performances in works and because the sequential fusion is much more simple to implement and to test. The *optimal* (global) PCR5 fusion rule formula for k sources is possible and has also been proposed [23] but is much more difficult to implement and has not been tested yet. A more efficient PCR rule (denoted PCR6) proposed very recently by Martin and Osswald in [15], which outperforms PCR5, could be advantageously used in the fusion machine instead PCR5. Such idea is currently under investigation and new results will be reported in a forthcoming publication. We present in more details in next section the DSmT-based fusion machine.

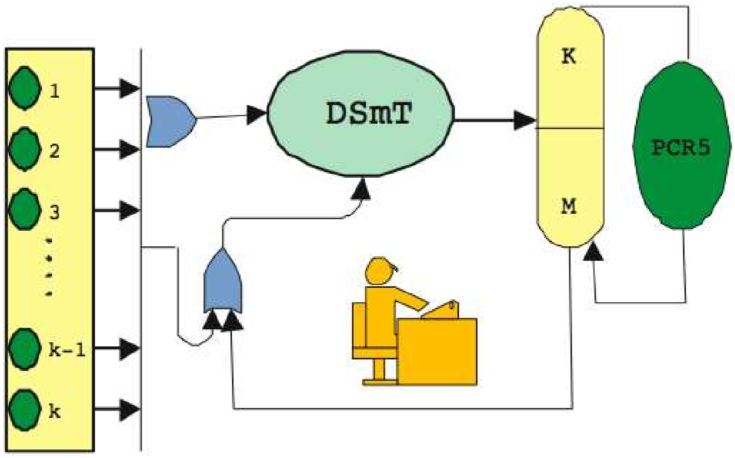

Figure 14.1: A kind of sequential fusion machine

14.2.2 Basis of DSmT

DSmT (Dezert-Smarandache Theory) is a new, general and flexible arithmetic of fusion, which can solve the fusion problem of different tiers including data-tier, feature-tier and decision-tier, and even, not only can solve the static problem of fusion, but also can solve the dynamic one. Especially, it has a prominent merit that it can deal with uncertain and highly conflicting information [5, 6, 22].

14.2.2.1 Simple review of DSmT

1) Let $\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}$, be the frame of discernment, which includes n finite focal elements $\theta_i (i = 1, \cdots, n)$. Because the focal elements are not precisely defined and separated, so that no refinement of Θ in a new larger set Ω_{ref} of disjoint elementary hypotheses is possible.
2) The hyper-power set D^Θ is defined as the set of all compositions built from elements of Θ with $\cup$ and $\cap$ (Θ generates D^Θ under operators $\cup$ and $\cap$) operators such that

a) $\emptyset, \theta_1, \theta_2, \cdots, \theta_n \in D^\Theta$.

b) If $A, B \in D^\Theta$, then $A \cap B \in D^\Theta$ and $A \cup B \in D^\Theta$.

c) No other elements belong to D^Θ, except those obtained by using rules a) or b).

3) General belief and plausibility functions
Let $\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}$ be the general frame of discernment. For every evidential source S, let us define a set of map of $m(\cdot) : D^\Theta \to [0, 1]$ associated to it (abandoning Shafer's model) by assuming here that the fuzzy/vague/relative nature of elements $\theta_i (i = 1, \cdots, n)$ can be non-exclusive, as well as no refinement of Θ into a new finer exclusive frame of discernment Θ_{ref} is possible. The mapping $m(\cdot)$ is called a generalized basic belief assignment function if it satisfies

$$m(\emptyset) = 0 \qquad \text{and} \qquad \sum_{A \in D^\Theta} m(A) = 1,$$

then $m(A)$ is called A's generalized basic belief assignment function (gbbaf). The general belief function and plausibility function are defined respectively in almost the same manner as within the DST, i.e.

$$\text{Bel}(A) = \sum_{B \in D^\Theta, B \subseteq A} m(B) \tag{14.1}$$

$$\text{Pl}(A) = \sum_{B \in D^\Theta, B \cap A \neq \emptyset} m(B) \tag{14.2}$$

4) Classical (free) DSm rule of combination
Let $\mathcal{M}^f(\Theta)$ be a free DSm model. The classical (free) DSm rule of combination (denoted (DSmC) for short) for $k \geq 2$ sources is given $\forall A \neq \emptyset, andA \in D^\Theta$as follows:

$$m_{\mathcal{M}^f(\Theta)}(A) \cong [m_1 \oplus \cdots \oplus m_k](A) = \sum_{\substack{X_1, \cdots, X_k \in D^\Theta \\ X_1 \cap \cdots \cap X_k = A}} \prod_{i=1}^{k} m_i(X_i) \tag{14.3}$$

14.2.2.2 Fusion of unreliable sources

1) On the necessity of discounting sources
In fact, sources of information are unreliable in real systems due to the sources with different knowledge and experience. For example, from the point of view of the mobile robots' sensors, the metrical precision and resolution with laser range finder are both higher than that with sonar sensor. Even if they are the same sonar sensors, then they have also different precision due to the manufacturing and other factors. Under this condition, if we treat data of unreliable information sources as data of reliable sources to be fused, then the result is very unreliable

and even reverses. Thus, unreliable resources must be considered, and then DSmT based on the discounting method [7, 12, 21, 26] does well in dealing with unreliable sensors.

2) Principle of discounting method

Let's consider k evidential sources of information $(S_1, S_2, \ldots, S_k)$, here we work out a uniform way in dealing with the homogeneous and heterogeneous information sources. So we get the discernment frame $\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}$, $m(\cdot)$ is the basic belief assignment, let $m_i(\cdot) : D^\Theta \to [0,1]$ be a set of maps, and let p_i represent reliable degree supported by S_i $(i = 1, 2, \ldots, k)$, considering $\sum_{A\in D^\Theta} m_i(A) = 1$, let $I_t = \theta_1 \cup \theta_2 \cup \cdots \cup \theta_n$ express the total ignorance, and then let $m_i^g(I_t) = 1 - p_i + p_i m_i(I_t)$ represent the belief assignment of the total ignorance for global system (*after discounting*), and then this is because of existing occurrence of malfunction, that is, $\sum_{A\in D^\Theta} m_i(A) = p_i$, we assign the quantity $1-p_i$ to the total ignorance again. Thus, the rule of combination for DSmT based on discounting method with $k \geq 2$ evidential sources is given as in the formula (14.3), i.e. the conjunctive consensus on the hyper-power set by $m^g_{\mathcal{M}^f(\Theta)}(\emptyset) = 0$ and $\forall A \neq \emptyset \in D^\Theta$,

$$m^g_{\mathcal{M}^f(\Theta)}(A) \cong [m_1^g \oplus \ldots \oplus m_k^g](A) = \sum_{\substack{X_1,\cdots,X_k \in D^\Theta \\ X_1\cap\cdots\cap X_k = A}} \prod_{i=1}^{k} p_i m_i(X_i) \tag{14.4}$$

14.2.3 The PCR5 fusion rule

When integrity constraints are introduced in the model, one has to deal with the conflicting masses, i.e. all the masses that would become assigned to the empty set through the DSmC rule. Many fusion rules (mostly based on Shafer's model) have been proposed [20] for managing the conflict. Among these rules, Dempster's rule [21] redistributes the total conflicting mass over all propositions of 2^Θ through a simple normalization step. This rule has been the source of debates and criticisms because of its unexpected/counter-intuitive behavior in some cases. Many alternatives have then been proposed [20, 22] for overcoming this drawback. In DSmT, we have first extended the Dubois & Prade's rule [7, 22] for taking into account any integrity constraints in the model and also the possible dynamicity of the model and the frame. This first general fusion rule, called DSmH (DSm Hybrid) rule, consists just in transferring the partial conflicts onto the partial ignorances[1]. The DSmH rule has been recently and advantageously replaced by the more sophisticated Proportional Conflict Redistribution rule no.5 (PCR5). According to Smarandache and Dezert, PCR5 does a better redistribution of the conflicting mass than Dempster's rule since PCR5 goes backwards on the tracks of the conjunctive rule and redistributes the partial conflicting masses only to the sets involved in the conflict and proportionally to their masses put in the conflict, considering the conjunctive normal form of the partial conflict. PCR5 is quasi-associative and preserves the neutral impact of the vacuous belief assignment. Since PCR5 is presented in details in [23], we just remind PCR5 rule for only two sources[2]: $m_{PCR5}(\emptyset) = 0$, and for all $X \in G \setminus \{\emptyset\}$,

$$m_{PCR5}(X) = m_{12}(X) + \sum_{\substack{Y\in G\setminus\{X\} \\ X\cap Y=\emptyset}} \left[\frac{m_1^2(X) m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2^2(X) m_1(Y)}{m_2(X) + m_1(Y)} \right], \tag{14.5}$$

[1] Partial ignorance being the disjunction of elements involved in the partial conflicts.

[2] A general expression of PCR5 for an arbitrary number ($s > 2$) of sources can be found in [23].

where all sets are in canonical form and $m_{12}(X) = \sum_{\substack{X_1,X_2 \in G^\Theta \\ X_1 \cap X_2}} m_1(X_1) \cdot m_2(X_2)$ corresponds to the conjunctive consensus on X between the two sources and where all denominators are different from zero. If a denominator is zero, that fraction is discarded.

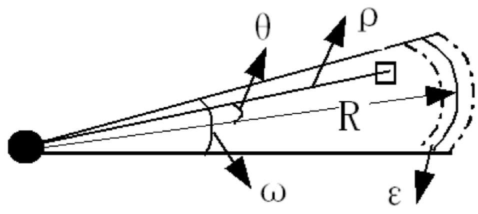

Figure 14.2: Sketch of the principle of sonar

14.3 Modeling of Sonar Grid Map Building Based on DSmT

Here we mainly discuss a sonar sensor, whose working principle (shown as Fig. 14.2) is: producing sheaves of cone-shaped wave and detecting the objects by receiving the reflected wave. Due to the restriction of sonar physical characteristic, metrical data has uncertainty as follows:
a) Beside its own error of making, the influence of external environment is also very great, for example, temperature, humidity, atmospheric pressure and so on.
b) Because the sound wave spreads outwards through a form of loudspeaker, and there exists a cone-shaped angle, we cannot know the true position of object detected among the fan-shaped area, with the enlargement of distance between sonar and it.
c) The use of many sonar sensors will result in interference with each other. For example, when the i-th sonar gives out detecting wave towards an object of irregular shape, if the angle of incidence is too large, the sonar wave might be reflected out of the receiving range of the i-th sonar sensor or also might be received by other sonar sensors.
d) Because sonar sensors utilize the reflection principle of sound wave, if the object absorbs most of heavy sound wave, the sonar sensor might be invalid.

Pointing to the characteristics of sonar's measurement, we construct a model of uncertain information acquired from grid map using sonar based on DSmT. Here we suppose there are two focal elements in system, that is, $\Theta = \{\theta_1, \theta_2\}$. Where θ_1 means grid is empty, θ_2 means occupied, and then we can get its hyper-power set $D^\Theta = \{\emptyset, \theta_1 \cap \theta_2, \theta_1, \theta_2, \theta_1 \cup \theta_2\}$. Every grid in environment is scanned $k \geq 5$ times, each of which is viewed as source of evidence. Then we may define a set of map aiming to every source of evidence and construct the general basic belief assignment functions (gbbaf) as follows: $m(\theta_1)$ is defined as the gbbaf for grid-unoccupied (empty); $m(\theta_2)$ is defined as the gbbaf for grid-occupied; $m(\theta_1 \cap \theta_2)$ is defined as the gbbaf for holding grid-unoccupied and occupied simultaneous (conflict). $m(\theta_1 \cup \theta_2)$ is defined as the gbbaf for grid-ignorance due to the restriction of knowledge and present experience (here referring to the gbbaf for these grids still not scanned presently), it reflects the degree of ignorance of grid-unoccupied or occupied.

The gbbaf of a set of map $m(\cdot) : D^\Theta \to [0, 1]$ is constructed by authors such as the formulae (14.6)∼(14.9) according to sonar physical characteristics.

$$m(\theta_1) = E(\rho)E(\theta) = \begin{cases} (1-(\rho/R)^2)\lambda \quad \text{if} & \begin{cases} R_{min} \leq \rho \leq R \leq R_{max} \\ 0 \leq \theta \leq \omega/2 \end{cases} \\ 0 \quad \text{otherwise} \end{cases} \tag{14.6}$$

$$m(\theta_2) = O(\rho)O(\theta) = \begin{cases} e^{-3\rho_v(\rho-R)^2}\lambda \quad \text{if} & \begin{cases} R_{min} \leq \rho \leq R+\epsilon \leq R_{max} \\ 0 \leq \theta \leq \omega/2 \end{cases} \\ 0 \quad \text{otherwise} \end{cases} \tag{14.7}$$

$$m(\theta_1 \cap \theta_2) = \begin{cases} [1-[\frac{2(\rho-R+2\epsilon)}{R}]^2]\lambda \quad \text{if} & \begin{cases} R_{min} \leq \rho \leq R \leq R_{max} \\ 0 \leq \theta \leq \omega/2 \end{cases} \\ 0 \quad \text{otherwise} \end{cases} \tag{14.8}$$

$$m(\theta_1 \cup \theta_2) = \begin{cases} \tanh(2(\rho-R))\lambda \quad \text{if} & \begin{cases} R \leq \rho \leq R_{max} \\ 0 \leq \theta \leq \omega/2 \end{cases} \\ 0 \quad \text{otherwise} \end{cases} \tag{14.9}$$

where $\lambda = E(\theta) = O(\theta)$ is given by (see [8] for justification)

$$\lambda = \begin{cases} 1-(2\theta/\omega)^2 \quad \text{if} \quad 0 \leq |\theta| \leq \omega/2 \\ 0 \quad \text{otherwise} \end{cases} \tag{14.10}$$

where ρ_v in formula (14.7) is defined as an environment adjusting variable, that is, the less the object is in environment, the greater the variable ρ_v is, and makes the function of $m(\theta_2)$ more sensitive. Here let ρ_v be one. $E(\cdot)$ and $O(\cdot)$ are expressed as the *Effect Function* of ρ, θ to grid's empty or occupancy. In order to insure the sum of all masses to be one, we must renormalize it. The analysis on the characteristics of gbbaf are shown as Fig. 14.3∼Fig. 14.7, when $R = 1.5m$.

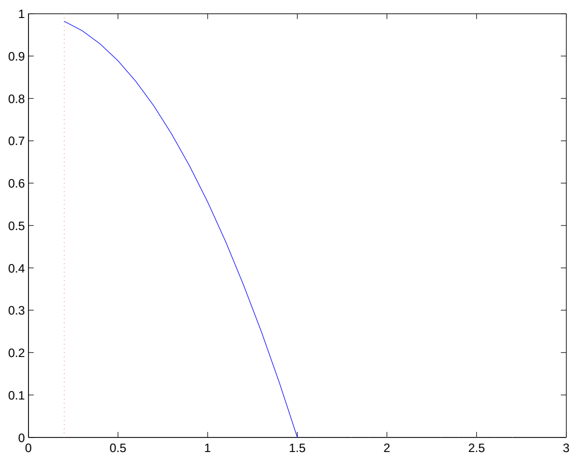

Figure 14.3: $m(\theta_1)$ as function of ρ given by (14.6)

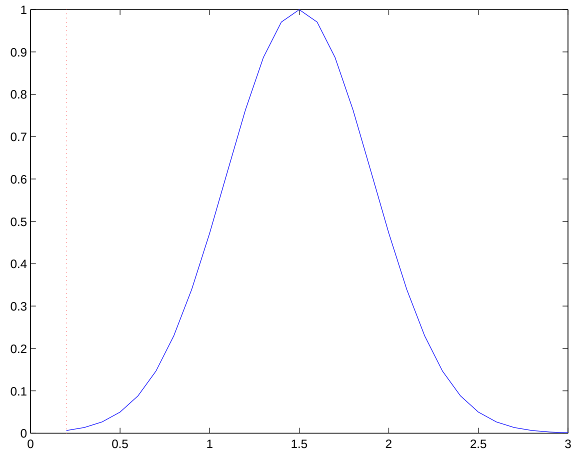

Figure 14.4: $m(\theta_2)$ as function of ρ given by (14.7)

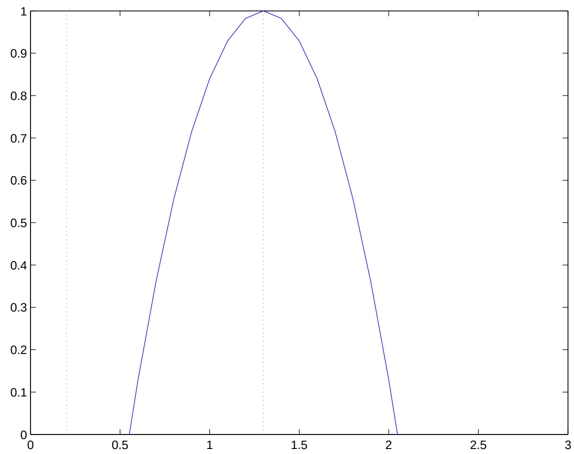

Figure 14.5: $m(\theta_1 \cap \theta_2)$ as function of ρ given by (14.8)

Seen from Fig. 14.3, $m(\theta_1)$ has a falling tendency with the increasing of distance between grid and sonar, and has the maximum at R_{min} and zero at R. From the point of view of the working principle of sonar, the more the distance between them approaches the measured value, the more that grid might be occupied. Thus the probability that grid indicated is empty is very low, of course the gbbaf of grid-unoccupied is given a low value.

From Fig. 14.4, $m(\theta_2)$ takes on the distribution of Gaussian function with respect to the addition of distance between them, has the maximum at R, which answers for the characteristic of sonar acquiring information.

From Fig. 14.5, $m(\theta_1 \cap \theta_2)$ takes on the distribution of a parabola function with respect to the addition of distance between them. In fact, when $m(\theta_1)$ equals $m(\theta_2)$, $m(\theta_1 \cap \theta_2)$ has the maximum there. But it is very difficult and unnecessary to find the point of intersection of the two functions. Generally, we let the position of $R - 2\varepsilon$ replace the point of intersection. Experience indicates that its approximate value is more rational.

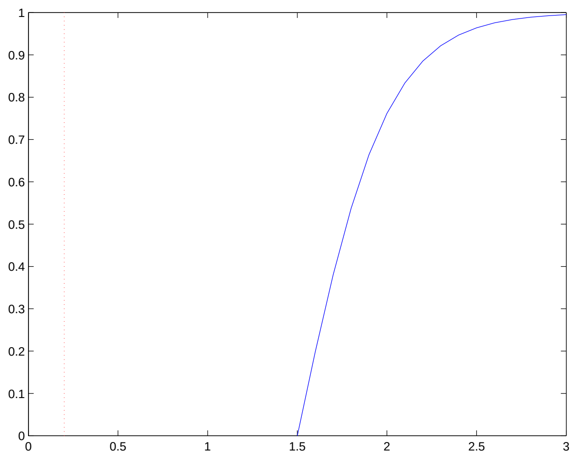

Figure 14.6: $m(\theta_1 \cup \theta_2)$ as function of ρ given by (14.9)

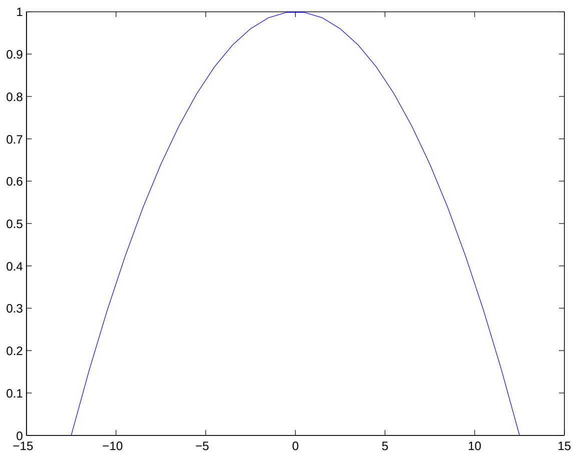

Figure 14.7: λ as function of θ given by (14.10)

From Fig. 14.6, $m(\theta_1 \cup \theta_2)$ takes on the distribution of the hyperbola function with respect to the addition of distance between them, and zero at R. This function reflects well the ignorance of grid information at $R \leq \rho \leq R_{max}$. The relation between θ and λ is reflected in Fig. 14.7, where the more the position of grid approaches the central axis, the greater λ becomes, that is, the greater the contribution to belief assignment is. Otherwise, the lower it is. In short, the general basic belief assignment functions (gbbaf) entirely fit with the characteristic of sonar acquiring information. This supplies a theoretic foundation for dealing with uncertain information in grid map building.

14.4 Sonar Grid Map Building Based on Other Methods

To apply the probability theory and fuzzy set theory to map building, at first, two functions of uncertainty are introduced. Here the working environment U of robot is separated into $m \times n$

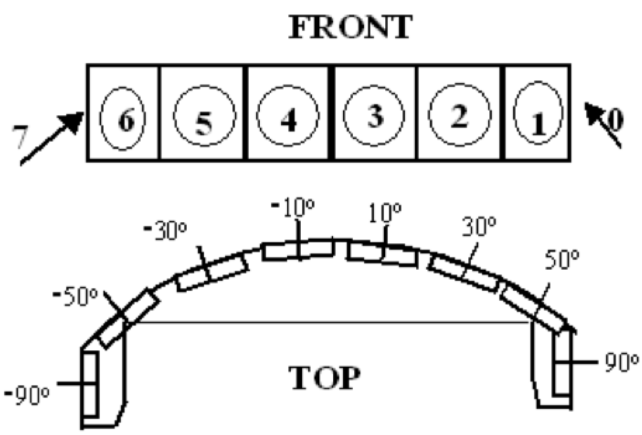

Figure 14.8: Sketch of the layout of sonars

rectangle grids of same size. Every grid is represented by G_{ij}, $U = \{G_{ij}|i \in [1,m], j \in [1,n]\}$, according to the reference [29]. Two functions are applied to represent the uncertainty of sonar as follows:

$$\Gamma(\theta) = \begin{cases} 1 - 21\left(\frac{\theta\pi}{180}\right)^2, & \text{if } 0 \leq |\theta| \leq 12.5^o, \\ 0, & \text{if } |\theta| > 12.5^o. \end{cases} \quad (14.11)$$

$$\Gamma(\rho) = 1 - (1 + \tanh(2(\rho - \rho_v)))/2, \quad (14.12)$$

where θ represents the angle between the center-axis and the spot (i,j) measured in Fig. 14.2. ρ_v is the pre-defined value, which reflects the smooth transferring point from the certainty to uncertainty. $\Gamma(\theta)$ shows that the nearer by center- axis is the spot (i,j), the larger is the density of the wave. $\Gamma(\rho)$ shows that the farther away from the sonar is it, the lower is the reliability, while the nearer by the sonar it is, the higher is the reliability of correct measurement.

1) Probability Theory

Elfes and Moravec [8, 9] firstly represented the probability of the grid occupied by obstacles with probability theory. Then Thrun, Fox and Burgard [27], Olson [17], Romero and Morales [19] also proposed the different methods of map reconstruction by themselves based on probability theory. According to the above methods, we give the general description of map building based on probability theory. To avoid an amount of computation, we suppose that all grids are independent. For every grid G_{ij}, let $s(G_{ij}) = E$ represent the grid empty, while $s(G_{ij}) = O$ represent the grid occupied and $P[s(G_{ij}) = E]$ and $P[s(G_{ij}) = O]$ the probabilities of these events with the constraint $P[s(G_{ij}) = E] + P[s(G_{ij}) = O] = 1$. According to the physical characteristics, the probability model to map the sonar perception datum is given by

$$P\left[s(G_{ij})=O|R\right]=P\left[s(\rho,\theta)=O|R\right]=\begin{cases}(1-\lambda')/2, & \text{if } 0\le\rho<R-2\varepsilon,\\ [1-\lambda'\left(1-(2+a)^2\right)]/2, & \text{if } R-2\varepsilon\le\rho<R-\varepsilon,\\ [1+\lambda'\left(1-a^2\right)]/2, & \text{if } R-\varepsilon\le\rho<R+\varepsilon,\\ 1/2, & \text{if } \rho\ge R+\varepsilon\end{cases} \tag{14.13}$$

where $\lambda'=\Gamma(\theta)\cdot\Gamma(\rho)$ and $a=(\rho-R)/\varepsilon$. Seen from Eq. (14.13), the mapping between the sonar data and probability answers for the physical characteristics of sonar. For the data outside the measurement range, the probability value is 0.5, that is, the uncertainty is the largest. When the distance between the grid in the range and the sonar is less than the measurement, the nearer by the sonar it is, the less is the possibility of grid occupied, while the nearer by the location of the measurement is the grid, and the larger is the possibility of grid occupied.

The fusion algorithm for multi-sensors is given according to the Bayesian estimate as follows:

$$P\left[s(G_{ij})=O|R_1,\cdots,R_{k+1}\right]=\frac{P\left[s(G_{ij})=O|R_{k+1}\right]\cdot P\left[s(G_{ij})=O|R_1,\cdots,R_k\right]}{\sum\limits_{X\in\{E,O\}}P\left[s(G_{ij})=X|R_{k+1}\right]\cdot P\left[s(G_{ij})=X|R_1,\cdots,R_k\right]} \tag{14.14}$$

Remark: In order to make the equation (14.14) hold, we must suppose at the beginning of map building

$$\forall G_{ij}\in U,\qquad P\left[s(G_{ij})=E\right]=P\left[s(G_{ij})=O\right]=0.5$$

2) Fuzzy Set Theory

Map building based on fuzzy logic is firstly proposed by Giuseppe Oriolo et al. [18]; they define two fuzzy sets Ψ (represents grid empty) and Ω (represents grid occupied), which of size all are equal to U, correspondingly, their membership functions are μ_Ψ and μ_Ω. Similarly, we can get fuzzy model to map the sonar perception datum.

$$\mu^{S(R)}\Psi(G_{ij})=\lambda'\cdot f_\Psi(\rho,R) \tag{14.15}$$

$$\mu^{S(R)}\Omega(G_{ij})=\lambda'\cdot f_\Omega(\rho,R), \tag{14.16}$$

where

$$f_\Psi(\rho,R)=\begin{cases}k_E, & \text{if } 0\le\rho<R-\varepsilon,\\ k_E((R-\rho)/\varepsilon)^2, & \text{if } R-\varepsilon\le\rho<R,\\ 0, & \text{if } \rho\ge R.\end{cases}$$

$$f_\Omega(\rho,R)=\begin{cases}k_O, & \text{if } 0\le\rho<R-\varepsilon,\\ k_O((R-\rho)/\varepsilon)^2, & \text{if } R-\varepsilon\le\rho<R+\varepsilon,\\ 0, & \text{if } \rho\ge R.\end{cases}$$

Here, $f_\Psi(\rho,R)$ represents the influence of ρ,R on the membership μ_Ψ of grid G_{ij}. $f_\Omega(\rho,R)$ represents the influence of ρ,R on the membership μ_Ω of grid G_{ij}. k_E and k_O are constants with $0<k_E\le1$ and $0<k_O\le1$. λ' is same as the definition in (14.13). Seen from the Eq.

(14.15) and (14.16), if the grid has the great possibility of occupation, then the membership is μ_Ψ small, while the membership μ_Ω is great. If the grid is outside the measuring range, then $\mu_\Psi = 0$ and $\mu_\Omega = 0$. The fusion algorithm for multi-sensors is given according to the union operator in the fuzzy theory as follow: $\forall X \in \{\Psi, \Omega\}$,

$$\mu_X^{S(R_1,\cdots,R_{k+1})}(G_{ij}) = \mu_X^{S(R_1,\cdots,R_k)}(G_{ij}) + \mu_X^{S(R_{k+1})(G_{ij})} - \mu_X^{S(R_1,\cdots,R_k)}(G_{ij}) \cdot \mu_X^{S(R_{k+1})}(G_{ij}). \quad (14.17)$$

Remark: Initially, we suppose $\mu_\Psi(G_{ij}) = \mu_\Omega(G_{ij}) = 0$ $(\forall G_{ij} \in U)$. According to membership of every grid, we can get the final map representation as follows:

$$M = \overline{\Psi^2 \cap \overline{\Omega} \cap \overline{A} \cap \overline{I}} \quad (14.18)$$

Here, $A = \Psi \cap \Omega$, $I = \overline{\Psi} \cap \overline{\Omega}$. At the same time, the rule of fuzzy intersection is

$$\mu_{i \cap j}(G_{ij}) = \mu_i(G_{ij}) \cdot \mu_j(G_{ij}),\ \forall\ G_{ij} \in U,$$

and the rule of fuzzy complementation is

$$\mu_{\overline{i}}(G_{ij}) = 1 - \mu_i(G_{ij}),\ \forall\ G_{ij} \in U.$$

The larger is the membership of G_{ij} belonging to the fuzzy set M, the greater is the possibility of the grid occupied.

3) Dempster-Shafer Theory (DST)

The idea of using belief functions for representing someone's subjective feeling of uncertainty was first proposed by Shafer [21], following the seminal work of Dempster [4] about upper and lower probabilities induced by a multi-valued mappings. The use of belief functions as an alternative to subjective probabilities for representing uncertainty was later justified axiomatically by Smets [26], who introduced the Transferable Belief Model (TBM), providing a clear and coherent interpretation of the various concepts underlying the theory.

Let θ_i $(i = 1, 2, \ldots, n)$ be some exhaustive and exclusive elements (hypotheses) of interest taking on values in a finite discrete set Θ, called the frame of discernment. Let us assume that an agent entertains beliefs concerning the value of θ_i, given a certain evidential corpus. We postulate that these beliefs may be represented by a belief structure (or belief assignment), i.e. a function from 2^Θ to $[0, 1]$ verifying $\sum_{A \subseteq \Theta} m(A) = 1$ and $m(\emptyset) = 0$ for all $A \subseteq \Theta$, the quantity $m(A)$ represents the mass of belief allocated to proposition "$\theta_i \subseteq A$", and that cannot be allocated to any strict sub-proposition because of lack of evidence. The subsets A of Θ such that $m(A) > 0$ are called the focal elements of m. The information contained in the belief structure may be equivalently represented as a belief function *bel*, or as a plausibility function *pl*, defined respectively as $bel(A) = \sum_{B \subseteq A} m(B)$ and $pl(A) = \sum_{B \cap A \neq \emptyset} m(B)$. The quantity $bel(A)$, called the belief in A, is interpreted as the total degree of belief in A (i.e. in the proposition "$\theta_i \subseteq A$"), whereas $pl(A)$ denotes plausibility of A, i.e. the amount of belief that could potentially be transferred to A, taking into account the evidence that does not contradict that hypothesis.

Now we assume the simplest situation that two distinct pieces of evidence induce two belief structures m_1 and m_2. The orthogonal sum of m_1 and m_2, denoted as $m = m_1 \oplus m_2$ is defined as:

$$m(A) = K^{-1} \sum_{B \cap C = A} m_1(B) m_2(C), \quad (14.19)$$

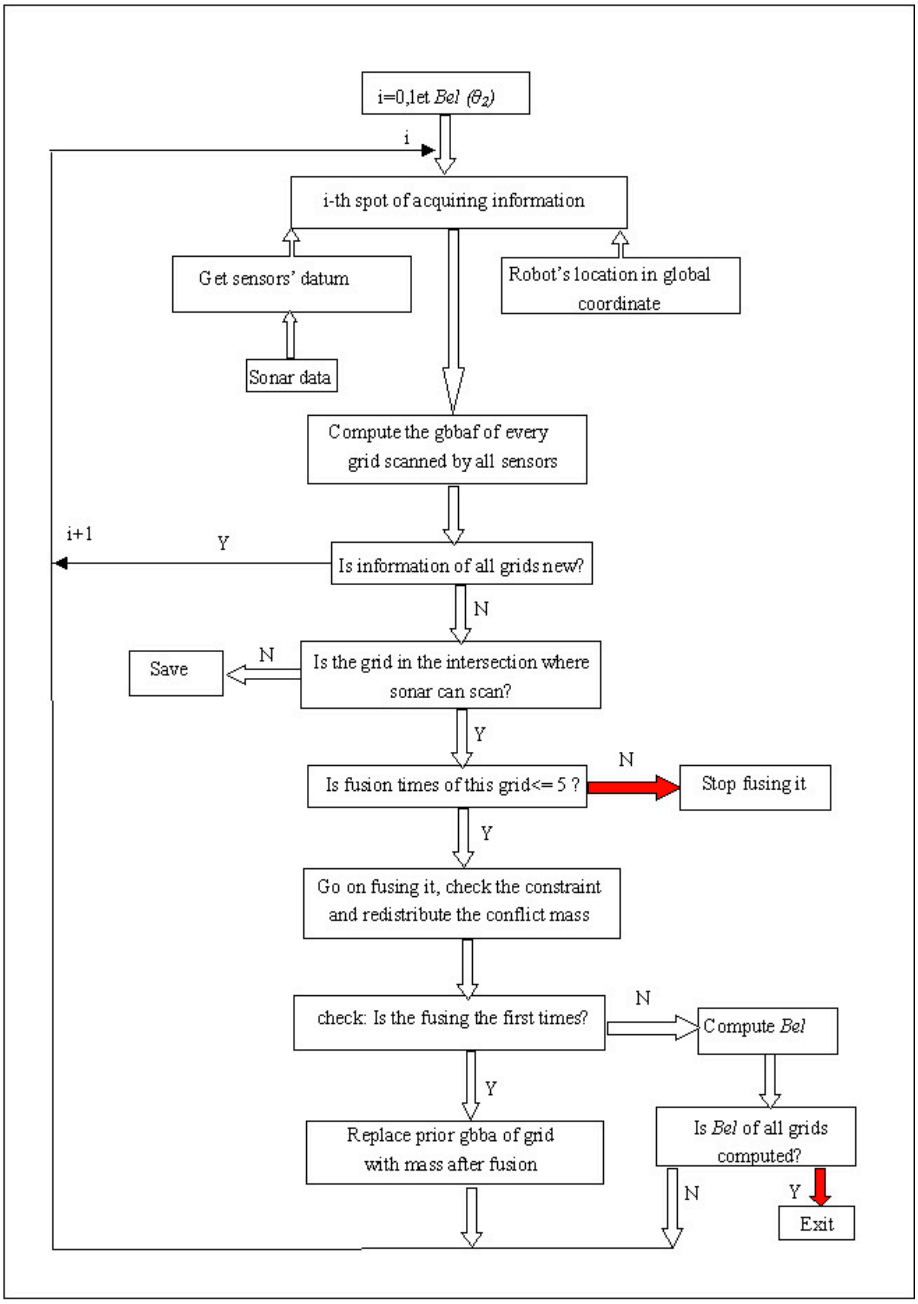

Fig.9: Flowchart of procedure of sonar map building based on DSmT

Figure 14.9: Flowchat of procedure of sonar map building based on DSmT and PCR5

Here $K = 1 - \sum\limits_{B\cap C=\emptyset} m_1(B)m_2(C)$ for $A \neq \emptyset$ and $m(\emptyset) = 0$. The orthogonal sum (also called Dempster's rule of combination) is commutative and associative. It plays a fundamental operation for combining different evidential sources in evidence theory. DST as an information fusion method has been applied to the environment exploration and map reconstruction [1, 13]. This method can assure to have a precise result in fusing the same or different multi-sources information from the sensors on the robot. According to the requirement of sonar grid map building, we need only to consider a simple 2D frame $\Theta = \{\theta_1, \theta_2\}$, where θ_1 means "the grid is empty", and θ_2 means "the grid is occupied", and then we work with basic belief assignments defined on its power set $2^\Theta = \{\emptyset, \theta_1, \theta_2, \theta_1 \cup \theta_2\}$. According to DST, let $m_{DST}(\emptyset) = 0$, here $m_{DST}(\theta_1)$ is defined as the basic belief assignment function (bbaf) for grid-empty, $m_{DST}(\theta_2)$ is defined as the basic belief assignment function (bbaf) for grid-occupied, $m_{DST}(\theta_1 \cup \theta_2)$ is defined as the basic belief assignment function for grid-ignorance. We may also construct basic belief assignment function such as $m_{DST}(\theta_1) = m(\theta_1)$, $m_{DST}(\theta_2) = m(\theta_2)$, $m_{DST}(\theta_1 \cup \theta_2) = m(\theta_1 \cup \theta_2)$. bbaf reflects still the characteristics of uncertainty for sonar grip map building in Fig. 14.2. Though here we define the same bbaf as DSmT, considering the definition of DST must be satisfied, we must renormalize them while acquiring sonar grid information [13, 14]. The new basic belief assignment after fusing two evidence sources from the sonar range finders can be obtained by the combination rule in Eq. (14.19).

14.5 Simulation Experiment

The experiment consists in simulating the autonomous navigation of a virtual Pioneer II Robot carrying 16 simulated sonar detectors in a 5000mm$\times$5000mm square array with an unknown obstacle/object. The map building with sonar sensors on the mobile robot is done from the simulator of SRIsim (shown in Fig.14.10) of ActivMedia company and our self-developing experimental or simulation platform together. (shown in Fig. 14.11). Here the platform developed with the tool software of visual c++ 6.0 and OpenGL servers as a client end, which can connect the server end (also developed by ourselves, which connects the SRIsim and the client). When the virtual robot runs in the virtual environment, the server end can collect many information (i.e. the location of robot, sensors reading, velocity .etc) from the SRIsim. Through the protocol of TCP/IP, the client end can get any information from the server end and fuse them. The Pioneer II Robot may begin to run at arbitrary location; here we choose the location (1500mm, 2700mm) with an 88 degrees angle the robot faces to. We let the robot move at speeds of transverse velocity 100mm/s and turning-velocity 50degree/s around the object in the *world* map plotted by the Mapper (a simple plotting software), which is opened in the SRIsim shown in Fig. 14.10.

We adopt grid method to build map. Here we assume that all the sonar sensors have the same reliability. The global environment is divided into 50×50 lattices (which of size are same). The object in Fig. 14.10 is taken as a regular rectangular box. When the virtual robot runs around the object, through its sonar sensors, we can clearly recognize the object and know its appearance, and even its location in the environment. To describe the experiment clearly, the flowchart of procedure of sonar map is given in Fig. 14.9. The main steps of procedure based on the new tool are given as follows:

1. Initialize the parameters of robot (i.e. initial location, moving velocity, etc.).

2. Acquire 16 sonar readings, and robot's location, when the robot is running (Here we set the first timer, of which interval is 100 ms.).

3. Compute gbbaf of the fan-form area detected by each sonar sensor.

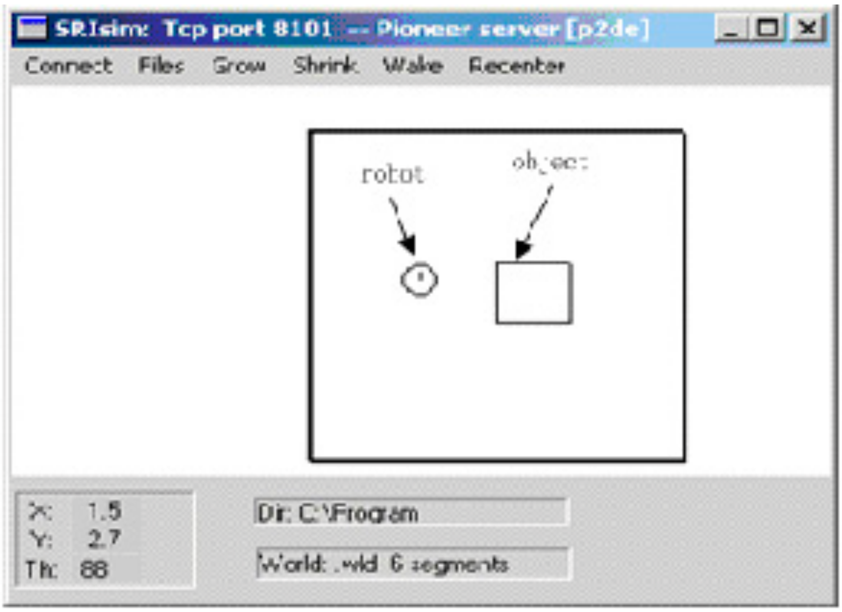

Figure 14.10: A world map opened in the SRIsim

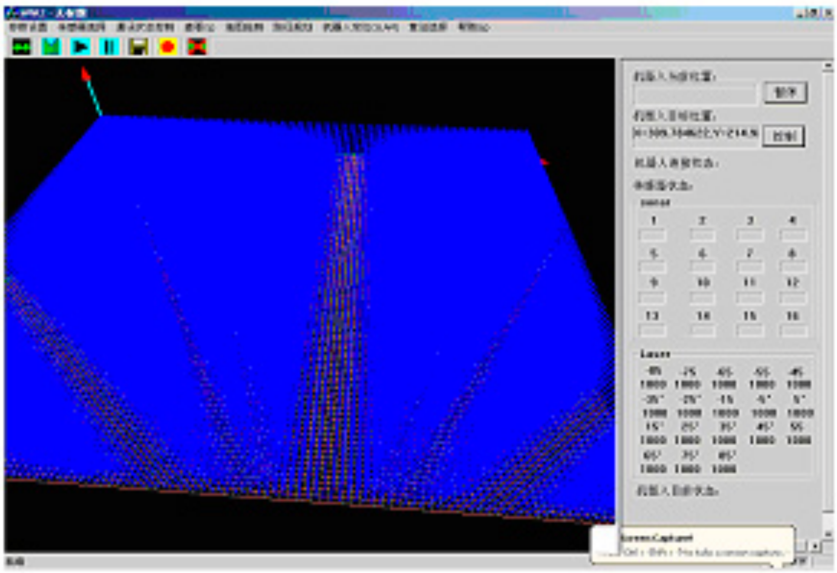

Figure 14.11: The platform for simulation or real experiment

4. Check whether some grids are scanned more than 5 times by sonar sensors (Same sonar in different location, or different sonar sensors. Of course, here we suppose each sonar sensor has the same characteristics.)? If "yes", then go to next step, otherwise, go to step 2.

5. According to the combination rule and the PCR5 in (14.3) and (14.5) respectively, we can get the new basic belief masses, and redistribute the conflicting mass to the new basic belief masses in the order of the sequential fusion, until all 5 times are over.

6. Compute the credibility/total belief of occupancy $bel(\theta_2)$ of some grids, which have been fused according to (14.1).

7. Update the map of the environment. (Here we set the second timer, of which interval is 100 ms) Check whether all the grids have been fused? If "yes", then stop robot and exit. Otherwise, go to step 2.

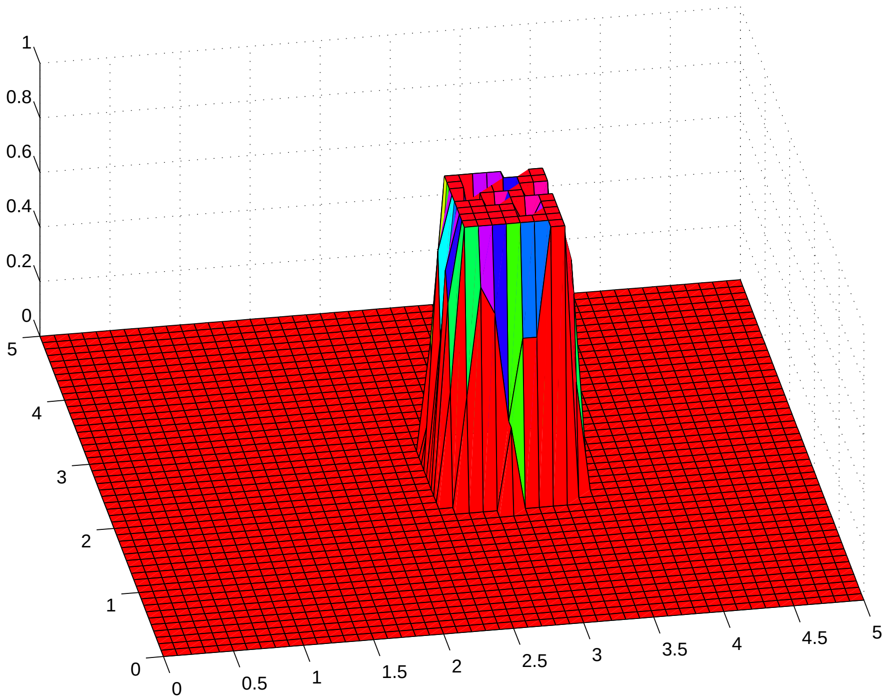

Figure 14.12: Map reconstruction with DSmT coupling with PCR5 (3D)

Finally, we rebuild the map shown in the Fig. 14.12 (3D) and Fig. 14.13 (2D) with the new tool. We also rebuild the map by other methods (i.e. probability theory, fuzzy theory, DST) in Fig. 14.14–Fig. 14.16. Because the other methods are not new, here we don't give the detailed steps. If the reader has some interest in them, please refer to their corresponding reference. We give the result of comparison in Table 14.5. Through the figure and table, it can be concluded that:
1) In Fig. 14.12, the Z axis shows the Belief of every grid occupied. The value 0 indicates that the grid is fully empty, and the value 1 indicates that this grid is fully occupied. This facilitates very much the development of human-computer interface of mobile robot exploring unknown, dangerous and sightless area.
2) Low coupling. Even if there are many objects in grid map, but there occurs no phenomenon of the apparently servered, but actually connected. Thus, it supplies with a powerful evidence for self-localization, path planning and navigation of mobile robot.
3) High validity of calculation. The fusion machine considering the restrained spreading arithmetic is adopted [29], and overcomes the shortcoming that the global grids in map must be reckoned once for sonar scanning every time, and improves the validity of calculation.

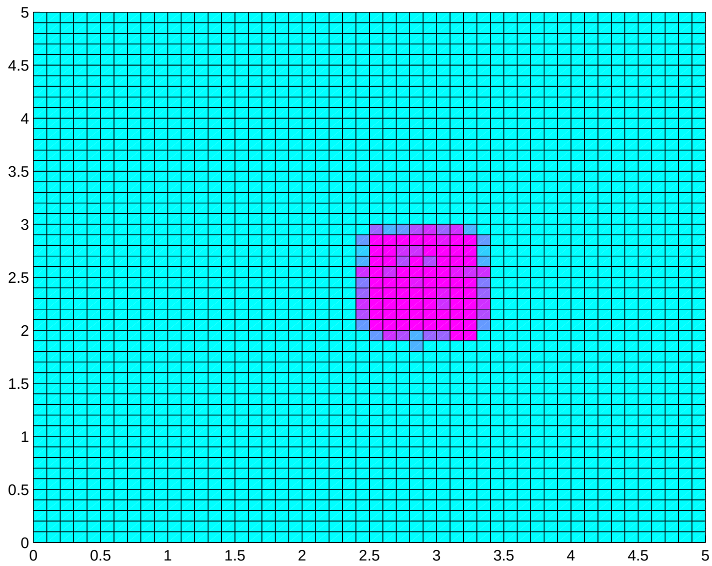

Figure 14.13: Map reconstruction with DSmT coupling with PCR5 (2D)

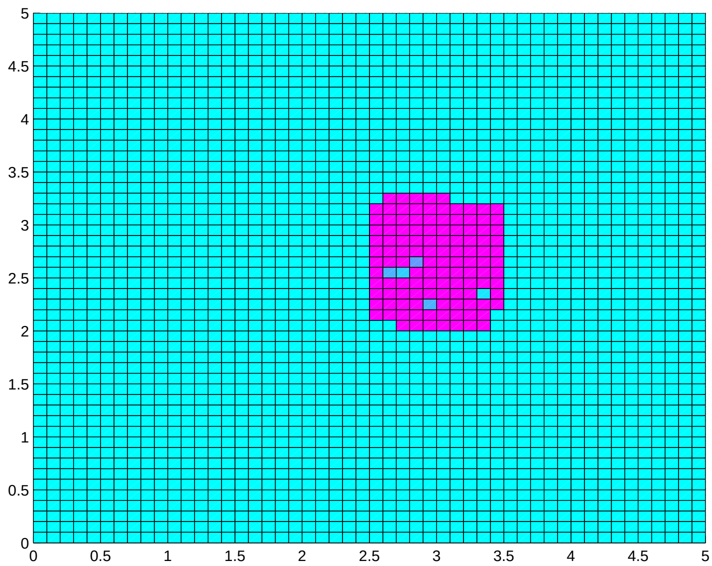

Figure 14.14: Map reconstruction with probability theory (2D)

4) Seen from the Fig. 14.12, the new tool has a better performance than just DSmT in building map, (see also [13, 14]), because of considering the conflict factor, and redistributing the conflict masses to other basic belief masses according to the PCR5.

5) Seen from the Table 14.5, Probability theory spends the least time, while Fuzzy theory spends the most time. But Map reconstruction with probability theory has very low precision and high mistaken judging rate shown in Fig. 14.14. Though the new tool spends a little more time than

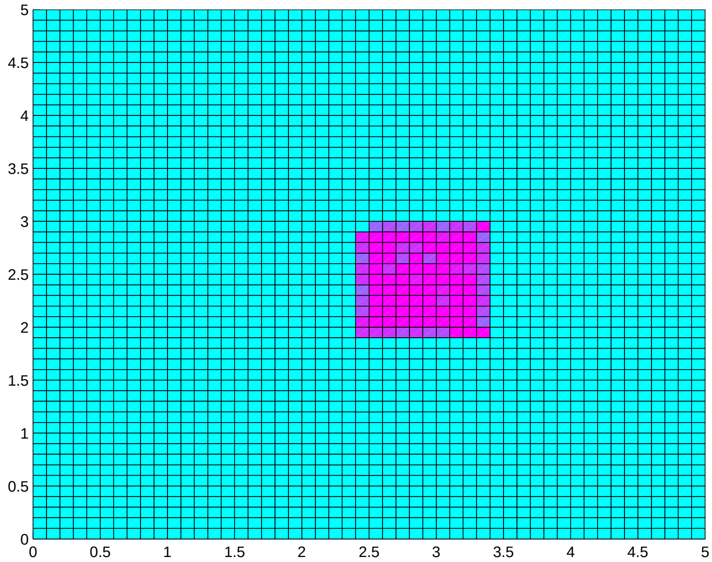

Figure 14.15: Map reconstruction with fuzzy theory (2D)

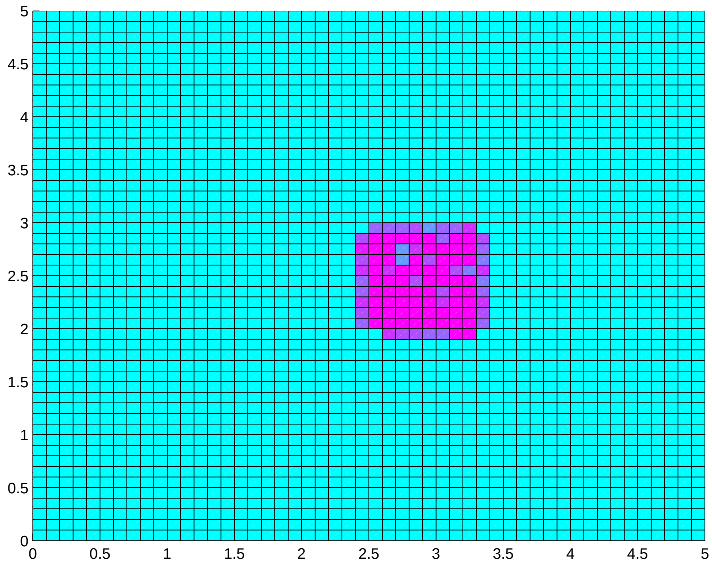

Figure 14.16: Map reconstruction with DS theory (2D)

probability theory. However, it has very high precision shown in Fig. 14.12 and Fig. 14.13 the same as the fuzzy theory and very low mistaken judging rate shown in Fig. 14.15. In fact, the comparison in map building between DST and DSmT have been made in details by us in [13]. Of course, here DST presents high precision and general mistaken rate shown in Fig. 14.16 without considering the PCR rules and other conflict redistribution rules. We don't compare the new tool from the fusion machine based on DST coupling with PCR5. Through the analysis

	Probability Theory	Fuzzy Set Theory	DST	DSmT
Total spent time (ms)	18146	35823	19673	19016
Map precision	Very low	Very high	High	Very high
Mistaken rate	High	Low	Medium	Very low

Table 14.1: Comparison of the performances of the different approaches

of comparison among the four tools in Table 14.5, we testify the new tool to play a better role in map building.

14.6 Conclusion

In this chapter, we have applied a fusion machine based on DSmT coupled with PCR5 for mobile robot's map building in a small environment. Then we have established the belief model for sonar grid map, and constructed the generalized basic belief assignment function. Through the simulation experiment, we also have compared the new tool with the other methods, and got much better performances for robot's map building. Since it is necessary to consider also robot's self-localization as soon as the size of environment becomes very large, complex, and irregular, we are currently doing some research works in Self-Localization And Mapping (SLAM) based on this new tool which improves the robustness and practicability of the fusion processing. In conclusion, our study has supplied a shortcut for human-computer interface for mobile robot exploring unknown environment and has established a firm foundation for the study of dynamic unknown environment and multi-robots' building map and SLAM together.

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