High School Math Problems
2017
Week 1
Problem and Solution

Find the minimum value of the function

$$f(x) = \frac{x^2 + x + 1}{x^2 + 2x + 1}$$

for x in the domain of f.

Solution:

The domain of the function f is $D = \{x : x \neq -1\}$.

We begin by re-writing the function f in the following way

$$f(x) = \frac{x^2 + x + 1}{x^2 + 2x + 1} = 1 - \frac{x}{(1+x)^2} = 1 - \frac{1}{1+x} + \frac{1}{(1+x)^2} = \left(\frac{1}{1+x} - \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4}.$$

Equality is achieved when $\dfrac{1}{1+x} - \dfrac{1}{2} = 0$, or, equivalently, when $x = 1$.

Since $1 \in D$, it follows that the minimum value of f over D is $\dfrac{3}{4}$.