LCP 3: The Physics of the Large and Small

…the mere fact that it is matter that makes the larger machine, built of the same material and in the same proportion as the smaller, correspond with exactness to the smaller in every respect except that it will not be so strong ...who does not know that a horse falling from a height of three or four cubits will break his bones, while a dog falling from the same height ... will suffer no injury? (Galileo, in the "Two New Sciences", 1638)

Fig. 1: Taken From Galileo's *Two New Sciences* (Book 1)

IL 1 *** (Galileo's "Two New Sciences": Discussion on scaling, free fall, trajectory motion)

IL 2 *** (Galileo's birthplace in Pisa)

… But yet it is easy to show that a hare could not be as large as a hippopotamus or a whale as small as a herring. For every type of animal there is a most convenient size, and a large change in size inevitably carries with it a change of form.

All warm blooded animals at rest lose the same amount of heat from a unit area of skin, for which purpose they need a food-supply proportional to their surface and not to their weight. Five thousand mice weigh as much as a man. Their combined surface and food or oxygen consumption are about seventeen times a man's. In fact a mouse eats about one quarter its own weight of food every day, which is mainly used in keeping it warm. (J.B.S. Haldane, in *On Being the Right Size*, 1928. See Appendix)

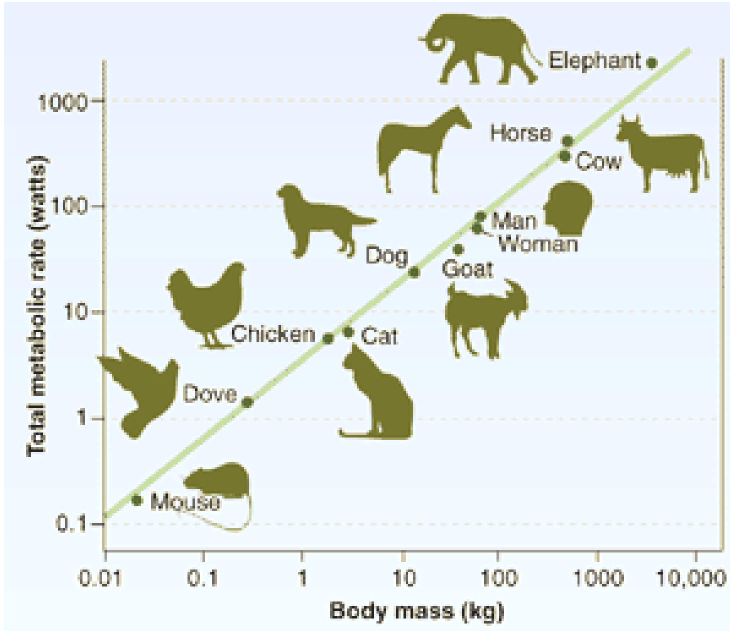


Fig. 2: Biology and Scaling

IL 3 *** (Picture taken from IL3: An advanced discussion of scaling in biology)

… consider a giant man sixty feet high—about the height of Giant Pope and Giant Pagan in the illustrated Pilgrim's Progress of my childhood. These monsters were not only ten times as high as Christian, but ten times as wide and ten times as thick, so that their total weight was a thousand times his, or about eighty to ninety tons. Unfortunately the cross sections of their bones were only a hundred times those of Christian, so that every square inch of giant bone had to support ten times the weight borne by a square inch of human bone. (J.B.S. Haldane, in *On Being the Right Size*, 1928).

Fig. 3: Gulliver in the Land of Lilliput

IL 4 **** (An excellent site for problems of scaling. Source of above picture)

Two generalities rule the design of both living and engineered structures and devices:
1) big is weak, small is strong, and
2) horses eat like birds and birds eat like horses
(Mel Siegel, Professor, Robotics Institute – School of Computer Science Carnegie Mellon University, 2004).

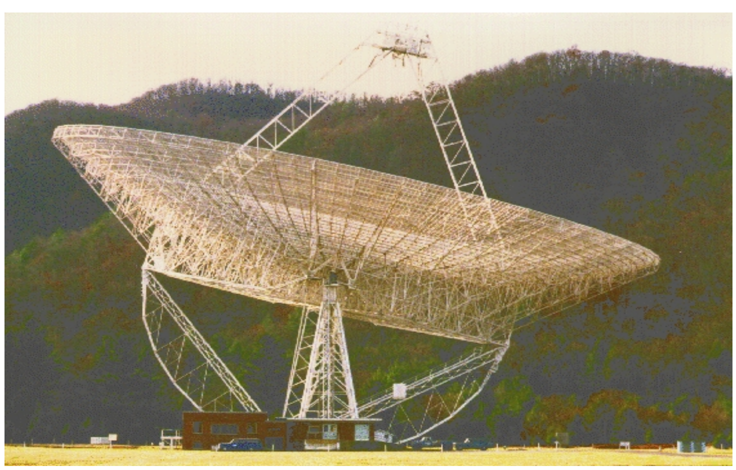


Fig. 4: The Collapse of a Giant Radio Telescope

IL 5 ** (Source of pictures above)

At 9:43 p.m. EST on Tuesday the 15th of November 1988, the 300 foot telescope in Green Bank West Virginia, USA collapsed. The collapse was due to the sudden failure of a key structural element—a large gusset plate in the box girder assembly that formed the main support for the antenna.

> *When two biologists and a physicist, recently joined forces at the Santa Fe Institute, an interdisciplinary research center in northern New Mexico, the result was an advance in a problem that has bothered scientists for decades: the origin of biological scaling.*
>
> *How is one to explain the subtle ways in which various characteristics of living creatures—their life spans, their pulse rates, how fast they burn energy—change according to their body size?* **(**George Johnson, science writer, The New York Times, 1999)

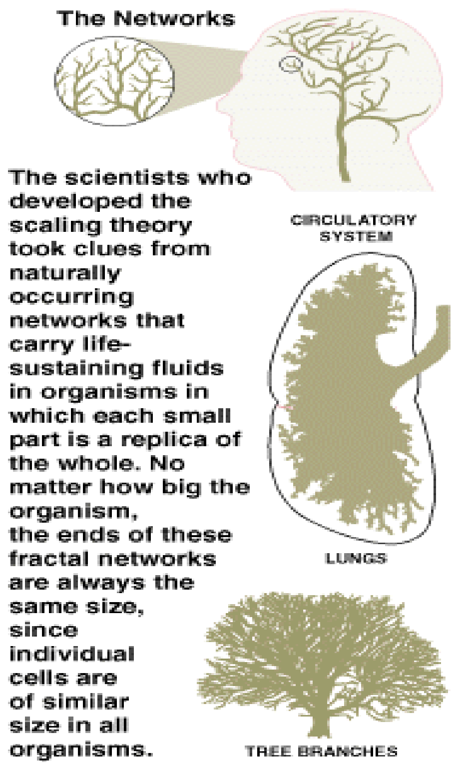


Fig. 5: Scaling and Small Biological Networks

IL 6 ** (A modern look at physics, biology and scaling "Of Mice and Elephants: A Matter of Scale")

Note: websites (Internet Links or IL's) in this document are evaluated according to the scheme
=good, **=very good, ***=excellent * = outstanding**

THE MAIN IDEA

The elementary physics of materials and of mechanics determine the limits of structures and the motion bodies are capable of. The physical principles of strengths of materials goes back to Galileo, and the dynamics of motion we need to apply is based on an elementary understanding of Newtonian mechanics, and the mathematics of scaling required depends only on an elementary understanding of ratio and proportionality. Finally, the main ideas developed here are intimately connected to architecture, biology, bionics, and robotics. It is hard to imagine a more motivating large context to teach the foundations of statics and dynamics with a strong link to the world around us.

The guiding idea for this LCP will be based on the idea that the science of materials and the physics of motion determine the limits of structures and the motion bodies are capable of.

We will also discover that the energy consumption for robots as well as animals and humans is critically connected to the laws of thermodynamics.

However, we will find that it is necessary to go beyond Galileo and Haldane to understand contemporary empirical evidence for new scaling laws describing metabolic rates and mass of animals. The range of the length and the mass of the smallest organism that we can see, say a small insect, about 1 mm long, and a mass of about 10^{-9} kg, and a whale, about 30 m long, and a mass of about 100 tons (10^5 kg) is 5 orders of magnitude in length and 14 orders of magnitude in mass. The scaling laws, however, we will find, are different for small things (micro systems) and large things (macrosystems).

Finally, in an effort to make contact with sizes we can see with our unaided eyes (between 10^{-3}m and 10^{-4}m) an attempt will be made to guess the size of molecules. This will be done by calculating the thickness of a soap film and by estimating the size of a molecule, describing the method of the French mathematician and physicist Pierre Laplace who estimated the size of a molecule using measurements of surface tension and the latent heat of water.

Since the size of a bacterium is about 10^{-6} m , we can extend our range for organisms from 10^{-6} m to 10 m, and their mass from about 10^{-16} kg (bacterium) to 10^{5} kg (whale), or about 7 orders of magnitude in size (length) and about 21 orders of magnitude in mass.

THE DESCRIPTION OF THE CONTEXT

In LCP 2 we used the ubiquitous pendulum as our guide to study both kinematics and dynamics, from Galileo to the present. This context also deals with bionics, robotics and the physics that is at the foundation of these disciplines. The context is based on five sources. For the first source we again turn to Galileo, namely his *The Two New Sciences*, published in 1638: the second reference is a classic and much admired article by the noted British biologist Haldane, published in 1928, The third topic is based on the work as described in an article by Dr. Mel Siegel, a robotics researcher, that was published in 2004. The fourth topic will refer to on the very informative and entertaining article “Fleas, Catapults and Bows”, by David Watson, followed by the article “Of Mice and Elephants, a matter of Scale”, by George Johnson The last source comes from contemporary research that is based on the question of *how one is to explain the subtle ways in which various characteristics of living creatures—their life spans, their pulse rates, how fast they burn energy—change according to their body size.*

This LCP concludes with the article “Physics and the Bionic Man” by the author and is available in PDF. These sources can all be found in the Appendix.

Appendix texts:

Click on Appendix I: Galileo’s *Two New Sciences*

Click on Appendix II: *Haldane’s article*

Click on Appendix III: *Mel Siegel’s Article*

Click on Appendix IV: *Energy storage and energy changes in Fleas, Catapults, and Bows*

Click on Appendix V: *Of Mice and Elephants: A Matter of Scale*

Click on Appendix VI: *Physics and the Bionic Man*

Most students are well aware of Galileo setting the stage for the study of motion, specifically kinematics. They may even realize that his studies paved the way for Newton's dynamics, and his three laws of motion. But few know that Galileo's ground-breaking book, *The Two New Sciences*, begins with a discussion of scaling and strength of materials and ends with description of motion along an inclined plane, the motion of a projectile (as propelled by rolling off an inclined plane), and the general study of pendulum motion. What will interest us especially from this work is Galileo's "square-cube" law, that is, the fact that when geometrically and materially similar structures are compared, their strength to weight ratio decreases inversely with their linear size. In his book, *The Two New Sciences, Day 1*, Galileo explains to his friends Sagredo and Simplicio:

> *Who does not know that a horse falling from a height of three or four cubits will break his bones, while a dog falling from the same height or a cat from a height of eight or ten cubits will suffer no injury? Equally harmless would be the fall of a grasshopper from a tower or the fall of an ant from the distance of the moon.*
>
> *Do not children fall with impunity from heights which would cost their elders a broken leg or perhaps a fractured skull? And just as smaller animals are proportionately stronger and more robust than the larger, so also smaller plants are able to stand up better than larger.*
>
> *I am certain you both know that an oak two hundred cubits high , would not be able to sustain its own branches if they were distributed as in a tree of ordinary size; and that nature cannot produce a horse as large as twenty ordinary horses or a giant ten times taller than an ordinary man. ..*
>
> *... Thus, for example, a small obelisk or column or other solid figure can certainly be laid down or set up without danger of breaking, while the large ones will go to pieces under the slightest provocation, and that purely on account of their own weight.* **See IL 1**

The second source for the context is based on J.B.S. Haldane's famous article, contained in a volume called *Possible Worlds* and other essays. The wonderful title: "On Being the Right Size." Haldane was a famous British theoretical biologist, and a tireless champion of Darwinian evolution.

IL 7 ** (Biography of Haldane)

IL8 **** (An excellent article: "When Physics Rules Biology". This article should be downloaded and kept as resource material. The article is also available in the Appendix).

Haldane, in a fascinating way, explored the argument that any animal whose body cells multiplied indefinitely would grow to such a size as to come to an end by other means than the mere process of aging. There is one exceptional circumstance, namely, where the animal is supported with respect to its body weight in a fluid medium—a circumstance which is borne out by the extraordinary size of some of the prehistoric monsters who lived mostly in the water, by whales at the present time (whales weigh up to 140 tons, compared with an elephant's mere 5 tons), and by the very long life of some fishes. Sturgeons, for example, live up to 100 years and halibut up to 70 years, and quite recently a turtle taken from the sea may have an age of 1000 years.

Haldane begins his essay by noting that differences of size are the most obvious differences among animals, but that little scientific attention seems to be paid to them. He shows that a consideration of the constraints of physics on form and function yields some surprising insights, including the answer to a question posed by a recent reader of New Scientist magazine who wondered if it was true "that you can drop a cat from any height and it will land unhurt because its terminal velocity is lower than the speed at which it can land unhurt." Haldane said you can drop a mouse down a thousand-foot mineshaft and it will walk away, "so long as the ground is fairly soft." Not so with a rat, or any larger animal, if you were wondering. He says:

> *To the mouse and any smaller animal it [gravity] presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and on arriving at the bottom, it gets a slight shock and walks away. A rat is killed, a man is broken, a horse splashes.*

Haldane claims that for every type of animal there is an optimum size. He goes on to argue that a person, for example, could not be 60 feet (about 20 m) tall. Giants may exist in literature, but not on *terra firma*. We will see that scaling up a person to 60 feet in height would increase his weight by about a thousand times, and increase the pressure on each square inch of bone by a factor of 10. But human thigh bones will break trying to carry ten times human weight, so giants couldn't walk without breaking their thighs with each step.

The aerodynamics of flying quickly imposes limits on the size of birds. The muscle power necessary to flap wings inhibits how big a bird can be and still stay aloft. Very large birds, such as eagles or condors, mostly soar, flapping their wings relatively rarely. Hummingbirds, in contrast, can flap their wings faster than our eyes can register, because of their very small size. The constraints that physics imposes on form and function are sometimes useful to us. "Were this not the case, eagles might be as large as tigers and as formidable to man as hostile aero planes," Haldane observes.

Considerations such as these soon 'show that for every type of animal there is an optimum size.' Haldane was writing about the physics of biology, about the limits of systems that are constituted in particular ways, or which are organized to solve specific problems, such as flying.

His point was that the basic nature of the world imposes limits on our ability to operate within it. If we are going to fly, we have to obey the laws of aerodynamics. The laws of optics, and the nature of light waves, have implications about how eyes must be constructed. We can apply the mathematics of scaling to study how a number of animal characteristics (e.g., metabolic rates) vary with size, to discuss "variations in design" in animal species, and even to consider how large diving mammals can be and how high animals in general can jump. We can then compare his back-of-the-envelope results with real data. This is all quite elementary, and at the same time quite fascinating.

At the end of his essay, he says: "…and just as there is a best size for every animal, so the same is true for every human institution." He argued that the reason the Greeks thought a small city was the largest size for a functioning democracy was that democracy required that all citizens be able to listen to debates about issues and vote on legislation. A large geographic area makes this method of governance unwieldy and unworkable.

We will next consider the work done in robotics today by looking at the basic writing and research of the American researcher Mel Siegel, a professor of robotics. As a preliminary exercise look at his website and other suggested links. This will give you an idea of the wide ranging work he is involved in.

We will concentrate on the article "When Physics Rules Robotics", by Mel Siegel, published in 2004 (See Appendix). He begins his paper by paying tribute to Galileo and his discussion of his "square-cube" law, that is, the fact that when *geometrically and materially similar structures* are compared, their strength to weight ratio decreases inversely with their linear size. According to Siegel, this law based on a simple scaling argument produces "two generalities, both at first counterintuitive, but straightforwardly physics-based, rule the design of both living and engineering structures and devices:

(1) big is weak, small is strong, and

(2) horses eat like birds, and birds eat like horses

That is, large structures that collapse under their own weight, large animals that break their legs when they stumble, etc; whereas small structures and animals are practically unaware of gravity; and small animals, like a mouse, must eat a large amount of food (almost equal to their body mass) per day to survive: and large animals, like an elephant, eats only a small amount (relative to the mass of the large animal).

Siegel goes on to say that a large animal or machine stores relatively larger quantities of energy and dissipates relatively smaller quantities of energy than a small animal or machine. The critical consequence of (1) is that

> *… it is hard to build large structures and easy to build small structures that easily support their own weight,* and

The critical consequence of (2) is that

> *…it is hard to build small structures and easy to build large structures that easily long enough and travel far enough to do any sort of an interesting job.*

Siegel then discusses the implications of Galileo's law for designing robots, small and large. He uses the term "fundamental issues", e.g., in the abstract, to mean

> *…opportunities provided by and restrictions imposed by the most basic laws of physics as they relate to things like the strengths of structures, the internal and external motions of the structures, the energy requirements associated with their basal metabolisms, the mechanical work they do, the energy they dissipate to friction associated with their mobility and the work they do, as well as some communication issues relating to energy cost and signal range, and the relationship between the size of an antenna and the efficiency with which it couples to the environment at any particular communication frequency.*

The student should "unpack" this long sentence and present it in parts so that it is more understandable.

We will insert a section based on the author's article "Physics and the Bionic Man" . The TV series with the same name was very popular in the late 1970s, and it still provides us with an interesting study of the physics of bionic body parts. An updated version of the of the content of the article lends itself to a discussion of the bionic parts today.

Finally, it is necessary to go beyond Galileo and Haldane to understand contemporary empirical evidence for *new scaling laws* describing metabolic rates and mass of animals.

A modern look at physics, biology and scaling is described in "Of Mice and Elephants: A Matter of Scale".

THE PRESENTATION OF THE CONTEXT

The presentation of the context will be roughly in three parts. The first part will be based on Galileo's square-cube law, taken from his "Two New Sciences", on the British biologist's J.B.S. Haldane's famous article of 1928 "On being the right size", and on "When Physics Rules Robotics", a very comprehensive review article written by the robotics researcher Mel Spiegel, published in 2004. These works will be the background for the main portion of the presentation.

This will be followed by more contemporary work based on the discussion of the article "Of Mice and Elephants: a Matter of Scale" by George Johnson, the noted science writer of the *New York Times*. The article discusses the effort made by a team of biologists and physicists to answer the question

> *How is one to explain the subtle ways in which various characteristics of living creatures—their life spans, their pulse rates, how fast they burn energy—change according to their body size?*

This question clearly takes us back to Galileo, showing that the discoveries of the 17th century physicist about how scaling affects everything around us and the later elaboration of the 20th biologist Haldane is being extended and given a new meaning in the context of the collaboration between 21st century physicists and biologists. All of these texts are available in the Appendix.

We will conclude with an updated version of "Physics and the Bionic Man", written by the author and published in *The Physics Teacher* in 1980 and also in the British journal *New Scientist* special Christmas edition in 1981. This article was quite popular to a generation of students in the 1980s.

Galileo and His Square-Cube Law: Physics and Structures

You can read part of the *Two New Sciences* by Galileo in IL 1. Galileo argued that when *geometrically and materially similar structures* are compared, their strength to weight ratio decreases inversely with their linear size. This means that if I compare two cubes made of the same material, one with sides of 1 cm and the other with sides of 2 cm, the larger one will have a mass (weight) 8 times of the smaller one. Galileo argued that the strength of the cubical structure, however, changes with the cross sectional area. So that when we compare the ratio of strength to weight, we compare the ratio of cross sectional area to volume.

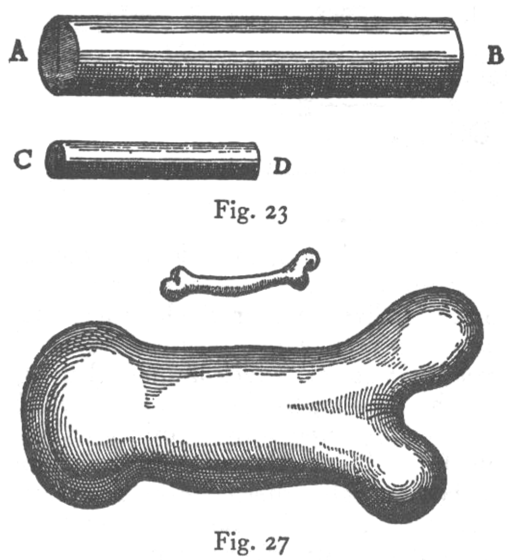


Fig. 6: Galileo's Scaling Law Illustrated, Taken from His Book

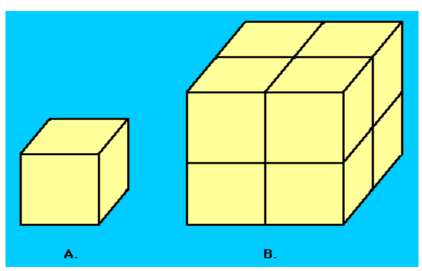


Fig. 7: Comparing the Volumes and Areas of Two Cubes

For the first cube this ratio is simply 1 and for the second cube it is 0.5. This means that, as far as the ability to hold up the second cube against gravity goes, it is only ½ as strong as the other. To make this effect a little more dramatic: If I have a column 10 cm with an area of 1 cm^2, the strength to weight ratio is only 0.1.

IL 9 *** (Wikipedia's "square-cube" law presentation)

IL 10 **** (An excellent detailed discussion of scaling, especially relevant for this LCP)

Make sure you read the texts by Galileo and the article by Haldane, and also study the suggested internet links before attempting to answer the questions below, study especially **IL10**.

(See Appendix texts above)

Questions for the student

1. What did Galileo understand by *geometrically and materially similar structures***?** Discuss, using examples.
2. How would you define strength to weight ratio? Use the fact that weight goes up as the cube of the cross section and strength depends on the area of the structure of the structure considered. See Fig 7.
3. In Fig. 6. Galileo is comparing two "identical" bones where one is twice as long as the other. Show that, even though the shape of the bones is irregular, the same argument applies to the comparison of these bones as did for the comparison of the two cylinders above.
4. What would be the weight of a 200 pound man if he were twice as tall?
5. A weight lifter in the light-weight category (60 kg) is able to lift above his head a mass of 120 kg (which physicists would call a weight of about 1200 N). Another weight lifter in the heavy-weight class (100 kg) is able to lift 150 kg above his weight. Which one is stronger, considering his strength to weight ratio.
6. The following problem is taken fro IL10:

   If a 73-kg (160 lb.) person is 1.8 m (6 ft) tall, how tall is a 37-kg (80 lb.) person? Since we are assuming the density of the two persons is the same and since mass is proportional to the density times the volume and the volume is proportional to a length (or height) cubed…. Show that the height is 1.4 m and then check the calculations on IL10.

IL 11 ** (Gulliver's Travels: A discussion of the book. Very informative and entertaining.)

Gulliver Twist.

Fig. 8: Cartoon of Gulliver in the Land of the Lilliputians

Fig. 9: Gulliver in the Land of Lilliput (movie). Taken from IL11.

Using Haldane's example we will consider a giant man sixty feet tall, about the height of Giant Pope and Giant Pagan in the illustrated *Pilgrim's Progress* by the 17th century writer Alexander Pope. These monsters were not only ten times as high as Christian, but ten times as wide and ten times as thick, so that their total weight was a thousand times his, or about eighty to ninety tons. Another example of large and small people is found in the 18th century masterpiece "Gulliver's Travels", by Jonathan Swift (see Fig. 3).

7. Verify the calculations of Haldane, mentioned above. (Since you are using ratios it is alright to work with feet or any other unit).

Unfortunately the cross sections of their bones were only a hundred times those of Christian, so that every square inch of giant bone had to support ten times the weight borne by a square inch of human bone. As the human thigh-bone breaks under about ten times the human weight, Pope and Pagan would have broken their thighs every time they took a step.

In movies we have many examples of clear violation of Galileo's square-cube law. In some movies we can accept this violation when it is adequately covered by an "artistic license". Giant insects and mile high buildings in science fiction movies clearly violate this law. The famous figure of King Kong could be considered an example of a scientifically impossible situation, but acceptable because of an artistic license. But we can still ask questions and show that physics would forbid the existence of such a creature.

Fig. 10: King Kong in the Movie of the Same Name

IL 12 ** (Source of Fig. 10)

8. Consider King Kong (KK) who is supposed to be a little over 60 feet, or about 20 m tall, and King Kong's young son (KKS), who is about 3 feet (or about 1 m tall m). They are identical in all respects, and KKS can be considered to have a *geometrically and materially similar structure* to his father. Assume the mass of KKS to be 50 kg.
   a. What is the mass of KK? Compare his mass to that of a large elephant (5000 kg).
   b. If the surface area of KKS is about 30 m^2, what is the approximate surface area of KK?
   c. Find the strength to weight ratio of KKS and that of KK.
   d. Which one is "stronger"? Discuss.

Research problems for the student

1. Investigate world record weight lifters' mass compared to the weight they were able to lift. In the light of our discussion, discuss your findings.
2. Specifically, go to the link below and compare the weight lifted to mass ratio of the lowest (53 kg) and highest (105+kg) participant mass.
3. Compare the height and weights of high jumpers and shot putters. Imagine a shot putter high jumping and a high jumper shot putting. What values for their maximum performance would you guess?

IL 13 ** (Statistics of world record holders in weight lifting)

Haldane writes:

> *Gravity, a mere nuisance to Christian, was a terror to Pope, Pagan, and Despair. To the mouse and any smaller animal it presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away, provided that the ground is fairly soft. A rat is killed, a man is broken, a horse splashes. For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal's length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force.*

IL 14 ** (A video of a cat falling 80 feet from a tree)

We will now look at the problem of a falling metallic sphere. The solution to this simple problem will guide us in calculating the terminal velocity of falling objects in general. The terminal velocity of the sphere is reached when the weight of the sphere is equal to the frictional force (drag of the atmosphere) opposing it. Look at fig. 11 and identify the forces.

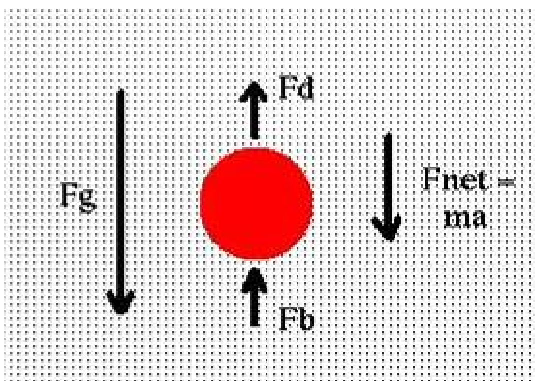


Fig. 11: Forces Acting On a Sphere in Free Fall

The weight (mass) of a sphere is proportional to volume, or $\mathbf{w} \propto \mathbf{V}$, and $\mathbf{V}$ is proportional the cube of the radius, or $\mathbf{V} \propto \mathbf{r}^3$.

So we have $\mathbf{w} \propto \mathbf{r}^3$.

Based on experiments, the drag force is known to be proportional to the square of the velocity and the area. When terminal velocity is reached, the drag force and the weight balance. Therefore we can write: $\mathbf{w} \propto \mathbf{r}^3 \propto \mathbf{A}\,\mathbf{v}^2$**,** where $\mathbf{v}$ is the terminal velocity of the sphere. But

$\mathbf{A} \propto \mathbf{r}^2$.

Therefore, $\mathbf{r}^3 \propto \mathbf{r}^2\,\mathbf{v}^2$, or

$\mathbf{v} \propto \mathbf{r}^{1/2}$.

The terminal velocity then is proportional to the square root of the radius or diameter. We can generalize from this:

The terminal speed of an object is proportional to the square root of the cross sectional area.

Problem 6 below illustrates how this generalization works. We can now develop the complete formula for moving through air in general. This formula will apply to cars, freely falling objects, including raindrops.

The resistance of the air on an object as it moving is proportional to the density of the air and the velocity of the object, is expressed as $\mathbf{D \propto A\,\rho\,v^2}$. This is conventionally expressed in an equation as

$\mathbf{D = \frac{1}{2} A\,\rho\,v^2 C_D}$,

where **D = Force of the drag (N)**

ρ = Density of air ($\mathbf{kg/m^3}$)

v = velocity (m/s)

A = Area ($\mathbf{m^2}$) and

$\mathbf{C_D}$ = Coefficient of drag (Dimensionless).

The unbalanced (or net) force on an object moving through air is given by

$\mathbf{F_{net} = D - W = \frac{1}{2} A\,\rho\,v^2 C_D - W}$

where **W** is the weight in Newtons (actually **mg**). According to Newton's second law, $\mathbf{F_u = ma}$, therefore, the acceleration of the object is
$\mathbf{a = F_{net} / m = (\frac{1}{2} A\,\rho\,v^2 C_D - W) / m}$.

The motion of an object falling through the atmosphere is therefore a category 3 motion: a changing acceleration that can be mathematically expressed.

The terminal velocity of an object then occurs when the unbalanced force $\mathbf{F_u}$ is zero. Therefore, the terminal velocity can be expressed as

$\mathbf{V_T = \{2\,W / C_D\,\rho A\}^{1/2} = \{2\,mg / C_D\,\rho A\}^{1/2}}$

where $\mathbf{V_T}$ is the terminal velocity.

(Note: The drag coefficient $\mathbf{C_D}$ is experimentally determined.

IL 15 *** (Drag coefficients)

Here are measured drag coefficients for some basic shapes. These numbers come from tests of shapes with known cross sectional areas. You blow air over them and measure the force on the shape. That's what wind tunnels do. The arrow in front of the shape gives the direction of the air blowing over the shape. The cone shape, for example, would have a lower C_D if it were rotated so the air saw the flat end first.

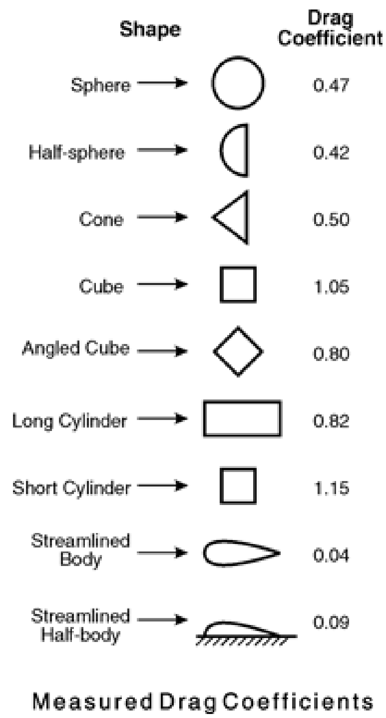


Fig. 12: Drag Coefficients

However, it may be easier to express the terminal velocity $\mathbf{V_T}$ by a proportionality statement:

> *For two similar objects (two spheres, for example), made of the same material (steel, for example) the terminal velocity is directly proportional to the square root of the weight and inversely to the square root of the effective area:* $\mathbf{V_T \propto \{W / A\}^{1/2}}$.

The terminal velocity of a freely falling object is directly proportional to the square root of the mass and inversely to the square root of the area.

You can now show that if two spheres, as described above, fall through the air side by side, and if the second has a radius twice that of the first then the terminal velocity of the second is simply: $\mathbf{V_{T2} \propto V_{T1} \{ r_2 / r_1 \}^{1/2}}$.

Here we assume that we compare two geometrically and materially similar objects made and falling in the same medium.

IL 16 *** (Good discussion of the equation of drag given above)

IL 17 **** (Terminal velocity calculator and terminal velocity table)

IL 18 **** (Free fall in air).

IL 19 *** (An excellent but advanced discussion of modeling the free fall of rain drops, a historical component)

Problems for the student

(Using the free-fall calculator in **IL 17**).

1. Show, using an argument based on ratios that the terminal velocity of a metallic sphere of radius 2 cm and a density of 8 g/cm^3 is about 81 m/s and that the a sphere made of the same metal whose radius is 1 cm has a terminal velocity of about 57 m/s.
2. Now, given the value of the terminal velocity of the small sphere, use the proportionality relationship to calculate the value of the other terminal velocity.
3. Continue to use the formula directly to calculate the terminal velocity of the small sphere and compare it to the value obtained using the free fall calculator. Comment.
4. Imagine that a skydiver whose mass is 82 kg (180 lb) is falling freely until the terminal velocity is reached. As an approximation, assume that a sphere whose density is the same as that of the skydiver (about 1 g/cm^3) and has a radius of 0.27 m is falling freely. Show that the terminal velocity of this sphere would be about 105 m/s.
5. However, the terminal velocity of a skydiver is typically about 60 m/s. So a sphere that has the same mass and density is a poor approximation for the free fall of a skydiver. Using the formula for terminal velocity, show that if we assume an area of 0.34 m^2, a density of 1 g/cm^3, and a drag coefficient of 1.0, the skydiver will reach a terminal velocity of about 50 m/s. Now determine the terminal velocity using the calculator.
6. Find the terminal velocity of a freely falling mouse if it is known that of a freely falling man has a terminal velocity of about 60 m/s (close to the earth's surface). Assume that a mouse is a "small man" and has a dimension of about 5 cm, compared to that of the man of about 180 cm. Was Haldane right when he said that the mouse would survive if the mouse falls on soft ground?
7. How fast does a raindrop fall? Check the accuracy of the following claim, using a calculator:

   "An average raindrop is about 2 millimeters in diameter and has a maximum fall rate of about 14.5 miles per hour or 21 feet per second. A large raindrop, 5 mm in diameter, falls at 20 mph (29 feet/second), but drops of this size tend to fall apart into smaller drops. Drizzle, which has a diameter of 0.5 mm, has a fall rate of 4.5 mph (7 feet/second)".

Would Galileo be surprised?

Read and study this IL 17 carefully, and especially pay attention to the terminal velocity examples and the terminal velocity calculator (TVC). Use the TVC to answer the following questions.

1. Two spheres made of iron are dropped from a hovering helicopter, from a height of 110 m, about twice the height of the Leaning Tower of Pisa. The density of the iron is 7000 kg/m^3. The small sphere has a diameter of 1 cm and the large sphere (about the size of a cannon ball) a diameter of 10 cm.

   a. Find the terminal velocities of the spheres.

b. What is the drag force on the spheres when they reach terminal velocity?

c. Assume that the deceleration is linear, from about 10 m/s^2 to 0.

   Using a graphical argument estimate the distance and the times the spheres fall before reaching terminal velocity.

d. Estimate the time it takes the sphere to reach the ground and also the distance between them at the time the heavier hits the ground.

e. Now imagine Galileo dropping these spheres from the Leaning Tower of Pisa. The height of the Leaning Tower of Pisa is 56 m. Would they have fallen together? Discuss.

f. Assuming that the terminal velocities for the spheres are reached before falling 56 m, estimate the difference in height of the two spheres for the case of falling through a height of 56 m and a height of 112 m.

Note: The correct solution to the last three parts would involve *differential equations*, See IL 27. But you can draw a velocity-time graph and a corresponding distance-time graph and make an estimate.

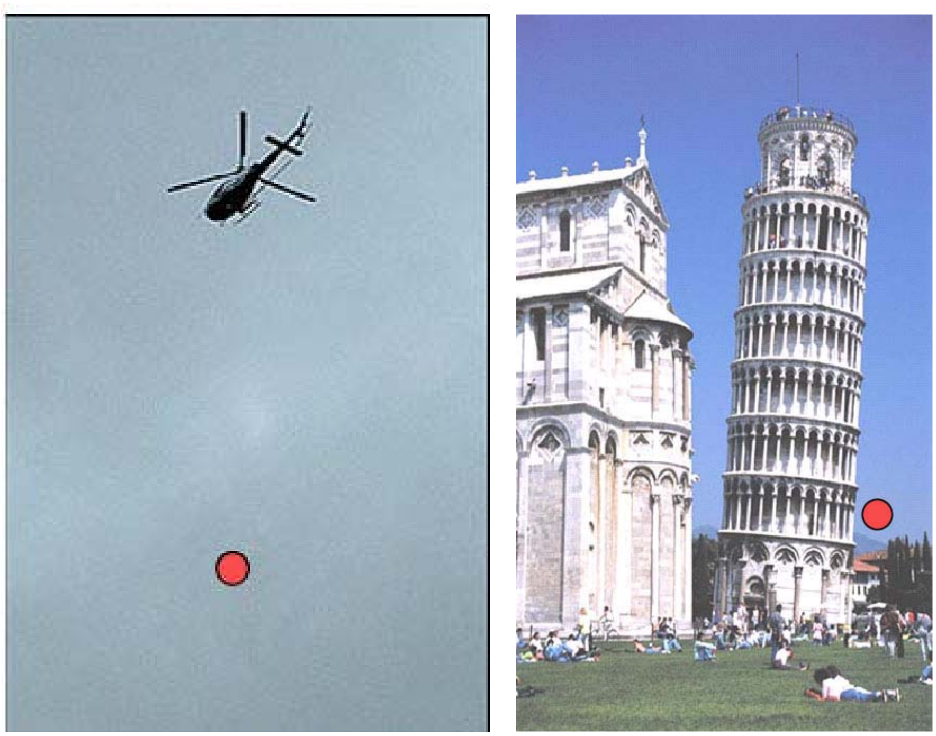


Fig. 13: Dropping Objects from a Helicopter and From the Leaning Tower of Pisa

An advanced problem for students

This is a special problem, looking ahead to LCP 9 where we will discuss Earth-Asteroid collisions.

> A small asteroid that that is shaped like a boulder and can be approximated by a cube is about 50 m in "diameter. The asteroid has a density of about 3000 kg/m^3 and is colliding with the atmosphere directly with a speed of 15 km/ s. Assume that the atmosphere is too small to affect the motion until the asteroid reaches about 35, 000 m. Also assume that the value of the gravitational pull doe not change,

a. What is the approximate mass of the asteroid?

b. Calculate the kinetic energy of the asteroid upon entering the earth's atmosphere. Compare this energy to the energy released by a Hiroshima size bomb. Comment.

c. Look up the table of values for the density of the atmosphere and decide on an "average" value from 35,000m to sea level.

d. Calculate the "average" force that the asteroid experiences, using the drag formula we have studied.

e. Apply Newton's second law to find the "average" deceleration and hence the velocity of the asteroid just before hitting the ocean.

f. Does the asteroid reach the terminal velocity? Comment

g. Compare the energy of the asteroid with that of the recent Tsunami. Speculate on the destructive powers it would have on a beach about 100 km away. See **IL 17.**

Fig. 14: An Asteroid Colliding with the Earth

IL 20 ** (Table of densities for the atmosphere)

IL 21 ** (Energy of a Tsunami)

IL 22 ** (The energy yield of a Hiroshima size atom bomb)

IL 23 ** (Ideas for weather fans)

A special research problem for the student

On 16 August 1960, US Air Force Captain Joseph Kittinger entered the record books when he stepped from the gondola of a helium balloon floating at an altitude of 31,330 m (102,800 feet) and took the longest skydive in history. As of the writing of this supplement 39 years later, his record remains unbroken. This event is described in great detail in the websites below. Read the first website carefully.

According the website below:

> *The highest-altitude parachute jump was made by Joseph Kittinger of the US air force, who jumped from a balloon at 31,333 metres on 16 August 1960. He was in free fall for 4 minutes 36 seconds, reaching an estimated speed of 1150 kilometers per hour. He opened his parachute at 5500 metres.*


Fig. 15: Free Fall from Great Heights

IL 24 ** (Picture of freely falling person in a space suit)

IL 25 *** (Tables for terminal speeds of various objects)

You can follow the description of free fall using the tables in **IL 25**. You may notice that many of the speeds reached are dubious and highly questionable. For example, the claim that Kittinger reaches and even surpassed the speed of sound (about 310 m/s, or bout 1100 km/h) seems exaggerated. Look at the following claim:

> *In freefall for 4.5 minutes at speeds up to 714 mph and temperatures as low as -94 degrees Fahrenheit, Kittinger opened his parachute at 18,000 feet. In addition to the altitude record, he set records for longest freefall and fastest speed by a man (without an aircraft!)*

1. Try to reconstruct this motion by sketching d-t, v-t, and a-t graph. Describe the motion in your own word. Look up the density of the atmosphere at his height, as well as the temperature.

Based on the following description, answer the questions below. You should learn how to convert these readings into the SI system:

> *An hour and thirty-one minutes after launch, my pressure altimeter halts at 103,300 feet. At ground control the radar altimeters also have stopped-on readings of 102,800 feet, the figure that we later agree upon as the more reliable. It is 7 o'clock in the morning, and I have reached float altitude …. Though my stabilization chute opens at 96,000 feet, I accelerate for 6,000 feet more before hitting a peak of 614 miles an hour, nine-tenths the speed of sound at my altitude.*

1. Assuming that the density of the atmosphere is too low to produce an appreciable drag to about 90,000 feet and that the gravity in this region is not significantly smaller than 9.8 m/s^2, calculate the maximum velocity and the time it took to fall to the height of 96000 feet.
2. Estimate the deceleration (clearly this increases and increases as the parachute descends) and also estimate the distance fallen before the parachute descends with a terminal velocity. (This problem should be solved graphically).
3. Now plot a more realistic set of kinematic graphs, for distance, velocity, and acceleration versus time, and
   a. Estimate the total time of descent and compare with the time of descent claimed.
   b. Compare your estimates with the claim made below and comment.
   c. Kittinger had to wear a space suit (1960 style). Why?

IL 26 *** (A nice IA for free fall)

IL 27 *** (Detailed calculations of the free fall by Kittinger)

IL 28 ** (Study this detailed description of the story of Kittinger's free fall with details about terminal velocity, maximum height, etc…)
(A terminal velocity calculator and an advanced level discussion of free fall in air)

<u>Other sites describing free fall</u>

IL 29 ** (An amusing applet of an elephant and a feather falling from a tall building. It also has a "true and false test".)

IL 30 *** (Accounts of survivals of free fall)

IL 31 *** (Interesting statistics about free fall from the US air force) **IL 32** *** (A short discussion of free fall in the atmosphere)

IL 33 ** (Nice animation of free fall and a parachute)

IL 34 *** (A thorough advanced discussion of free fall in air)

IL 35 *** (A video of free fall from a tower unto a rebounding net)

The following is taken from **IL 34**:

> *Approximating terminal velocity is much more easily done than calculating the terminal velocity because of the difficulty in finding the value of C_d. One simple small scale method is to hang an object out of a car window by a small string. The terminal velocity of the object is the speed of the car when the object hangs at a 45° angle. This can be easily proven mathematically because it is when the atmospheric drag (in the horizontal direction) is equal to the force of gravity. It is when air resistance and gravity are the same. When gravity is greater then the terminal velocity is greater.*

Try to show the reasoning behind this. Exercising caution, you could suspend from a long stick (and placing it outside the car from the passenger side) a baseball and a ping pong ball from a short string and find its terminal speed in a car traveling on a highway. (First, find the mass, the radius and the density of the baseball and the ping pong ball and check the terminal speed of the baseball, using the terminal velocity calculator in **IL 17.**

Fig. 16: Using the Speed of a Car and a Pendulum to Estimate the Drag Coefficient

A similar simple experiment can be performed by hanging a pendulum from the ceiling of a car. The angle the string makes with the vertical will measure the acceleration of the car. What angle would you expect for an acceleration of 0.3 g, or about 3 m/s^2, as claimed for one of the cars we describe later? Another approach to measure acceleration is shown in Fig. 17 below.

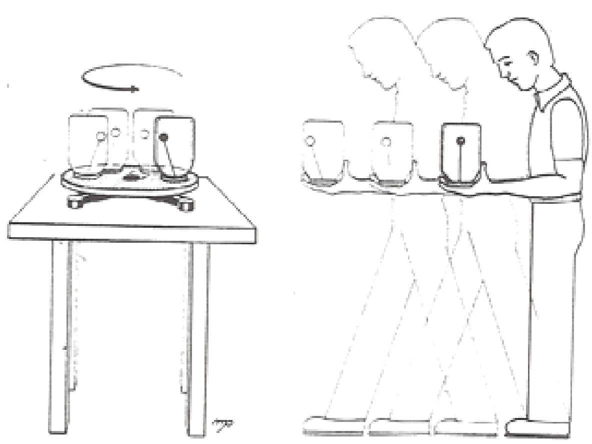


Fig. 17: A Simple Device to Measure Acceleration

IL 36 ** (Source of Fig. 17)

This device consists of a tethered ball floating in a jar of glycerin. To establish its operation, hold it in front of you, and begin rapidly walking across the lecture hall. The ball will move forward in the direction of your acceleration at first, and then return to the vertical position as your velocity becomes constant. When you stop walking, the ball will move back towards you showing a deceleration.

Physics and Biology

Haldane continues and discusses some of the advantages of size:

An insect, therefore, is not afraid of gravity; it can fall without danger, and can cling to the ceiling with remarkably little trouble. It can go in for elegant and fantastic forms of support like that of the daddy-longlegs. But there is a force which is as formidable to an insect as gravitation to a mammal. This is surface tension.

Research for the student

You've seen examples of surface tension in action: water spiders walking on water, soap bubbles, or perhaps water creeping up inside a thin tube. One way to define surface tension is:

The amount of energy required to increase the surface area of a liquid by a unit amount. So the units can be expressed in joules per square meter (J/m^2).

You can also think of it as a force per unit length, pulling on an object. In this case, the units would be in Newtons / meter (N/m).

1. Show that J/m^2 and N/m are equivalent.
2. Study surface tension using the following approaches:
   a. Float a needle on water.

      This is quite easily done by laying a piece of tissue paper gently on top of the water in a glass and then placing a needle on the paper. Then, with another needle carefully push the paper down into the water. It is obviously held up by surface tension since needles are made of steel which is almost 8 times as dense as water.

   b. Estimate the surface tension of water using a simple thought experiment (TE) that makes the following plausible assumptions:
      i. An iron needle, 1mm in cross section and 1 cm long floats in water.
      ii. About half of the needle is immersed in water.
      iii. The density of water is 1.00 g / cm^3 and the density of iron is about 8 g / cm^3. The surface tension of water is about 0.7 N/m. How close is your value based on this simple calculation? Discuss.

   c. You can also blow a soap bubble using a soda straw.

Notice that after you blow a bubble, it tries to contract if you let the air escape. This is because the surface tension of the water creates a pressure inside the bubble which is 2s/r greater that the pressure outside. Here s is the surface tension (for water, about 0.07 N/m) and r is the radius of the bubble. (The 2 is there because a soap bubble has two surfaces: the inside and the outside).

d. Similar effects can be seen when a thin glass tube is put halfway into water. The water climbs up the walls of the tube because the water molecules are attracted to the glass molecules more strongly that to other water molecules. Mercury shows the opposite effect and the mercury level is depressed inside a thin glass tube. You might have a mercury thermometer where you can observe this effect.

Students should study this strange phenomenon and then offer an explanation.

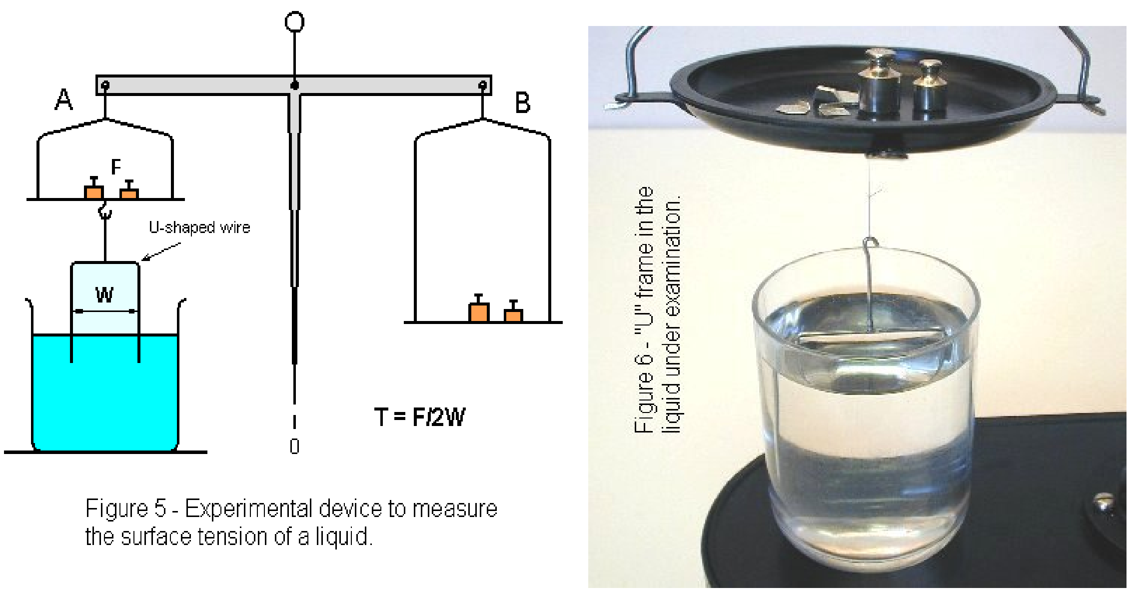


Fig. 18: Measuring the Strength of Surface Tension

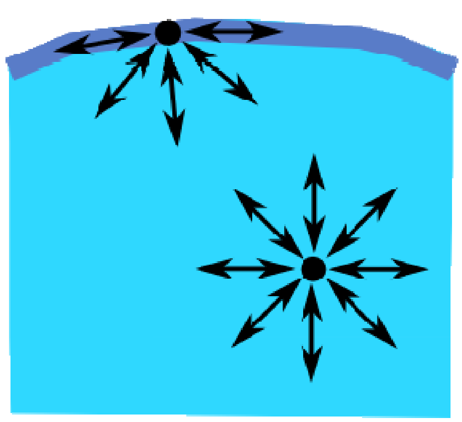


Fig. 19: Diagram of the forces on a molecule of liquid.

Fig. 20—A Floating Needle Demonstration

IL 37 ** (A floating needle)

Laplace Estimates the Size of a Molecule

The great physicist Pierre Laplace, already in the early 1800s, estimated the size of a molecule using the latent heat of vaporization and the surface tension of water. His argument went like this:

*The solution will be in two parts: First we consider **latent heat**, and then we look at **surface tension.***

1. *Latent heat*:

*Consider the transfer of a molecule from within the body of the liquid (water) to a point, a distance **d** above the surface (Fig 19, upper part). When the molecule is evaporated it has to move a distance of **2d** against an average molecular attraction **f.** Thus the work per molecule is = **2fd***

We can write: **L = 2 fd / m**

where **L** is the work done per gram, and **m** is the mass of one molecule.

2. Surface tension:

Here we shall find the work done in bringing up molecules from within the body of the liquid to create a 1 cm^2 of new free surface. So long as the molecule is more than a distance d from the surface it is not, on the average, subject to a force in any direction.

*But, as it enters the surface layer, it begins to experience a net force pulling it back into the main body of the liquid (see Fig 19, bottom part. In considering a 1 cm^2 of boundary layer we have to up **ρd / m** molecules, and the average distance moved by each molecule is ½ **d** against a force of **f.***

*Thus the work per molecule to bring it into the surface layer is ½ **fd.** Therefore the work required to produce 1 cm^2 of surface (S) is given by*

$$S = \rho f d^2 / 2m$$

From these two results we have: $d = 4S / \rho L$

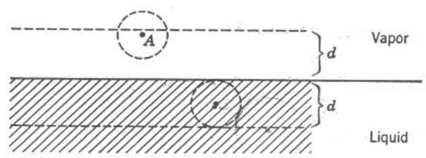
Laplace knew that for a liquid at room temperature,

S = 75 dynes /cm

L = 500 cal / g = 2.1x 10^3 Joules per gram

$\rho = 1\ g / cm^3$

It follows then that the size of a molecule is about 1.5 x 10^{-8} cm. This is a very good estimate, when you consider that it was made over 200 years ago.



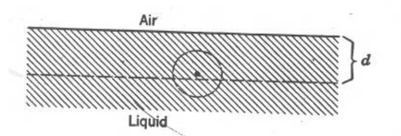


Fig. 21: To illustrate the latent heat of vaporization of a liquid in terms of short-range forces. Molecule A (considered a sphere) is about to escape from all attractions due to the molecules within the liquid (upper). A molecule entering the surface layer of thickness d, and so beginning to acquire surface energy

LI 38 ** (A simple experiment to estimate the size of an oil molecule)

LI 39 ** (Lord Rayleigh's experiment: Taken from IL above)

Lord Rayleigh Estimates the Size of an Oil Molecule

Lord Rayleigh (English physicist and Nobel prize winner, 1842 to 1919) made a guess, one of the earliest good ones, by doing an experiment in which he put a little oil on clean water and watched it spread. He bought a big tub nearly a meter across, cleaned it carefully, filled it with water and then put a tiny droplet of olive oil on the surface. He tried it again and again until he found the amount of oil that would just cover the whole surface, by using crumbs of camphor. Where the water was oily, the camphor did not move, but where it was clean, the camphor rushed around.

Lord Rayleigh knew that the oil molecule consisted of long chains of atoms with one end clinging to the water. He expected the oil to spread until it did not spread any more; until it was one molecule thick. It was a risky guess, but this has since been verified with alternative measurements.

Measurements of the diameter of the oil drop and the diameter of the oil patch on the water are obviously very rough but they gave an order of magnitude size of 10^{-8} cm, the same order of magnitude Laplace obtained with his method of measurement

Questions and problems for the students

1. Try to follow Laplace's argument to estimate the size of a molecule. Do this with a friend and then discuss the concepts and the mathematics with your instructor.
2. If molecules of water were actually micro spheres of diameter of about 1.5 x 10^{-8} cm, about how many molecules of water would you find in 1 cm^3 of water?

3. In chemistry you have learned that 1 gram-equivalent weight of water (18 g) would contain 1 Avogadro number of water molecules (about 6×10^{23}). Using the value of Laplace for the size of a water molecule, how many little spherical water molecules would there be in 18g of water? Discuss your answer.
4. When oil is spilled over water you may have noticed that soon colour patterns will form on the surface (See Fig.). We will see in the next section that these patterns also form on soap films. The formation of color patterns indicates that the thickness is of the order of the wave length of light, or about 5×10^{-7} m, or 5×10^{-4} cm. Here is the problem:

   A large oil tanker spills 1000 tons of oil. How much surface area of the water will the oil cover? Discuss.

Soap Bubbles and Soap Films

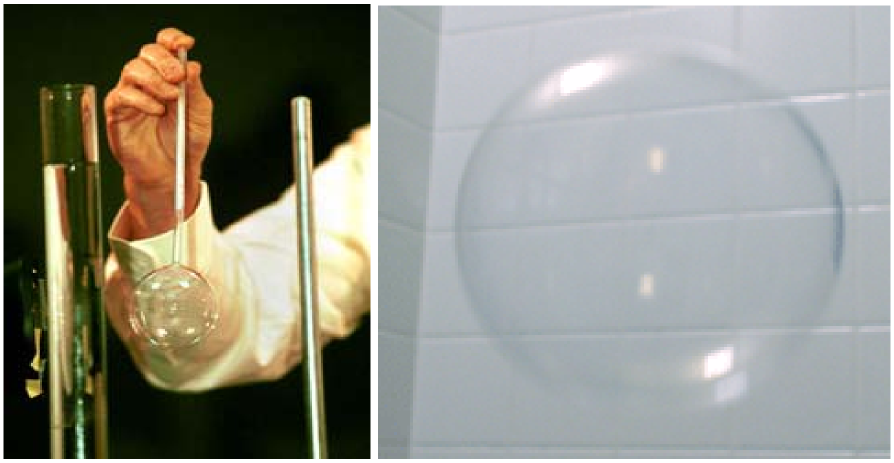


Fig. 22: Making a perfect sphere
a. A soap bubble before it falls: oval b. A soap bubble falling: a perfect sphere

LI 40 *** (Excellent description of the physics of soap films)

IL 41 *** (An advanced and excellent discussion of surface tension)

Finding the thickness of a soap film

Dip a round wire (see Fig.) into a soap solution and carefully lift the wire with the film attached to it, as shown in IL. Hold the wire so that the film is pulled down by gravity, forcing the film into a wedge shape, as shown in Fig. Using the formula or the calculator determine the thickness of the film for various colors. Note that the upper part of the film is black.

IL 42 *** (Calculating the thickness of a soap film)

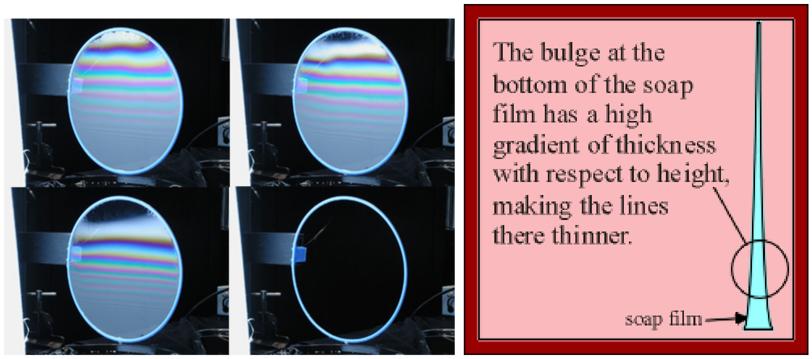


Fig. 23: The wedge shape of a soap film produces different color interference a. Interference pattern seen b. Shape of the film

Fig. 24: Interference Pattern when Held Horizontally

A soap film has a thickness L and an index of refraction n. Light is incident on the film. The wavelength of light in air is λ_0. When an incident ray of light strikes the film, some of it is reflected (the **reflected ray** r_1) and some of it is transmitted (the **transmitted ray** r_2). What happens to each ray?

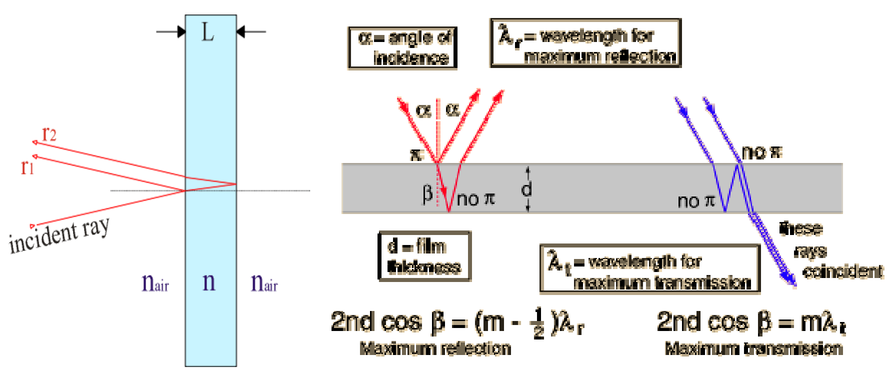


Fig. 25: Measuring the Thickness of a Soap Film

The reflected ray: Since $n > n_{air}$, the index of refraction of air, when the light ray r_1 reflects off the surface of the soap, its phase is shifted by λ_0

The transmitted ray: The ray r_2 travels through the soap, reflects off the back of the film at the soap-air interface, then travels back through the soap. At the first interface again, some of r_2 is transmitted out to the air where it can **interfere** with r_1.

IL 43 ** (measuring the thickness of a soap film)

IL 44 ** (measuring the thickness of a soap film)

Questions and problems for the student

1. A curious discrepant event: The following is a popular parlor game. Peanuts are placed in a glass of beer (actually any carbonated drink will do) and their motion is observed. What one sees is rather discrepant (unexpected): The peanuts will slowly sink to the bottom of the glass and after a short time will rise again, only to sink back to the bottom. The motion continues for a long time and then it stops. Most of the peanuts will settle at the top, floating on the surface. Try to explain this motion.
2. In an interference pattern seen on a soap film (See Fig.) the top part is usually black, followed by strips of blue, green, to yellow, to red.
    a. Looking up the wavelength of light for these colors, estimate the thickness of the film and show that the film must be wedge-shaped.
    b. The top part, which is black, must have a thickness that is considerably smaller than the wavelength of blue light. Why?
    c. Estimate this thickness.
    d. Now use the size of Laplace's molecules estimate the number of water molecules across the thickness at the top of the wedge.

How Can Soap Films Help to Optimize Road Layout?

A soap film will form the shape with the minimum surface area, to minimize the energy associated with surface tension. Soap films always try to form a surface with minimum area, given of course that they meet the other constraints on the system. A bubble must enclose a specific volume of trapped air so the minimum surface required to do this is that of a sphere.

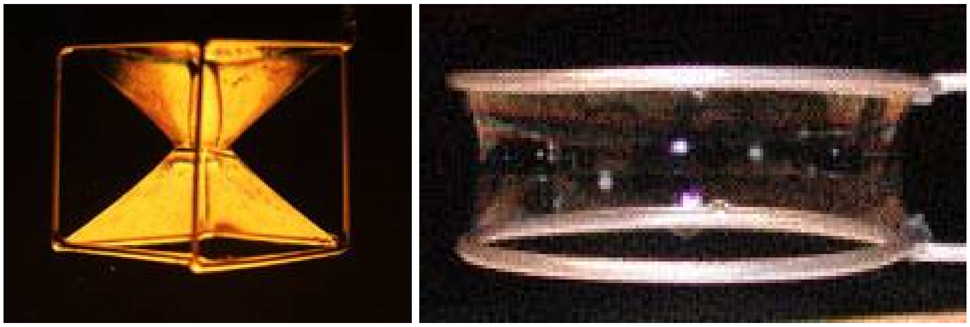
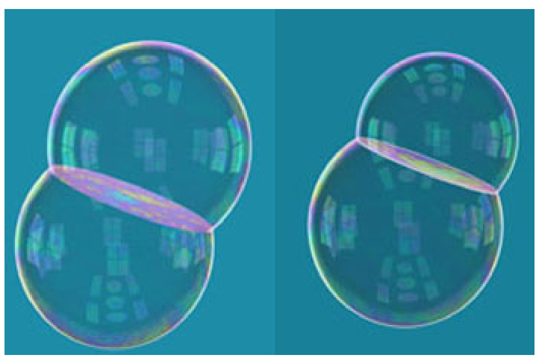


Fig . 26: Soap film arrangements to find minimal surface area
a. Cubic thread framework b. Soap film between two parallel circular rings

Double Soap Bubbles: Proof Positive of Optimal Geometry

Taken from Double Soap Bubbles

What do dish soap, an ancient question, a team of mathematicians and their ingenious proof of the Double Bubble Conjecture have to do with solving 21st century optimization problems? Plenty**.**



Side-by-side images of double-bubbles of (a) equal and (b) unequal volume chambers

Ask Frank Morgan, a leading researcher in optimal geometry, "What happens when one soap bubble likes another soap bubble?" and he'll answer with effervescent enthusiasm. "They meet to make a double bubble. And they always meet at angles of 120 degrees."

Although the question may sound like a riddle, it involves complex mathematics and science. Every time two soap bubbles form a double bubble, they demonstrate the best -- or optimal -- geometric figure for enclosing two separate volumes of air within the least amount of surface area. It took mathematicians centuries to arrive at the proof, which was announced in 2000. Further research using techniques from that proof could enhance our understanding of the physical properties of structures ranging in size from the nanoscale to the galactic.

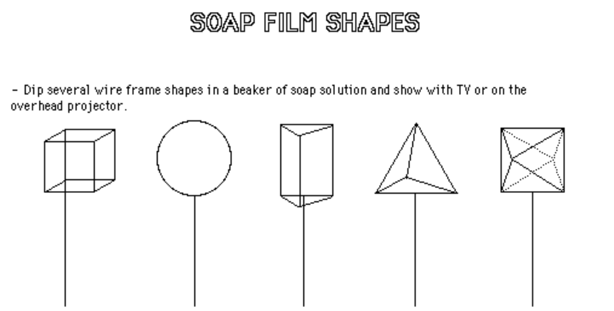


Fig. 27: Different wire shapes to find the minimal area of a 3-D configuration

Student activity

1. Make wire frames like those suggested in Fig. 27 and dip them into a soap bubble solution. Study the surfaces produced and discuss with your friends and the instructor.
2. A soap bubble always becomes a perfect sphere in free fall (Fig. 22). Why is this so?
3. Try to have two soap bubbles come together as shown above. Discuss the statement by the scientist Frank Morgan:

   Every time two soap bubbles form a double bubble, they demonstrate the best -- or optimal -- geometric figure for enclosing two separate volumes of air within the least amount of surface area. It took mathematicians centuries to arrive at the proof, which was announced in 2000.

IL 45 ** (Fig. 27 taken from here). Also see:

IL 46 **** (An excellent source of history of soap bubbles and the physics of surface tension and films

IL 47 *** (The link for the problem below)

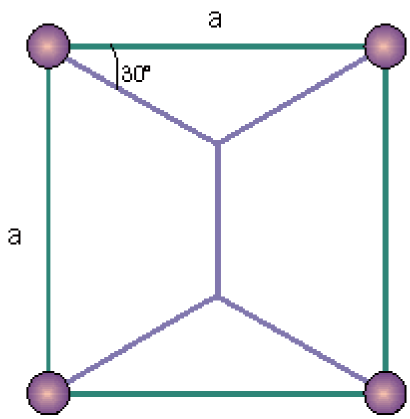
The following is taken from **IL 47** :

To find the minimum length of road that needs to be built to connect a number of towns and cities one only needs to create a map of the towns and cities using two transparent plates. These plates are joined together with pins at the location of the towns and cities. An air gap between the plates is left for the soap film which is supported by the plates and the pins. As the soap film forms a surface of minimum area the roads should be planned according to the lines of contact between the film and one of the plates.

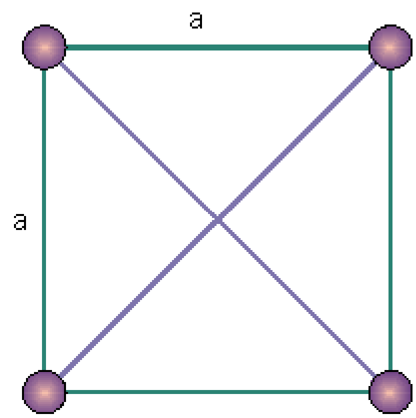
For complex arrays of pins one has to be careful as there may be many local minima for the surface area.

Question: What would be the minimum road layout joining four towns each at the corner of a square?

Try to solve this problem before looking at the solution below in detail.



The optimal route, total road length 2.232a



The simple cross has road length 2.828a

Fig. 28: An Optimization Problem that a Soap Film "Solves" Naturally

Questions and problems

1. Study the solution to the optimization problem above and discuss with other students.

More to come …

Soap Bubbles, Surface Tension and the Mystery of Beer Foam

Fig. 29: The Mystery of Beer Foam

The following is taken from the internet, entitled **The beer froth equation** (April 2007):

> *Scientists in the United States say they have devised an equation which could help solve the age-old question: why does the foam on a pint of lager quickly disappear, but the head on a pint of stout linger?*
>
> *Writing in the British science journal* Nature, *Robert MacPherson from the Institute for Advanced Study in Princeton, New Jersey, and David Srolovitz from Yeshiva University, New York, say they have devised an equation to describe beer froth.*

> *The breakthrough will not only settle the vexatious lager vs stout debate, it will also help the quest to pour a perfect pint every time.*
>
> *Beer foam is a microstructure with complex interfaces.*
>
> *In other words, it is a cellular structure comprising networks of gas-filled bubbles separated by liquid.*
>
> *The walls of these bubbles move as a result of surface tension and the speed at which they move is related to the curvature of the bubbles.*
>
> *As a result of this movement, the bubbles merge and the structure "coarsens," meaning that the foam settles and eventually disappears.*
>
> *Three-dimensional equations to calculate the movement have been made by Professor MacPherson, a mathematician, and Professor Srolovitz, a physicist.*
>
> *They build on work by a computer pioneer, John von Neumann, who in 1952 devised an equation in two dimensions.*
>
> *The mathematics of beer-bubble behaviour are similar to the granular structure in metals and ceramics, so the equation also has an outlet in metallurgy and manufacturing as well as in pubs.*

This report should be discussed in the class room.

<u>A discussion question</u>

The following is a quote by the physicist Lord Kelvin:

> *Blow a soap bubble and observe it. You may study it all your life and draw one lesson after another in physics from it.*

What lessons have you so far drawn from your study of soap bubbles?

IL 48 ** (Whales blowing soap bubbles video)

IL 49 ** (History of "soapbubbling")

Now back to Haldane. He continues:

> *A man coming out of a bath carries with him a film of water of about one-fiftieth of an inch in thickness. This weighs roughly a pound. A wet mouse has to carry about its own weight of water. A wet fly has to lift many times its own weight and, as everyone knows, a fly once wetted by water or any other liquid is in a very serious position indeed. An insect going for a drink is in as great danger as a man leaning out over a precipice in search of food. If it once falls into the grip of the surface tension of the water—that is to say, gets wet—it is likely to remain so until it drowns. A few insects, such as water-beetles, con-*

trive to be unwettable; the majority keep well away from their drink by means of a long proboscis.

All warm-blooded animals at rest lose the same amount of heat from a unit area of skin, for which purpose they need a food-supply proportional to their surface and not to their weight. Five thousand mice weigh as much as a man. Their combined surface and food or oxygen consumption are about seventeen times a man's. In fact a mouse eats about one quarter its own weight of food every day, which is mainly used in keeping it warm.

Problems for the student

1. You can check Haldane's claim by assuming that a mouse has a mass of about 12g, is about 10 cm long. Assume that the mouse and the man are "geometrically and materially similar". The man is about 175cm tall.
   a. Estimate the mass of the man.
   b. Estimate the ratio of the surface areas of the man and the mouse.
   c. Compare the rate at which heat is lost by their bodies.
   d. How does their food intake compare if this rate is to be maintained?
2. King Kong's son (KKS) needs about 2000 food calories (kilocalories) to maintain his health and a constant body temperature. How many kilocalories would KK need? KK is 20 m tall and KKS is 1 m tall.
3. Compare the amount of food per body unit mass KK needs to that of KKS.

Jumping

Haldane remarks that, although Galileo demonstrated the contrary more than three hundred years ago, people still believe that if a flea were as large as a man it could jump a thousand feet into the air. As a matter of fact, *the height to which an animal can jump is more nearly independent of its size than proportional to it.* This is a surprising claim.

We can compare the heights to which a cat and a tiger can jump. A young adult cat can jump to a height of about 5 feet. A tiger can jump to a height of about 8 feet. However, if we count the height that they jump from their *center of mass*, the cat and the tiger jump to about the same height.

IL 50 ** (Source of Fig. 21)

Fig. 30: "Cats" Jumping

Questions for the student

1. What argument can you give for believing "the height to which an animal can jump is more nearly independent of its size than proportional to it "?
2. How would you convince someone that if a flea were as large as a man it could jump a thousand feet into the air is not possible?
3. An elephant (5 tons) eats about 100 kg of food a day. If the elephant were the size of a mouse (about 50g), how much food would the "mouse" need to sustain life? Assume that all the energy of the food is ultimately lost to the environment through the surface area. Compare the food that is consumed by each to their body weight. Comment.
4. The following is taken from the website: "The Flea, the Catapult and the Bow". Based on the website below, the text is also placed in the Appendix.

IL 51 **** (A fascinating and amusing discussion: The Flea, the Catapult and the Bow" of how insects (flees) manage to jump so high in comparison to their size).

The author of the previous discussion, an engineer, is talking about how the jumping technique of fleas uses the same basic "technology" as that used by an archery bow, cross bows, and ancient catapults (and lots of other things.

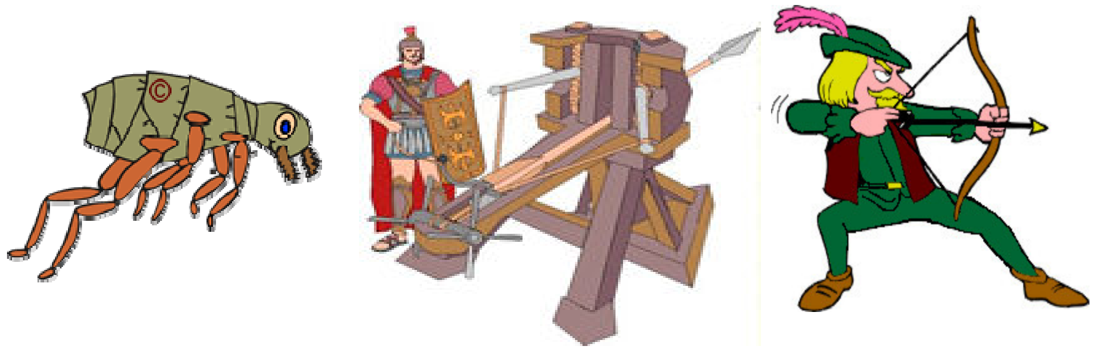


Fig. 31: Taken from "The Flea, the Catapult, and the Bow".

Read : "How a Flea Changes Muscle Energy into a High Speed Jump" in the website above, or in the Appendix. The essential argument is as follows.

Click on Appendix: *Energy storage and energy changes in Fleas, Catapults, and Bows*

1. *The first problem is air resistance. Air resistance slows small things a lot more than big things. For an animal the size of a flea, air resistance is a huge problem. Nothing can be done about this except, of course, go somewhere where there is no air, like on the moon. The flea could jump significantly higher in a vacuum (except that he'd be dead).*

Question for the student

1. Compare the air resistance (expressed in Newtons /m^2 for a large bird and for a small insect).

2. *The second problem is that **muscle moves too slow**. How high an animal jumps depends on how fast it is traveling when it leaves the ground (and of course, on how much air resistance slows it down afterward). The flea's short legs only allow it an acceleration distance of a fraction of a millimeter. In order to reach an acceptable take-off velocity (speed) the flea has to accelerate (speed up) very quickly. There are real physical limits on how fast muscles can move and how much power they can generate. There is no way the flea's muscles (or any animal's) muscles, can achieve the necessary speed. They just can't generate that kind of power.*

The author continues:

> *But we all know that fleas can jump pretty well. This means they are speeding up (accelerating) during the jump much faster than should be possible if they were using their muscles during the jump.*
>
> *So how do they do it? How do they jump higher than it's possible for muscles to jump. Is it magic? Nah. They just cheat a little. They use their muscles, not to jump, but to slowly store energy in an efficient springy material called resilin. Then, when they are ready to let loose, they release the energy quickly in a burst of power that literally springs them into the air like a...well...like a spring. It's pretty much just like a catapult.*

<u>Questions and problems for the student</u>

An early example of supplementing muscle power: storing energy in a device to propel an object is the bow and arrow:

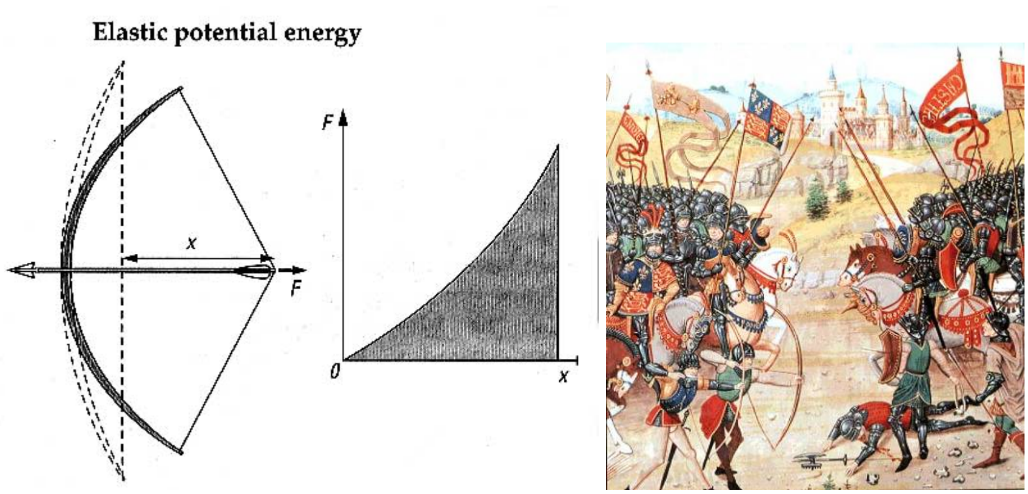


Fig. 32: a. The Physics of Shooting an Arrow. b. A Medieval Battle.

IL 52 *** (A detailed discussion of the physics of the bow and arrow. A good description of warfare using the bow and arrow in the middle ages.).

IL 53 *** (A detailed discussion of the modern archery)

<u>Questions and problems for the student</u>

1. Explain in your own words how a bow transforms energy.
2. There is good evidence (see website above) to believe that a good archer in the 15th century could shoot as far 250 m. The maximum force on the bow was as high as 150 pounds, or about 700 N and the mass of the bow about 40 grains or about 60 gm. The arrow is held at an angle of 45 degrees to insure a maximum range. Assuming that the bow behaved like spring, and that the bow is pulled out to a distance of 30 cm, calculate:
   a. The average force on the bow.
   b. The total potential energy stored in the bow.
   c. The initial velocity of the arrow.

   It is known that about 70 % of the energy of the arrow will be lost to friction.

   d. Estimate the height to which the arrow will rise
   e. The maximum range of the arrow, if the ground is level.

f. The final velocity of the arrow just before it hits the ground. Neglect the height of the archer.

Now let us look at the physics of a contemporary bow and arrow, as discussed in the website below:

IL 54 *** (Detailed description of arrow flight)

An advanced problem

The following is a quote taken from the above website, **IL 54**

> *For what I think my bow does, the 90 meters trajectory was found to be like this: initial velocity 56 m/s, launch angle 11 degrees. Arrow reaches max. height of 6.70 m after 0.96 s at 47 m before plunging into the target after 2.02 s at a speed of 35 m/s. So for the 90 m distance this says my arrow looses 40 % of his speed and that about 60 % of it's energy was consumed by drag.*

1. Show that the energy loss is about 60%.
2. Calculate the range of the arrow if there had been no frictional losses.
3. Calculate the height of the arrow if there had been no frictional losses.
4. Estimate the maximum range if the arrow is shot at angle of 45 degrees.
5. What would be the maximum range if there were no air resistance?

The Physics of Flying

Haldane states that it is an elementary principle of aeronautics that the minimum speed needed to keep an airplane of a given shape in the air varies as the *square root of its length*. That means that if its linear dimensions are increased four times, it must fly twice as fast. It is also known that the power needed for the minimum speed increases more rapidly than the weight of the machine. So the larger airplane, which weighs sixty-four times as much as the smaller, needs one hundred and twenty-eight times its horsepower to keep up.

Problems for the student

1. Give an argument to show that "the minimum speed needed to keep an airplane of a given shape in the air varies as the *square root of its length*"
2. Discuss the claim that "the larger airplane, which weighs sixty-four times as much as the smaller, needs one hundred and twenty-eight times its horsepower to keep up".

Research problems for the student

The following problems are based on the specifications of two air aeroplanes, one old and the other very new. We are assuming that the smaller aircraft, the Boeing 737 is similar to the large aircraft, the A380 airbus. The similarities are only approximate, as you can see for yourself when you look at the pictures of the two aeroplanes. Therefore the simple proportionality relations we have developed will only apply approximately.

Study the table below and find out how good the statement that the minimum speed needed to keep an airplane of a given shape in the air varies as the square root of its length is. (These technical specifications were taken from the ILs listed below.)

Fig. 33: Airbus 380

Fig. 34: Boeing 737

IL 55 *** (The A 380 Airbus size and technical details, a comparison with a Boeing 474)

IL 56 *** (Detail of Airbus, outside and inside)

IL 57 ** (The Boeing A 380 Airbus: Picture in Pictures LCP)

Comparing a Boeing 737 and the new giant A380 Airbus:

Specifications	Boeing 737	A380 Airbus
Length	35 m	73 m
Takeoff Weight	60,000 kg	520,000 kg
Thrust	110,000 N per engine	1200 kN per engine
Takeoff speed	210 km/h	300 km/h
Cruising speed	850 km/h	900 km/h
Range	5000 km	14,000 km
Fuel consumption	2.1 liters (0.55 gallon) per 100 seat kilometers (62.5 miles)	2.9 liters (0.76 gallon) per 100 seat kilometers (62.5 miles).
Area of wings	125 m^2	845 m^2
Fuel Capacity	24,000 1	310,000 l
No of passengers	150 (189 max.)	550 (800 max.)
Cabin width	About 4 m	About 7 m
Wing span	39 m	80 m

1. Compare the weights (masses) of the two airplanes, using the proportionality between length and weight for similar object. How good is the comparison? Comment.
2. Compare the approximate take-off speed of the A380 airbus and that of the Boeing 737. Does the scaling law hold here. Discuss.
3. Check the last claim made by Haldane: that the minimum speed needed to keep an airplane of a given shape in the air varies as the square root of its length. Comment.
4. Estimate the weight of a scaled-down model of a Boeing 737 as well as that of an A380 airbus, to a length of 1 m. Also estimate the take-off speed for these models. Comment.

   Use the equilibrium condition for constant velocity and the proportionality statements.

IL 58 ** (Pictures and explanation of flying characteristics of various types of wings)

IL 59 ** (Basic physics of flight and flight of birds)

IL 60 ** (An advanced level of study of the flight of insects, a long power point presentation)

IL 61 *** (A very comprehensive presentation of elementary flight theory, very visual)

IL 62 ** (A report by British Airways)

5. According to British Airways, a 747-400 plane cruises at 576 mph (927 km/h), burns 12,788 liters (3378 US gallons) of fuel per hour, and carries 409 passengers when full
   a. (From **IL 62**.) If the plane is 100% full, what is the "efficiency" of the plane expressed as the number of kilometers a passenger is flying for each liter of fuel burned"?
   b. Consider the following flight, using the Boeing 747 data above: The distance from Chicago to Milan, according to the Global Distance calculator (see IL 53), is 7315 km (4545 miles), making a return journey of 14630 km (9090 miles).
      i. How many liters of fuel are used for each passenger?
      ii. What is the distance that one passenger is flying for each liter of fuel?
   c. Assume that the average fuel consumption 13 km/ l (about 30 miles per gallon). Compare traveling in a car with traveling by air, based on this calculation. Comment.
6. Using our table, estimate the fuel efficiency of the two planes, the Boing 737 and the 380A airbus, assuming the planes travel maximum distance with a full load of passengers. Compare these fuel efficiencies and comment.

IL 63 ** (Distance calculator)

Special problems for the student

1. Lift on an airplane (see **IL 61**) is proportional to speed, density of air, and the area of the wings. Assuming the same conditions for both planes described in the table above compare their weight to wing area ratio, as Newtons per square meter. Comment.
2. What is the maximum acceleration for the two planes on take-off? Express this both in m/s^2 and in terms of the gravitational constant g taken as about 10.0 m/s^2.
3. Compare this acceleration with the maximum acceleration possible for the Mercedes-Benz GL in the example below, from rest to 100 km/h.
4. Compare the drag on the Boeing 737, when travelling at low altitude, say about 300 m above sea level, just before landing, at about 300 km/h with the drag at cruising altitude and travelling at cruising speed. Comment.
5. Imagine the Boeing 737 to fly at cruising speed (850 km/h) at an altitude of 300m. Compare the drag force now with that experienced when travelling at cruising altitude and speed.
6. Estimate the ratio of the rate of fuel consumption (litres per second) for the when they climb to cruising height and when they cruise at a high altitude.

7. First compare the drag on the two planes at cruising speeds and then find the drag force on the planes at takeoff speeds and then during the cruising period.
8. Now estimate the acceleration during takeoff, taking into account the drag force acting on the planes.

Research problems for the student

In conclusion to this section we will compare the performance of two "units", one mechanical and the other biological. For the case of the cars, the dimensions will be about 2:1 and for the dogs about 1.75:1.

First, we will compare two dogs which are "geometrically similar" and "physiologically similar." We will choose a large dog, a Doberman, and a small dog, a Fox Terrier.

Next we will look at the performance of two cars, a Mercedes-Benz (2007) GL car and a Mercedes-Benz, the Smart car that is bout is "half size". Cars are arguably also, roughly speaking, geometrically and physiologically similar. For the purpose of our argument, we will imagine that a correspondingly small person will drive the small car.

Part 1: The performance of the two dogs, one large and the other small.

Fig. 35: Comparing a Fox Terrier with a Doberman

IL 64 ** (Source of Doberman in Fig. 26)

IL 65 ** (Description of a Doberman)

IL 66 ** (Source of Fox Terrier in Fig. 26)

IL 67 ** (Description and pictures of a Fox Terrier)

The following data will be required:

- The mass of the large dog, dog 1 (Doberman): 43 kg

- The mass of the small dog, dog 2 (Fox Terrier): 8 kg
- The height dog 1: 70 cm
- The height of dog 2: 40 cm
- The running speed for both dogs is 4 m/s, or about 15 km/h
- The power output for the large dog running is about 100 J/s
- The food energy recommended for the large dog per day is 1600 Kcal/day, or 6700 KJ/day.
- The power output for the large dog while running comfortably is about 100 W (J/s)
- The maximum range of the big dog, running at 4 m/s, is found to be 1 km.

1. Show that the masses of the dogs are approximately given by the proportionality of $\{(\text{height of dog 1}) / (\text{height of dog 2})\}^3$.
2. Approximately how do the surface areas of the two dogs compare?
3. Confirm that the food energy recommended for the small dog is about 300 Kcal/day.
4. Confirm that the power output of the small dog while running comfortably is about 20 W (J/s).
5. Estimate the maximum range of the small dog.
6. Compare the energy consumption of the two dogs.

Part 2: The performance of two cars, one large and the other small

The performance of the Mercedes Benz (2007) GL and the Mercedes Benz Smart car will be compared. We will consider these cars to be made of sufficiently "geometrically and materially similar structures" and therefore valid for comparison, using our scaling laws.

Fig. 36: Comparing two Mercedes Benz cars.

The following data will be required:

- The mass of the large car, The Mercedes Benz (2007) GL: 2000 kg

- The mass of the small car, The Mercedes- Benz Smart car: 500 kg
- The fuel capacity of the large car: 80 l (gasoline)
- The fuel capacity of the small car: 20 l (gasoline)
- Fuel economy for the large car: 20 MPG in traffic, 25 MPG on the highway.
- Fuel economy for the small car: 40 MPG in traffic, 67 MPG on the highway
- The cruising speed on the highway for both cars: 100 km/h, or about 30 m/s
- The total energy available in 1 l of gasoline: 32 MJ
- The energy actually used for driving the car is about 12%.

1. Show that the fuel capacity of the small car should be about 20l.
2. Determine the total energy available to the large car when the fuel tank is filled to capacity.
3. Determine the total energy available to the small car when the fuel tank is filled to capacity.
4 Calculate the maximum range of the large car.
5. Estimate the maximum range of the small car.
6. Compare the fuel consumption of the two cars.

(Note: the rest of the energy are: thermal (60 %), friction, about 20%, and other losses).

The gasoline consumption for the large car traveling on a level highway at a100 km/h is about 1 l/ 10 km

IL 68 ** (Technical specs of the Mercedes Benz GL)

IL 69 ** (Picture of Mercedes Benz Smart car)

The following specs are for two cars that you could actually buy, one large and the other small. The large car is the Mercedes -Benz (2007) GL class and the small one is a Mercedes-Benz Smart car. The dimensions for these cars are almost exactly 2:1 and they are sufficiently "geometrically similar" as well as "physiologically similar".

In this case the idealized assumption made earlier when doubling the size of "geometrically similar" units does not apply any more. Why not?

Type	Length m	Mass kg	Power HP kW	Fuel Capacity l	Fuel Consumption Km / l	Est. Maximum range km	Price $ (Am)

							15000
MB GL	5.08	2500	335 250	33	7.0	693	
Smart Car	2.50	730	61 45	120	21	840	60,000

We could add the following specs :

Type	Acceleration. Time (s), from 0-100 km/ h	Top speed Km / h	Width Height (m)	Drag Coefficient (Nm/kg) Area (m^2)	Economy Price/ km?
MB GL	9.0	240	1.92 1.84 1.51	0.30 1.00 0.30	
Smart Car	15.5	134	1.55	0.50	

<u>Questions based on the tables above</u>

1. What is the acceleration (actually the 'average acceleration) of both cars, from rest to 100 km/h? Express these in terms of g, the acceleration due to gravity.
2. What is the average force acting on the cars to accomplish this acceleration?
3. Power is defined as rate of doing work. Express this as force times velocity and calculate the power that would be necessary to maintain an acceleration that you calculated at 100 km/h . *Notice that we are neglecting the drag force acting on the car.*

The Drag Force Acting on a Car

The drag force acting on a car, due to air resistance, is small for low speeds but becomes important after the car reaches speed of over about 50 km/h . We will now apply our knowledge of drag on a freely falling object to the motion of cars.

As before, for a freely falling sphere, drag force on a car: proportional to the effective Area, density of air and the square of the velocity:

$\mathbf{D \propto A\, \rho\, v^2}$

or $\mathbf{D = A\, \rho\, v^2\, C_D}$

where **D = Force of the drag (N)**

ρ = Density of air (kg/m^3)

v = velocity (m/s)

A = Area (m^2)

C_D = Coefficient of drag (Dimensionless).

<u>A "guided" research problems for the student</u>

It is interesting to use the specs given above for a car and, using elementary physics, and make some interesting calculations. We will use the Mercedes-Benz GL as an example.

1. Calculate the force required to accelerate the car from 0-100 km/h in 9.0 seconds. Show that this force is about 8300 N. (We have already made this calculation earlier)
2. The force you calculated, however, must overcome the inertia of the car as well as the drag force that acts on a car. Calculate this force, expressed as an average. Show that the values given in the table below are correct.
3. The total force, however, must be equal to the accelerating force and the drag force. You will notice that even at 100 km/h the drag force is negligible when compared to the accelerating force. So taking 8300 N 100 km/ h should give us a good idea of the effective power output of the engine.
4. Show that the power output is about 250 kW,. This value is the same as the value of the power of the engine given by the specs above.
5. We will now find the energy requirement for driving the car on a level road at a constant speed of 30 m/s , or about 108 km/h. for 10 km We have chosen 10 km because the consumption of the engine at this speed and in these conditions is about 1 liter. It is known that 1 liter of gasoline has an energy capacity o 30 MJ.
6. Let us assume that the all the frictional forces (including drag) acting on the car at this speed is about 350 N, including the drag. Show that the energy required to drive the car under the conditions specified it would be about 3.0 MJ. The overall efficiency then is about 10%.

Calculating the Drag Force on a Mercedes-Benz GL

V (km/h)	V (m/s)	Drag force (N)	Power Required for that constant velocity to overcome drag alone (P = Fv = Dv)
36	10	20	. 0.20 kW 0.27 HP
54	15	45	0.67 0.90

72	20	80	1.6 2.14
90	25	125	3.12 4.18
108	30	180	5.40 7.23
126	35	245	8.57 11.5
144	40	320	12.8 17.2
180	50	500	25.0 …………33.5

Student activities

1. Plot a graph of velocity versus drag force.
2. Plot a graph of "power required" against drag force.
3. Compare the two graphs. For example, they are both exponential graphs.

Energy, Power and the Physics of the Internal Combustion Engine

IL 70 *** (A comprehensive short discussion of efficiency and the second law of thermodynamics).

The laws of thermodynamics describe the energy exchange and determine the efficiency involved in heat engines in general. Thermodynamics can also be defined as the *study of the inter-relation between heat, work and internal energy of a system.*

There are two main laws plus a third one that forbids reaching absolute zero temperature. The Laws of Thermodynamics dictate the specifics for the movement of heat and work. Basically, the First Law of Thermodynamics is a statement of the conservation of energy - the Second Law is a statement about the direction of that conservation - and the Third Law is a statement about the impossibility of reaching absolute zero (-273° C, or 0 K).

The first law is simply an expression of the general law of conservation of energy. This law (or principle) can be expressed verbally and symbolically (mathematically) on several levels of complexity. We will give two simple verbal definitions, followed by two simple symbolic representations. Note that in the second version we have a statement of the first law of thermodynamics.

Verbal

The law of conservation of energy states that:

1. *Energy cannot be created (made from nothing), or destroyed (made to disappear) and that energy can be changed from one form to another (such as heat energy into mechanical work, or electrical energy into heat energy).*
2. *The change in the internal energy of a system equals the difference between the heat taken in by the system and the work done by the system.*

Symbolic (mathematical)

For the first law:

$$\mathbf{Q = W + \Delta U,}$$

where **Q** is equal to the net amount of heat flowing *into* a system during a given process, **W** is the net work done *by* the system, and **Δ U** is the change in the system's internal energy.

The second law describes and prescribes the restrictions placed on the first law. These restrictions were suggested by such empirical evidence as the observation that no work can be done unless heat can be taken in at one temperature and exhausted at a lower temperature. In addition, we know from elementary physics that gravitational potential energy problems, work can only be done if a body from one height is taken to a lower height. Does this contradict the law of the conservation of energy?

We will also express the second law verbally first and then symbolically.

Verbal:

1. *Heat cannot, by itself, pass from a colder to warmer body.*
2. *It is not possible for heat to be transferred from one body to another body that is at a higher temperature with no other change taking place.*

Another way to state the second law is:

1. *There are no perfect engines.*
2. *There are no perfect refrigerators.*

Symbolic (mathematical):

There is a standard mathematical expression that relates to the second law that uses the concept of entropy, which is a complex concept that relates to the so called *disorder* of a system. But we will only use the idea of efficiency to demonstrate the how the first and the second laws are related.

The second law forbids that any energy transformation is 100% efficient. For example, it is not possible to convert all the heat energy in an internal combustion engine to driving the car. You already know that efficiency is defined as the energy put into the system divided by the work derived from the system. For a car engine, for example, the purpose is to transform as much of the extracted heat $\mathbf{Q_H}$ as possible and transform it to mechanical energy.

Look at Fig. During every cycle, energy is extracted as heat $\mathbf{Q_H}$ from a reservoir at temperature $\mathbf{T_H}$, a portion is diverted for useful work. **W**, and the rest is lost as $\mathbf{Q_C}$. to a reservoir at a lower temperature $\mathbf{T_C}$.

The first law says that

$$\mathbf{Q = W + \Delta U.}$$

Rewriting, we have: $\mathbf{W = Q - \Delta U.}$

Clearly, the change of the internal energy **Δ U,** is zero here, because the gas in the cylinder returns to its original value. From the first law then we can write for the engine

W = Q. But **Q** here is given by $\mathbf{Q_H - Q_C}$.

Since efficiency **e** is given by $\mathbf{W_{in} / W_{out}}$, we have

$\mathbf{e = (Q_H - Q_C) / Q_H}$

The efficiency can also be written in terms of the temperatures of the gas in the cylinder and the temperature of the gas at the exhaust:

$\mathbf{e = (T_H - T_C) / T_H}$

(Remember, the temperature must be expressed in degrees Kelvin)

(It is interesting to note that the French engineer Sadi Carnot discovered this relationshipin 1824, 25 years before the laws of thermodynamics were established by von Helmholtz and Kelvin). This equation was applied to finding the efficiencies of steam engines before the internal combustion engine was developed. It was known that the theoretical efficiency of a steam engine was determined by the difference of the intake temperature and the exhaust temperature of the gases involved. We can write this famous equation this way:

% efficiency = 100 x (1 –Temp. of the exhaust gas / Temp. of the exploding gas)

The temperatures must be expressed in the Kelvin scale.

It is important to note that this is an *ideal efficiency* - real engines also lose some efficiency due to friction, etc., but this is above this theoretical limit. Thus, a heat engine would operate with 100% efficiency in converting heat into useful work only if the cool reservoir was at 0º K (-273 K), and if their were no frictional losses. This impossible to achieve.

For example, a steam powered electrical generating plant which operates between 500 K and 300 K (room temperature) has a maximum possible efficiency of 40%. You can check this using the equation for efficiency. Similar considerations hold for an internal combustion engine, the basic operation of which is illustrated below

IL 71 **** (An especially well presented (very visual) discussion of the two laws)

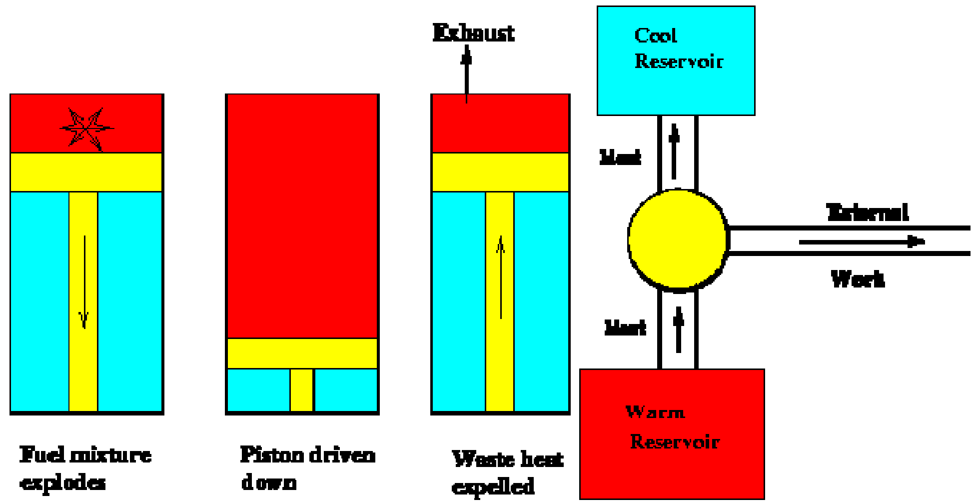


Fig. 37: The Heat Exchange for an Internal Combustion Engine

In this segment of the cycle, the fuel mixture explodes, either from a spark plug for a gas engine or from the high pressure for a diesel engine. This drives the piston downwards, which subsequently turns the crankshaft and eventually the wheels - this is the part which converts the energy of heat into useful work. The piston then rises, expelling the exhaust gases which carry away the waste heat. The cycle then goes on to draw in more fuel mixture to repeat the cycle. The major point here is that the exhaust gases carry with them excess heat which could not be converted into useful work.

The important message here is that some ``waste heat'' is always expelled into the cooler reservoir; and that no heat engine could operate without such expulsion. This is why, for example, one notices in the winter near a steam powered electrical generating plant that nearby ice on a river is melted - this comes from the waste heat of the plant being expelled into the river.

The British scientist and author, C.P. Snow, had an excellent way of remembering the three laws:

1. ***You cannot win*** *(that is, you cannot get something for nothing, because matter and energy are conserved).*
2. ***You cannot break even*** *(you cannot return to the same energy state, because there is always heat loss to work done, or there is always an increase in disorder; entropy always increases).*
3. ***You cannot get out of the game*** *(because absolute zero is unattainable).*

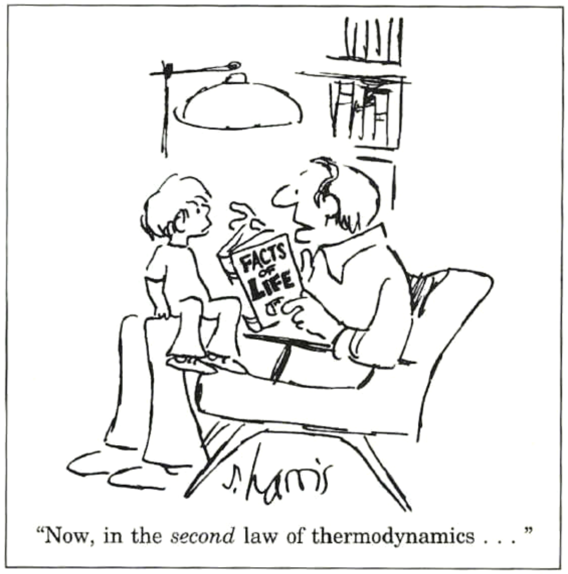


Fig. 38: The student should try to complete this sentence

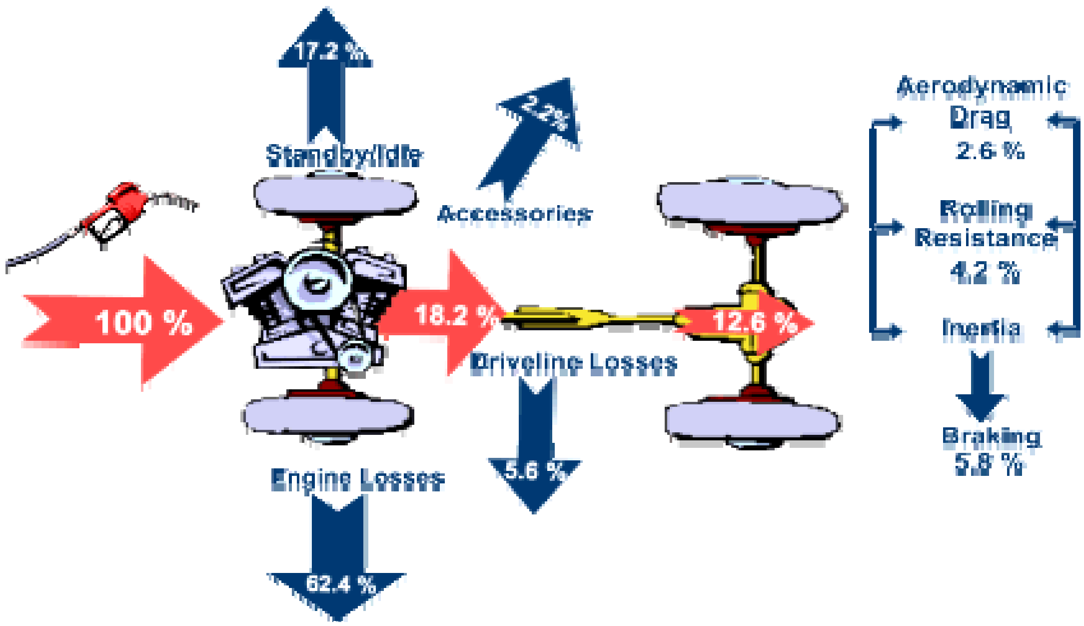
The Internal Combustion Engine

The following gives you the energy distribution for an internal combustion-driven car:

Input energy: (Heat content of the gasoline) 100%

Engine heat (heat lost in exhaust gases, heat lost o coolant , heat lost to air): 62%

Drive Train: Overcoming friction in transmission, differential, and wheel bearings,): 10%, overcoming inertia, air resistance and gravity: 12%



Idling Losses – 17.2% Rolling Resistance – 4.2%

Aerodynamic Drag – 2.6% Driveline Losses - 5.6%

Accessories – 2.2% Engine Losses – 62.4%

Overcoming Inertia; Braking Losses– 5.8%

Fig. 39: The Energy Losses of a Car Engine

(For detail see IL below)

The Age of "Green Cars"

Scientific evidence strongly suggests that the buildup of greenhouse gases in the atmosphere is raising the earth's temperature and changing the earth's climate - both have many potentially serious consequences. Transportation, specifically the combustion of fossil fuels in our vehicles, is the single largest source of human-made greenhouse gases. The more fuel your vehicle burns the more greenhouse gases it emits.

You may be surprised to know that most vehicles produce several times their weight in greenhouse gases each year. Not only does most of the fuel you put in your tank become greenhouse gas emissions, but the carbon in the fuel combines with oxygen in the air, almost tripling the weight of the fuel itself.

A major contributor to the green house effect by human activity (sometimes referred to as *anthropogenic)* is carbon dioxide. (The other major contributor is water vapor). Carbon dioxide is produced from the combustion of fossil fuels (coal, oil, gas) in vehicles, industrial boilers and residential furnaces. The average car pumps two to three times its own weight in carbon dioxide into the atmosphere each year. About 30% of all carbon dioxide emissions in Canada are from road vehicles and mostly from personal and commercial light-duty vehicles.

The greenhouse gas estimates presented here are "full fuel-cycle estimates" and include the three major greenhouse gases emitted by motor vehicles: carbon dioxide, nitrous oxide, and methane (notice the absence of water vapor). Full fuel-cycle estimates consider all steps in the use of a fuel, from production and refining to distribution and final use. This gives a more complete picture of how using a particular fuel contributes to climate change.

If you want to help the environment by driving the most environmentally positive or "greenest" vehicle, think small or think hybrid.

IL 72 ** (List of anthropogenic green house gases)

IL 73 *** (Good general discussion of hydrogen economy)

IL 74 ** * (A discussion of green house gases)

IL 75 *** (Discussion of hydrogen economy)

IL 76 *** (Description of all makes and models of cars and their fuel economy)

IL 77 *** (A comprehensive discussion of the green house effect and the contribution to this effect by emissions by cars)

IL78 **** (An excellent discussion and a comprehensive list of environment friendly cars)

IL 79 *** (A list of the "greenest cars" available in 2007)

IL 80 *** (Description and explanation of "Green Rating")

IL 81 *** (A detailed discussion of the green house effect)

The following is taken from **IL 76**:

The major natural greenhouse gases are water vapour, which causes about 36-70% of the greenhouse effect on Earth (not including clouds); carbon dioxide, which causes 9- 26%; methane, which causes 4-9%, and ozone, which causes 3-7%. It is not possible to state that a certain gas causes a certain percentage of the greenhouse effect, because the influences of the various gases are not additive. (The higher ends of the ranges quoted are for the gas alone; the lower ends, for the gas counting overlaps.)[2][3] Other greenhouse gases include, but are not limited to, nitrous oxide, sulfur hexafluoride, hydrofluorocarbons, perfluorocarbons and chlorofluorocarbons (see IPCC list of greenhouse gases).

The major atmospheric constituents (nitrogen, N_2 and oxygen, O_2) are not greenhouse gases. This is because homonuclear diatomic molecules such as N_2 and O_2 neither absorb nor emit infrared radiation, as there is no net change in the dipole moment of these molecules when they vibrate. Molecular vibrations occur at energies that are of the same magnitude as the energy of the photons on infrared light.

Note that:

a. The major contributor to the green house effect is water vapor

b. The effect of the green house gases is not additive.

c. Nitrogen, (N_2) and Oxygen, (O_2) are not greenhouse gases.

Discussion:

a. In almost all cases of reporting to the public in the media about global warming, the fact that the major contributing green house gas is water vapor, is not mentioned. Reporting to the public should always mention that, first, the major contributor to the green house effect is water vapor, and secondly, that human activity has little influence on the amount and global distribution of water vapor.

b. The reason for the large variation of the percentage of green house gases given in most tables is that their effect is complicated. To find the combined effect of the gases is not a simple additive process. For example 1 g of methane is as effective as 7 g of carbon dioxide in causing global warming.

c. It has been known since the late 19^{th} century that Nitrogen and oxygen, the two major gases of the atmosphere, are not green house gases; that is, they do not absorb infrared radiation reflected back from the earth.

Questions and problems for the student

1. One of the best way to demonstrate the *green house effect* in is to measure the temperature difference between the outside and the inside of a closed car on a cold, sunny day. You can test this by parking your car for about an hour on a cold sunny day in winter and measure the temperatures inside and outside. You should then compare this to the temperature difference you find on a similar cold day, but when it is overcast.
2. Read the following statement taken from IL and then discuss the implications for global warming.

Methane (CH_4) is the primary component of natural gas and an important energy source. Methane is also a greenhouse gas, meaning that its presence in the atmosphere affects the Earth's temperature and climate system. Due to its relatively short life time in the atmosphere (9-15 years) and its global warming potency — 20 times more effective than carbon dioxide (CO_2) in trapping heat in the atmosphere — reducing methane emissions should be an effective means to reduce climate warming on a relatively short timescale.

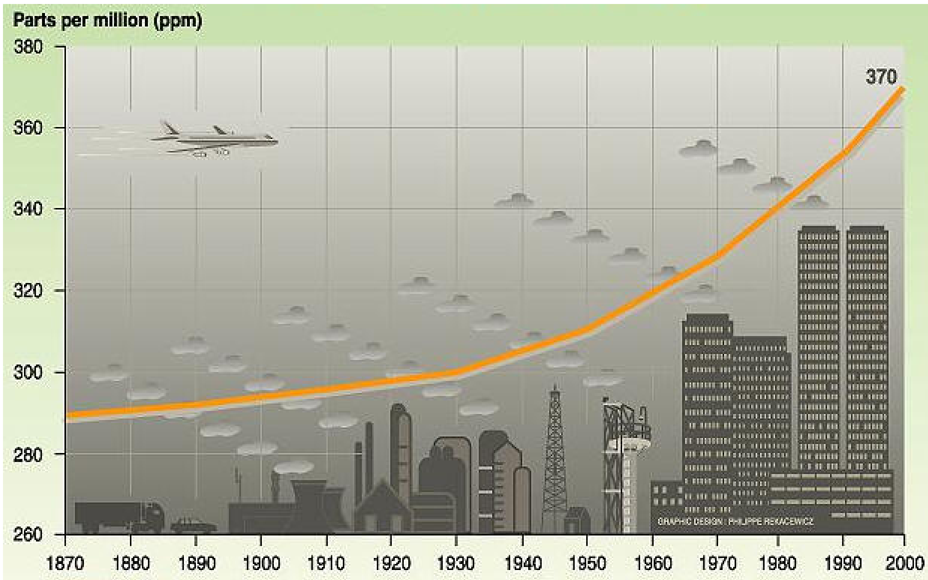


Fig. 40: The Atmospheric CO_2 Concentration in the Past Century

Figure shows the atmospheric CO_2 concentration in the past century. CO_2 is the greenhouse gas with the highest anthropogenic emission. The main cause of anthropogenic CO_2 emissions to the atmosphere is fossil fuel combustion. Fossil fuels are oil, coal and natural gas. These are exploited by humans to gain energy.

IL 82 **** (An excellent source of information about the greenhouse gases.)

3. Refer to Fig. in answering the questions below.
   a. Estimate the rate of increase of CO_2 , at the beginning of the industrial revolution (Circa 1870) and now. Express this rate in ppm/year, and try to express this change as an exponential increase.
   b. The total current CO_2 is about 368 ppm (parts per million). What percentage of the atmosphere does this represent?

IL 83 ** (An advanced discussion of the effect of greenhouse gases on global warming)

IL 84 ** (A comprehensive and visual discussion of global per capita emission of carbon)

4. The impact of greenhouse gases on the enhanced greenhouse effect is determined by their residence time in the atmosphere. When atmospheric residence time is greater, the total impact of a greenhouse gas on global warming is larger. Atmospheric residence time represents the average amount of time the molecules of a greenhouse gas exist in air before it is somehow removed. The average residence time of the greenhouse gases carbon dioxide and nitrous oxide is more than a century. Consequen-

tially, these greenhouse gases will impact global warming long after emission cutbacks are achieved. Methane, on the other hand, only has a residence time of one decade.

Assuming that carbon dioxide stays in the atmosphere for about 100 years, and methane only for about 10 years, compare the greenhouse effect of injecting 1kg of carbon dioxide with injecting 1kg of methane every year for 10 years,

IL 85 ** (Methane, a greenhouse gas)

5. The per capita contribution of carbon to the atmosphere is the highest in the US the total amount representing about 30% of the world's total. The second largest contribution per capita are the Canadians. The estimated amount per capita is 5.4 metric tons per person in the US and 4.2 metric tons per person in Canada.
   a. Estimate the total carbon contribution of the world due to man-made sources.
   b. The average North American drives a car about 20, 000 km (or 12,500 miles) a year. Use the carbon pollution calculator to calculate the amount of carbon and carbon dioxide emitted by a car that is rated at 25 miles a gallon (0r about 11 km/l). Compare this amount to the annual per capita contribution of carbon and carbon dioxide.

IL 86 ** (Carbon pollution calculator).

6. All hydrocarbons form the same products when they are burned. They react with oxygen to produce water and carbon dioxide. Gasoline is a complicated mixture of hydrocarbons with chemical formulas between C_6H_{14} and $C_{12}H_{26}$. A good "average" compound is C_8H_{18}. These react in an ideal situation to produce carbon dioxide and water, but in an actual automobile engine they also produce some amount of undesirable compounds, including carbon monoxide, oxides of nitrogen, compounds containing sulfur.

Using the reaction: $\mathbf{2\ C_8H_{18} + 25\ O_2 \ldots\ldots > 18\ H_2O + 16\ CO_2}$,

   a. Show that the equation is balanced.
   b. Calculate the mass of CO_2 that each liter of gasoline produces. Show that for every liter of gasoline you produce about 2.3 kg of CO_2 .
   c. Show that for every kilogram of gasoline the combustion produces about 3.1 kg of CO_2

   (Note: The density of gasoline is 0.737 kg / l.)

   d. Refer back to problem 4b. Estimate the carbon and carbon dioxide contribution if a person drives 20,000 km in a car that is uses on the average 11 km/l. Compare you result with the one you obtained using the carbon pollution calculator. Comment.

IL 87 ** (Estimates of global emissions of carbon dioxide)

LP 88 ** (Total emission calculator)

Comparing the "environment friendliness" of two popular cars.

The two cars we have chosen are two popular cars, the Honda Civic Hybrid, and the other the Honda Odyssey, a very popular SUV.

The following is an advertisement for the Honda Civic hybrid:

The Hybrid's Integrated Motor Assist (IMA) system pairs a super-efficient 1.3-litre gasoline engine with a lightweight, high-output electric motor that delivers an estimated highway fuel consumption of only 4.3 L/100 km. How is this possible?

At steady speeds below 60 km/h, on level roads and under light throttle, fuel injection can cease and the car can be propelled solely by the IMA system's electric motor. At higher cruising speeds, the gasoline engine provides the power. A marvel of low-friction engineering, it provides plenty of horsepower while still sipping fuel and having a near-zero effect on air quality.

Thanks to a whole new generation of regenerative-braking technology, the 2006 Civic Hybrid's IMA system can tap into the kinetic energy of the car more efficiently than ever. The system's motor turns itself into a generator during braking, helping to slow the car while it builds up the energy stored in the batteries.

Come to a stop, and the auto-idle stop feature shuts down the gasoline engine to reduce fuel consumption and overall emissions. Step on the accelerator and both gas and electric power are immediately available for a brisk restart.

The following is an advertisement for the Honda Odyssey:

Honda also boasts about the Odyssey's engine, a 255-horsepower V-6 that's the most powerful you can get in a minivan. Soccer moms won't care about the specs or the bragging rights, but the boost from the V-6—standard on all versions, from the base model to the $35,000 "Touring" line—provides lots of built-in reassurance. It allows quick acceleration when getting up to speed on the highway and keeps the Odyssey sprightly when fully loaded down with people or stuff. The Odyssey may also have the best handling you'll find in a minivan, crisp steering, and sedanlike agility that defy the van's hefty size and its higher center of gravity.

Fuel consumption is 12.0 L/100km (24 mpg) in the city and 7.7 L/100km (37 mpg) on the highway is claimed.

IL 89 ** (Advertisement above taken from here)

Questions and problems

More to come …

Physics and Six Million Dollar Man (SMDM)

IL 90 *** (The six million dollar man: A complete description and history of the series)

IL 91 ** (The Six Million Dollar Man TV show)

The following is taken from **IL 90**:

> *In medicine, Bionics means the replacement or enhancement of organs or other body parts by mechanical versions. Bionic implants differ from mere prostheses by mimicking the original function very closely, or even surpassing it. This definition of bionics is best known to the general public in reference to the television series The Six Million Dollar Man, in which the titular cyborg character is referred to as a "bionic man". In the mid-1970s, when scientists in a popular TV series rebuilt a wounded, barely-living test pilot into the world's first bionic man, making him "better, stronger, faster," the field of medical bionics was the stuff of science fiction.*
>
> *No longer. On April 3, at Experimental Biology 2006, some of the leading scientists in the rapidly expanding field of bionics explain how much of what was once fiction is today at least partial reality - including electronically-powered legs, arms, and eyes like those given TV's Six Million Dollar Man 30-plus years ago. The symposium on "The $6 Billion (Hu)Man" is part of the scientific program of the American Association of Anatomists.*

The opening narration for each episode of the SMDM has become part of American pop culture After an early version of the narration was tested in the *Solid Gold Kidnapping* TV film, the most famous version was introduced in the weekly series:

Narrator [series producer Harvey Bennett]:

"*Steve Austin: astronaut. A man barely alive.*"

Oscar Goldman : "Gentlemen, we can rebuild him. We have the technology. We have the capability to make the world's first bionic man. Steve Austin will be that man. Better than he was before. Better... stronger... faster."

In 1972 the ABC television company turned Martin Caidin's science fiction novel Cyborg into the popular program The Six Million Dollar Man. Colonel Austin, the hero of the series, is an astronaut by training, whose body suffers irreparable damage in a rocket sled experiment.

In the popular 1970s television series, actor Lee Majors played Col. Steve Austin, an astronaut and NASA test pilot horribly injured in a plane crash.

He survived and was fitted with $6 million worth of bionic parts an arm, an eye and both legs — which give him super-human strength and speed.

After bionic reconstruction he is able to perform superhuman feats. The setting and the story line of the episodes strongly suggests that what the viewers see is just a dramatized version of the lives of real bionic men now living within the confines of top secret high-tech laboratories. Moreover, the program implies that the material presented is to be regarded as realistic within the limitations of modern technology and the laws of physics.

The main objective of this assignment is not to debunk but, rather, to look critically at the extraordinary physical feats performed by the Bionic Man *within the framework of Newtonian mechanics*. The message to the student should be clear: If we do not grant poetic license then even the Bionic Man is unable to supersede the limits imposed by the laws of motion or the conservation of momentum and energy principles.

The Six Million Man performs many amazing feats of strength and agility. He lifts cars, runs at 100 km/h, jumps out of and into speeding vehicles and leaps to heights exceeding 20 m with the greatest ease. Most of the feats of strength and agility are dramatically acceptable, but they are often physically impossible. They are impossible in the world in which we live because:

a. known material will not withstand the forces required for their successful execution,

and/or

b. the laws of physics are violated.

Let us consider critically a few situations from the popular TV series of the late 1970's.

(Note: Read the article: Physics and the Bionic Man (Published in *The Physics Teacher*, in 1980.

The article is found in the Appendix. See references).

IL 92 ** (DVDs of the TV series are available here)

Fig. 41: A Poster of Steve Austin, the SMDM in 1974.

IL 93 ** (Comments by people who loved the series)

IL 94 ** (The "new" SMDM

IL 95 ** (New prosthetic parts…)

IL 96 ** (The SMDM today….)

Kinematics and the Bionic Man

Kinematics is the study of motion without explicitly considering the forces involved. You are asked to study each situation carefully and identify those that you think may be impossible. Be ready to support your claim.

Research problems for the student

1. Colonel Austin is capable of awesome accelerated motion in times of critical emergency. During one episode he accelerates from rest to 100 km/h in 2.0 s, in order to overtake a car driven by a foreign agent. Calculate his average acceleration.

A simple calculation should show that the Colonel accelerated at about 14 m/s^2 . Compare this to the sustained acceleration of the fastest animal known, namely the cheetah. A cheetah is able cover a distance of 100 m from rest in about 4.0 s. Find the average acceleration of the cheetah and compare it to that Colonel Austin.

Note: It seems that acceleration much over one g (or about 10 m/s^2 is not achievable, either by animals or cars accelerating. Why this is so will be discussed in the dynamics section.)

Fig. 42: The SMDM runs at a high speed to catch a speeding car

2. In almost every episode the Bionic Man either jumps to or leaps from great heights.
   a. If he jumps from a height of 20 m, what is his speed just before he reaches the ground? Remember that air resistance plays an important role here. See the supplementary section).
   b. The bionic man now jumps to a height of 20 m, by propelling his body (actually his centre of mass) upward through a distance of 0.5 m in order to reach the necessary velocity. What must be his average acceleration while he is pushing off?

   Note: You should find a very large value for the average acceleration. You may suspect that accelerations of this magnitude, even if only for a short time, are not possible. We will discuss the reason for suspecting this in the dynamics section.

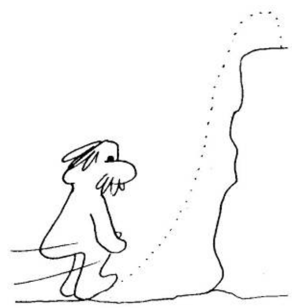


Fig. 43: The SMDM Jumps to Great Heights

3. Jumping over vehicles is a routine task for the Colonel.

On one occasion he jumps over a large truck 3 m high, just clearing the top of it. Approximately how long does he stay in the air?

Note: You may be puzzled by the fact that the width of the jump is not specified. Does it matter where he jumps from, assuming that the ground is level? Discuss.

4. On the only occasion that fans of Colonel Austin can recall when his foe escaped, the colonel attempted to catch a foreign agent making his getaway in a Porsche 911. The distance between them was 100 m when the Porsche began to accelerate with a constant acceleration of 5 m/s^2. Colonel Austin was running with a constant speed of 100 km/h in hot pursuit. Show that the colonel is unable to catch the Porsche and calculate the “frustration distance”, that is, the distance of closest approach.

Note: You can solve this challenging problem using three different approaches by : a. using the relationship between the d-t, v-t, and a-t graphs, b. applying the equations of motion (kinematics) and solve it algebraically, and c. using the method of the calculus. What you should do is compare your solutions with other students and discuss the merits of the three approaches.

Dynamics and the SMDM

We saw in chapter one that dynamics is the study of the forces involved in motion. You can apply Newton’s laws of motion and /or the conservation principles of momentum and mechanical energy. Assume that Colonel Austin has a mass of 80 kg.

Identify situations you think are possible and those you think are impossible. Support your claims with a good argument. Also refer back to part I and discuss the situation now in the light of the forces involved.

<u>Problems for the student</u>

1. In a tug of war between the colonel and an elephant, the good colonel wins. Explain why this is impossible, no matter how strong the colonel is. Assume that the ground is ordinary is of ordinary gravel-top or grass composition.

Note: This is an excellent problem to show how Newton's third and second laws are related. Assume a maximum force of friction in terms of the bionic man's weight

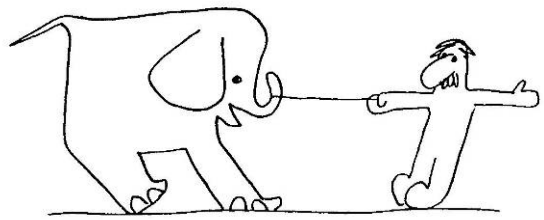


Fig. 44: The SMDM pulls an Elephant

2. In part I, problem 1, you found that the acceleration of the colonel is considerably higher than the acceleration of free fall, g. What must have been the minimum frictional force between his feet and the ground?

Note: Consider the wearing of spiked shoes on soft ground. Do you think this acceleration could now be achieved? Discuss.

3. You have already calculated the acceleration of the bionic man's body in order for him to jump to a height of 10 m. Now calculate the average force on his bionic knees during the acceleration period.

Note: Consider what would happen to the blood supply to the brain of the bionic man during this acceleration.

4. The bionic man often hurls large objects in order to destroy his enemies. In one episode he picks up a boulder of about 1000 kg mass and throws it at a Sasquatch. The boulder leaves the hands the bionic man with a speed of 10 m/s.
   a. According to the conservation of momentum principle, what would be the effect on the bionic man?
   b. What would be the average force acting on him if he supported his body by standing against a tree for 0.1 s?

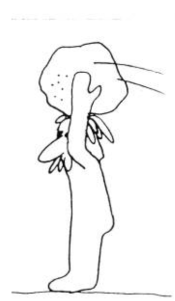


Fig. 45: The SMDM Hurls a Large Object

5. Colonel Austin is often seen performing great feats of strength such as the bending of iron of iron bars in windows and the breaking of chains embedded i walls. He often accomplishes this while standing on the ground with his feet apart. Draw a sketch of the bionic man as he would stand if he respected Newton's third law.
6. In one episode the Colonel opens the door to a large chamber that functions as a simulator for planetary atmospheres. In this particular scene he is coming to the aid of an astronaut who is trapped in the chamber where the pressure is effectively zero. Calculate the force that would be necessary to open a 4 square meter door. Assume that the atmospheric pressure is 100 kPa.

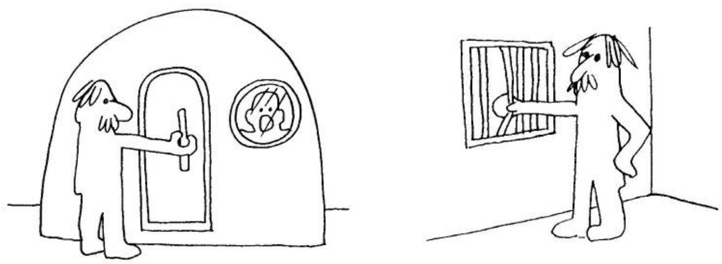


Fig. 46 The SMDM Performs Feats of Strength

7. In one memorable episode the Colonel, while sitting in the passenger's seat, stops a car by making contact with the ground using his right foot through the open car door. He stopped the car in 10 seconds. The mass of the car is 1000 kg and the initial speed is 20 m/s.
   a. What must be the average force exerted by his foot in the horizontal direction in order to stop the car?
   b. What is the actual force along his leg if the angle of the leg is 60 degrees to the horizontal?
   c. How much work is done in stopping the car?
   d. What is his power output?
   e. speculate about what would happen to his shoes, assuming that they did not slip off his feet?
   f. Assume that bionic man is wearing metallic shoes of 1.0 kg mass with a specific heat of 50 J/kg C. To what approximate temperature would his shoes rise?
   g. In fact it would be impossible to stop the car in this fashion, even if a steady force of the appropriate magnitude and direction could be applied. The car would quickly rotate and leave the road. Why?

Research questions for the student

1. Look up the meaning and the origin of the word "bionic". Is Colonel Austin really bionic? Now speculate on what consequence one million bionic men and women would have on athletics, crime, social behaviour and occupational opportunities.
2. Write a short dramatic episode in which the bionic man performs great feats of strengths but stays within the confines the laws of physics. For every situation that calls for physical action, describe in detail (with sketches and calculations appropriate to the situation) how the scene should be worked out. How do you think the viewing public would react?
3. Write a short report o new discoveries and developments in bionic parts.

Beyond Galileo and Haldane

We will conclude with the discussion of the article "Of Mice and Elephants: a Matter of Scale" by George Johnson, the noted science writer of the *New York Times*. The article talks about the contemporary effort made by a team of biologists and physicists to answer the question:

> *How is one to explain the subtle ways in which various characteristics of living creatures—their life spans, their pulse rates, how fast they burn energy—change according to their body size?*

This question clearly takes us back to Galileo, showing that the discoveries of the 17th century physicist Galileo about how scaling affects everything around us and the later elaboration of the 20th biologist Haldane is being extended and given a new meaning in the context of the collaboration between 21th century physicists and biologists. Please read this article before going on. It is available in the Appendix.

IL 97 **** (Text for "Of Mice and Elephants")

IL 98 *** (Scaling picture taken from here)

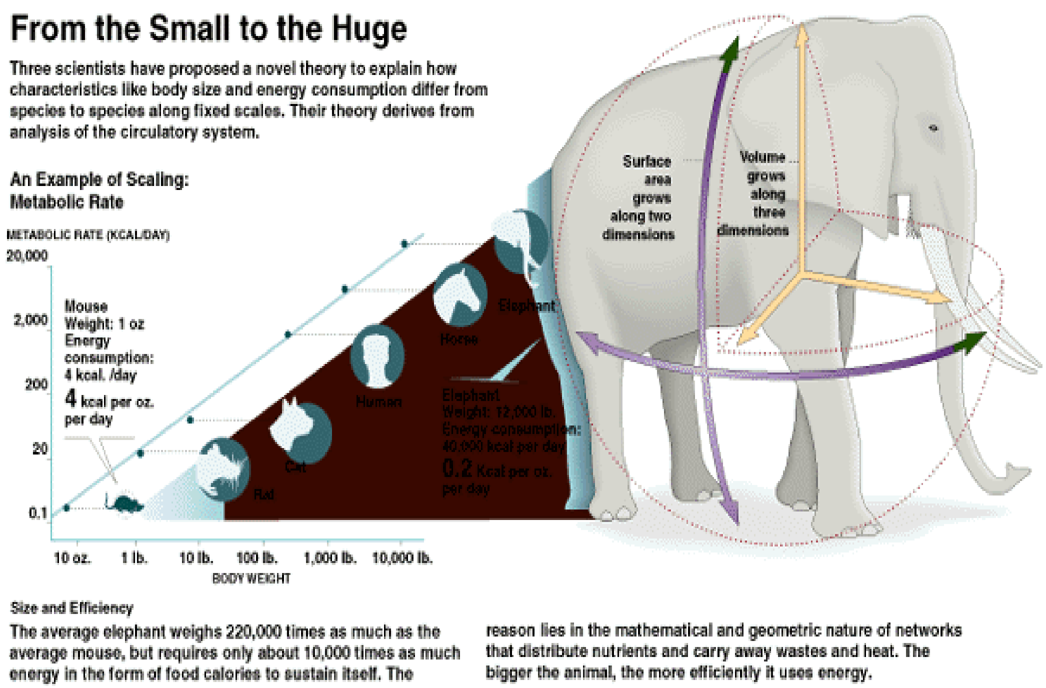


Fig. 47: Scaling Picture Taken From Above

In the article above, Johnson says, (referring to the ecologist James Brown):

> *The lesson he took away from this was that you cannot just naively scale things up. He liked to illustrate the idea with Superman. In two panels labeled "A Scientific Explanation of Clark Kent's Amazing Strength," from Superman's first comic book appearance in 1938, the artists invoked a scaling law: "The lowly ant can support weights hundreds of times its own. The grasshopper leaps what to man would be the space of several city blocks." The implication was that on the planet Krypton, Superman's home, strength scaled to body mass in a simple linear manner: If an ant could carry a twig, a Superman or Superwoman could carry a giant ponderosa pine.*

Johnson continues:

> *But in the rest of the universe, the scaling is actually much slower. Body mass increases along three dimensions, but the strength of legs and arms, which is proportional to their cross-sectional area, increases along just two dimensions. If a man is a million times more massive than an ant, he will be only 1,000,000 to the two-thirds power stronger: about 10,000 times, allowing him to lift objects weighing up to a hundred pounds, not thousands.*

IL 99 **

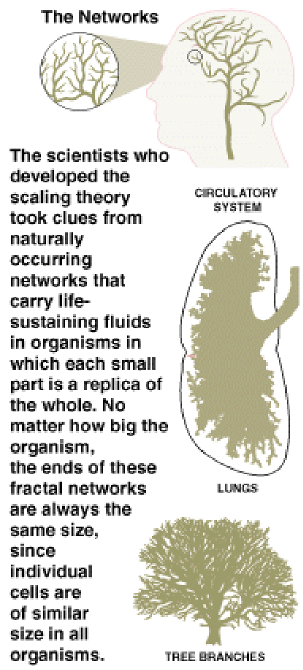


Fig. 48: The New Scaling Theory

Questions for the student

1. Criticize the text of "the physics of superman".
2. Show that, so far there is nothing beyond what we have discussed so far for scaling.
3. Johnson mentions that the agricultural scientist Max Kleiber in the 1930s measured the metabolic rates of animals, from the size of mice, cats, dogs, to men, and elephants, and concluded that the metabolic rate P (Kcal/day) and mass M did not quite follow the classical scaling rules we have used so far, that is, he found that P did not follow the classical scaling law

$P \propto M^{2/3}$ (Classical law of scaling)

but followed the scaling law that

$P \propto M^{3/4}$ (Kleiber's law)

Johnson then says:

Evolution seems to have found a way to overcome in part the limitations imposed by pure geometric scaling; the fact that surface area grows more slowly than size. For decades no one could plausibly say why.

Kleiber's "mouse-to-elephant" curve

In the nineteenth century biologists established the classical law of scaling shown in Figure 2 and again above. Max Kleiber in the 1930s, however, established empirically that the law is actually a ¾ power law and not a 2/3 power law, as it was classically expected. This discovery, of course, was a great surprise for scientists and today there is still an ongoing effort to explain this major deviation from the classical law.

IL 100 *** (The new scaling theory of Max Kleiber's work)

Problems for students

The following is taken from **IL 100.** The student should be made to realize that the classical laws that relate structure, anatomy and physiology to size are "idealized".

1. The magnitude of many body processes changes in a regular fashion as the size of the organism changes. A surprising number of such processes can be described in a very simple fashion by:

$\mathbf{M = a \cdot x^k}$

where **M** is the *body process* in question (for example metabolic rate), **x** is a measure of the size of the organism, and **a** and **k** are constants. For example, a large number of measurements suggest that the relationship between metabolic rate (let's call this $\mathbf{P_{met}}$) and mass ($\mathbf{m_b}$) is:

$\mathbf{P_{met} = 73.3 \cdot m_b^{0.75}}$

where **P** is measured in kcal/day and $\mathbf{m_b}$ is the weight in kilograms. This equation, first proposed in 1932 by Max Kleiber, has been amply justified, and is good for men, mice and elephants! There is a small amount of uncertainty about the exact exponent (his original equation actually had an exponent of 0.74) but the value is certainly extremely close to ¾, or 0.75.

To get an idea of the complexity facing scientists who attempt to build a theory that predicts the empirical result of Kleiber, consider the following examples of constraints that have scaling implications:

- Diffusion is hopeless for transporting anything over more than a tiny distance.
- The solubility of oxygen in water is poor.
- Nerve conduction speeds are limited by physical factors although "higher" organisms have overcome this to a degree by "re-designing" - developing myelin sheaths).
- Air resistance becomes important as you get smaller.
- Don't forget blood viscosity!
- For complex reasons, it appears difficult to "design" a heart that beats at more than about 1300 beats per minute.
- It is intriguing to determine what the physical constraints are on a complex biological system, but it's even more fascinating observing how nature manages to sidestep such obstacles!

It is intriguing to determine what the physical constraints are on a complex biological system, but it's even more fascinating observing how nature manages to sidestep such obstacles!

Students should try to discuss each of these and present them to class.

Finally, as argued in the article "Scaling" (see below)

> …*organisms effectively live in four spatial dimensions. They have exploited fractal geometry so that critical linear dimensions and surface areas scale as the 1/4 and 3/4 powers of body mass, respectively, rather than the 1/3 and 2/3 powers expected from conventional Euclidean geometry.*

Summarizing then, the model that was developed is based on the following assumptions:

1. Biological rates and times are ultimately limited by the rates at which limited energy and materials can be supplied to cells through a hierarchical branching network.
2. The distribution system has three attributes:
    a. it is space-filling (i.e., it reaches all parts of the organism);
    b. it minimizes the energy required for distribution; and
    b. it has size-invariant terminal units (e.g., capillaries or terminal xylem).

From these assumptions the scientists derived a quantitative model for the geometry and physics of the entire distribution system. The model predicts:

a. a fractal-like branching network with scaling laws governing the sizes of the branches;
b. the whole-organism metabolic rate scales as M 3/4; and
c. many other anatomical and physiological characteristics of mammalian cardiovascular and respiratory systems.

This model claims to solve the longstanding problem of quarter-power scaling in biology.

Constructing a Model or a Theory to Explain New Empirical Finding

Scientists are always trying to find a theory or a model (unfortunately, often used interchangeably by scientists themselves) to explain significant empirical findings. A good example of a theory in science is Newton's Gravitational Theory and Bohr's hydrogen model is a good example of a model. However, in addition to explaining empirical findings, a good theory or model also makes testable predictions.

A theory, like Newton's theory of gravity, contains laws, principles, definitions and rules of inference (accepted methods of reasoning) that explains such empirical findings as the kinematics of free fall, the period of a pendulum, the motion of the planets, and the occurrence of the tides. A model, like the Bohr model of the hydrogen atom, explains the empirical findings of the hydrogen spectrum, the value of the empirically established so Rydberg constant and the ionization potential of hydrogen. However, the model later became part of the larger quantum theory.

In biology, a theory or model can be seen as what we may call an *internal mechanism* from which the empirical findings can be "deduced". A good theory in biology is the germ theory of disease and a good model is the cell model.

Reading the articles above will allow you to understand how scientist went about recently in finding an internal mechanism for explaining the deviation of the empirical findings from the scaling laws. These were based on the "classical" model using geometric reasoning founded on Euclidean geometry. The new model, takes into account the fractal nature of blood vessels. It seems that there are two sets of scaling laws. The classical, which still holds for structures and comparing strength to weigh ration and the new scaling law by Kleiber.

The section below will summarize how this was done.

Questions and problems based on the text on contemporary research in bionics

It is assumed that the student has carefully read the article "Of Mice and Elephants: A Matter of_Scale", found in the Appendix, as well as the article "Scaling" found in **IL 3** nd **Appendix V**: *Of Mice and Elephants: A Matter of Scale.*

1. Discuss in class the validity of the distinction made by Johnson between "lumpers" (physicists) and "splitters" (biologists).
2. The research teem, consisting of two biologists and a physicists, were collaborating

In answering the question:

How is one to explain the subtle ways in which various characteristics of living creatures -- their life spans, their pulse rates, how fast they burn energy -- change according to their body size?

Why is it necessary here to have biologists and physicists collaborate?

3. Discuss and confirm the flowing, taken from the article:

Mysteriously, these and a large variety of other phenomena change with body size according to a precise mathematical principle called quarter-power scaling. A cat, 100 times more massive than a mouse, lives about 100 to the one-quarter power, or about three times, longer. (To calculate this number take the square root of 100, which is 10 and then take the square root of 10, which is 3.2.) Heartbeat scales to mass to the ***minus*** *one-quarter power. The cat's heart thus beats a third as fast as a mouse's.*

4. Notice and then discuss how Johnson uses the concept of model and the idea of theory, almost interchangeably (theory and model underlined, not found in the original text):

In their <u>theory</u>, scaling emerges from the geometrical and statistical properties of the internal networks animals and plants use to distribute nutrients. But almost as interesting as the details of this <u>model</u>, is the collaboration itself. It is rare enough for scientists of such different persuasions to come together, rarer still that the result is hailed as an important development.

5. One of the biologists (John Gittleman) says:

Scaling is interesting because, aside from natural selection, it is one of the few laws we really have in biology. He then adds the comment:

What is so elegant is that the work makes very clear predictions about causal mechanisms. That's what had been missing in the field.

a. Can you think of another "law" in biology?

b. Do you agree with Johnson that natural selection is a "law"? Discuss.

c. Discuss the added comment by considering the following:

In physics, say Newton's gravitational theory, we can predict accurately the position of a comet or a planet in the future, but we do not really understand the underlying mechanism of gravity. In biology, on the other hand, we understand the underlying mechanism of say, how viruses and bacteria cause disease, but it difficult to make predictions.

6. Discuss the following statement made by Johnson, referring to metabolism in animals and the scaling laws: The size of the biological radiator cannot possibly keep up with the size of the metabolic engine.

7. Show that the classical scaling law for metabolic rate should be given by

$\mathbf{P = k\, m^{2/3}}$

8. Refer to the graph in Fig. 2. Notice that the graph uses units of watts (J/s) for P. and the formula above uses Kcal/ day. You should convince yourself that 1 kcal/day is equivalent to about 0.055 J/s.

   a. Using the classical scaling law above, find the metabolic rate P (J/s) of a horse, assuming that P for a mouse id about 2 J/s. Assume that the mass of a mouse is 30 g and that of a horse 700 kg,

   b. Check this figure by referring to the graph in Fig.2. You will find that the reading of the graph is difficult.

   c. What would be the value of P, if the scaling law were linear?

9. You should have found in problem 8, part a. that the classical law predicts a P of about 170 watts for the horse.. .

10. You can now compare the metabolic rates between two animals, A and B. if P for an animal A compares with animal B as 100:1, using a linear scale, then show that:

    a. The classical scale would predict a ratio of about 100: 21 and

    b. The equation by Kleiber predicts a ratio of about 100:32.

We will conclude with a passage from the article:

The lesson he took away from this was that you cannot just naively scale things up. He liked to illustrate the idea with Superman. In two panels labeled "A Scientific Explanation of Clark Kent's Amazing Strength," from Superman's first comic book appearance in 1938, the artists invoked a scaling law: "The lowly ant can support weights hundreds

> *of times its own. The grasshopper leaps what to man would be the space of several city blocks." The implication was that on the planet Krypton, Superman's home, strength scaled to body mass in a simple linear manner: If an ant could carry a twig, a Superman or Superwoman could carry a giant ponderosa pine.*
>
> *But in the rest of the universe, the scaling is actually much slower. Body mass increases along three dimensions, but the strength of legs and arms, which is proportional to their cross-sectional area, increases along just two dimensions. If a man is a million times more massive than an ant, he will be only 1,000,000 to the two-thirds power stronger: about 10,000 times, allowing him to lift objects weighing up to a hundred pounds, not thousands.*
>
> *Things behave differently at different scales, but there are orderly ways -- scaling laws -- that connect one realm to another. "I found this enormously exciting," West said. "That's what got me thinking about scaling in biology."*

The above passage should be discussed in class.

Concluding remarks (To be added)

References

[1] Galileo Galilei, Discourses and Mathematical Demonstrations Relating to Two New Sciences, 1638.

[2] Dialogues Concerning Two New Sciences, translated by Henry Crew and Alfonso di Salvio, Prometheus Books, 1991. Identified as "the classic source in English, published in 1914" on the website

[3] C. Blythsway and I. Gilhespy, web pages and links to several published articles on

EcoBot I and II.

[4] See DARPA website for Grand Challenges: **DARPA**

[5] V. Hill, The dimensions of animals and their muscular *dynamics*, Science Progress, **38** 209-230. Frequently referenced as the first account of scaling theory for moving animals.

[6] V. Hill, The heat of shortening and the dynamic *constants of muscle*, Proceedings of the Royal Society, Series B, **126** 136-195.

[7] C. J. Pennycuick, Newton Rules Biology: A physical approach to biological problems, Oxford University.

[8] *Why Size Matters*, by John Tyler Bonner. An excellent little book for the general reader, written by a biologist. Highly recommended. See website **Tyler**

Appendix I (Galileo): Back

FIRST DAY

INTERLOCUTORS: SALVIATI, SAGREDO AND SIMPLICIO

SALV. The constant activity which you Venetians display in your famous arsenal suggests to the studious mind a large field for investigation, especially that part of the work which involves mechanics; for in this department all types of instruments and machines are constantly being constructed by many artisans, among whom there must be some who, partly by inherited experience and partly by their own observations, have become highly expert and clever in explanation.

SAGR. You are quite right. Indeed, I myself, being curious by nature, frequently visit this place for the mere pleasure of observing the work of those who, on account of their superiority over other artisans, we call "first rank men." Conference with them has often helped me in the investigation of certain effects including not only those which are striking, but also those which are recondite and almost incredible. At times also I have been put to confusion and driven to despair of ever explaining something for which I could not account, but which my senses told me to be true. And notwithstanding the fact that what the old man told us a little while ago is proverbial and commonly accepted, yet it seemed to me altogether false, like many another saying which is current among the ignorant; for I think they introduce these expressions in order to give the appearance of knowing something about matters which they do not understand.

SALV. You refer, perhaps, to that last remark of his when we asked the reason why they employed stocks, scaffolding and bracing of larger dimensions for launching a big vessel than they do for a small one; and he answered that they did this in order to avoid the danger of the ship parting under its own heavy weight, a danger to which small boats are not subject?

SAGR. Yes, that is what I mean; and I refer especially to his last assertion which I have always regarded as a false, though current, opinion; namely, that in speaking of these and other similar machines one cannot argue from the small to the large, because many devices which succeed on a small scale do not work on a large scale. Now, since mechanics has its foundation in geometry, where mere size cuts no figure, I do not see that the properties of circles, triangles, cylinders, cones and other solid figures will change with their size. If, therefore, a large machine be constructed in such a way that its parts bear to one another the same ratio as in a smaller one, and if the smaller is sufficiently strong for the purpose for which it was designed, I do not see why the larger also should not be able to withstand any severe and destructive tests to which it may be subjected.

SALV. The common opinion is here absolutely wrong. Indeed, it is so far wrong that precisely the opposite is true, namely, that many machines can be constructed even more perfectly on a large scale than on a small; thus, for instance, a clock which indicates and strikes the hour can be made more accurate on a large scale than on a small. There are some intelligent people who maintain this same opinion, but on more reasonable grounds, when they cut loose from geometry and argue that the better performance of the large machine is owing to the imperfections and variations of the material.

Here I trust you will not charge me with arrogance if I say that imperfections in the material, even those which are great enough to invalidate the clearest mathematical proof, are not sufficient to explain the deviations observed between machines in the concrete and in the abstract.

Yet I shall say it and will affirm that, even if the imperfections did not exist and matter were absolutely perfect, unalterable and free from all accidental variations, still the mere fact that it is matter makes the larger machine, built of the same material and in the same proportion as the smaller, correspond with exactness to the smaller in every respect except that it will not be so strong or so resistant against violent treatment; the larger the machine, the greater its weakness.

Since I assume matter to be unchangeable and always the same, it is clear that we are no less able to treat this constant and invariable property in a rigid manner than if it belonged to simple and pure mathematics. Therefore, Sagredo, you would do well to change the opinion which you, and perhaps also many other students of mechanics, have entertained concerning the ability of machines and structures to resist external disturbances, thinking that when they are built of the same material and maintain the same ratio between parts, they are able equally, or rather proportionally, to resist or yield to such external disturbances and blows. For we can demonstrate by geometry that the large machine is not proportionately stronger than the small. Finally, we may say that, for every machine and structure, whether artificial or natural, there is set a necessary limit beyond which neither art nor nature can pass; it is here understood, of course, that the material is the same and the proportion preserved.

SAGR. My brain already reels. My mind, like a cloud momentarily illuminated by a lightning-flash, is for an instant filled with an unusual light, which now beckons to me and which now suddenly mingles and obscures strange, crude ideas. From what you have said it appears to me impossible to build two similar structures of the same material, but of different sizes and have them proportionately strong; and if this were so, it would not be possible to find two single poles made of the same wood which shall be alike in strength and resistance but unlike in size.

SALV**.** So it is, Sagredo. And to make sure that we understand each other, I say that if we take a wooden rod of a certain length and size, fitted, say, into a wall at right angles, i. e., parallel to the horizon, it may be reduced to such a length that it will just support itself; so that if a hair's breadth be added to its length it will break under its own weight and will be the only rod of the kind in the world.* Thus if, for instance, its length be a hundred times its breadth, you will not be able to find another rod whose length is also a hundred times its breadth and which, like the former, is just able to sustain its own weight and no more: all the larger ones will break while all the shorter ones will be strong enough to support something more than their own weight. And this which I have said about the ability to support itself must be understood to apply also to other tests; so that if a piece of scantling will carry the weight of ten similar to itself, a beam having the same proportions will not be able to support ten similar beams.

Please observe, gentlemen, how facts which at first seem improbable will, even on scant explanation, drop the cloak which has hidden them and stand forth in naked and simple beauty. Who does not know that a horse falling from a height of three or four cubits will break his bones, while a dog falling from the same height or a cat from a height of eight or ten cubits will suffer no injury? Equally harmless would be the fall of a grasshopper from a tower or the fall of an ant from the distance of the moon.

Do not children fall with impunity from heights which would cost their elders a broken leg or perhaps a fractured skull? And just as smaller animals are proportionately stronger and more robust than the larger, so also smaller plants are able to stand up better than larger. I am certain you both know that an oak two hundred cubits high ,would not be able to sustain its own branches if they were distributed as in a tree of ordinary size; and that nature cannot produce a horse as large as twenty ordinary horses or a giant ten times taller than an ordinary man unless by miracle (*note this phrase - Galileo is trying to cover himself*) or by greatly altering the proportions of his limbs and especially of his bones, which would have to be considerably enlarged over the ordinary.

Likewise the current belief that, in the case of artificial machines the very large and the very small are equally feasible and lasting is a manifest error. Thus, for example, a small obelisk or column or other solid figure can certainly be laid down or set up without danger of breaking, while the large ones will go to pieces under the slightest provocation, and that purely on account of their own weight.

*The author here apparently means that the solution is unique.

Appendix II (Haldane) : Back

On Being the Right Size
J. B. S. Haldane

Note:

This essay was originally published in 1928 (long before computer networks were invented :-)) and discussed size in the natural (biological) world and systems. As you read it, think about whether there is a "right size" for a network (or a piece of a network such as an Autonomous System), and what aspects of a network determine the "right size." You might also find the political statements at the end of interest.

The Essay

The most obvious differences between different animals are differences of size, but for some reason the zoologists have paid singularly little attention to them. In a large textbook of zoology before me I find no indication that the eagle is larger than the sparrow, or the hippopotamus bigger than the hare, though some grudging admissions are made in the case of the mouse and the whale. But yet it is easy to show that a hare could not be as large as a hippopotamus, or a whale as small as a herring. For every type of animal there is a most convenient size, and a large change in size inevitably carries with it a change of form. Let us take the most obvious of possible cases, and consider a giant man sixty feet high—about the height of Giant Pope and Giant Pagan in the illustrated Pilgrim's Progress of my childhood. These monsters were not only ten times as high as Christian, but ten times as wide and ten times as thick, so that their total weight was a thousand times his, or about eighty to ninety tons. Unfortunately the cross sections of their bones were only a hundred times those of Christian, so that every square inch of giant bone had to support ten times the weight borne by a square inch of human bone. As the human thigh-bone breaks under about ten times the human weight, Pope and Pagan would have broken their thighs every time they took a step. This was doubtless why they were sitting down in the picture I remember. But it lessens one's respect for Christian and Jack the Giant Killer.

To turn to zoology, suppose that a gazelle, a graceful little creature with long thin legs, is to become large, it will break its bones unless it does one of two things. It may make its legs short and thick, like the rhinoceros, so that every pound of weight has still about the same area of bone to support it. Or it can compress its body and stretch out its legs obliquely to gain stability, like the giraffe. I mention these two beasts because they happen to belong to the same order as the gazelle, and both are quite successful mechanically, being remarkably fast runners.

Gravity, a mere nuisance to Christian, was a terror to Pope, Pagan, and Despair. To the mouse and any smaller animal it presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away, provided that the ground is fairly soft. A rat is killed, a man is broken, a horse splashes. For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal's length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force.

An insect, therefore, is not afraid of gravity; it can fall without danger, and can cling to the ceiling with remarkably little trouble. It can go in for elegant and fantastic forms of support like that of the daddy-longlegs. But there is a force which is as formidable to an insect as gravitation to a mammal. This is surface tension. A man coming out of a bath carries with him a film of water of about one-fiftieth of an inch in thickness. This weighs roughly a pound. A wet mouse has to carry about its own weight of water. A wet fly has to lift many times its own weight and, as everyone knows, a fly once wetted by water or any other liquid is in a very serious position indeed. An insect going for a drink is in as great danger as a man leaning out over a precipice in search of food. If it once falls into the grip of the surface tension of the water—that is to say, gets wet—it is likely to remain so until it drowns. A few insects, such as water-beetles, contrive to be unwettable; the majority keep well away from their drink by means of a long proboscis.

Of course tall land animals have other difficulties. They have to pump their blood to greater heights than a man, and, therefore, require a larger blood pressure and tougher blood-vessels. A great many men die from burst arteries, greater for an elephant or a giraffe. But animals of all kinds find difficulties in size for the following reason. A typical small animal, say a microscopic worm or rotifer, has a smooth skin through which all the oxygen it requires can soak in, a straight gut with sufficient surface to absorb its food, and a single kidney. Increase its dimensions tenfold in every direction, and its weight is increased a thousand times, so that if it is to use its muscles as efficiently as its miniature counterpart, it will need a thousand times as much food and oxygen per day and will excrete a thousand times as much of waste products.

Now if its shape is unaltered its surface will be increased only a hundredfold, and ten times as much oxygen must enter per minute through each square millimetre of skin, ten times as much food through each square millimetre of intestine. When a limit is reached to their absorptive powers their surface has to be increased by some special device. For example, a part of the skin may be drawn out into tufts to make gills or pushed in to make lungs, thus increasing the oxygen-absorbing surface in proportion to the animal's bulk. A man, for example, has a hundred square yards of lung. Similarly, the gut, instead of being smooth and straight, becomes coiled and develops a velvety surface, and other organs increase in complication. The higher animals are not larger than the lower because they are more complicated. They are more complicated because they are larger. Just the same is true of plants. The simplest plants, such as the green algae growing in stagnant water or on the bark of trees, are mere round cells. The higher plants increase their surface by putting out leaves and roots. Comparative anatomy is largely the story of the struggle to increase surface in proportion to volume. Some of the methods of increasing the surface are useful up to a point, but not capable of a very wide adaptation. For example, while vertebrates carry the oxygen from the gills or lungs all over the body in the blood, insects take air directly to every part of their body by tiny blind tubes called tracheae which open to the surface at many different points. Now, although by their breathing movements they can renew the air in the outer part of the tracheal system, the oxygen has to penetrate the finer branches by means of diffusion. Gases can diffuse easily through very small distances, not many times larger than the average length traveled by a gas molecule between collisions with other molecules. But when such vast journeys—from the point of view of a molecule—as a quarter of an inch have to be made, the process becomes slow. So the portions of an insect's body more than a quarter of an inch from the air would always be short of oxygen. In consequence hardly any insects are much more than half an inch thick. Land crabs are built on the same general plan as insects, but are much clumsier. Yet like ourselves they carry oxygen around in their blood, and are therefore able to grow far larger than any insects. If the insects had hit on a plan for driving air through their tissues instead of letting it soak in, they might well have become as large as lobsters, though other considerations would have prevented them from becoming as large as man.

Exactly the same difficulties attach to flying. It is an elementary principle of aeronautics that the minimum speed needed to keep an aeroplane of a given shape in the air varies as the square root of its length. If its linear dimensions are increased four times, it must fly twice as fast. Now the power needed for the minimum speed increases more rapidly than the weight of the machine. So the larger aeroplane, which weighs sixty-four times as much as the smaller, needs one hundred and twenty-eight times its horsepower to keep up.

Applying the same principle to the birds, we find that the limit to their size is soon reached. An angel whose muscles developed no more power weight for weight than those of an eagle or a pigeon would require a breast projecting for about four feet to house the muscles engaged in working its wings, while to economize in weight, its legs would have to be reduced to mere stilts. Actually a large bird such as an eagle or kite does not keep in the air mainly by moving its wings. It is generally to be seen soaring, that is to say balanced on a rising column of air. And even soaring becomes more and more difficult with increasing size. Were this not the case eagles might be as large as tigers and as formidable to man as hostile aeroplanes.

But it is time that we pass to some of the advantages of size. One of the most obvious is that it enables one to keep warm. All warm blooded animals at rest lose the same amount of heat from a unit area of skin, for which purpose they need a food-supply proportional to their surface and not to their weight. Five thousand mice weigh as much as a man. Their combined surface and food or oxygen consumption are about seventeen times a man's. In fact a mouse eats about one quarter its own weight of food every day, which is mainly used in keeping it warm. For the same reason small animals cannot live in cold countries. In the arctic regions there are no reptiles or amphibians, and no small mammals. The smallest mammal in Spitzbergen is the fox. The small birds fly away in winter, while the insects die, though their eggs can survive six months or more of frost. The most successful mammals are bears, seals, and walruses.

Similarly, the eye is a rather inefficient organ until it reaches a large size. The back of the human eye on which an image of the outside world is thrown, and which corresponds to the film of a camera, is composed of a mosaic of "rods and cones" whose diameter is little more than a length of an average light wave. Each eye has about a half a million, and for two objects to be distinguishable their images must fall on separate rods or cones. It is obvious that with fewer but larger rods and cones we should see less distinctly. If they were twice as broad two points would have to be twice as far apart before we could distinguish them at a given distance. But if their size were diminished and their number increased we should see no better. For it is impossible to form a definite image smaller than a wave-length of light. Hence a mouse's eye is not a small-scale model of a human eye. Its rods and cones are not much smaller than ours, and therefore there are far fewer of them. A mouse could not distinguish one human face from another six feet away. In order that they should be of any use at all the eyes of small animals have to be much larger in proportion to their bodies than our own. Large animals on the other hand only require relatively small eyes, and those of the whale and elephant are little larger than our own. For rather more recondite reasons the same general principle holds true of the brain. If we compare the brain-weights of a set of very similar animals such as the cat, cheetah, leopard, and tiger, we find that as we quadruple the body-weight the brain-weight is only doubled. The larger animal with proportionately larger bones can economize on brain, eyes, and certain other organs.

Such are a very few of the considerations which show that for every type of animal there is an optimum size. Yet although Galileo demonstrated the contrary more than three hundred years ago, people still believe that if a flea were as large as a man it could jump a thousand feet into the air. As a matter of fact the height to which an animal can jump is more nearly independent of its size than proportional to it. A flea can jump about two feet, a man about five. To jump a given height, if we neglect the resistance of air, requires an expenditure of energy proportional to the jumper's weight. But if the jumping muscles form a constant fraction of the animal's body, the energy developed per ounce of muscle is independent of the size, provided it can be developed quickly enough in the small animal. As a matter of fact an insect's muscles, although they can contract more quickly than our own, appear to be less efficient; as otherwise a flea or grasshopper could rise six feet into the air.

And just as there is a best size for every animal, so the same is true for every human institution. In the Greek type of democracy all the citizens could listen to a series of orators and vote directly on questions of legislation. Hence their philosophers held that a small city was the largest possible democratic state. The English invention of representative government made a democratic nation possible, and the possibility was first realized in the United States, and later elsewhere. With the development of broadcasting it has once more become possible for every citizen to listen to the political views of representative orators, and the future may perhaps see the return of the national state to the Greek form of democracy. Even the referendum has been made possible only by the institution of daily newspapers.

To the biologist the problem of socialism appears largely as a problem of size. The extreme socialists desire to run every nation as a single business concern. I do not suppose that Henry Ford would find much difficulty in running Andorra or Luxembourg on a socialistic basis. He has already more men on his pay-roll than their population. It is conceivable that a syndicate of Fords, if we could find them, would make Belgium Ltd or Denmark Inc. pay their way. But while nationalization of certain industries is an obvious possibility in the largest of states, I find it no easier to picture a completely socialized British Empire or United States than an elephant turning somersaults or a hippopotamus jumping a hedge.

Economists have not drawn much inspiration from biology, with some few exceptions, despite Alfred Marshall's noting that the economy is much more like a biological system than a mechanical one. The paradigm of the modern economist has been physics, not biology.

Darwin, though, drew his inspiration from economics. It was his reading of Malthus that spurred his thinking about the struggle for existence. He thought of nature as an economy, where each area of competitive advantage would be occupied by some organism. ''Wedges in the economy of nature'' was the phrase he used.

Appendix III: Back

When Physics Rules Robotics

Mel Siegel

Robotics Institute – School of Computer Science

Carnegie Mellon University, Pittsburgh PA USA

This is a lecture that Professor Siegel gave at a conference bionics (ICARA) in New Zealand, in December of 2004.

Scaling was the first of the *Two New Sciences* revealed in Galileo's *Discourses and Mathematical Demonstrations* (1638); physics was the second[1][2].The practical connection between them is materials: *big is weak, small is strong* is a consequence of the impossibility of altering the strength of matter in parallel with altering the size of the structures made of that matter. Galileo understood this: "... the mere fact that it is matter makes the larger machine, built of the same material and in the same proportion as the smaller, correspond with exactness to the smaller in every respect except that it will not be so strong ...who does not know that a horse falling from a height of three or four cubits will break his bones, while a dog falling from the same height ... will suffer no injury?" Similarly, the ultimate inalterability of achievable energy storage density – also a consequence of the fundamental strength of matter –links every mobile machine's range to its size, profoundly limiting the prospects for building arbitrarily small robots that will operate in arbitrarily low available energy environments. The large and small ends of robotics come full circle in the development of large networks of small robots, where geometrical scale issues again both enable and constrain the practicality of the internal communications essential to network functionality. Two generalities, both at first counterintuitive but both straightforwardly physics-based, rule the design of both living and engineered structures and devices:(1) *big is weak*, *small is strong*, i.e., it is large structures that collapse under their own weight, large animals that break their legs when they stumble, etc., whereas small structures and animals are practically unaware of gravity, and (2) horses *eat like birds* and *birds eat like horses*, i.e., a large animal or machine stores relatively larger quantities of energy and dissipates relatively smaller quantities of energy than a small animal or machine.

The critical consequence of (1) is that it is hard to build large structures and easy to build small structures that easily support their own weight. The critical consequence of (2) is that it is hard to build small structures and easy to build large structures that easily operate long enough and travel far enough to do any sort of interesting job.

1.2 Strength

Strength related scaling is not yet much of a problem in robotics. Big-end robots – e.g., radio telescopes are designed by mechanical engineers who know how to build structures that only rarely collapse under their own weight (see figure 1). And the mechanical over design of the present generation of small-end robots –e.g., prototype fly-on-the-wall nano-robot spies – does not significantly decrease their already minuscule functionality. Over design of small machines helps relax some manufacturing challenges; it is apparent even in robots built to near-human scale, e.g., Honda's tour-de-force humanoid Asimo, whose body proportions are those of a three- or four-meter man. This unnatural scaling causes disturbing perceptual dissonance when Asimo's actual 1.2 meter height is revealed by pictures of him with humans (see figure 2). EPFL's Alice isan example of a state-of-the-art over-designed mini-robot; it employs a geometrical scale that seems appropriate to a much larger human scale vehicle, e.g., a wheelchair (see figure3).

Figure 1: A large robot that later collapsed under its own weight. [Image courtesy of NRAO/AUI; see http://ftp.gb.nrao.edu/imagegallery.]

Figure 2: Perceptual dissonance due to over design of Honda's ASIMO Humanoid. [From http://www.honda.co.jp/ASIMO/technology/tech_09.html]

Figure 3: EPFL's Alice mini-robot, based on a watch motor. [Floreano et al, Evolutionary Bits 'n' Spikes, Artificial Life VIII, MIT 2002, pp. 335-344.]

1.3 Energy

The most serious scaling problem for present day robotics relates not to strength but to energy: universal enthusiasm for applications of tiny robots is untempered by the should-be-obvious fact that a bug cannot pack in enough calories to do much more than look for its next meal. The temporal endurance of any machine is its stored energy divided by its minimum power requirement. Stored energy obviously scales s the cube of a characteristic length. There are innumerable scenarios for minimum power, several of them analyzed in some detail in Section 3. As discussed in Section 3.8, the most useful model for a machine whose purpose is to move is probably that drag is proportional to the product of frontal area and velocity. Time-between-meals is thus proportional to length divided by velocity, and range is proportional to length. With step size proportional to characteristic length, all machines have the same range in steps and the same running time in step times. From the same sort of argument – even for machines with very different minimum power models – it invariably emerges that small robots on useful missions must either run on energy beamed in from the outside or must forage for it in their environments. It's a good thing too, otherwise the air we breathe would probably be as densely populated with microorganisms as is the energy-rich liquid environment running through the sewers under our feet. Apropos of this observation, a sewage-powered robot was recently described [3].

1.4 Communication

The public is fascinated by visions of smart microrobots; the roboticists are fascinated by visions of huge armies of not-so-smart nano-robots organizing themselves into super-brains and mega-bodies that adapt themselves to any task. Robots were classically defined as machines that sense, think, and act. When roboticists realized that what makes robots interesting is their mobility I added communicate to my personal version of this paradigm. Societies of many robots will need to communicate with each other even more than they will need to communicate with us. High-density robot societies – those in which inter-robot distance is typically no more than a few tens of robot characteristic dimensions – will be able to use the same sorts of one-spatial-dimension communication channels we use in our bodies, our machines, and most of our telecommunications. But low density societies of highly mobile individuals – those like the contemplated global environmental monitoring network, 1010 nodes seeded 1 per km3 to an altitude of 20 km over the surface of the earth – will need somehow to contend with 1/r2 communications in an uncertain direction at least for signal acquisition, and – unless unlikely sophisticated pointing technology emerges – nodes in a turbulent viscous medium will probably need always to broadcast into large solid angles. Intriguing solutions can be contemplated via device scales that are macroscopic in some dimensions and microscopic in others, e.g., decimeter-long filaments of dekamicron diameter, making them good antennas – and good sails – whose volume nevertheless fits into the 1-mm cube that is the practical upper size limit for manufacturing 1010 devices without making impossible demands on the world's annual production of silicon wafers.

1.5 Scope

When I use the term "fundamental issues", e.g., in the abstract, I mean opportunities provided by and restrictions imposed by the most basic laws of physics as they relate to things like the strengths of structures, the internal and external motions of the structures, the energy requirements associated with their basal metabolisms, the mechanical work they do, the energy they dissipate to friction associated with their mobility and the work they do, as well as some communication issues relating to energy cost and signal range, and the relationship between the size of an antenna and the efficiency with which it couples to the environment at any particular communication frequency. This being an exercise in reality, what I will not consider is hypothetical possibilities that are contingent on finding construction materials, energy storage principles, operating environments, etc., whose physical parameters, differ substantially from materials that actually exist or that we can realistically imagine developing. On the other hand, it is sensible for us to consider environmental parameters that are outside familiar ranges, e.g., extra-terrestrial, subterranean, deep-ocean, etc., environments in which temperatures, pressures, gravitational acceleration, etc., would be very different. Although I will not discuss the consequences of any of these in detail, it should be apparent that different environmental parameters lead to different expectations.

1.6 Organization

The subsequent sections provide additional discussion and analysis (Sections 2 and 3), conclusions (Section 4), acknowledgements (Section 5), and references (Section 6). Section 2, begins with subsections on size and strength (2.1), energy (2.2), and force (1.3)), the later introducing some dynamic issues that are familiar in animal efficiency studies but not yet particularly relevant to robotics because current generation robots are generally over designed in the strength domain and underperforming in the dynamics domain. Section 3 discusses a hypothetical family of robot vacuum cleaners that differ from each other in scale. It addresses performance – primarily range and operating time on stored energy – in several alternative maintenance power scenarios including one dominated by the power needed to move air (3.3), one dominated by the power needed to overcome brush friction (3.4), a "constant cleaning power" model (3.5), and a power loss to body drag model (3.6), argued in Sections 3.7 and 3.8 to be the most relevant for general robotic vehicle scenarios.

2 Size, Strength, Energy, and Force

Good examples of robots that are much larger than human scale are radio telescopes like the one shown in figure 1, extraterrestrial structures like the International Space Station, and modern buildings that incorporate large dynamic elements that actively compensate for wind and earthquake forces. Good examples of robots that are much smaller than human scale are the mobile devices contemplated for applications like exploring and treating ailments of the human body from the inside out, dust-particle sized active nano-sensors for global scale environmental monitoring, and the micro-scale active components of advanced airfoil surfaces.

2.1 Strength

Interesting issues with important practical consequences arise even for size – and consequent strength – decisions about structures that are only a little different from human-sized. For example, I would be inclined to make my entry in DARPA's Grand Challenge [4] a shoebox or smaller sized vehicle. It would give me extra leeway for staying on the road and passing obstacles, it would be rugged in a tip-over, it would be easy to right if it did tip, and it would be difficult for an adversary to detect and target. But my strategy has a fatal flaw: a jeep-sized vehicle can easily carry enough fuel to cover the 200 km course – and return home too – but even an extremely efficient shoe-box sized vehicle would be hard pressed to cover just a few kilometers. The occasional collapse of radio telescopes, bridges, and medieval cathedrals notwithstanding, it is scales smaller than ordinary human experience that typically thwart robotics applications by constraining robot run time and range. We will subsequently demonstrate quantitatively in several maintenance-power scenarios that the unavoidably limited energy carrying capacity of small structures requires that, below a critical size it inevitably becomes necessary to extract energy more-or-less continuously from the environment vs. carrying energy for the duration of the mission.

2.2 Energy

We can acquire an intuitive feeling for the absolute scales at which energy carrying capacity becomes, at the small end, an insurmountable barrier, and, at the large end, an issue only at intercontinental distances, by looking at some examples from the animal world at the small end and some examples from the engineering world at the large end.

At the small end, fly-sized insects crawl and even fly substantial distances between feedings, but mites that get down to barely visible size are pretty much constrained to live parasitically on food-bearing surfaces. Bacteria-sized microorganisms usually perish rapidly – the germ-fear-exploiting advertising of household cleanser purveyors notwithstanding – when removed from the energy-rich three dimensional soup in which they are normally bathed. At the large end we recognize that long distance transportation is most economically provided by a small number of very large vessels vs. a large number of very small vessels, fuel capacity inevitably winning over the many other considerations – some mentioned previously – that favor smaller vehicles. Absolute scales at this end are already quite intuitive to us: small airplanes and large cars have ranges in the 500 m regime, medium sized airplanes can cross the atantic economically, but only the largest airplanes –often retrofitted with auxiliary fuel bladders – can cross the Pacific expanse. Like small organisms, small boats can travel large distances only by foraging for energy en route, e.g., by sailing; they can carry enough fuel only for maneuvering in port and for short excursions. A petroleum tanker, on the other hand, could probably run on its own fuel load until it wore out its engines.

2.3 Force

The foregoing considers the factors that determine how big big structures can be before there is no material strong enough to keep them from collapsing under their own weight – if you don't believe it ask yourself why planet-size objects are invariably near spherical – and how small small structures can be before there is no energy storage medium dense enough to sustain a useful run time. Structural integrity and the energy to get from here to there are both crucial, but neither says more than a little about the ability to do useful mechanical work. Of course many useful robotic tasks can be accomplished without doing any mechanical work beyond what it takes to get from here to there; for example, they can simply carry sensors that convey enormously valuable data to remotely located people. Still, if for no reason but completeness, it is important to ask and understand what matters in this respect.

Static scaling issues have been discussed since the dawn of modern science, but dynamic scaling issues – relating mostly to how fast animals can run, how high they can jump, etc., – seem not to have been discussed until A. V. Hill's *The dimensions of animals and their muscular dynamics* [5] was published in 1950, though Hill had laid the groundwork in 1938 when he published *The heat of shortening and the dynamic constants of muscle* [6], excellently summarized by Pennycuick's *Newton Rules Biology: A physical approach to biological problems* [7]. The interesting constraint across the entire animal kingdom is that all muscle is essentially the same, and only a small variety of energy sources, range-of-motion transformers, and power integrators are available to animals. In contrast the forces that electromechanical actuators can exert, their ranges-of-motion, and their speeds are very flexible: mechanical and electrical transformers can convert between whatever the prime mover delivers and whatever the application requires. Power issues per se are also relatively minor for robots, since energy from a low power source can usually can be integrated by springs or capacitors and delivered as rapidly as may be required, albeit for a limited time. To efficiently drive a repetitive motion – a flapping wing or a running leg – it is necessary for the period of the muscle action, the length of the muscle divided by a limiting velocity characteristic of the muscle material, to match the period of the motion. Also recognizing that when these motions are efficient they are essentially pendulous – for terrestrial but not for aquatic animals – it can be shown that that the cruising speed of geometrically similar animals – e.g., members of the cat family – increases with their size, but their top speed running flat out is independent of size. This is confirmed by observational data. Similar considerations lead to the conclusion that all animals – actuated, as stated, by essentially identical muscle material – should be able to jump to the same height. The flea, often credited with being theworld's champion jumper, actually does much worse than this analysis suggests, primarily because it is too small to push against the floor long enough to realize its potential. Robotic mechanisms have more leeway than animals because they are not constrained to use one kind of muscle for every job.

3 Robot Vacuum Cleaner Family

We can imagine a family of robotic vacuum cleaners, all of the same design, but implemented at various scales from the huge aircraft hangar model down to the standard residential model, then further down to the mouse-sized model for cleaning under furniture, and even further down to the ant-sized model for cleaning, say, the crevices between bathroom tiles. By focusing on this hypothetical family of robots related to each other only by scale we can pose a broad set of questions whose answers provide us with comprehensive quantitative insight.

3.1 Energy, Power, and Running Time

The quantitative relationship between size and energy carrying capacity is easy: for any given energy storage medium – batteries, liquid fuel, etc. – the stored energy increases as the volume, i.e., as the cube of the linear dimension h. The running time between recharging, refueling, etc., is thus proportional to h3/P, where P is the power demand, i.e. time = energy/power. There are innumerable scenarios for how P might scale with h. A simple model that is adequate to introduce the topic is to say that it is simply proportional to the robot's surface area h2, from which we conclude that the machine's potential running time is proportional to h. In this particular scenario a robotic vacuum cleaner design that runs for 30 minutes when its diameter is 30 cm could run for only 1 minute when its diameter is reduced to 1 cm. We will subsequently consider several alternative models or the dependence of P on h, how they play out, and what can be concluded from the outcomes.

3.2 Baseline Energy Demand

First, under what circumstances is the initial illustrative assumption that P is proportional to h2 the correct model? If the vacuum cleaner is not a vacuum cleaner but, say, a mouse, then the body heat loss rate to the environment, i.e., the power required to maintain the body temperature, is proportional to h2, and the time during which the stored energy can keep the mouse above any specified threshold temperature is proportional to h. The same is true if the on-board energy is expended to keep the mouse – or robot – cool, e.g., it absorbs energy from solar illumination or from a hot atmosphere that it needs to dissipate to prevent overheating of its delicate organs or electronics. An on-board heat pump must operate at a rate proportional to the surface area, h2, hence the time during which stored energy can keep the heat pump running is proportional to h. Of course, exactly how to do this – perhaps by making a part of the surface a radiator that is actually hotter than the surroundings in order to cool the bulk of the volume, or by sucking in some of the hot atmosphere, making it even hotter, and expelling it – might be an engineering challenge, but it is certainly possible.

3.3 Energy Lost Moving Air

So what is the right model for a vacuum cleaner – a machine that needs to cover some ground vs. an animal that needs to keep itself warm or cool? It may depend on how efficient the vacuum cleaner is. If it really is a "vacuum cleaner", an awfully inefficient machine that wastes most of its power on blowing air and making noise, and if the important issue is to maintain a constant air velocity at the intake irrespective of the machine's scale, then the power required is again proportional to h2, and the running time is still proportional to h. This assumes that we don't make it so small that its dimensions become comparable to the mean-free-path of the air molecules, in which case it would probably not be possible to s satisfy the goal of maintaining an arbitrary air intake velocity.

3.4 Energy Lost to Brush Friction

So what if it is a more efficient sort of "vacuum cleaner", one that actually picks up dirt with a rotating brush rather than by sucking it in with a high-velocity air flow? Most of the power might then be expended in the friction of the brush on the floor. To determine the running time we must ask some more about the model. The width of the brush in contact with the floor obviously scales as h. To maintain strict proportionality the front-to-back length of the brush in contact with the floor should also scale as h. But a better model for "constant cleaning power" would be for the front-to-back length to be determined by the interaction of the brush with the floor, independent of the brush width. Constant cleaning power would also require that the rotational speed of the brush against the floor be independent of the scale of the machine.

Under these assumptions P would scale as h, and the running time would be proportional to h2. Now scaling down would be *really* costly: the 30 cm diameter machine that ran for 30 minutes, when scaled down to 1 cm, would run for only 2 seconds.

3.5 Constant Cleaning Power

Note that "constant cleaning power" was defined in terms of having a constant front-to-back length of brush in contact with the floor and a constant rotational speed of the brush with respect to the floor. But what about the forward velocity of the machine over the floor? Again, strict proportionality would say the forward velocity should scale as h, but for the machine to be really useful it is probably more realistic for the forward velocity, like the front-to back length of the brush in contact with the floor, to be independent of the scale of the machine. The area of floor cleaned per unit time would then depend only on the width of the brush, which scales as h. Since the running time scales as h2, the area cleaned in the machine's running time scales as h3. The 30 cm diameter machine scaled down to 1 cm would clean only (1/30)3 of the floor area before running its batteries down. This might not actually be as bad as it sounds, inasmuch as an alternative reasonable expectation might be that a machine 1/30 as wide would clean only 1/30 as much floor area, in which case the smaller machine would fall short of our

expectation by only a factor of (1/30)2 versus (1/30)3.

3.6 Energy Lost to Body Drag

We could go on for a long time examining different scenarios and assumptions, but let's do just one more. Let's assume the machine picks up dirt in some undisclosed but very efficient way that consumes practically no power. The power cost of using the machine is then its frictional drag across the floor and through the air. Both frictional costs scale, to a good approximation, as the product of the area and velocity, h2 v. The machine running time thus scales as h/v. If we assume a constant-velocity-over-the floor model then the running time still scales as h, so the 30 cm / 30 minute machine scales down to a 1 cm / 1 minute machine. But if we take another alternative reasonable assumption, one that says our expectation is for a the smaller machine to move across the floor more slowly in proportion to its diameter, i.e., v is proportional to h, then the running time is independent of scale: all members of this family of vacuum cleaners run for 30 minutes. However the area cleaned in running time t scales as h v t, so with v proportional to h the area cleaned scales as h2, still falling short of our expectation that the area cleaned in the machine's running time might reasonable scale as h.

3.7 Total Cleaning Power

Finally, we can start with the goal of building a family of machines each of which cleans an area proportional to its diameter in whatever its running time may be and ask what is th corresponding power consumption model. The area cleaned is h v t, and we will be satisfied if it is proportional to h, i.e., if v t – equal to the linear range of the machine – is constant. From our very first analysis we have t proportional to h3/P, so we require h3 v / P to be constant, or the power consumed to be proportional to h3 v. Since the machine's mass is proportional to h3, this is exactly the power cost of hill climbing: a robotic vacuum cleaner most of whose energy is spent on going uphill at constant speed will clean an area proportional to its linear dimension, will traverse an altitude change independent of its scale, and will do it in time that is independent of scale and (obviously) inversely proportional to speed, which may be chosen arbitrary. This result is similar to but not exactly the same as the "all geometrically similar animals jump to the same height" observation, inasmuch as in the running up hill case we have specified that the velocity is arbitrary but constant, whereas in the jumping case the velocity is linear in the time.

3.8 Best General Answer

There is no best answer. There is not even a single answer, because all of the models discuss are to some extent simultaneously realized in every device; the real question for any particular device is what is the relative weighting of these and other energy loss mechanisms. For a mobile robot whose main job is to provide remote human observers with sensor information obtained by sensors mounted on the robot the energy requirement is likely to consist of a constant component related to information processing and communication, an h2 component related to maintaining a suitable operating temperature, and an h2v component relating to viscous drag. The last may be the most interesting, as on the practical side it will be the dominant term for high performance high speed robots, and on the theoretical side it leads to an interesting invariance worth keeping in mind. This invariance is derived in Section 3.6, but it bears repeating and a high-level interpretation here. The model is that the dominant energy loss term is viscous drag, power proportional to the product of frontal area and velocity – the mechanical equivalent of Ohm's Law. With P thus proportional to h2 v and carried energy proportional to h3 we obtain running time t proportional to h/v. It is useful to think of h/v, the time it takes to robot to move one body length, as a step time; robot running time *measured in step times* is thus independent of robot scale, and robot range *measured in steps* is also independent of robot scale, with the same proportionality factor.

4 Conclusion

After setting up the background context so as to give the reader a concrete scenario in which a variety of performance expectations and scaling issues could be considered, discussion focused on a hypothetical family of geometrically similar robotic vacuum cleaners. No doubt the reader will appreciate the underlying universality of the principles and the approach, and with this appreciation be able to pose and answer questions about the range, running time, and a variety of other performance considerations for mobile robots in general. For mobile robots of characteristic dimension h and velocity v in which the dominant energy loss mechanism is drag, if we think of h as a step length and h/v as a step time, the range in steps and the running time in step times are both independent of robot scale. This is probably the most realistic single-term model for modern vehicles, e.g., automobiles, aircraft, and ships. By comparison current generation mobile robots are over designed and underperforming; it is nevertheless entirely reasonable to expect that what is now the best model for high performance transportation will in the future also be the best model for high performance robots.

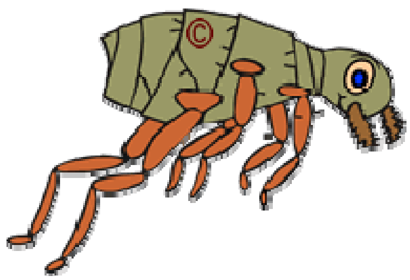
Appendix IV: Back

Energy storage and energy changes in Fleas, Catapults, and Bows

Robin Hood Revisited

If Robin Hood had been Written by an Engineer....

And Robin didst slowly and with great determination put potential energy equal to the work of his muscles into an elastic storage device, much as the lowly and pesky flea hath been known to store it's slow muscle calories into a compressed pad of most springy and efficient material inside the leg of this very same flea.



And therewith Robin the Bold and Valiant didst convert this stored energy most quickly, efficiently, and accurately into the velocity of a sturdy and pointed dart (oft called arrow) such that almost all of its former potential energy didst become kinetic. Then this speedy dart didst split an arrow (oft called dart) already buried in most distant target, having been previously hurled there at an equally great speed by a similar conversion of stored energy.

This having been done much in the same manner as dost the flea convert stored muscle energy into the velocity of its own body, being hurled then into the air to a height many times greater than its own length (though this is not so impressive a feat as many wouldst have us believe).

Both these feats having been impossible by mere mortal strength alone. Yea, it is manifestly plain that conversion of stored muscle energy by an elastic storage device hath made these miracles described herein possible.

Then didst bold Sir Robin kiss Maid Marion, she being most impressed, unto the point almost of swooning, by a man that understandeth in such thorough, noble, and practical fashion, the workings of energy conversion devices. (Happy Ending)

Okay, that's not really how it happened.
Too bad, we think. But it makes for a good introduction to our subject.
Aren't you curious? Don't you wonder what the heck we're talking about?

We're talking about how the jumping technique of fleas uses the same basic "technology" as that used by an archery bow, cross bows, and ancient catapults (and lots of other things).

Both the jumping fleas and the weapons of war use this technology to overcome the same basic

problem - the limitations of muscle power. Okay, that's not really how it happened.
Too bad, we think. But it makes for a good introduction to our subject.
Aren't you curious? Don't you wonder what the heck we're talking about?

We're talking about how the jumping technique of fleas uses the same basic "technology" as that used by an archery bow, cross bows, and ancient catapults (and lots of other things).

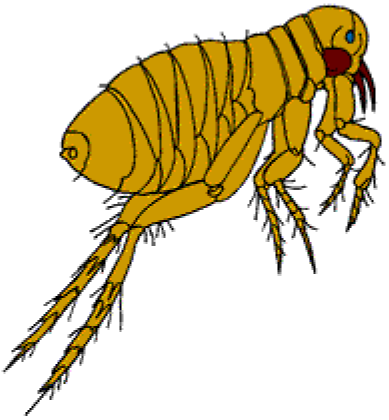
Both the jumping fleas and the weapons of war use this technology to overcome the same basic problem - the limitations of muscle power.

Problem 1 - Little Animals can't Jump

Two things make it hard for small critters to jump very high.

The **first problem** is **air resistance**. Air resistance slows small things a lot more than big things. For an animal the size of a flea, air resistance is a huge problem. Nothing can be done about this except, of course, go somewhere where there is no air, like on the moon. The flea could jump significantly higher in a vacuum (except that he'd be dead).

The **second problem** is that **muscle moves too slow**. How high an animal jumps depends on how fast it is traveling when it leaves the ground (and of course, on how much air resistance slows it down afterward). The flea's **short legs** only allow it an acceleration distance of a fraction of a millimeter. In order to reach an acceptable take-off velocity (speed) the flea has to accelerate (speed up) very quickly. There are real physical limits on how fast muscles can move and how much power they can generate. There is no way the flea's muscles (or any animal's) muscles, can achieve the necessary speed. They just can't generate that kind of power.



But we all know that fleas can jump pretty well. This means they are speeding up (accelerating) during the jump much faster than should be possible if they were using their muscles during the jump.

Solution to Problem

So how do they do it? How do they jump higher than it's possible for muscles to jump. Is it magic? Nah. They just cheat a little. They use their muscles, not to jump, but to slowly store energy in an efficient springy material called resilin. Then, when they are ready to let loose, they release the energy quickly in a burst of power that literaly springs them into the air like a...well...like a spring. It's pretty much just like a catapult.

The shorter an animal's legs are, the faster it has to accelerate (or speed up) to jump the same height. Some fleas have to accelerate to over 140 gravity forces, or 50 times the acceleration rate of the space shuttle, in order to jump just a few inches into the air. This sounds impressive but actually the stresses in such a small animal are not particularly high, and it is the stresses that matter. If a flea grew to human size it would probably not be able to accelerate as fast as us.

Problem 2 - Humans Can't Throw Rocks and Pointed Sticks as Fast or as Far as they would like to

It's those **slow muscles** again. Human muscles are about as slow as insect muscles (fleas are insects).

Just as with jumping, how far an arrow or rock or spear travels, or how deeply it sinks into it's target, depends first on how fast it is going when it leaves the throwing device. That's why javelin throwers get running pretty fast before they throw (their arm and body are the throwing devices). Just as with the jumping flea, it must be accelerated very quickly in a short distance. We just can't generate the power needed to get our arms moving fast enough.

And of course, when people started to want to throw really big rocks and darts at each other, then they had the problem of their muscles being not only too slow, but also too weak.

Solution to Problem

So what did we do? We improvised, like the flea, and used a springy material that could store slow but steady muscle energy and then release it much faster. We invented archery, then catapults and crossbows.

How Slow is Slow?

What do we mean muscles move too slow? Randy Johnson can throw a baseball a hundred miles an hour. Fleas can jump several times a second. And didn't you see Maurice Greene and Marion Jones in the Olympics? Their legs were moving pretty fast!

Well it is, as they say, all relative.

It only takes the flea about 50 milliseconds (or about one twentieth of a second) to cock its leg in preparation for a jump. That must mean he could jump at least that fast.

Well it ain't fast enough. To achieve the lofty heights a flea reaches requires the little buggers to take off in about 0.7 milliseconds (7 ten thousandths of a second!), or about 70 times faster than 50 milliseconds.

If the Greeks and Romans had several thousand Randy Johnsons throwing rocks and spears for them, they might not have been in such a hurry to develop catapults. Not many people can even come close to throwing as hard as Randy Johnson, but a simple hand held shield could stop one of his fastballs pretty easily. The same could not be said of stones hurled by catapults.

The shorter an animal's legs are, the faster it has to accelerate (or speed up) to jump the same height. Some fleas have to accelerate to over 140 gravity forces, or 50 times the acceleration rate of the space shuttle, in order to jump just a few inches into the air. This sounds impressive but actually the stresses in such a small animal are not particularly high, and it is the stresses that matter. If a flea grew to human size it would probably not be able to accelerate as fast as us.

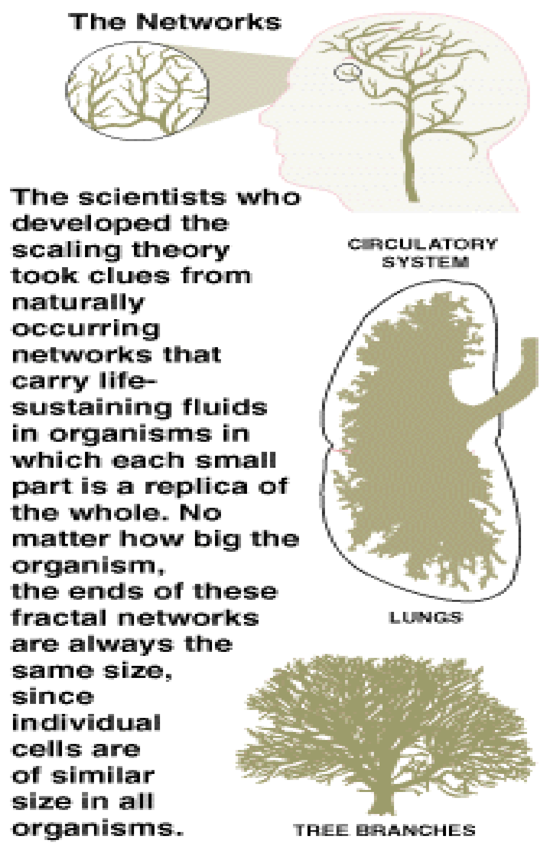
Appendix V: Back

Of Mice and Elephants: A Matter of Scale

By GEORGE JOHNSON

Scientists, intent on categorizing everything around them, sometimes divide themselves into the lumpers and the splitters. The lumpers, many of whom flock to the unifying field of theoretical physics, search for hidden laws uniting the most seemingly diverse phenomena: Blur your vision a little and lightning bolts and static cling are really the same thing.

The splitters, often drawn to the biological sciences, are more taken with diversity, reveling in the 34,000 variations on the theme spider, or the 550 species of conifer trees.



Juan Velasco/The New York Times

Source: Dr. Geoffrey West, Los Alamos National Laboratory

But there are exceptions to the rule. When two biologists and a physicist, all three of the lumpier persuasion, recently joined forces at the Santa Fe Institute, an interdisciplinary research center in northern New Mexico, the result was an advance in a problem that has bothered scientists for decades: the origin of biological scaling. How is one to explain the subtle ways in which various characteristics of living creatures -- their life spans, their pulse rates, how fast they burn energy -- change according to their body size?

As animals get bigger, from tiny shrew to huge blue whale, pulse rates slow down and life spans stretch out longer, conspiring so that the number of heartbeats during an average stay on Earth tends to be roughly the same, around a billion. A mouse just uses them up more quickly than an elephant.

Mysteriously, these and a large variety of other phenomena change with body size according to a precise mathematical principle called quarter-power scaling. A cat, 100 times more massive than a mouse, lives about 100 to the one-quarter power, or about three times, longer. (To calculate this number take the square root of 100, which is 10 and then take the square root of 10, which is 3.2.) Heartbeat scales to mass to the **minus** one-quarter power. The cat's heart thus beats a third as fast as a mouse's.

The Santa Fe Institute collaborators -- Geoffrey West, a physicist at Los Alamos National Laboratory, and two biologists at the University of New Mexico, Jim Brown and Brian Enquist -- have drawn on their different kinds of expertise to propose a model for what causes certain kinds of quarter-power scaling, which they have extended to the plant kingdom as well.

In their theory, scaling emerges from the geometrical and statistical properties of the internal networks animals and plants use to distribute nutrients. But almost as interesting as the details of this model, is the collaboration itself. It is rare enough for scientists of such different persuasions to come together, rarer still that the result is hailed as an important development.

"Scaling is interesting because, aside from natural selection, it is one of the few laws we really have in biology," said John Gittleman, an evolutionary biologist at the University of Virginia. "What is so elegant is that the work makes very clear predictions about causal mechanisms. That's what had been missing in the field."

Brown said: "None of us could have done it by himself. It is one of the most exciting things I've been involved in."

It might seem that because a cat is a hundred times more massive than a mouse, its metabolic rate, the intensity with which it burns energy, would be a hundred times greater -- what mathematicians call a linear relationship. After all, the cat has a hundred times more cells to feed.

But if this were so, the animal would quickly be consumed by a fit of spontaneous feline combustion, or at least a very bad fever. The reason: the surface area a creature uses to dissipate the heat of the metabolic fires does not grow as fast as its body mass. To see this, consider (like a good lumper) a mouse as an approximation of a small sphere. As the sphere grows larger, to cat size, the surface area increases along two dimensions but the volume increases along three dimensions. The size of the biological radiator cannot possibly keep up with the size of the metabolic engine.

If this was the only factor involved, metabolic rate would scale to body mass to the two-thirds power, more slowly than in a simple one-to-one relationship. The cat's metabolic rate would be not 100 times greater than the mouse's but 100 to the power of two-thirds, or about 21.5 times greater.

But biologists, beginning with Max Kleiber in the early 1930s, found that the situation was much more complex. For an amazing range of creatures, spanning in size from bacteria to blue whales, metabolic rate scales with body mass not to the two-thirds power but slightly faster -- to the three-quarter power.

Evolution seems to have found a way to overcome in part the limitations imposed by pure geometric scaling, the fact that surface area grows more slowly than size. For decades no one could plausibly say why.

Kleiber's law means that a cat's metabolic rate is not a hundred or 21.5 times greater than a mouse's, but about 31.6 -- 100 to the three-quarter power. This relationship seems to hold across the animal kingdom, from shrew to blue whale, and it has since been extended all the way down to single-celled organisms, and possibly within the cells themselves to the internal structures called mitochondria that turn nutrients into energy.

Long before meeting Brown and Enquist, West was interested in how scaling manifests itself in the world of subatomic particles. The strong nuclear force, which binds quarks into neutrons, protons and other particles, is weaker, paradoxically, when the quarks are closer together, but stronger as they are pulled farther apart -- the opposite of what happens with gravity or electromagnetism.

Scaling also shows up in Heisenberg's Uncertainty Principle: the more finely you measure the position of a particle, viewing it on a smaller and smaller scale, the more uncertain its momentum becomes.

"Everything around us is scale dependent," West said. "It's woven into the fabric of the universe."

The lesson he took away from this was that you cannot just naively scale things up. He liked to illustrate the idea with Superman. In two panels labeled "A Scientific Explanation of Clark Kent's Amazing Strength," from Superman's first comic book appearance in 1938, the artists invoked a scaling law: "The lowly ant can support weights hundreds of times its own. The grasshopper leaps what to man would be the space of several city blocks." The implication was that on the planet Krypton, Superman's home, strength scaled to body mass in a simple linear manner: If an ant could carry a twig, a Superman or Superwoman could carry a giant ponderosa pine.

But in the rest of the universe, the scaling is actually much slower. Body mass increases along three dimensions, but the strength of legs and arms, which is proportional to their cross-sectional area, increases along just two dimensions. If a man is a million times more massive than an ant, he will be only 1,000,000 to the two-thirds power stronger: about 10,000 times, allowing him to lift objects weighing up to a hundred pounds, not thousands.

Things behave differently at different scales, but there are orderly ways -- scaling laws -- that connect one realm to another. "I found this enormously exciting," West said. "That's what got me thinking about scaling in biology."

At some point he ran across Kleiber's law. "It is truly amazing because life is easily the most complex of complex systems," West said. "But in spite of this, it has this absurdly simple scaling law. Something universal is going on."

Enquist became hooked on scaling as a student at Colorado College in Colorado Springs in 1988. When he was looking for a graduate school to study ecology, he chose the University of New Mexico in Albuquerque partly because a professor there, Brown, specialized in how scaling occurred in ecosystems.

There are obviously very few large species, like elephants and whales, and a countless number of small species. But who would have expected, as Enquist learned in one of Brown's classes, that if one drew a graph with the size of animals on one axis and the number of species on the other axis, the slope of the resulting line would reveal another quarter-power scaling law? Population density, the average number of offspring, the time until reproduction -- all are dependent on body size scaled to quarter-powers.

"As an ecologist you are used to dealing with complexity -- you're essentially embedded in it," Enquist said. "But all these quarter-power scaling laws hinted that something very general and simple was going on."

The examples Brown had given all involved mammals. "Has anyone found similar laws with plants?" Enquist asked. Brown said, "I have no idea. Why don't you find out?"

After sifting through piles of data compiled over the years in agricultural and forestry reports, Enquist found that the same kinds of quarter-power scaling happened in the plant world. He even uncovered an equivalent to Kleiber's law.

It was surprising enough that these laws held among all kinds of animals. That they seemed to apply to plants as well was astonishing. What was the common mechanism involved? "I asked Jim whether or not we could figure it out," Enquist recalled. "He kind of rubbed his head and said, 'Do you know how long this is going to take?"'

They assumed that Kleiber's law, and maybe the other scaling relationships, arose because of the mathematical nature of the networks both animals and trees used to transport nutrients to all their cells and carry away the wastes. A silhouette of the human circulatory system and of the roots and branches of a tree look remarkably similar.

But they knew that precisely modeling the systems would require some very difficult mathematics and physics. And they wanted to talk to someone who was used to trafficking in the idea of general laws.

"Physicists tend to look for universals and invariants whereas biologists often get preoccupied with all the variations in nature," Brown said. He knew that the Santa Fe Institute had been established to encourage broad-ranging collaborations. He asked Mike Simmons, then an institute administrator, whether he knew of a physicist interested in tackling biological scaling laws.

West liked to joke that if Galileo had been a biologist, he would have written volumes cataloging how objects of different shapes fall from the leaning tower of Pisa at slightly different velocities. He would not have seen through the distracting details to the underlying truth: if you ignore air resistance, all objects fall at the same rate regardless of their weight.

But at their first meeting in Santa Fe, he was impressed that Brown and Enquist were interested in big, all-embracing theories. And they were impressed that West seemed like a biologist at heart. He wanted to know how life worked.

It took them a while to learn each other's languages, but before long they were meeting every week at the Santa Fe Institute. West would show the biologists how to translate the qualitative ideas of biology into precise equations. And Brown and Enquist would make sure West was true to the biology. Sometimes he would show up with a neat model, a physicist's dream. No, Brown and Enquist would tell him, real organisms do not work that way.

"When collaborating across that wide a gulf of disciplines, you're never going to learn everything the collaborator knows," Brown said. "You have to develop an implicit trust in the quality of their science. On the other hand, you learn enough to be sure there are not miscommunications."

They started by assuming that the nutrient supply networks in both animals and plants worked according to three basic principles: the networks branched to reach every part of the organism and the ends of the branches (the capillaries and their botanical equivalent) were all about the same size. After all, whatever the species, the sizes of cells being fed were all roughly equivalent. Finally they assumed that evolution would have tuned the systems to work in the most efficient possible manner.

What emerged closely approximated a so-called fractal network, in which each tiny part is a replica of the whole. Magnify the network of blood vessels in a hand and the image resembles one of an entire circulatory system. And to be as efficient as possible, the network also had to be "area-preserving."

If a branch split into three daughter branches, their cross-sectional areas had to add up to that of the parent branch. This would insure that blood or sap would continue to move at the same speed throughout the organism.

The scientists were delighted to see that the model gave rise to three-quarter-power scaling between metabolic rate and body mass. But the system worked only for plants. "We worked through the model and made clear predictions about mammals," Brown said, "every single one of which was wrong."

In making the model as simple as possible, the scientists had hoped they could ignore the fact that blood is pumped by the heart in pulses and treat mammals as though they were trees. After studying hydrodynamics, the nature of liquid flow, they realized they needed a way to slow the pulsing blood as the vessels got tinier and tinier.

These finer parts of the network would not be area-preserving but area-increasing: the cross sections of the daughter branches would add up to a sum greater than the parent branch, spreading the blood over a larger area.

After adding these and other complications, they found that the model also predicted three-quarter-power scaling in mammals. Other quarter-power scaling laws also emerged naturally from the equations. Evolution, it seemed, has overcome the natural limitations of simple geometric scaling by developing these very efficient fractal-like webs.

Sometimes it all seemed too good to be true. One Friday night, West was at home playing with the equations when he realized to his chagrin that the model predicted that all mammals must have about the same blood pressure. That could not be right, he thought. After a restless weekend, he called Brown, who told him that indeed this was so.

The model was revealed, about two years after the collaboration began, on April 4, 1997, in an article in Science. A follow-up last fall in Nature extended the ideas further into the plant world.

More recently the three collaborators have been puzzling over the fact that a version of Kleiber's law also seems to apply to single cells and even to the energy-burning mitochondria inside cells.

They assume this is because the mitochondria inside the cytoplasm and even the respiratory components inside the mitochondria are arranged in fractal-like networks.

For all the excitement the model has caused, there are still skeptics. A paper published last year in American Naturalist by two scientists in Poland, Dr. Jan Kozlowski and Dr. January Weiner, suggests the possibility that quarter-power scaling across species could be nothing more than a statistical illusion. And biologists persist in confronting the collaborators with single species in which quarter-power scaling laws do not seem to hold.

West is not too bothered by these seeming exceptions. The history of physics is replete with cases where an elegant model came up against some recalcitrant data, and the model eventually won. He is now working with other collaborators to see whether river systems, which look remarkably like circulatory systems, and even the hierarchical structure of corporations obey the same kind of scaling laws.

The overarching lesson, West says, is that as organisms grow in size they become more efficient. “That is why nature has evolved large animals,” he said. “It’s a much better way of utilizing energy. This might also explain the drive for corporations to merge. Small may be beautiful but it is more efficient to be big.”

Appendix VI: Back

The text of the article: "Physics and the Bionic Man", available in PDF