Where to Place a Post? Engaging Students in the Mathematical Modeling Process

Ahmad Alhammouri, *Ohio University*
Jami Durkee, *Cambridge High School*
Gregory D. Foley, *Ohio University*

Abstract: *In this article, we explain how to involve students in the mathematical modeling process. We discuss the modeling framework from the* Common Core State Standards for Mathematics*. In particular, we share findings from a post-placing activity that engaged high school students in the modeling process.*

Keywords: *mathematical modeling, rich tasks, modeling process*

1 Introduction

Modeling is an essential ingredient for meaningful learning in today's mathematics classroom. When students engage in mathematical modeling, they solve rich real-world tasks while learning to value mathematics and realizing its importance in their daily lives. Furthermore, when students engage in mathematical modeling, they become active learners. As they explore open-ended problems, they collect data, use mathematics, and reflect on their own work and the work of others. Despite the many benefits of modeling for school mathematics, too often teachers struggle to enact mathematical modeling effectively. Confusion continues to surround the mathematical modeling process. In this article, we explain the stages of the mathematical modeling process and how to engage students in the process. To provide a specific context, we analyze a modeling task that one of the authors enacted with high school students.

2 The Mathematical Modeling Process

Mathematical modeling is an iterative process. Students begin by considering a real-world problem to be solved. Next, they apply mathematics to solve the problem. Finally, they reflect back on the original real-world situation and assess the feasibility of their solution. There are many frameworks that represent the mathematical modeling process. In this article, we use the framework presented in the *Common Core State Standards for Mathematics* (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010) while drawing on ideas from Bliss, Fowler, and Galluzzo (2014). Figure 1 illustrates the iterative nature of the mathematical modeling process in six stages. We will explore each stage in depth in the next section.

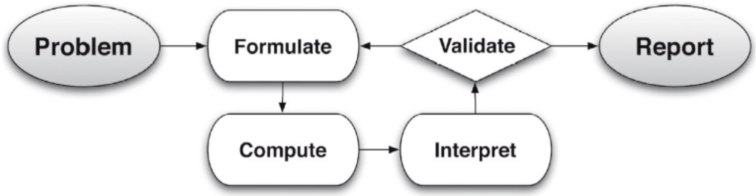

Fig. 1: *A schematic diagram of mathematical modeling (NGA Center & CCSS, 2010, p. 72).*

2.1 Problem

To start the process, the teacher presents the students with a real-world situation, and the students are given time to think about the situation, explore it, and discuss it. The students need to understand the nature of the problem, or even create their own problems out of the given situation (Butts 1980/2013; Pollak, 1966). In either case, students should be encouraged to conduct a background search about the real-world situation (Felton, Anhalt, & Cortez, 2015). After the students understand the problem, they should identify the nature of the expected outcomes.

2.2 Formulate

To build a mathematical model, the students must make assumptions and define variables (Bliss et al., 2014):

- **Make assumptions:** Once they understand the real-world situation, the students brainstorm about which factors are important, which can be ignored, and what needed information is missing. This brainstorming leads the students to make informed assumptions concerning the major factors that affect the real-world situation. (These assumptions can be changed on the next iteration of modeling, so initial assumptions are made to keep the model as simple as is reasonable.)
- **Define variables:** After the students identify the major factors that affect the real-world situation, they quantify these as variables. The students determine which variables are dependent and which are independent. They also should decide on the appropriate units for each variable (e.g., inches or miles; centimeters or kilometers; seconds or years).
- **Create a mathematical model:** The students develop a mathematical model that involves the variables and that can be used to obtain the expected outcomes that were identified in the Problem stage. The model can be algebraic, geometric, graphical, or a combination of these.

These three Formulate steps can be done in any order. For example, the students may start by identifying the mathematical model and then define its variables while making assumptions along the way. Furthermore, the students can—and often do—refine their assumptions, variables, and mathematical models on subsequent iterations of the modeling process.

2.3 Compute

At this stage of the process, the students perform mathematical procedures using the model they formulated. Getting a mathematical solution "may involve pencil-and-paper calculations, evaluating a function, running simulations, or solving an equation" (Bliss et al., 2014, p. 7).

2.4 Interpret

Next, the students relate the results of the Compute stage to the real-world situation. Thus, they obtain outcomes interpreted within the original problem context.

2.5 Validate

The students then test the quality of their model and process. They "reflect on whether the mathematical answer makes sense in terms of the original situation" (Felton et al., 2015, p. 344). For example, they should identify and eliminate mathematical answers, such as negative lengths, that do not make sense in the context of the problem. In addition, the students consider how the outcomes would be influenced if they changed their assumptions or variables (Bliss et al., 2014). After validation, the students decide whether to reformulate their model or report their findings. In our experience and other research, we have found that validation can occur at any stage of the modeling process (Alhammouri, 2016).

2.6 Report

Once they are satisfied with their solution, the students share their findings and the reasons behind them (NGA Center & CCSS, 2010). The students explain the assumptions that they have made, how they validated their models, and what challenges they faced during the modeling process.

3 Where to Place a Post?

In this section, we present details of a modeling activity as it was enacted by a class of high school students. The students used trigonometry, but geometry students might set up a scale model using dynamic geometry software to solve the same problem. We discuss how the students addressed the six stages of the modeling process.

In addition to developing modeling know-how, the goal of the activity was to help students use "trigonometric ratios [to] solve problems involving right triangles" (NGA Center & CCSS, 2010, p. 75). The teacher started the lesson with a quick review of trigonometric ratios for right triangles. Next, the teacher asked the students to work in groups on the following problem:

> Assume that you are an engineer working for an airport. How far should you place a 100-ft post from the end of the airport's runway?

The teacher asked the students to think about the problem and develop questions that they had concerning the problem situation.

3.1 Problem

In groups, the students started thinking, "Why would we need to consider the distance from the end of the runway to the post?" Almost all realized that the distance mattered to prevent airplanes from colliding with the post during takeoff and landing. For example, as shown in Figure 2, one student drew a picture to understand the situation and see why the distance would matter.

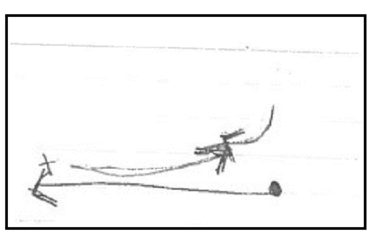

Fig. 2: *Initial student diagram of the problem situation.*

3.2 Formulate

Once the students understood the problem situation, they developed a mathematical model for the situation. Typically, the model was a right triangle with the post as the opposite leg, the horizontal distance from the end of the runway to the base of the post as the adjacent leg, and the slant distance from the end of the runway to the top of the post as the hypotenuse. Figure 3 shows one group's diagram.

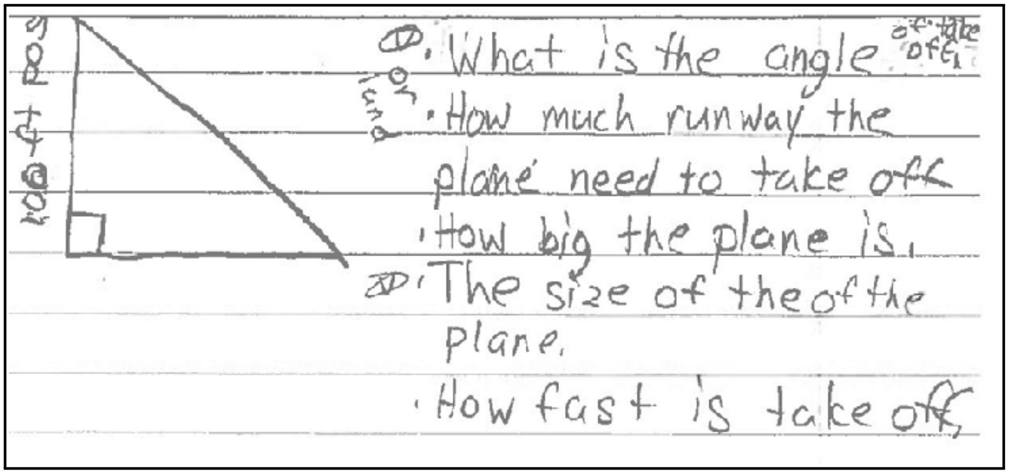

Fig. 3: *A geometric model and some related questions.*

The students posed questions to identify the factors for the model and to aid in developing assumptions. Notice that the students created a mathematical model before making assumptions. Because the teacher had just reviewed the trigonometric ratios of sine, cosine, and tangent, many students naturally used a right triangle as their model. Then, the students came together as a class to discuss some of the questions that they created in their groups. Each group had to share at least one question with the rest of the class. The teacher led a discussion to decide which of the questions related to the right-triangle model. Some questions were excluded because they did not relate to the agreed-upon model, shown in Figure 3. Some of the excluded questions were,

- How fast does the airplane takeoff?
- How long is the runway?

In the whole-class discussion, the students focused on the takeoff angle of the airplane, the type (size) of the airplane, and takeoff point on the runway. By the end of the discussion, the students agreed that they needed to find the takeoff angle and whether there was a relation between the size of the airplane and its takeoff angle. In addition, the students assumed that the airplanes would takeoff and land at the end of the runway, the worst-case scenario for airplane safety. The students also identified known quantities in their models. They noticed, for example, that the height of the post was given.

Once back to their groups, the students worked on finding the takeoff angles for the airplanes and determining whether there was relation between the angle and the size of the airplanes. After the students conducted an Internet search, they found that the takeoff angle was usually between $10°$ to $15°$ regardless of the size or type of the airplane. This was the major independent variable for their model.

Figure 4 shows how a group indicated the variables in their geometric model. This particular group assumed the takeoff angle would be 10°. During the class discussion, the teacher asked them why they assumed an angle of 10°. The group explained that if they assumed an angle of 15° and an airplane actually took off at 10°, the airplane might collide with the post. Another group assumed the angle was 15°, and realized their faulty reasoning during class discussion. A third group assumed the angle would be 7° to make takeoff even safer. A fourth group used the average of 10° and 15°: 12.5°. This variety enriched subsequent class discussion.

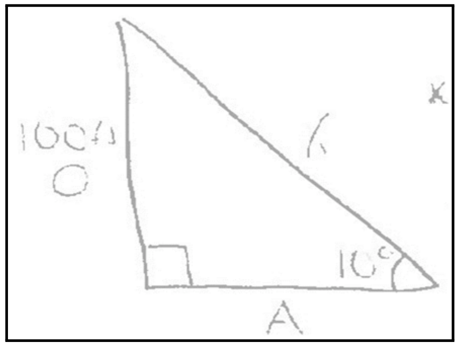

Fig. 4: *Student defining variables on a geometrical model.*

Next, as shown in Figure 5, the students labeled their triangles and sought a trigonometric ratio to find how far the post needs be from the end of the runway. There were three trigonometric ratios for the students to consider: sine, cosine, and tangent. All groups agreed to use the tangent equation.

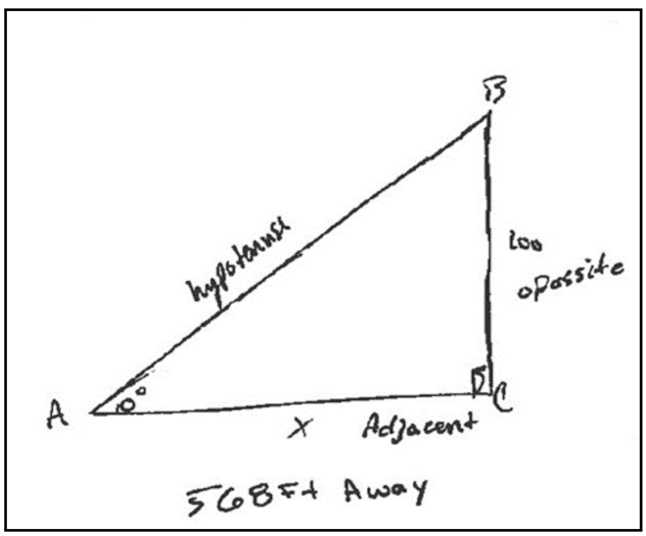

Fig. 5: *One student's labeling of the sides of a right-triangle model.*

3.3 Compute and Interpret

After the lengthy formulate stage, the students performed mathematical procedures using the tangent ratio to find the distance from the end of the runway to the post, as shown in Figure 6. The groups obtained varying answers based on their assumptions for the takeoff angle. Then, the students related their mathematical answers to the problem situation by indicating that their answers represented the location of the post, and they provided units (feet) for their answers.

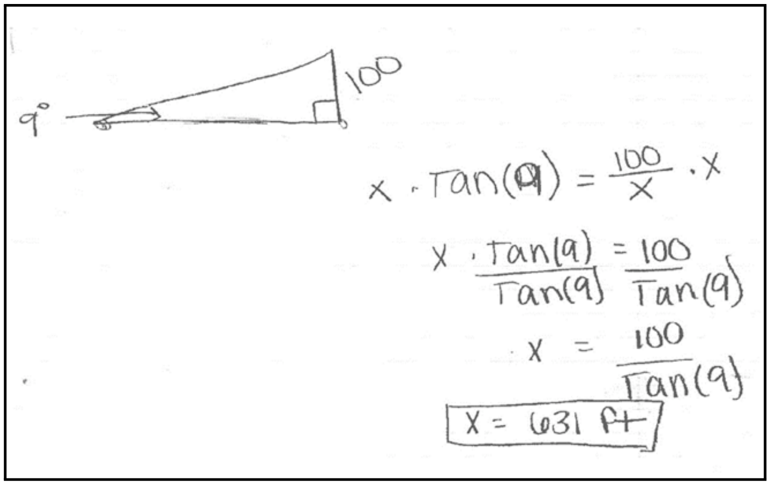

Fig. 6: *Procedures to obtain a mathematical answer.*

3.4 Validate

Instead of waiting until the end, the students validated their work throughout the mathematical modeling process. For example, when the students were making assumptions, they excluded some assumptions that did not relate to the geometric model. In addition, when the students were defining variables, some of them decided to assume the takeoff angle was 10°, which would be safer. Furthermore, some students revised their models after they interpreted their answers. After a discussion with other group members, one student decided to assume that the angle should be 9° instead of 10° to make the takeoff even safer.

3.5 Report

Initially, the students shared and explained their results in small-group and whole-class discussions. They discussed how the model could be improved. For example, one student commented that an airplane takes off at a certain angle but then the angle increases after a few seconds. The students realized that they could have improved their model and results by taking this into account. Ultimately, each student submitted a brief written report explaining his or her methods and findings.

4 Reflections

As already noted, modeling is an iterative process. Students should be gently eased into this process. Just as mathematical modeling is new for most teachers, it is also new for most students. By posing the post-placing problem within the context of trigonometry, this gave the students some scaffolding that helped them greatly. Once students become more accustomed to the modeling process, even more open-ended problems should be given—and given without hints about which mathematics might apply to the situation.

After the students have completed the work as described in this article, a next step could be to ask them to consider the landing of an airplane. In particular, they could be asked to do further Internet research. This likely would lead them to consider the Rule of Three (`https://en.wikipedia.org/wiki/Rule_of_three_(aeronautics)`). The 3:1 rule of descent calls for 3 mi of horizontal travel for each 1,000 ft of descent. What implications would this rule have for the placement of the post?

5 Conclusion

In this article, we explained the process of mathematical modeling. To provide a context, we presented a post-placing problem for high school students and explained how the students used the modeling process to solve this problem. Along the way, the students addressed the six stages of mathematical modeling and deepened their (and our!) understanding of the modeling process. Some of the major takeaways from this classroom-based action research include the following:

- In mathematical modeling, students enact mathematics to solve rich real-world tasks.
- Students may use more than one mathematical model (e.g., a geometry and an algebraic model) to solve a single modeling problem.
- The real-world problems are open ended, and student answers naturally vary based on the assumptions they make and the models they use.
- Students often refine their initial mathematical model during subsequent modeling cycles, and they should be encouraged to do so.
- Validation can occur at any stage of the modeling process.

References

Alhammouri, A. (2016, April). *How do modeling practitioners conceptualize the modeling process?* Poster presented at the Student Research and Creative Activity Expo, Ohio University, Athens.

Bliss, K. M., Fowler, K. R., & Galluzzo, B. J. (2014). *Math modeling: Getting started and getting solutions*. Philadelphia, PA: Society for Industrial and Applied Mathematics.

Butts, T. (1980). Posing problems properly. In S. Krulik & R. E. Reys (Eds.), *Problem solving in school mathematics: 1980 yearbook* (pp. 23–33). Reston, VA: National Council of Teachers of Mathematics. (Reprinted in 2013 in F. Fennell & W. R. Speer (Eds.), *Defining mathematics education: 75th yearbook* [pp. 257–268]. Same publisher.)

Felton-Koestler, M. D., Anhalt, C. O., & Cortez, R. (2015). Going with the flow: Challenging students to make assumptions. *Mathematics Teaching in the Middle School, 20*, 342–349.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Author. Retrieved from `http://corestandards.org/assets/CCSSI_Math%20Standards.pdf`.

Pollak, H. O. (1966). On individual exploration in mathematics education. In E. G. Begle (Ed.), *The role of axiomatics and problem solving in mathematics* (pp. 117–122). Washington, DC: Conference Board of the Mathematical Sciences (published by Ginn).

Ahmad Alhammouri, `aa282812@ohio.edu`, is a PhD candidate at Ohio University, majoring in mathematics education. Currently, Mr. Alhammouri coteaches modeling and quantitative reasoning at Alexander High School. His interests include the teaching and learning of mathematical modeling and designing professional development activities for mathematics teachers. Mr. Alhammouri is a research associate for the Mathematical Modeling and Spatial Reasoning (Modspar) project.

Jami Durkee, jami.durkee@cambridgecityschools.org, graduated from Muskingum University with a BS in Mathematics and currently teaches mathematics at Cambridge High School. Her interests include teaching Algebra and Geometry and using modeling to help students to see the real-life value of mathematics. Ms. Durkee is a Modspar scholar.

Gregory D. Foley, foleyg@ohio.edu, is the Robert L. Morton Professor of Mathematics Education at Ohio University. His scholarship focuses on learning mathematics through inquiry and the transition from secondary to postsecondary mathematics. Dr. Foley directs the Modspar project.