Average and Standard Deviation of Demand over Replenishment Lead Time

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This document describes how to calculate the demand over the lead time, and the standard deviation of demand over the lead time. First, a little terminology from the book:

$\hat{x}_L$	Expected demand over the lead time.
σL	Standard deviation of demand over LT.
D	Demand over the whole year.
LT	Lead time (assumed to always be the same)

We want to figure out the average and standard deviation of the total demand over the lead time. We begin with the assumption that demand each day is a random variable that has a normal distribution, and that every day has the same average and standard deviation. When we talk about the demand over the lead time, we are really talking about what the sum of the daily random demands will be. What will they sum up to?

1 Calculating $\hat{x}_L$

Regarding the sum of random variables, your statistics professor at one time may have said:

> "the average of the sum is the sum of the averages."

So to get the average demand over the lead time, we just sum up the average demands of each day of the LT. Since all the days are assumed to be the same, we just multiply the daily average by the number of days. If demand per day has an average of 50, and the LT is 5 days, expected demand over the lead time is

$$\hat{x}_L = 5 * 50 = 250. \tag{1}$$

If we are given the demand per year, we just figure out how much of a year the LT represents. If the LT is one and a half months, then

$$\hat{x}_L = 1.5 * D/12, \tag{2}$$

where D is annual demand. If the LT is 5 weeks, then

$$\hat{x}_L = 5 * D/52. \tag{3}$$

Obviously, if D is per month or some other period, this all changes slightly.

2 Calculating σ_L

Your statistics prof may have gone on to say:

> "the variance of the sum is the sum of the variances."

That means to get the variance of the demand over the LT, we need to sum up the variances of the daily demands. If σ_{day} is the standard deviation of demand per day, and LT_{days} is the lead time expressed in days, then

$$\sigma_L^2 = LT_{days} * \sigma_{day}^2 \tag{4}$$

To get the standard deviation of the LT demand (instead of the variance), we have to take the square root:

$$\sigma_L = \sqrt{LT_{days}} * \sigma_{day}. \tag{5}$$

Depending on whether we have the LT in days, weeks, or months, we can use the same logic:

$$\sigma_L = \sqrt{LT_{days}} * \sigma_{day}, \tag{6}$$

$$\sigma_L = \sqrt{LT_{weeks}} * \sigma_{week}, \tag{7}$$

$$\sigma_L = \sqrt{LT_{months}} * \sigma_{month}. \tag{8}$$

If we are given the standard deviation of demand per month, we can calculate the standard deviation per day as follows (assuming 30 days per month):

$$\sigma_{day} = \frac{\sigma_{month}}{\sqrt{30}}. \tag{9}$$

Although this looks funny, to convince yourself its ok, multiply both sides by $\sqrt{30}$. You get: $\sigma_{month} = \sqrt{30} * \sigma_{day}$, which should make sense, because it looks a lot like equation (5). Using the same logic, we can get the standard deviation of demand over months and days the same way:

$$\sigma_{month} = \frac{\sigma_{year}}{\sqrt{12}} = \sigma_{week} * \sqrt{4.29} = \sigma_{day} * \sqrt{30} \tag{10}$$

$$\sigma_{week} = \frac{\sigma_{year}}{\sqrt{52}} = \frac{\sigma_{month}}{\sqrt{4.29}} = \sigma_{day} * \sqrt{7} \tag{11}$$

$$\sigma_{day} = \frac{\sigma_{year}}{\sqrt{365}} = \frac{\sigma_{month}}{\sqrt{30}} = \frac{\sigma_{week}}{\sqrt{7}} \tag{12}$$

We're assuming a month has 30 days, which is $30/7 = 4.2857$ weeks, which we rounded to 4.29. So, for example, assume the standard deviation per year is $\sigma_{year} = 1{,}000$. From equation (10), the standard deviation per month is $\sigma_{month} = \frac{1{,}000}{\sqrt{12}} = \frac{1{,}000}{3.464} = 288.67$. From equation (11), we can calculate the standard deviation per week: $\sigma_{week} = \frac{1{,}000}{\sqrt{52}} = \frac{1{,}000}{7.211} = 138.675$.

Now, to double-check, let's convert $\sigma_{week} = 138.675$ into a monthly value, using the second part of (10), and see how well it compares with the value we already got. So we take $\sigma_{month} = \sigma_{week} * \sqrt{4.29} = 138.675 * 2.0712 = 287.2$. This is not the same as 288.7, because of rounding.

(The other reason they don't match perfectly is because if you really divided the year into 12 even months, they would have 30.41667 days, not 30. But we're not going worry about this, because then we'd also have to include the fact that a year is 52.167 weeks. The amount of error introduced by these roundings is bound to be much smaller than the amount of error in all of our estimated quantities, so these errors are not worth trying to fix.)

3 The Thing to Remember

The thing to remember is that first you have to figure out the standard deviation per day, per week, or per month, using equations (10)-(12). Which one you use depends on how you want to write the Lead Time.

Then you can use (6)-(8) to find σ_L.