

A Comparison of Math Teaching and Learning in China and the United States -: Problem Solving Skills in Geometry of Chinese and U.S. Students

THESIS

Presented in Partial Fulfillment of the Requirements for the Degree Master of Mathematical Sciences in the Graduate School of The Ohio State University

By

Yaomingxin Lu

Graduate Program in Mathematical Sciences

The Ohio State University

2016

Dissertation Committee:

Michael Battista, Rodica Costin

Copyrighted by
Yaomingxin Lu
2016

Abstract

According to the TIMMS report, Chinese students tend to have better performance in solving geometry problems than U.S. students. What is the explanation for the superior mathematics achievement of Chinese students? What are the possible contributing factors? The purpose of this study is to compare and examine US and Chinese students' problem solving skills in geometry and analyze the contributing factors through interviews with students from both countries. The interviews show that the Chinese students were more successful than the US students in creating auxiliary elements for helping them solve geometry problems as well as more comfortable switching to different problem solving strategies. This better performance in solving geometry problems may be due to Chinese teachers' ability to apply their geometric conceptualizations flexibly in the classrooms because of their experience solving problems using different strategies as well as several non-school-related factors including things such as students' self-expectation and efforts, family pressure, and cultural value. This kind of research will be useful to develop a deeper understanding of why Chinese students perform better in solving geometry problems compared with their American peers and the factors which contribute to their performances.

Dedication

I want to dedicate this work to my boyfriend, Qing Zang. I could not have completed this effort without his assistance, love and support throughout this period.

Acknowledgments

I would like to express my sincere gratitude to my advisor Prof. Michael Battista for the continuous support of my study and research, for his patience, motivation, and immense knowledge. His guidance helped me in all the time of research and writing of this thesis.

Besides my advisor, I would like to thank the rest of my thesis committee: Prof. Rodica Costin, for her encouragement, insightful comments, and questions.

Vita

2010.....Red Cross Nordic United World College, Norway
2014 B.A. Mathematics, Skidmore College, NY
2014 M.S. Mathematics, Ohio State University, OH
2014 to present Graduate Teaching Associate, Department of
Mathematics, The Ohio State University

Fields of Study

Major Field: Mathematical Sciences

Table of Contents

Abstract	ii
Dedication	iv
Acknowledgments.....	iv
Vita.....	v
List of Figures	viii
List of Tables	ix
Chapter 1: Introduction and Background.....	1
1.1 Chinese Math Education System	
1.2 United States Math Education System	
1.3 Teaching Differences	
Chapter 2: Special Education related Background of Chinese Math Education	9
2.1 Chinese Teachers	
2.2 Chinese Students	
2.3 Chinese Parents	
Chapter 3: Problem Solving Skills in Geometry of Chinese and U.S. Students.....	13
3.1 Literature Reviews.....	13
3.2 Students' Interviews	17
3.3 Discussion and Analysis	56
3.4 Evaluation	60

Chapter 4: Conclusion.....	63
References.....	65

List of Figures

Figure 1. Interview Problem 1 Graph	19
Figure 2. Interview Problem 2 Graph	19
Figure 3. Suggested Solution for Problem 1	20
Figure 4. Suggested Solution for Problem 2	22
Figure 5 Student's Work.....	27
Figure 6. Student's Work.....	28
Figure 7. Student's Work.....	29
Figure 8. Student's Work.....	32
Figure 9. Student's Work.....	33
Figure 10.Student's Work.....	37
Figure 11.Student's Work.....	38
Figure 12.Student's Work.....	39
Figure 13. Student's Work.....	42
Figure 14. Student's Work.....	44
Figure 15. Student's Work.....	47
Figure 16. Student's Work.....	48
Figure 17. Student's Work.....	50
Figure 18. Student's Work.....	52
Figure 19. Studeent's Work	53

List of Tables

Table 1. Comparison of mathematics instruction between two countries	8
Table 2. Summary of U.S. students' interview	54
Table 3. Summary of Chinese students's interview.....	55

Chapter 1. Introduction and Background

The 2011 Trends in International Math and Science Study (TIMSS) reveals that the U.S. is lagging behind leading countries in math achievement. Fourth grade Chinese (Hong Kong) students, producing an average score of 602, rank 3rd out of the 57 countries that participated, while their counterparts from the United States rank 11th with an average score of 541. In the U.S., teaching of mathematics tends to emphasize too much procedural skills. Because of this, many students arriving at college have only experienced mathematics with a focus on symbol manipulation skills (Weber, 2005). After teaching a college level pre-calculus class in the U.S., I have found that students can typically solve mathematical equations, but as Weber argues, most of them are unable to apply the mathematics to meaningful situations and have little conceptual understanding of the mathematics concepts as well (2005). From all these, one may ask, what is the explanation for superior mathematics achievement of Chinese students? What are the possible contributing factors? What can be learned from the Chinese way of mathematics teaching and learning so as to improve mathematics education in the U.S.?

The purpose of this study is to compare and examine US and Chinese students' problem solving skills in geometry, and analyze the contributing factors. In Chapter 1 and 2, we will briefly introduce some math education background in both countries. This will be useful to develop a deeper understanding of how Chinese students perform better in solving geometry problems compared with their American peers and the factors which

contributed to their performances in Chapter 3. The goal is to have some ideas of what we can learn from each other and answer some of the above questions.

1.1 Chinese Math Education System

From the *Senior Secondary Curriculum Standards (2003)*, the period of schooling in China is six years for primary school and six years for middle school (three years for junior middle and three years for senior middle). Children usually enter the educational system at the age of six (or seven in some country area) and graduate from senior middle school at the age of 18.

In elementary school (grade 1-6), students study Chinese, mathematics and natural science. They begin to study foreign language, usually English, at grade 5. After entering into middle school, in addition to Chinese and mathematics, they begin to study physics, chemistry and biology. Mathematics is not in this situation; they keep learning new knowledge.

Uniformity is the biggest characteristic of Chinese education. Because of the uniform National University Entrance Exam, which is held once a year, the syllabus and textbooks are the same in many provinces in China. In the United States, which has the least uniform educational system, curriculums are different in different states.

China's Math Curriculum Standards:

All schools in China subscribe to one major curriculum document, the *Mathematics Curriculum Standard* (2003), prepared by the Ministry of Education. The document is divided into four parts. The first two parts outline the theoretical framework, the aims and objectives, the structures, and implementation of the general primary and secondary education in China. The third part is *Content Standards* subdivided into three sections: (a) Compulsory Curriculum, (b) Optional Curriculum, and (c) Mathematics Exploration, Modeling and Culture. The last part provides some recommendations for teaching.

In addition, the curriculum standard has divided the content into five key stages: (a) Primary Grades 1-2, (b) Primary Grades 3-5, (c) Middle school Grades 6-7, (d) Middle school Grades 8-9, and (e) High school Grades 10-11. For each key stage, the content is further divided into basic and extension. The basic content includes numbers and operations, algebra and equations, graphs and geometry, statistics and probability, and function and analysis. The extension section covers the same content areas, but each area is extended in breath and depth. Each content area or topic is displayed in a table listing the learning content and displaying the detailed content demand and suggestions for teaching activities. Teachers and students are encouraged to teach and learn the extended content through various innovative and creative activities, such as investigation projects, solving daily life problems, mathematical puzzles and games, as well as the history of mathematics.

One significant feature of this curriculum document was that the document explains in clear detail how each of the objectives and content could be taught. These descriptions are supplemented with suggestions and examples. However, mathematics teachers are not required to follow the document strictly; instead they are very much encouraged to expand and explore further mathematical content as needed.

Testing

Because of the uniform National University Entrance Exam, which is held once a year, the syllabus and textbooks are the same in many provinces in China. In the United States, which has the least uniform educational system, curriculums are different in different states.

1.2 United States Math Education System

Common Core:

The math curriculum standard is different for different states in the United States since the U.S. is a decentralized education system—states may vary in math content taught. In the state of Ohio, for example, the Common Core concentrates on a clear set of math skills and concepts. The standard is subdivided into three sections: (a) Mathematical Practice, (b) K-8, and (c) High School. In addition, it specifically emphasizes the content

students need to know in high school, including numbers and quality, algebra, function, geometry, statistics and probability.

Using the common core, it is argued that students will learn concepts in a more organized way both during the school year and across grades. The standards will encourage students to solve real-world problems. The knowledge and skills students need to be prepared for mathematics in college, career, and life are woven throughout the mathematics standards. These standards define what students should understand and be able to do in their study of mathematics.

As one compares content of both educational systems, one sees that the Chinese standard includes numbers and operations, algebra and equations, graphs and geometry, statistics and probability, and function and analysis. Also, it emphasizes the entire school year for students. In the U.S.-based Common Core, the basic five parts are similar, but they miss the key word “operation”, “equations”, “graphs” and “analysis”. Also, we only see this emphasis at the high school stage.

School Structure

The United States School system consists of: elementary education, secondary education (middle school and high school), and a four stage higher learning degree system (Associate, Bachelor, Masters, and Doctorate). The first twelve years of U.S. schooling is compulsory and consists of grades 1 through 12.

Elementary school typically starts when children are 5 or 6 years old (Sydney 2006). Elementary school encompasses, depending on the school district, grades 1 to 5 or 6. The curriculum of elementary schools varies among states and districts but there is typically an emphasis on arithmetic, English proficiency, and basic knowledge of other subjects (social studies and science). Middle School or Junior High School typically begins in grade 6 and ends in grade 8 (upper level primary schooling and lower level secondary schooling). Students in middle school usually have several different teachers, unlike in elementary school where students only have 1 or 2 teachers. The middle school curriculum characteristically places emphasis on science, mathematics, English, and social studies.

High school is the last stage of public education before higher learning education begins. In high school, students are given the freedom to choose the core classes that they wish to take. Upon completion of secondary education students receive a high school diploma, which represents their successful completion of state mandated curricula. Moreover, high schools have a variety of electives available for students to choose from. These electives usually fall under the categories of: the arts, technology, computer science, foreign language, and athletics.

Testing

The American school system currently uses testing for two purposes. First, testing is used to assess the progress of students. Second, testing is used for admission

into higher education. For students who wish to continue their education beyond high school they must either take the Scholastic Aptitude Test (SAT) or the American College Testing (ACT). The results from these two standardized tests are usually, but not always, considered when students apply to colleges or universities.

1.3 Math Teaching Differences between China and the United States

Mullis (2000) summarized several main factors in the difference on mathematics instruction as depicted below:

China (Hong Kong)	United States
<ul style="list-style-type: none"> • Direct Instruction • Teach mathematics by showing students how to solve problems • Use of worksheets • Hong Kong high stakes testing • Causes more time to be spent outside of school practicing math because of exams • Hong Kong students spend more time on mathematics than U.S. students 	<ul style="list-style-type: none"> • Direct Instruction & Constructivist approaches • Teach mathematics by showing students how to solve problems • Use of worksheets and textbook in instruction • United States testing mainly for assessment purposes • The U.S. teaches many more math topics but at a low depth compared to other TIMSS countries (Barth, 2002)

Table 1. Comparison of mathematics instruction between two countries

From this chart, we can see that, Chinese students (Hong Kong) generally spend more time on mathematics compared to students from the United States. We will discuss some reasons for this difference in the next section to develop a better understanding of the influential factors contributing to these differences.

Chapter 2. Education Related Background of Chinese Math Education

2.1 Chinese Teachers

Teachers are viewed as models of good conduct and learning for students in the Chinese tradition. As a country with an extensive teaching tradition, teaching practices in China continue to be influenced by the ideas of Confucius (551-479 BCE) and by texts written in subsequent centuries (An et al., 2002). For example, the distinctive character of Confucianism in respect to learning is to ask questions constantly and to review previous knowledge frequently (Jones, 2006).

Teaching and Learning environment

The physical layout of Chinese classrooms differs from that of U.S. schools. Chinese teachers have much larger classes: typically around twice the size of U.S. classes. When I was an elementary school student in China, I had classrooms with as many as 60 students. Also, classes in China typically have all of the desks facing the teacher, whereas in the United States, desks may be clustered into groups so that students can work together (though many U.S. classrooms are still organized along traditional lines, as in China). Also, in Chinese schools, students tend to stay in one room and teachers travel to that room to teach. In the United States, teachers can decorate and be creative in their homeroom; Ma (1999) explains this in her book about knowing and

teaching math in China, because they “own” the room. But in China, students own their rooms and teachers travel to them. One-to-one or one-to-small group interactions between teacher and students, or among students, are limited as compared with classrooms in the United States (Dong, 2008).

Instruction and Textbooks

China primarily uses unified textbooks to define the mathematics for which the Chinese students would be taught. It is clear this is caused by the NUEE (The National University Entrance Examination). But because U.S. curriculum decisions are made at the state and local levels, the U.S. has never used a unified set of textbooks. In addition to using textbooks, for the sake of NUEE, the Chinese also use many supplementary materials and outside-class reading materials written by mathematicians and experienced mathematics teachers. In contrast, in the U.S. there is a limited number of supplementary materials.

As is common in education, examinations play a critical role in school mathematics curricula and thus, according to Li (2002), mathematics teachers in China are likely to carefully select a considerable quantity of exercises as one of their main teaching strategies (Jones, 2006).

2.2 Chinese Students

Motivation of learning

For many decades, the Chinese, inspired by Confucius, viewed schooling as a way to educate government officers. Since that time, schooling has become an official and glorious ladder to reach the top of society. It has encouraged many ancient students and scholars from average or poor families to study diligently and consistently. Nowadays in China, education is still an important and effective way to raise one's social and economic status. Success in school, especially if one can pass the national university entrance examination, and then graduate from the university, means that he or she could expect a better career with security and high income.

Exam Pressure

The once a year National University Entrance Examination forces students, especially high school students, to spend more time on study compared with American students.

Parents Pressure

For most families, as the only child, children get much more pressure from their parents and even grandparents to be successful and achieve their parents' dreams.

2.3 Chinese Parents

Chinese parents are very keen on the results of schooling, especially the exam marks of their children. Most parents in China will reward their children if they get high marks on examinations, or will punish them for poor marks. Some of the parents even employ home tutors for their children during the last year of senior secondary schooling. All parents aim at improving their children's examination performances.

One child policy

This policy was established in the late 1980s. Because of that, many parents pay all their attention on their only child.

Chapter 3. Problem Solving Skills in Geometry of Chinese and U.S. Students

3.1 Introduction and Literature Review

As the study of spatial objects, relationships, and transformations, geometry is recognized as not only one of the most important components of the school mathematics curriculum but also, alongside algebra, as one of most important elements of mathematics itself (Royal Society, 2001). This makes geometry a demanding element of mathematics to teach well. However, the results of the recent TIMSS (2011) shows that students in the United States do not learn much geometry from grade to grade and students generally have lower achievement compared with their Chinese peers. From the report, 8th graders from the United States scored 55% compared with 84% for those from China (Hong Kong). What factors contribute to this difference?

Ding (2006) reports some findings of a study of geometry teaching at the lower secondary level in Shanghai. The analysis is based on observation of a geometry lesson and students' performance on the exams. Ding specifically chose a Grade 8 geometry class, since Grade 8 is a critical year for students to begin making the transition from practical geometry to theoretical geometry. They paid special attention to lesson structure and the development of students' geometrical thinking. A common lesson structure is divided into 5 parts: Introduction and review, introduce new content, exercises, summary

and homework. The first three parts usually took the major part of the lesson. They observed that it was difficult for most students to write a full proof for many geometry proof types of questions. For most of the parts, teaching is direct. New knowledge is learned by students through observations and exercises. Chinese teachers tend to draw out graphs and explain from the graph when teaching geometry problems.

Educators know from their teaching experience that students very often face difficulties in coping with geometrical problems — more than when attempting to solve algebraic ones. One of the reasons is probably that while in algebra one may usually rely on some standard operational rules, in synthetic geometry we are asked to invent specific, adapted strategies. A second reason refers to the particular interactions between concepts and images in geometrical reasoning. It is this aspect which inspired the previous research in this area (Fishbein, 1990).

The purpose of chapter 3 is to examine US and Chinese students' generalization skills in solving geometry problems, their generative thinking in the problems, and the students' performance on geometry problem solving.

Background

Chinese students consistently outperform US students across grade levels and mathematical topics. This general finding from cross-national studies are echoed by the heavy emphasis of basic skills and knowledge in Chinese Education. Chinese students are more likely to use generalized strategies, which is generalized by their teachers, and

symbolic representations (Cai, 2007). Math education in China puts a heavy emphasis on mathematical methodology; students are required to master problem-solving methods through repetitive learning. Many secondary math teachers have joined the effort of studying mathematical methodology, with a focus on studying mathematical problem-solving strategies. Many teachers require students to memorize specific techniques for solving different problems, so that students can quickly recognize the types of problems and solve them immediately (Cai, 2007). In addition, in Chinese classrooms, teachers usually present a problem and provide opportunities for students to solve it in different ways. This trains students to think and solve the same problem from different perspectives, which will be helpful for them to switch to different strategies once they get stuck.

From my interviews with American students and Chinese students from my class, I have noticed that compared with the American students, the Chinese students I have interviewed have a better knowledge of constructing auxiliary lines to geometry problems for helping with proofs. Students told me that they have learned various ways of making auxiliary lines in middle school and high school classes in China.

Before we start, what do we mean by auxiliary lines in geometry problems?

The use of Auxiliary Lines in Geometry Proofs

In geometry, it may be necessary to add a line or segment to a diagram to help in solving a problem or proving a concept. Such an added line or segment is called an

auxiliary line. The word "auxiliary" means providing additional help or support. While it is never acceptable to change any of the original parts of a diagram, it is acceptable to add new lines or segments. These new lines are drawn "dashed" or "dotted" to indicate that they were not part of the original diagram. The goal is to add something to the diagram that will help you solve the problem.

Students who add auxiliary lines to diagrams indicate by their action that for them diagrams are not an untouchable, final product, but rather the result of a process which can be added to or changed. Scholars have pointed out the importance of adding auxiliary lines in solving geometry problems. Polya (1973) presents several problem situations in which the construction and use of auxiliary lines indicates an understanding, or learning, of geometry that transcends having been merely "taught" geometry. In Polya's view, adding auxiliary lines helps the problem solver access prior knowledge. "Having recollected a formerly solved related problem and wishing to use it for our present one, we must often ask: Should we introduce some auxiliary element in order to make its use possible?" (p47). Auxiliary lines have yet another function, a creative one. Adding an auxiliary line, and focusing on parts of the diagram which include this new line, can generate new insights into the diagram, both questions and conjectures (Chazan, 1990).

In the following problems, especially Problem 1 (see section 3.2 below), students are expected to determine what auxiliary line might be drawn in order to solve the problem. This is not a common problem posed to students in the U.S. because, typically, teachers either construct the auxiliary lines for their students or a hint is provided in the

textbook that helps students determine where this line should be drawn (Herbst & Brach, 2006).

However, in middle school in China, the strategy of constructing auxiliary lines in geometry problems has been emphasized. For instance, from the Chinese *Math Curriculum Standards for Geometry (2003)*, the students are taught that there are a series of postulates that are used to support the addition of an auxiliary line to a proof, such as:

- Two points determine one unique line or segment.
- Each angle has one unique angle bisector.
- Through a point not on a line (segment), only one line can be drawn parallel to the given line (segment). (Parallel Postulate)
- Through a point not on a line (segment), only one line can be drawn perpendicular to the given line (segment).
- Through a point not on a segment, only one line (segment) can be drawn to the midpoint of the given segment.

Let's see more clearly about these through our interviews.

3.2 Student Interview Problems

I chose two problems shown below, to test students' geometry knowledge. Both problems can be completed with the help of auxiliary lines and I also wanted to see students' ability to use such a strategy.

These questions are good practice for students' geometry skills which are supposed to be learned in high school. From my point of view, these problems aim to test several aspects in math. For Problem 1, students need to be familiar with the properties of special triangles, squares, similar triangles and other geometry objects, as well as knowing how to use these properties to help them to solve the problem. Problem 2 also tested certain math equations or laws, like the law of cosines. This question can be really relevant for both a geometry learner and a trigonometry learner. Lastly, which I think is the key to solving this problem, is whether the student can jump out from the box that we can only use something they have already provided instead of creating something of their own. Both problems are not straight forward problems, they need students to have some insights into the problems, especially when they didn't see these type of questions before. In that sense, these problems might be a challenge for the solver.

As I have noticed, students tend to rely too much on graphs diagrams??, which may not be a bad thing in Problem 1. Problem 2 is heavily relying on the construction of auxiliary lines, while Problem 1 can be also solved without constructing auxiliary lines. Therefore, compared with Problem 1, Problem 2 will be more challenging for a student at this level, which we will discuss later.

Interview Problems

Problem 1: The problem asked for the shaded area captured between oblique segments as illustrated in Figure 1. Our data consisted of; ABCD a square of side 2 inches, and G the midpoint of AB. The oblique segments were connected from each vertex to the midpoint of one non-adjacent side.

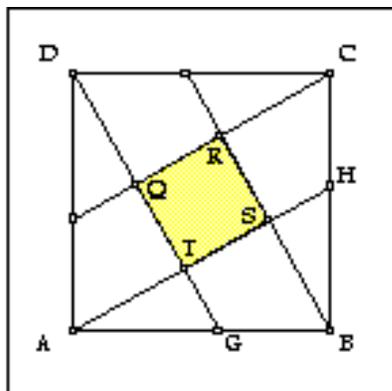


Figure 1. Interview Problem 1 Graph

Problem 2: Four roads form a square $ACDE$ with side length s . A barn B is 5 miles from A , 8 miles from C , and 13 miles from D . What is the length of a side of the square?

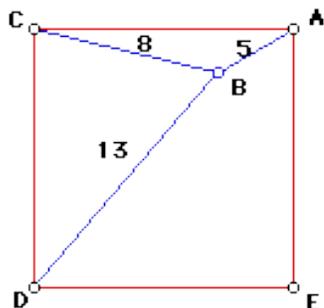


Figure 2. Interview Problem 2 Graph

Do you need to find the distance from the barn to the fourth corner of the square?

Suggested Solution to the Problems:

Problem 1:

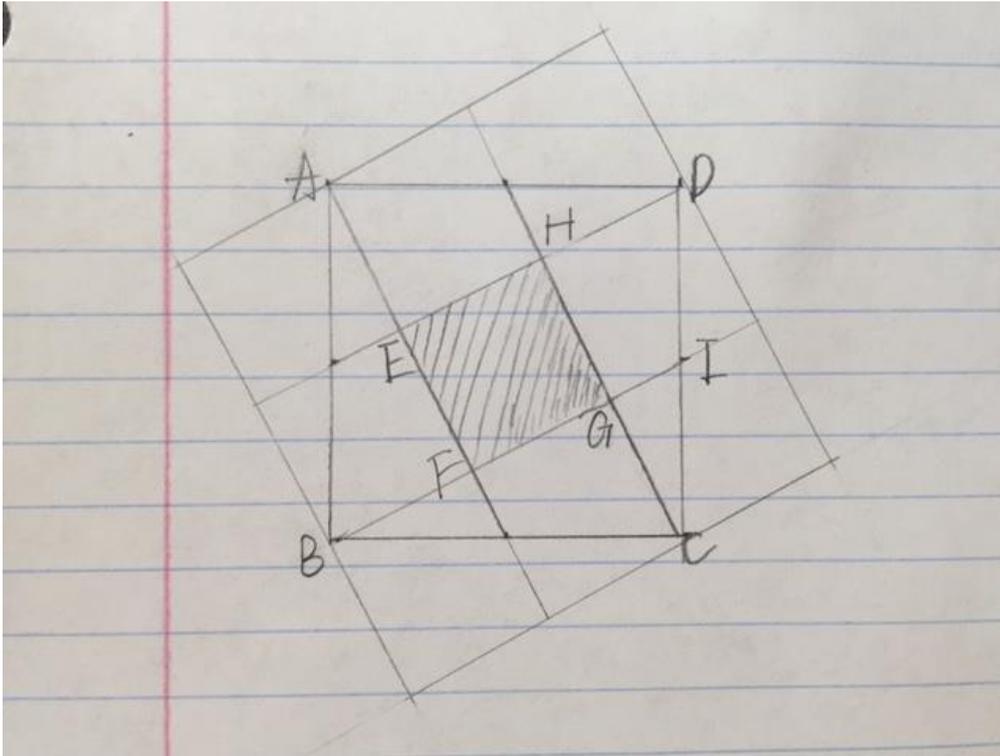


Figure 3. Suggested Solution for Problem 1

Solution 1: (Geometry approach)

Extended the square by drawing lines along and parallel to the segment within the square.

If a represent the area of the shaded region, then $9a$ represents the area of the circumscribing square with sides parallel to the shaded region. Each side of the original square has a right angle triangle, i.e. ΔAED , with area also a . Since there're 4 of them,

$$A_{\text{square } ABCD} = 4\Delta AED + \text{shaded} = 4a + a = 5a.$$

Thus, the ratio of the area of shaded square to original is $\frac{1}{5}$ (There're also other ways to look at it geometrically).

Solution 2: (particularize the data)

Let $BC=2b$, $CI=b$, so $BI= \sqrt{5}b$. BI is divided with ratio $2:2:1$. Then, $FG= \frac{2}{5}BI$.

Area of shaded square: $FG^2 = (\frac{2}{5} \cdot \sqrt{5}b)^2 = \frac{4}{5}b^2$

Area of ABCD: $BC^2 = 4b^2$

Thus, $\frac{FG^2}{BC^2} = \frac{4}{5}b^2 \cdot \frac{1}{4b^2} = \frac{1}{5}$

There are multiple ways of solving this problem. Students' first impression when they saw this problem could be to try to solve it geometrically by looking at the relationship between the areas. Just by looking at the original square and the shaded one, it's not that straightforward. They could extend the square to help them better see the relationships between the two. They can then circumscribe a larger square with sides parallel to the shaded square (as shown). With this, it's pretty clear that the ratio of area is $1/5$.

The other way is to particularize the value of each side of the original square by $2b$. Since the points are middle points of the sides, they could compute the ratio of the area by first comparing the ratio of the sides of each square. The ratio will also be $1/5$.

After two ways of solving the original problems, I was thinking what ways student may use to solve this problem. I thought some of them will start by counting the number of grid units in the squares. This approach might be easy for students to

understand, and it does become useful in this problem. Students may learn from this problem that their previous knowledge could work well if they know how to apply them correctly.

This problem does not require the use of auxiliary lines in order to be solved. But by using the similar triangle method, students need to highlight and identify which triangles are useful, and these are not given in the problem. Also, by using the grid square method, students need to complete the square by themselves.

Problem 2:

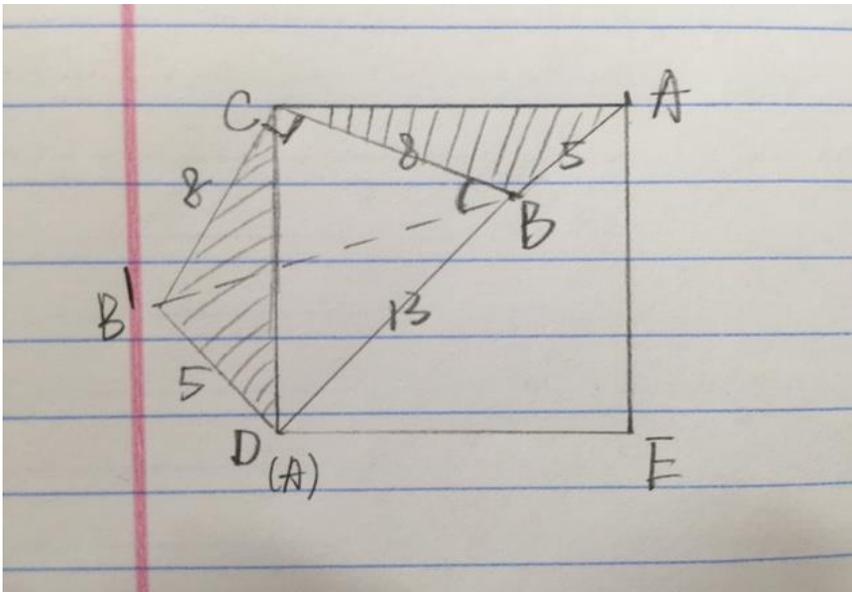


Figure 4. Suggested Solution for Problem 2

Rotating $\triangle CBA$ 90° to $\triangle CB'D$ and connecting BB' .

In $\triangle CB'B$, $\angle B'CB = 90^\circ$, $CB=CB'=8$, thus $BB' = 8\sqrt{2}$.

Now we know all the sides' length for $\triangle CB'B$.

By the law of cosines,

$$\begin{aligned}\cos(\angle B'BD) &= \frac{(8\sqrt{2})^2 + 13^2 - 5^2}{2 \cdot 8\sqrt{2} \cdot 13} \\ &= \frac{64 \times 2 + 169 - 25}{16\sqrt{2} \cdot 13} \\ &= \frac{272}{208\sqrt{2}} \approx 0.9247\end{aligned}$$

Therefore, $\angle B'BD = \cos^{-1}(0.9247) \approx 22.38^\circ$.

$$\begin{aligned}\text{In } \triangle CBD, \quad \angle CBD &= \angle CBB' + \angle B'BD \\ &= 45^\circ + 22.38^\circ \\ &= 67.38^\circ\end{aligned}$$

In $\triangle CBD$, we use the law of cosines again,

$$\begin{aligned}CD^2 &= CB^2 + BD^2 - 2CB \cdot BD \cos(\angle CBD) \\ &= 8^2 + 13^2 - 2 \cdot 8 \cdot 13 \cdot \cos(67.38^\circ) \\ &= 64 + 169 - 208 \times 0.385 \\ &= 153\end{aligned}$$

Therefore, $CD = \sqrt{153} \approx 12.37$.

When students first see this problem, they may start by drawing a picture and try to put all the given information together. Since the distance from B to each vertex is given, students could think about how to put the three known lengths into a same triangle. The next step is a bit harder. Students need to jump out of the box and think about how to connect to a point which is outside the square. This thought could bring them to a different lens of viewing this problem. They can always make something up to help them figure out what they want. The right way is to think about rotating one of the triangles 90 degrees. After doing that, they will realize that they have actually create a totally known triangle by connecting B to B'. Then use the sides of this known triangle to complete another triangle B'BD, which is the key for solving this problem. After applying the law of cosine twice, the problem is solved.

All these variations are very good practices for students' geometry skills. To my point of view, the problem aims to test several aspects in math. Students need to be familiar with the properties of special triangles, squares and other geometry objects, as well as knowing how to use these properties to help them to solve the problem. It also tested certain math equations or laws, like the law of cosine. Lastly, which I think is the key solving this problem, is whether the student can jump up from the box that they can only use something that has been already provided instead of creating something of their own. For students who may not be comfortable with this way of doing math problems, this problem might be a challenge for them.

Interviews

I interviewed eight students, all from the pre-calculus class at OSU that I am teaching. Some of them are freshman who are placed straight into this class, others are sophomores, juniors and seniors who need to complete certain math classes before taking this one. Based on that, we should assume that they will have similar foundation in math knowledge. In addition, they have all completed their high school, either in the U.S. or in China. Therefore, they are expected to have all the pre-requisite knowledge like the similar triangles and the Law of Cosines for solving these two problem. I intended to choose two students from each country that are above the average in class, two of them are around average, to make sure that it will be possible for them to solve the problems.

All the name used below are pseudo name.

American student, Betty

Betty is a college student in pre-calculus. This is her third semester at OSU. She was in the military for a while, so she said she finished her high school ten years ago. She was first placed in MA1075, and now is in my pre-calculus class. So far, she has been receiving A's on exams and quizzes. She told me that she plans to be a math major, then a school math teacher.

She was a bit worried in the beginning, and she kept asking me questions like “Do I need a calculator?”, “What type of questions are they going to be?”, “What if I can't solve any

of them?”, “I have been away from high school for too long.” I told her that it’s not important for her to solve the problem, I just want to know her process of thinking and problem solving, and how she thinks in each step.

She read problem 1 out loud. Her first reaction is, “Can we trust the graph?” She said teachers in her high school told them to not trust the pictures, sometimes it may not be correct from what you see. I said you can trust the graph this time.

Then she asked, “Can we assume the shaded area is a square?”

Y: Why do you make this assumption?

B: I don’t know how to prove it, but you told me I can trust the graph. It looks like.

Her intuition is that the shaded area is a square.

Then she told me that we can use the way we learned in class. When we want to find the shaded area, we can find the area of the big square, then subtract the unshaded area.

Y: Good thinking. How should we calculate the unshaded area?

B: I can get the value for DG by Pythagorean Theorem. Since it’s the middle point, one side is 1, one side is 2, and then the other one is $\sqrt{5}$ in triangle ADG. But that’s all the information we know.

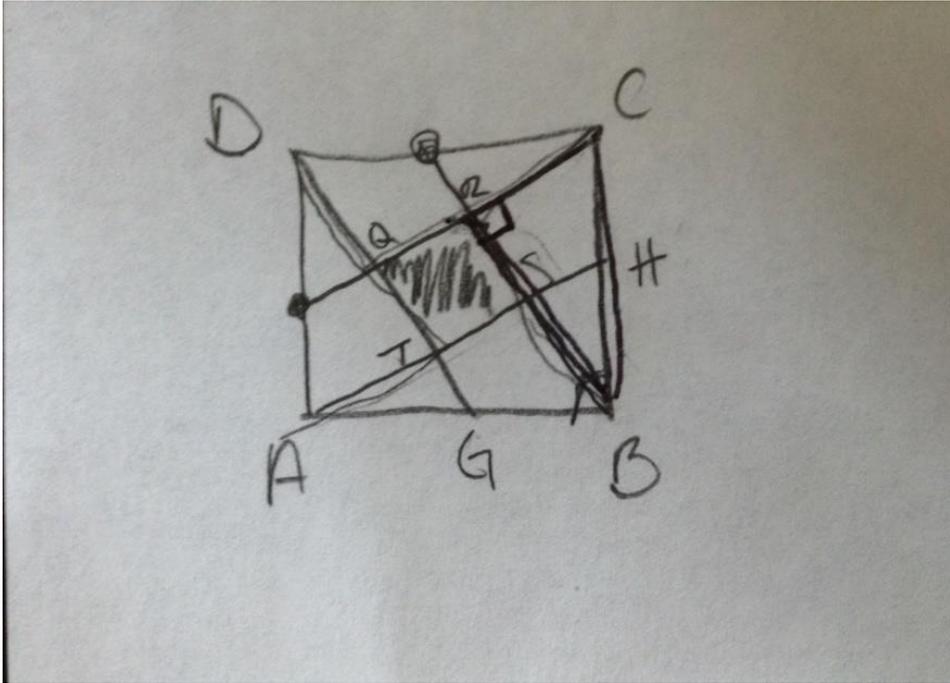


Figure 5. Student' Work

She stopped and thought for several minutes. Then there seems to be a sudden enlightenment, she said she knew what to do now.

B: We can find the area of one of the triangles, such as ADI, then the unshaded area is a sum of 4 of them. But how we figure out the other side AI and DI?

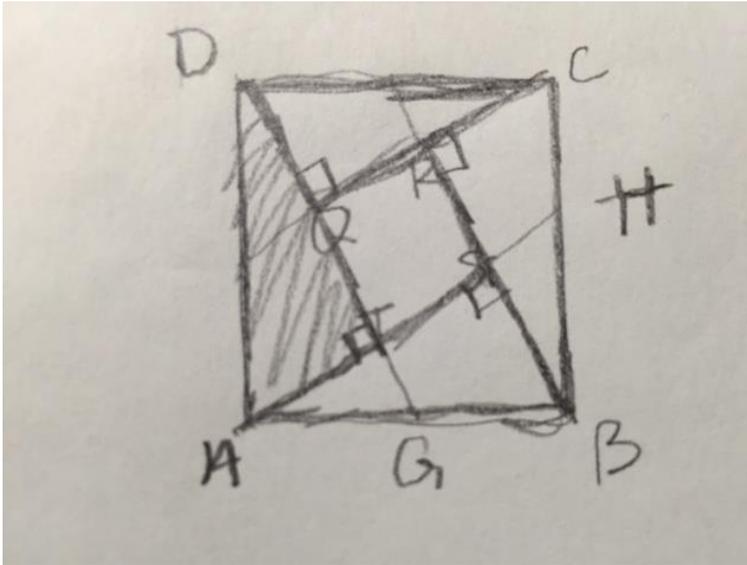


Figure 6. Student's Work

She soon got stuck again after this realization.

She started to draw out the graph again on her scratch paper, and tried to figure out how she can go further. For a minute or two, she gave up.

B: I don't think I can go any further. Can I have a hint?

Y: How about similar triangles? Can we find any similar triangle to determine the sides of the triangles you are looking for?

B: Similar Triangle? I learned it in high school, it has been long, I quite forget.

Y: We can use similar a triangle to determine the ratio of two triangles.

B: Sorry, I don't remember anything about that. I don't think I can go any further. Can we try another one?

Then we moved on to the second problem.

After explaining the problem, I asked her, "Do you think we need to find the distance from the barn to the fourth corner of the square in order to find the length of a side of the square?"

Her answer was pretty positive. She thought we already have enough information. We can solve the triangle, BCD or ABC, to find the other side, which is the side of the square. I was surprised by her quick reaction. She explained that she related this with the first problem we just did, which also required us to solve triangles.

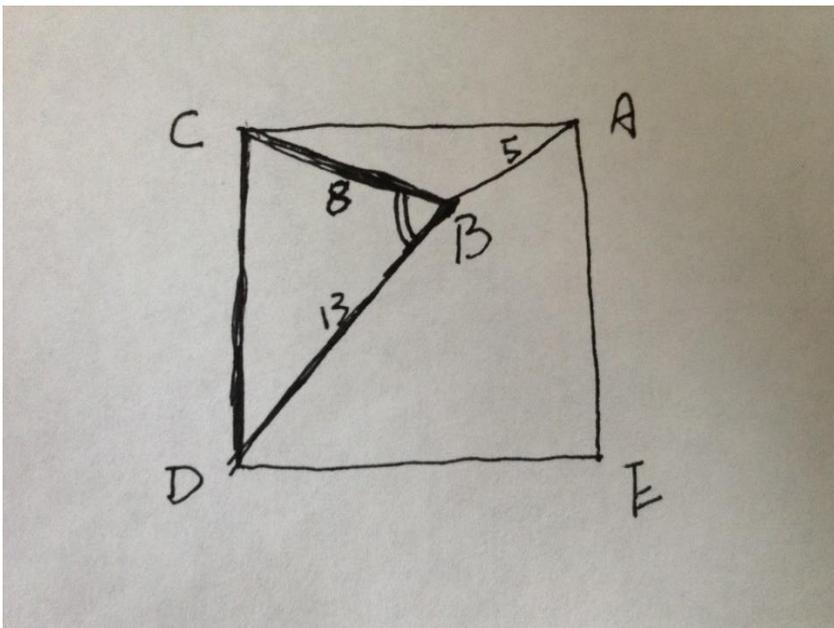


Figure 7. Student's Work

She highlighted triangle BCD and told me that we can use what we just learned in class to solve this, the law of cosine. She wrote out the formula for the law of cosine, but soon get stuck.

B: We are missing out one angle. We need to figure out what is angle CBD in order to find the side. How can we do that?

She thought for a while, then concluded that we are missing some information, we need to know the angle CBD in order to figure out the program.

Y: How about if I told you we have enough information?

B: Then I don't know what to do next.

She started to try other things.

B: But we can't do anything else, since AB and AD are not in the same line and none of the lines are angled bisector. This is so hard. I don't think I can figure it out.

She seems a bit tired after these two problems. We then finished up the interview.

Notes:

I realized she mastered well the knowledge that she has just learned, and was trying to apply the techniques and formulas that we did in class. However, some of her previous knowledge in geometry seems to be forgotten. That's why she soon realized she should use the law of cosine and went a bit further into the second problem compared with the first one.

American student, Mike

I can't really say much about Mike's performance in my class. He is a freshman, placed straight into pre-calculus. He participates well in class and always asks me questions after classes. He also comes to my office hours pretty often. It seems like he really tries to learn it, but it turned out that his grades for quizzes and midterms are not that good. I really would like to find a bit evidence about what is lacking in his mathematical thinking and problem solving skills.

He asked me once he saw the problem 1, "The shaded area is a square?"

Y: Why do you make this assumption?

M: I think so. Otherwise how can we calculate it?

Y: If we know it's a square, how can we find its area?

M: Find one side of the square? Is that half of the side length of the bigger square ABCD?

Y: Why do you think it is half the side length?

M: Because we connected each middle point, otherwise, what can it be?

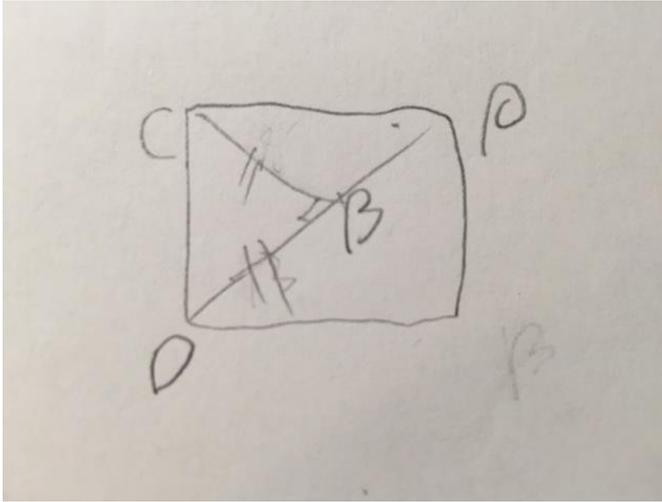


Figure 8. Student's Work

He continues, "So if the side length of the bigger square is 2, then the area is 4. Then the side of the square in the middle has area one."

M: I got it. The area is $\frac{1}{4}$.

Y: What if I told you this is not the right answer?

M: Really? How can it be? I am pretty sure it is right...

Then we moved on to the second problem.

He looked at the problem again, and told me, "This is one is easy."

I was a bit surprised by his reaction.

Y: Why do you say that?

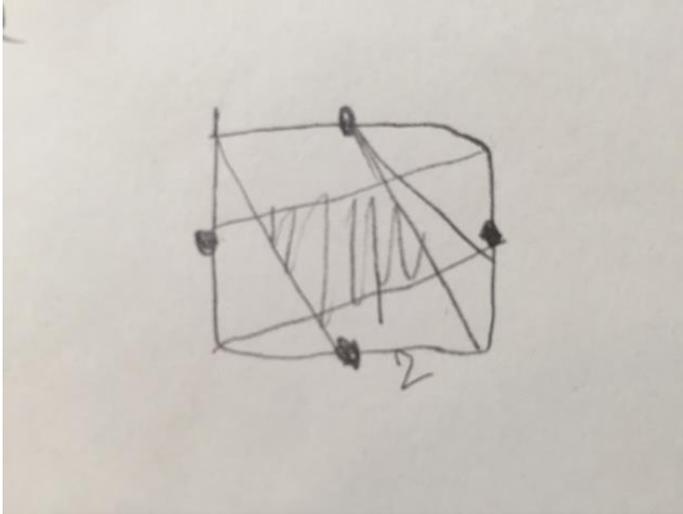


Figure 9. Student's Work

M: It's clear, in the right angle triangle BCD, we have two sides with lengths 8 and 13 given, so the other one can just be calculated by using Pythagorean Theorem, which is the side of the square.

Y: But unfortunately, angle BCD is not 90 degrees, we actually don't know how big it is. And one bit of information they give which you didn't get to use.

M: Really? Sometimes not all the information is going to get in used, they just want to fool you. If it is not 90 degrees...then I don't have a clue.

Note:

Mike tends to have a lot of assumptions which are not correct. These false assumption lead to his failure with these two problems.

American student, Emma

Emma is a sophomore student who had completed MATH 1148 and 1149 before taking the college level pre-calculus class. She didn't perform too well in the midterm and quizzes. She was surprised when I asked her for an interview on two math problems.

"I should be the last one you want to ask for that," she says, "you will be disappointed."

I told her that don't worry, this is just for my own study, I am more interested in how you think instead of whether you will be able to solve the problem.

We started with problem 1. She looked at the problem for a minute, and turned to me, "I don't know how to do it."

Y: Did you think about it? How you know you can't do it?

E: I have never seen a problem like this. I can only do problems which teachers already taught, like a similar one. I can't come up with something by myself.

Y: So is there anything in this problem that looks a bit familiar to you?

E: Let me see...It looks a bit like the grid square we used in high school. Like just counting how many grid squares are there. Maybe I can count how many grid squares are there inside the bigger one, then we can find the ratio of the bigger square and smaller square?

Y: Really good! See, you can actually do more than you think.

E: By counting the grid squares, I think there are 5 smaller square in total inside the bigger square. So if we take the area of the bigger one as 5 and the shaded one as 1, then the ratio is just $1/5$?

Y: Good! You got it.

I think she was a bit motivated, we then moved on to the second problem.

Again, she looked at the problem for a while, and told me, “I am not kidding, but this one is really hard.”

Y: Why do you say that?

E: I don't even think we have enough information, how can I figure it out?

Y: What do you mean?

E: By just giving this information, I can't figure out the sides of the square. We never learned something like this in high school or even college.

Y: Can you put all the information together and go from there?

E: Sorry...I don't want to think about it anymore, math is killing me...

Note:

Emma didn't write down much in her thinking process. She doesn't seem to like to push herself too much in the whole math thinking process.

American student, Kevin

Kevin, is a freshman who placed straight into this class without taking any requisite. He did well in the exams and quizzes, and always answers my questions in class. Kevin seems really calm when we start our interview.

I told him the same as I told to all the other students. I just want to know his process of thinking and problem solving, and how he thinks in each step.

After handing out the problem to him, Kevin was silent for a while. He then started to draw out the graph on the paper.

K: I think the shaded area is also a square.

Y: Why did you make that assumption?

K: Because each opposing sides is parallel.

Y: Really good. So then what do we need to find the area?

K: Generally, when we want to find the area of the square, we need to find at least one side. How can we find that? Let me think.

He had really clear thinking. Then he kept silent for a few minutes. And started to highlight the right angle triangles inside the bigger square.

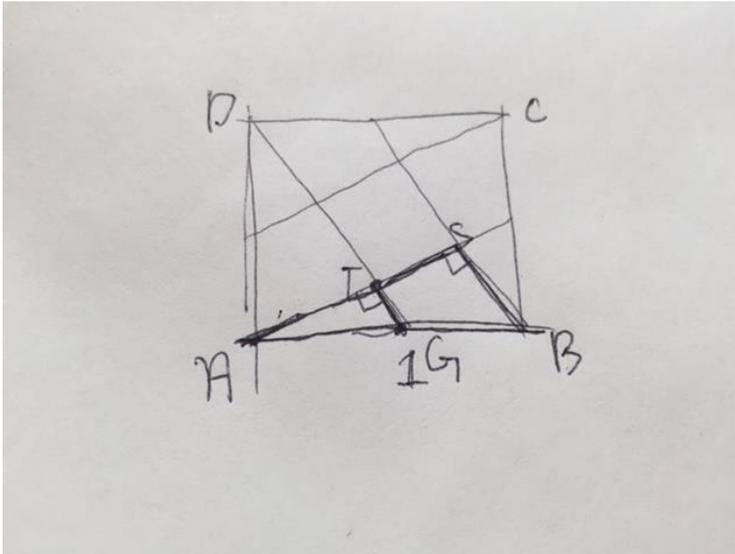


Figure 10. Student's Work

Y: Can you tell me what you are doing now?

K: I am thinking about using similar triangles. Use the ratio of sides on similar triangles to determine the value of the side we are looking for.

Y: Great! But we don't know the value of the sides...

K: Can we just assign some values...like the sides of the bigger square is all 1...

Y: We can do that! The ratio will still be the same.

K: Then I know how to do it now!

Kevin went on writing out the equations and finally almost solved the problem. His result was still not there yet, but very close. As we can see from Figure 11, he just need to go one more step and figure out the relationship between IS and IG. Then he will be able to solve the problem.

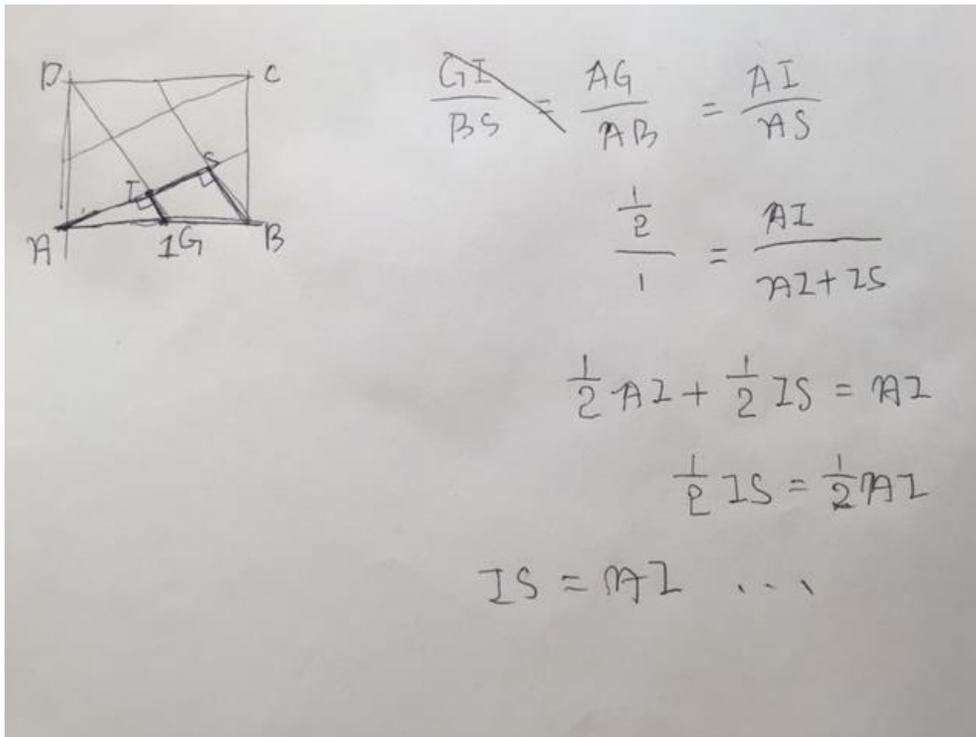


Figure 11. Student's Work

Then we moved on to the second problem.

In the second problem, I asked him, the same as others, "Do you think we need to find the distance from the barn to the fourth corner of the square in order to find the length of a side of the square?"

K: We can't make a certain conclusion at this stage. I will try the problem first and tell you whether we need it or not.

He then thought for a while, and said.

K: Can we solve one of the triangles, BCD or ABC, to find the unknown side, which is the side of the square?

Y: Good thinking. How?

K: For each of the triangles, we only know two sides, no angles are given, so we can't use the Law of Sines...How about the Law of Cosines? But we are still missing an angle...If that's all the information we have, this one seems to be a hard problem. I probably need to think a bit more.

He tried a different thing inside the squares, like connecting BE, finding right angle triangles...but still can't get any further.

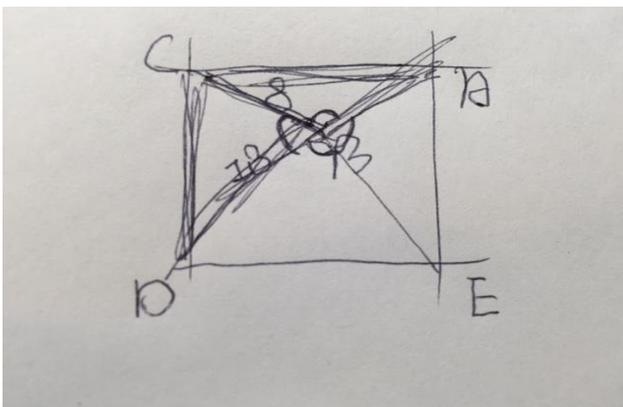


Figure 12. Student's Work

Y: Have you ever thought about adding some auxiliary elements?

K: Auxiliary elements? Like extra lines? Do we need to do this? We never used it in high school, and my high school teacher never taught us how to do it. If you tell me how to do it, maybe I will figure it out tomorrow.

Y: It's ok, you already did a great job.

Notes:

Kevin almost solved the first problem. Among all the four American students I have interviewed, he turned to be the best one in solving these two problems. But still, he couldn't figure out the second problem with all the existing information.

Now, let's compare them with the Chinese students.

Chinese student, Ping

Ping is a Chinese student who was also placed into pre-calculus in my class. He just came to the U.S. this year. All his previous education had been done in China. He asked me in the beginning whether he could tell me what he is thinking in Chinese, to reduce the nervousness. Therefore, some of the conversation below is a translation from Chinese.

Ping is one of the best students in our class. He told me that he was always confident with the course material because he learned most of the things in high school in China already. But he wasn't sure what type of geometry problems I would ask him about.

When he was ready, we started with the first problem. He read the question silently, then asked me whether he could draw auxiliary lines to the graph.

Y: Why do you want to create auxiliary lines?

P: I remembered in middle school, we were always taught to try to draw auxiliary lines if possible to help solving geometry problems.

Y: So which auxiliary line do you want to draw?

P: I want to draw something to cut the big squares into small squares which all equal to the one we are looking for.

He was then silent for a while. He didn't write down anything for a minute or so. I asked him what he was thinking. He said he couldn't find a way to draw the auxiliary lines which would help him to accomplish his goal. **He then switched his problem solving strategy.** "Maybe I should just use the normal way of using the similar triangle."

He then went on smoothly with the similar triangle method and solved the problem.

We moved on to problem 2. After explaining the problem, I asked him the same problem I asked Betty, "Do you think we need to find the distance from the barn to the fourth corner of the square in order to find the length of a side of the square?"

He had a totally different response. "Maybe we should connect B with E." He told me, that's the only vertex which is not connected with P. He connected BE as the auxiliary line.

Then, he went on trying other ways of drawing auxiliary lines.

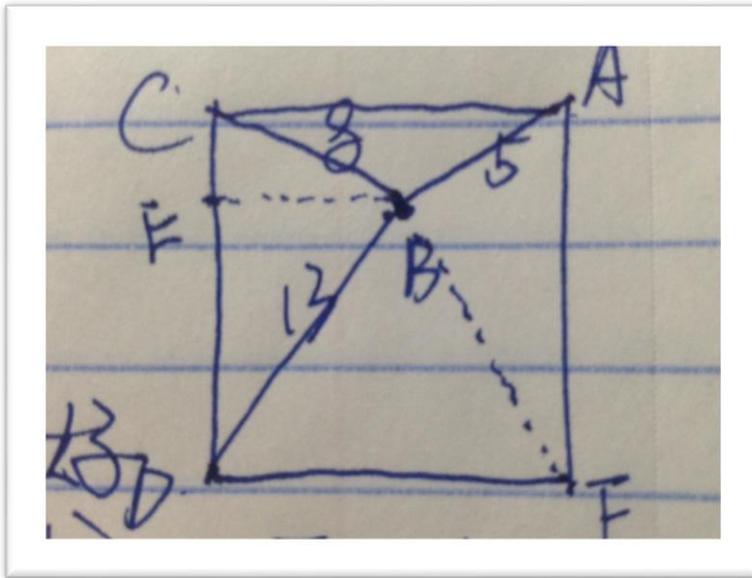


Figure 13. Student's Work

Y: What are you trying to do now?

P: I want to solve for triangle BCD, but I only know two sides and it's not a special triangle. I was thinking about creating the height to see whether it helps, but it actually does not. How can I solve this triangle with only two sides? Is there any information missing?

Y: How about using the Law of Cosines?

For problem 2, I was surprised to hear that he never learned the law of cosines in high school. He also skipped the lecture and recitation when we were talking about the law of cosines. Therefore, this was where he became stuck.

He tried multiple other ways without using the law of cosines, but couldn't get anywhere.

We then ended up the problem.

Notes:

Compared with Betty, Ping seems to be really quiet while doing problems. I asked him to tell me what he was thinking for each step. Then he told me, he couldn't keep thinking properly while speaking to me at the same time.

Chinese student, Zhao

Zhao is a Chinese student who was also placed straight into pre-calculus in my class. But he completed his high school studies at California. Before high school, all his previous education had been done in China. Zhao worked hard in our class, but still didn't do well especially for the first midterm. He told me once, people always say that Chinese students are good at math, but he is an exception.

We start with problem 1. He thought for a while, and asked me.

Z: I feel like something is missing, they didn't tell us anything about the sides of the bigger square, we don't even know the area of the bigger one, how can we figure out the smaller one?

Y: What if I told you we had enough information.

Z: Oh, I think I have an idea. I remembered in high school we used something called the grid square. We just need to count how many grid squares are there inside the bigger one, then we will know the ratio of the bigger square and smaller square.

Y: Good! Did you learn this in high school in the U.S.?

Z: Yeah, I think this really makes a lot of sense to me. I was really bad at math while I was in China, the teachers always talking about making auxiliary lines, adding elements and those problem solving strategies, but that never made sense to me.

Y: Your information is really important here. Thanks!

But after a while of solving this problem by his way, he only gave me a guess of the answer.

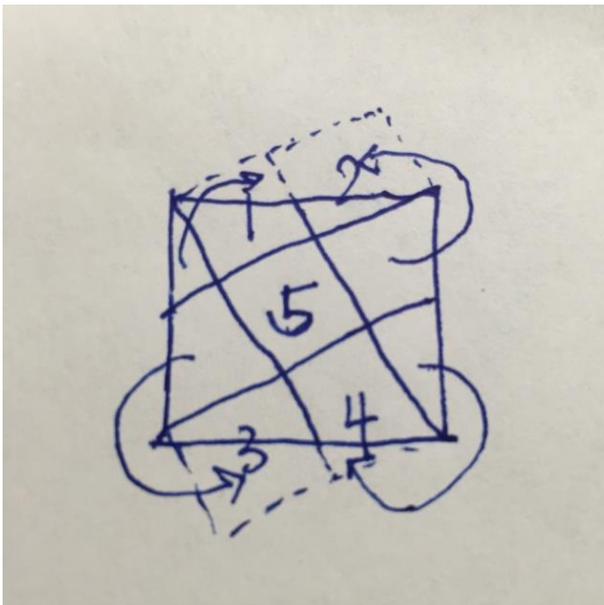


Figure 14. Student's Work

Z: I know it's $\frac{1}{5}$ by counting. Just don't want to write everything down.

We moved on to problem 2.

I asked him the same problem I asked everyone else, "Do you think we need to find the distance from the barn to the fourth corner of the square in order to find the length of a side of the square?"

"It could be", he says, "That's the only vertices which is not connected with B".

Z: But what has that to do with solving this problem?

Y: So what do you think we should do in order to solve this problem?

Z: Putting things in a triangle and solve it? But we only know two sides in each triangle, without knowing any angles...I don't know. I don't think I can solve this one.

Note:

I gave him a paper to write down what he thought and to work on graphs if he needed.

But after the interview, his paper was not much fulfilled.

Chinese student, Xinyue

Xinyue is a freshman who was placed straight into pre-calculus. She also just came to the U.S. this year. All her previous education had been done in China. She worked really

hard, and always asked me questions after class. She is kind of a bit above the average in class.

After reading the first problem, she asked me whether we know the length of the side of the bigger square ABCD.

Y: It's not provided, but you can always assign something by yourself, since we just need to find the ratio in the end.

X: I see. I remember this problem solving strategy which I learned in China. I think I can use a similar triangle now.

Y: Great! Keep going.

X: We did so many similar triangles in middle school, so I think I can do this.

She wrote out all her work on the paper, and almost solved the problem. But as we can see from her work, she got stuck after she found the relationship between one side of the smaller triangle and the square. She didn't realize that since all the sides of the square are equal, this can be used to determine the relationship of HS and AH also.

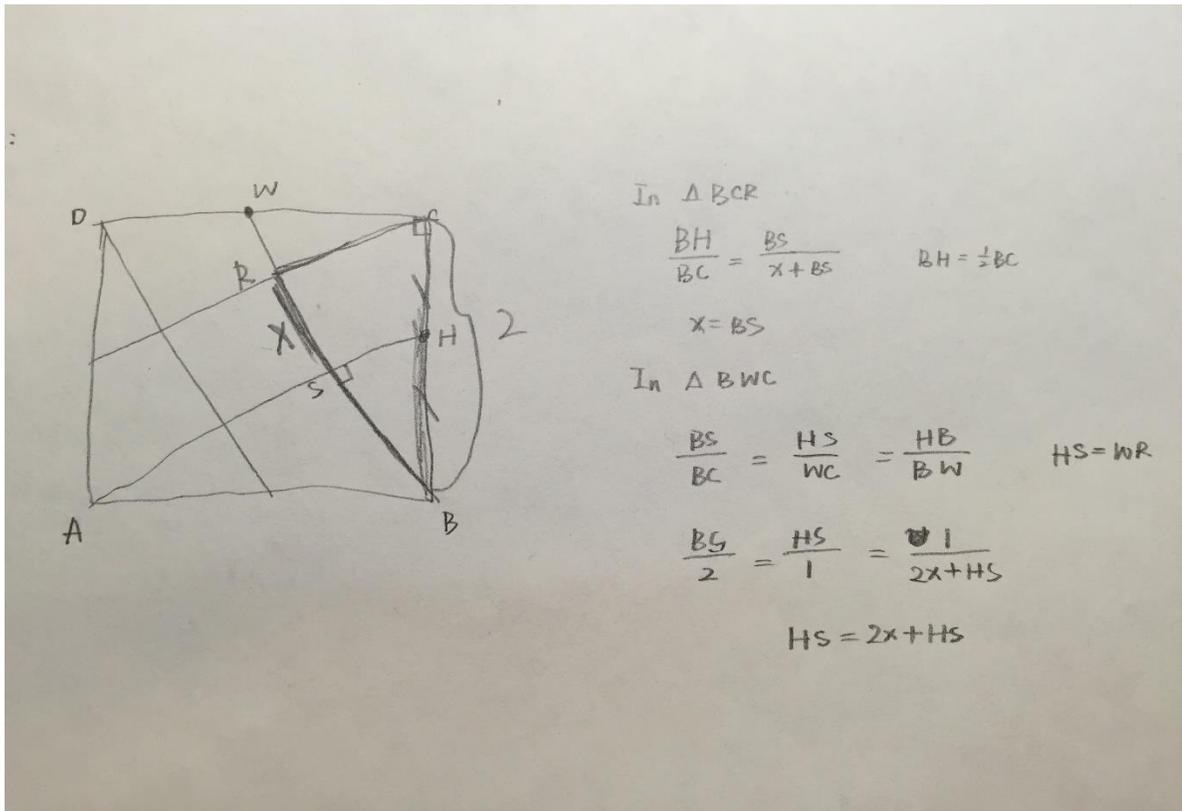


Figure 15. Student's Work

We then moved on to problem 2. After explaining the problem, I also asked her the same problem, "Do you think we need to find the distance from the barn to the fourth corner of the square in order to find the length of a side of the square?"

X: Yeah. Since it's the only vertex which is not connected to B.

Y: But does that help you to solve the problem? Is that because you always did that in middle school?

X: Yeah, teachers always told us to complete the graph first. Let me look at it carefully first without connecting BE.

Xinyue switched her strategy by not connecting BE first.

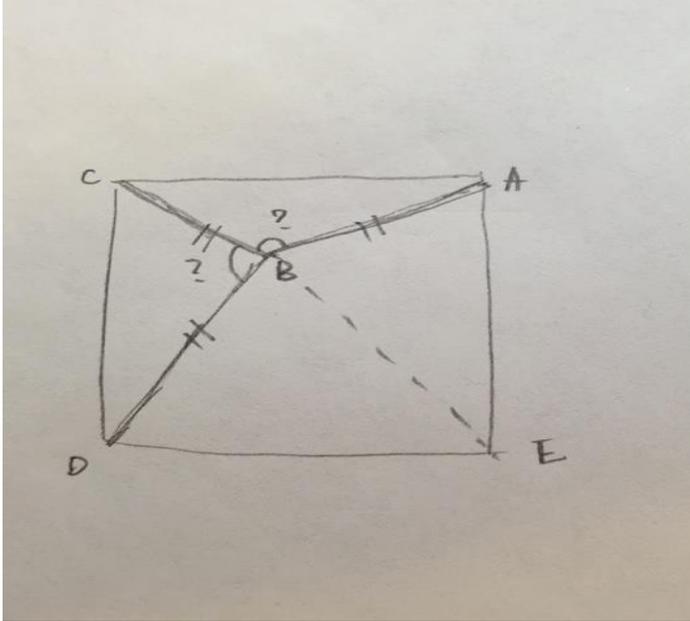


Figure 16. Student's Work

X: Without connecting BE. I have two triangles with only two sides. You taught us in class that in that case, we can use the law of cosines. But we are still missing an angle...I don't think we can solve it based on all the information already provided. Maybe I need to create something by myself.

Y: Do you mean drawing auxiliary lines?

X: Yes, I remembered doing auxiliary lines problems in middle school, but I kind of forget how to do it with a problem like this....we were always dealing with triangles....

Y: Do you think you can go any further?

X: Probably not.

Note:

Xinyue actually did a good job in solving these two problems, even though she didn't solve problem 2. She has also brought some interesting thinking here.

Chinese student, Jinan

Jinan is a sophomore who was placed straight into pre-calculus. But for some reason, she waited to take this class until this semester. All her previous education had also been done in China. Jinan is also one of the best students in our class, and she always gets one of the highest scores for quizzes and midterms. She also told me that she was always confident with the materials because she learned most of it in high school in China already.

When she was ready, we started with the first problem. She read the question silently, then started to draw auxiliary lines on the graph.

Y: Why do you do that?

J: There are four equal triangles in the bigger square. I was thinking maybe I can put them together in order to figure out the unshaded area. Once I have that, the shaded area is just the whole square minus the unshaded one.

Jinan was then silent for a while, she then told me, "Maybe it's not even necessary."

She then switched her problem solving strategies.

J: I over thought about it. We can just use the similar triangle to find the length of the sides of the shaded square, by letting the side of the bigger square be 2. Since the points are middle points of the sides, I could compute the ratio of the area by first comparing the ratio of the sides of each square.

Jinan went ahead and solved this problem.

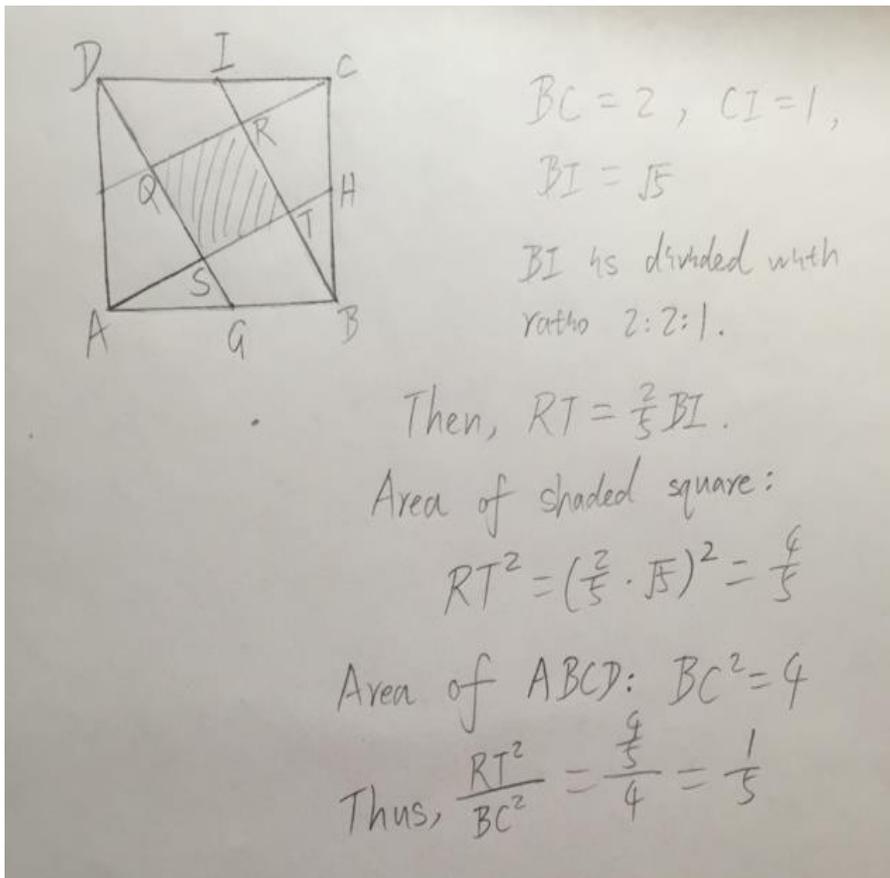


Figure 17. Student's Work

For the second problem, her first action was also to connect BE.

Y: Why do you do that?

J: Since that's the only vertex which is not connected to B...I have the impression that whenever the graph is not complete, we need to first complete it by adding auxiliary lines. That what we always do in middle school math classes in China.

Y: But do you think this will help?

J: Maybe not. But the information seems not complete, always in these cases, they told us to create auxiliary lines by ourselves to help solve the problem.

Y: Here "they" means your previous Chinese teachers?

J: Yeah, I remembered doing a lot of auxiliary line problems, but I actually don't remember doing one like this. I was thinking about drawing some auxiliary lines to put all the three lengths, 5, 8, 13 together...It has been long, I really can't remember.

She then started to try out all different possibilities of adding auxiliary lines, to keep searching for ways to solve this problem.

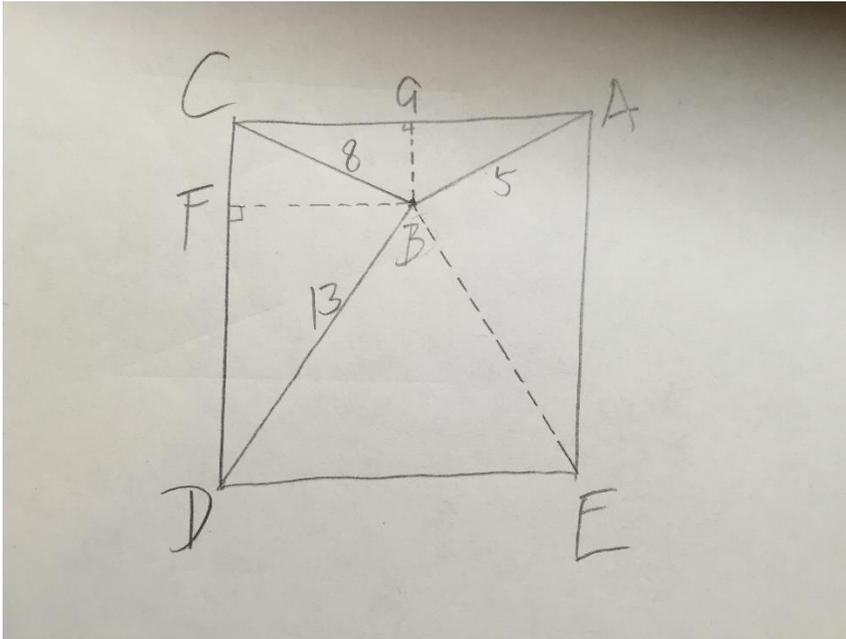


Figure 18. Student's Work

Y: Whenever you can't solve something with the first ways you used, you always turn to use some other ways?

J: Yeah, that's what I was trained I guess. I have a feeling that in high school and middle school math classes in China, in order to successes, you have to know all different ways of solving just a single problem.

Y: Very good! Have you thought about rotating triangle ABC around C until side CA touches side CD?

She thought for a while and said, "I see what you are saying, this is so smart."

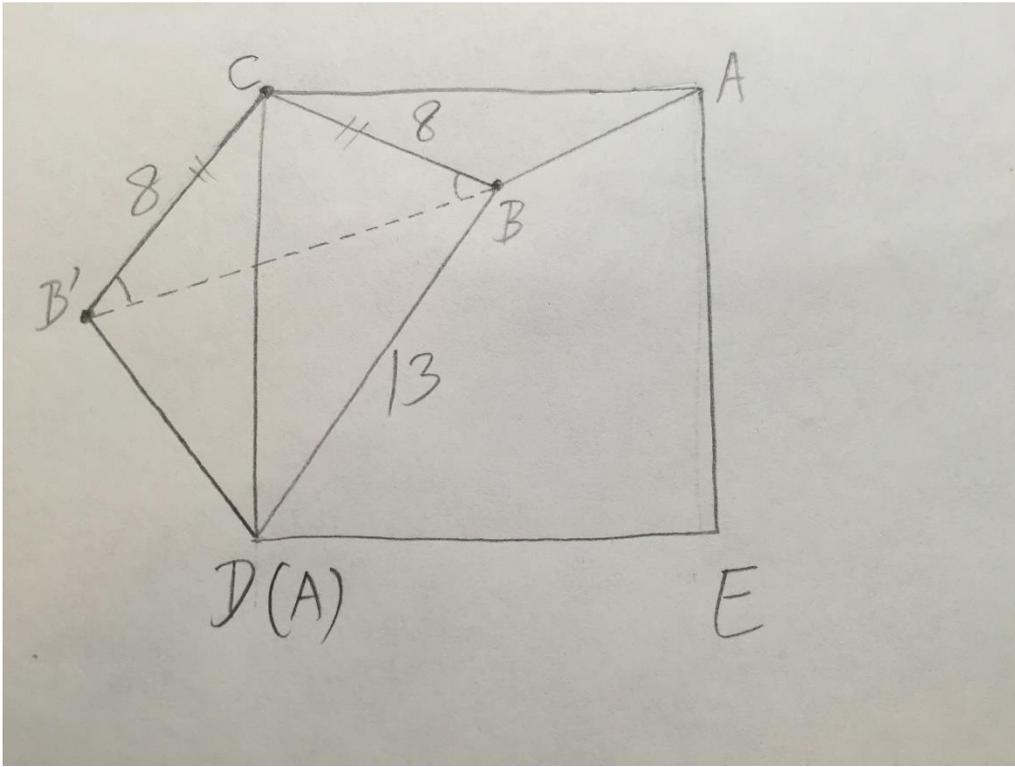


Figure 19. Student's Work

She then solved this problem by using the Law of Cosines.

Note:

I did notice a lot from the interview of Jinan. Some of her ways of solving problems supported some of my assumptions about Chinese students, which we will be talking about in the next section.

Interview Result Summary:

US Students

Student Name	Problem 1 correct or incorrect (1 or 0)	Problem 1 Hints Given	Problem 2 correct or incorrect (1 or 0)	Problem 2 Hints Given
Betty	0	Similar triangle	0	We have enough information
Mike	0	None	0	Angle BCD is not 90 degree
Emma	1	Think about something learned before	0	We have enough information
Kevin	1	None	0	Adding auxiliary lines
<i>Total</i>	2		0	

Table 2. Summary of U.S. students' interview

Chinese Students

Student Name	Problem 1 correct or incorrect (1 or 0)	Problem 1 Hints Given	Problem 2 correct or incorrect (1 or 0)	Problem 2 Hints Given
Ping	1	None	0	The Law of Cosines
Zhao	1	None	0	None
Xinyue	1	None	0	Connecting BE is not the right strategy here
Jinan	1	None	1	Rotating triangle ABC around C 90 degrees
<i>Total</i>	4		1	

Table 3. Summary of Chinese students' interview

From the table above, seems the most useful hint for problem 1 is either to use the similar triangle or grid squares. And the most useful hint for problem 2 is telling them how to construct the auxiliary lines for solving the problem. For the once who don't have hint given, either he or she could already been able to solve the problem by themselves or they are pretty certain about their wrong answer by holding some misconception. Only Jinan get the hint about the right angle triangle, since she was very close in thinking about all different ways of creating auxiliary lines.

Comparing the results we see here, from the scores, Chinese students did better both in Problem 1 and Problem 2. All Chinese students interviewed has successfully solved Problem 1 compared with only half of the American students who solved the same problem. Almost all the Chinese student interviewed thought about creating auxiliary

lines for Problem 2. And three of them actually used auxiliary lines even though only one of them successfully solved the problem. Compare with Chinese students, almost none of the American students has used or thinking about creating auxiliary lines for the same problem.

Even though Problem 1 doesn't require students to make auxiliary lines—they can solve it using similar triangles—they still need to highlight and identify which similar triangles are useful for them to determine the ratio of the sides of square. In general, for those students who did solve the problem, they drew and highlighted things on the diagram. In that sense, drawing anything on the diagram should help students to figure out the problem better.

3.3 Discussion and Analysis

As we can see from the interview, some of the students' formal geometry knowledge was not fully developed. In "Mathematical problem solving" (1985), Schoenfeld points out that students often can only partially recall relevant details which limits their problem solving processes. For example, in problem 2, Betty soon recalled the Law of Cosines, and believed that it could be used with a correct procedure for trying to figure out the side length by first figuring out the angle. However, how to deal with two unknowns in the same equation became her main problem. There is also the possibility that a student's previous knowledge in certain area of mathematics is missing. As in problem 1, even after I provided a hint for Betty about using a similar triangle, her

lack of knowledge about similar triangles resulted in her not going any further into the problem. Her assumption about not remembering high school geometry was also a factor. This will also apply to the Chinese student, Ping, when his knowledge of the Law of Cosines was missing. Even so, maybe students know how to solve a straight forward problem using the similar triangles and the Law of Cosines, but that doesn't mean they can take these ideas into problem solving. In that sense, they don't really understand the similar triangle and the Law of Cosines conceptually.

However, most students I interviewed had a pretty good sense of referring back to previous problems and knowledge he or she had learned. As Silver (1985) pointed out, exploiting a related problem is sometimes useful. I have noticed that, especially with Betty, when she was solving problems, she always related back to similar problems that either she had previously learned or just did in class. In problem 1, she assumed the shaded area was a square based on her intuition. She then related back to a "similar" problem that was done in the class, using the method that if one wants to find the shaded area, one can find the area of the larger one, then subtract the unshaded one. In problem 2, she related back to problem 1 which the class learned, and assumed that she could use the same strategies of solving triangles. Then she minimized her focus on just solving one triangle through figuring out one angle, then realized she can apply what she just learned in class, the Laws of Cosines. This can also be seen with many other students in the above interviews

For all of the students, problem 2 seemed more challenging than problem 1. There could be a lot of factors on that since they are testing different pieces of knowledges. But

both problems don't seem straight forward to most students. This should not be surprising since both problems require a more conceptual understanding. Besides these similarities, there were also many differences evidents in their problem solving processes, which will be discussed in the following paragraph. The disparity in the U.S. and Chinese students' problem solving success may be related to their use of different strategies.

Compared with the American student, the Chinese students I interviewed had a better knowledge of constructing auxiliary lines in a geometry problem for help in solving problems. Almost all of them told me that they had learned various ways of making auxiliary lines in middle school and high school classes in China. As Cai (2007) pointed out, Chinese students are more likely to use generalized strategies and symbolic representations like the strategy of making auxiliary lines for certain conditions and certain type of math problems. Since math education in China puts a heavy emphasis on mathematical methodology, students are required to master problem-solving methods through repetitive learning. This is due to the "Exam culture" in China as we have mentioned in Chapter 2.

China is a country in which, to a great extent, the scores on exams can determine one's opportunity for additional education and even further careers. Many parents believe that obtaining higher scores on exams means being intellectually elite. Therefore, one of the main goals of classroom instruction is to prepare for examinations and to ensure high scores in exams. Many teachers require students to memorize specific techniques for solving different problems, so that students can quickly recognize the types of problems and solve them immediately (Cai, 2007).

But sometimes this leads to overuse of a strategy like making unnecessary auxiliary lines. Take problem 2 as an example; the first thought of most of the Chinese students interviewed was to connect B and E, because he or she felt that all other vertices had been connected to the point B in the middle. However, this didn't help to solve the problem.

On the other hand, the American students did not have a strong sense of making auxiliary lines. From all the American students I have interviewed, none of them even mention the word “auxiliary”. Some of them told me that creating auxiliary lines in solving geometry problems is rarely used, and it is only seen in a really advanced high school math class. American teachers tend to not put enough emphasis on this.

In addition, the Chinese students I interviewed seemed to be more comfortable switching between different problem solving strategies. The students told me that in their middle and high school classrooms, teachers always encourage them to solve one problem with multiple ways. According to Cai, textbooks and teacher reference books in China contain worked examples showing how a problem can be solved using different strategies and representations (2000). In that sense, Chinese students are more successful in applying heuristics according to Poyla (1973).

The American students I interviewed tended to stick with a single strategy even when they got stuck with that strategy. Limiting thinking in only one way without trying other possibilities or strategies might result in their incomplete solutions to both of the problems. According to Cai, students tend to hold the misconception that there is only

one way to approach and solve a problem, which leads to the failure to develop the flexibility to change to different strategies (2000). This can be largely due to the lack of practice in trying to use different ways for solving one problem in the classroom environment.

Students' performance on problems could also be a reflection of the teaching they previously had. Chinese interviewees did mention that that was the way they learned from their Chinese teachers. Through analyzing Chinese middle school teachers' lessons on triangles, Wang (2006) also found that Chinese teachers were able to help students build connections between existing problems and new problems. They will then develop flexible connections between mathematics concepts and multiple solutions.

3.4 Evaluation

The interview showed that the Chinese students were more successful than the US students in creating auxiliary elements for helping them solve geometry problems as well as more comfortable switching to different problem solving strategies. This may be due to the "Exam culture" in China. Mathematics education in China puts a heavy emphasis on mathematical methodology, and students are required to master problem-solving methods through repetitive learning. Thus, Chinese students are more likely to use generalized strategies and symbolic representations.

On the other hand, in U.S. classrooms, the strategy of creating auxiliary elements and solving a problem with multiple ways is not heavily emphasized. Students tend to

hold the misconception that there is only one way to approach and solve a problem, which leads to the failure to develop flexibility to change to different strategies

As college students who passed the college entrance exam and placed into pre-calculus, the interviewed students should have known more than enough geometry concepts to solve the two problems. However, as Schoenfeld (1985) states, “the issue is not what the student knew, it is how they used that knowledge or failed to use it” (32). Both similar triangles and the Law of Cosines were taught in the interviewees' previous math education, but they still didn't use the knowledge conceptually to help them solving the problem.

Schoenfeld (1985) argues that individuals will develop a certain problem-solving strategy through math problems, and this strategy is somewhat uniform. For instance, the interviewee Betty tended to always relate back to previous similar problems, to use this kind of strategy she already “built” instead of the strategy to create some auxiliary element; while the interviewee Ping and Jinan tended to always try to create auxiliary lines in the graph to help solve the problem. I personally believed this kind of uniformity will depend on the type of math education that each individual had. For instance, the lack of emphasis about using auxiliary element in secondary education here in the U.S. influences the students' strategies when solving this type of geometry problem.

In general, most students from both countries which I interviewed had a limited knowledge about how to solve geometry problems. However, some of them were still unable to use what they had learned in solving the experimental problems. Students'

interviews illustrated that they did not understand geometry conceptually. Students remembered what we have done in class, therefore, they could do a similar type of problem. On the classroom test in our pre-calculus class, a lot of them were actually able to use the Laws of Cosines solving straightforward problems. This shows that they knew the process of doing a specific problem once they have been taught in that way. While solving problems, a lot of them relied too much on their memorization. It could cause misconceptions and mistakes once their memories for the specific concept is incomplete or incorrect. Also, students' ability of building connections between different concepts is lacking. Like we can see in the above interviews, a lot of the students who got interviewed had a brief idea of what to do and some level of understanding of the related concepts. However, their main difficulty is figuring out when to use these concepts, understanding how to apply and put them together. For instance, even if all of the Chinese students mentioned the strategy of making auxiliary lines, for most of them, the understanding towards this strategy is still limited.

Since this study used only two geometry problems with a limited number of interviewees, caution must be taken in interpreting the results regarding students' problem solving skills in geometry. Further research will help develop our understanding of the differences between students' procedural and conceptual understanding of geometry in helping develop their problem solving skills.

Chapter 4. Conclusion

From the literature reviews and the interviews in this study, the Chinese students were more successful than the US students in creating auxiliary elements for helping them solve geometry problems as well as more comfortable switching to different problem solving strategies. This better performance in solving geometry problems may be due to Chinese teachers' ability to apply their geometric conceptualizations flexibly in the classrooms because of their experience solving problems using different strategies. Chinese interviewees did mention that this was the way they learned from their Chinese teachers. In comparison with China, from our reviews in Chapter 1 and 2, U.S. curriculum materials are less focused and American teachers don't emphasis enough on problem solving strategies like creating auxiliary lines in geometry problems.

However, the better performance of Chinese students in solving geometry problems can't be attributed solely to the Chinese instruction methods. As what we have discussed in Chapter 1 and 2, several non-school-related factors are likely to also contribute to Chinese students' math performance. Those factors include things as students' self-expectation and efforts, family pressure and cultural value. Chinese students tend to get more pressure on study math from not only the society, but also their family and themselves; therefore, they put more efforts in studying mathematics compare with their American peers.

This kind of research will be useful to develop a deeper understanding of why Chinese students perform better in solving geometry problems compared with their American peers and the factors which contribute to their performances.

References

Cai, Jinfan (2000). "Understanding and Representing the Arithmetic Averaging Algorithm: an Analysis and Comparison of Us and Chinese Students' Responses." *International Journal of Mathematical Education in Science and Technology*. 31.6: 839-855.

Cai, Jinfan, and Bikai Nie (2007). "Problem Solving in Chinese Mathematics Education: Research and Practice." *Zdm : the International Journal on Mathematics Education*. 39: 459-473.

Cai, J., Lin, F., & Fan, L. (2004). How do Chinese learn mathematics? Some evidence based insights and needed directions. In L. Fan, N. Y. Wong, J. Cai, & S. Li (Eds.), *How Chinese learn mathematics: Perspectives from insiders* (pp. 535-554). Singapore: World Scientific.

Chen, Huabin (2001) Parents' Attitudes and Expectations Regarding Science Education: Comparisons among American, Chinese-American, and Chinese Families Adolescence v36 n142 p305-13

Ding, L. and Jones, K. (2006), Teaching geometry in lower secondary school in Shanghai, China. *Proceedings of the British Society for Research into Learning Mathematics*, 26(1), 41-46.

Dong, Ting, Richard C. Anderson, Il-Hee Kim, and Yuan Li. "Collaborative Reasoning in China and Korea." *Reading Research Quarterly*. 43.4 (2008): 400-424. Print.

Douglas. H. Clements (2003). "Teaching and Learning Geometry". A Research Companion to *Principles and Standards for School Mathematics*. Chapter 11.

Fan, Lianghuo, Ngai-Ying Wong, Jinfai Cai, and Shiqi Li. *How Chinese Teach Mathematics: Perspectives from Insiders*. , 2015. Internet resource.

Fan, Lianghuo, Ngai-Ying Wong, Jinfai Cai, and Shiqi Li. *How Chinese Learn Mathematics: Perspectives from Insiders*. , 2004. Internet resource.

Herbst, Patricio, and Catherine Brach. "Proving and Doing Proofs in High School Geometry Classes: What Is It That Is Going on for Students?" *Cognition & Instruction*. 24.1 (2006).

Li, Sh.Q. (2002) Practice Makes Perfect: a key belief in China. In: Pre-conference Proceedings of the ICMI Comparative Study Conference, 20th-25th October, 2002. Hong Kong: The University of Hong Kong, Faculty of Education.

Li, Yeping, and Rongjin Huang. *How Chinese Teach Mathematics and Improve Teaching*. New York: Routledge, 2013. Internet resource.

Lim, C.S. "Characteristics of Mathematics Teaching in Shanghai, China: Through the Lens of a Malaysian." *Mathematics Education Research Journal*. 19.1 (2007): 77-89. Print.

Jerry Wellington (1992) Physics Teaching and Teacher Training in China: A Western Perspective. *Physics Education* v27 n3 p130-33

Jiang Zhonghong (1995) A Brief Comparison of the U.S. and Chinese Middle School Mathematics Programs. *School Science and Mathematics* v95 n4 p187-94

Jones, Keith. "Windows on Mathematics Education Research in Mainland China: a Thematic Review." *Research in Mathematics Education*. 10.1 (2008): 107-113. Print.

Jones, K., Fujita, T. & Ding, L. (2006), Informing the pedagogy for geometry: learning from teaching approaches in China and Japan, *Proceedings of the British Society for Research into Learning Mathematics*, 26(2), 110-114.

Ma, Liping. *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States*. Mahwah, N.J: Lawrence Erlbaum Associates, 1999.

Math Curriculum Standards (2003). Ministry of Education of People Republic of China.

Math Curriculum Standards for Geometry (2003). Ministry of Education of People Republic of China.

Mullis, I.V.S., Martin, M.O., Gonzales, E.J., Gregory, K.D., Garden, R.A., O'Connor, K.M., Chrostowski, S.J., & Smith, T.A. (2000), TIMSS 1999 International Mathematics Report. Chestnut Hill, MA: Boston College.

Polya, G. (1973). *How to solve it*. Princeton, NJ: Princeton University Press. (Originally copyrighted in 1945).

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Orlando, FL: Academic Press.

TIMMS (2011). Mathematics Achievement for Fourth and Eighth-grader. Highlights from TIMMS.

Wang, Jian, and Emily Lin. "Comparative Studies on U. S. and Chinese Mathematics Learning and the Implications for Standards-Based Mathematics Teaching Reform." *Educational Researcher*. 34.5 (2005): 3-13.

Weber, K. (2005). Students' understanding of trigonometric functions. *Mathematics Education Research Journal*. 17(3), 91-112.

Yerushalmy, Michal, and Daniel Chazan. "Overcoming Visual Obstacles with the Aid of the Supposer." *Educational Studies in Mathematics : an International Journal*. 21.3 (1990): 199-219.