EXCEL SPREADSHEET MANUAL

for

APPLIED MATHEMATICS

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Chapter 0 Getting Started

In this part, we give an overview of the general concepts and tools that involve spreadsheets. The spreadsheet software we use is Microsoft's Excel 2013 in the MS-Windows 7 environment, but the principles should hold for any current software package.

a. Spreadsheets Introduction

One could argue that the advent of spreadsheet and word-processing software ushered in the PC as a primary tool in every workplace. In today's world, wherever there is a table of data, it is stored in a spreadsheet. Some of the things one can do with data via spreadsheets are:

- Data Arrangement
- Calculation
- Database Management
- Visualization
- Statistical Analysis
- Predictions
- Optimization

Spreadsheet packages also come with powerful built-in programming languages, making their versatility almost limitless. The best part about spreadsheets is that the initial learning curve is very short! We use Microsoft Excel 2013 for all demonstrations in this manual, but due to the standardization in today's spreadsheets, one should be able to apply this material to almost any spreadsheet package. Throughout this manual, spreadsheet and PC terminology will be used. In ***Figure 1***, the typical window one sees is displayed with many of the names we use in this manual.

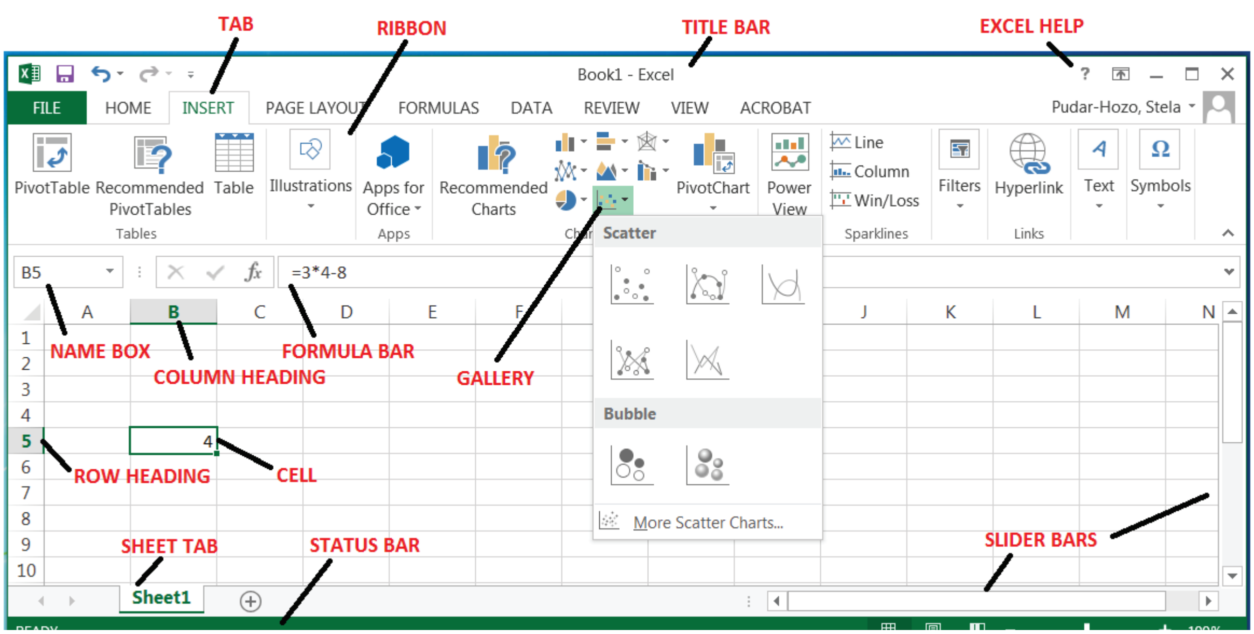


Figure 1

b. Files

Collections of sheets are stored in files called workbooks. When you open Excel, it starts with a brand new workbook called "Book1". ***Figure 1*** is very close to what you will see. Notice the title of the workbook in the title bar at the top of the window. If you want to open a previously saved workbook, you can do so through the "File" (top left corner), or you can open it directly by finding the file and double-clicking it.

Each workbook consists of sheets, accessed via the sheet tabs at the bottom of the screen (***Figure 1***). Each sheet can be either a chart or a worksheet. Charts are any kind of visualization of data, such as scatter plots or bar graphs. Worksheets are the "spreadsheets," the place where we (and the program) do all of the work.

c. Cells

Everything begins with the cell. If you can understand exactly what a cell is and what properties it has, the rest should be relatively easy. It is crucial that you understand this section.

A cell is one of the many rectangles you see on a worksheet. One or more cells are selected when they are surrounded by a bold outline with a fill handle in the lower right-hand corner of the outline (we will discuss the purpose of the fill handle later). You can select a single cell by clicking on it, and you can select a rectangular set of them by left-click-dragging. ***Figure 2*** shows the range B3:D6 selected.

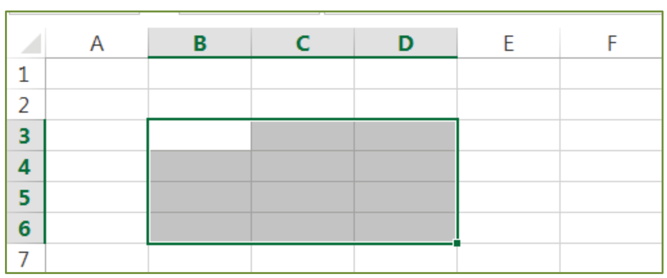


Figure 2

The cell's primary function is to hold and display "stuff". The cell has four major attributes, which you need to learn: address, content, value, and format.

d. Cell Address

The address of a cell is indicated by the row and column in which it sits. (If you have ever played Battleship, you know exactly how this works!) The horizontal column heading is always one or two letters (ranging from A to XFD) and the vertical row heading is always a number (from 1 to 1048576). A cell's address consists of the column letter(s) and the row number. For example, if a cell sits in column B and row 3, then its address is B3 (***Figure 3***). Note: The address of the selected cell is given in the name box.

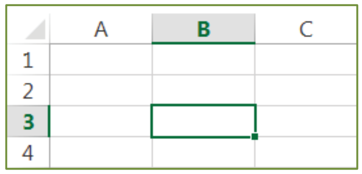


Figure 3

When a rectangular range (array) of cells is selected, the address of the range is given by the address of the upper left cell and the lower right cell, separated by a colon. In ***Figure 2*** the upper left cell of the selected range is B3 and the lower right cell is D6: thus, the range is denoted by B3:D6. Note: When a range of cells forms a single column or row, the two end cells are used to denote the range.

e. Cell Format

A cell can contain many things, like numbers and text, but how the cell displays those things can vary greatly. Suppose you type into every cell of a range (say B1:B5) the number 0.75. Without any formatting, the cells all display 0.75. By right-clicking a cell, you bring up a plethora of options, including "Format Cells". The first tab you get in the "Format Cells" pop-up window is "Number" (***Figure 4***). Here is where you can format the cell to display its value as anything from dates to currency. You can even dictate how many decimal places the cell will display (which can cause Excel to round the displayed value but not the actual value). In ***Figure 5*** you can see some of the possibilities, where all the cells contain the same value.

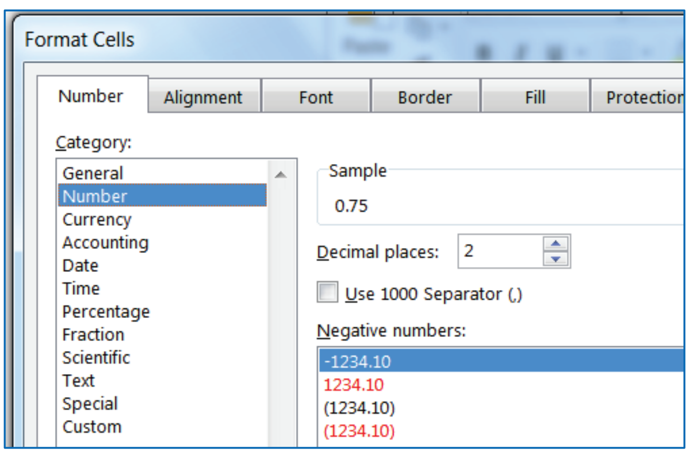


Figure 4

	A	B
1		0.75
2		75%
3		$0.75
4		3/4
5		7.5E-01

Figure 5

f. Cell Content

The content of a cell is whatever you type into it, which is usually a number, text, or a formula. The content of the cell is not necessarily what the cell displays! In other words, you may enter a formula into a cell, but then what the cell displays is the value of the formula, while the content of the cell is the formula. Whenever you select a cell, the content of the cell is shown in the formula bar. For example suppose you select a cell, type the formula "=0*3+3/4", and press "Enter". If you formatted the cell as a percentage, then the cell will display the value 75%. Now reselect the cell and look in the formula bar: you should see the formula "=0*3+3/4" (***Figure 6***).

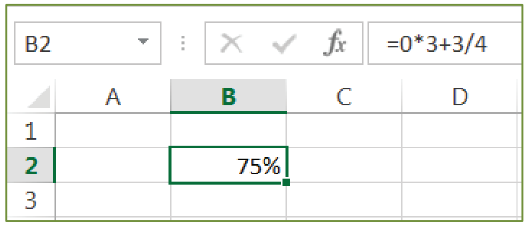


Figure 6

This is what makes spreadsheets so powerful. You will be able to enter formulas into cells that use values from other cells. In turn, these other cells may contain formulas themselves.

There is something very important to remember when using longer, more complicated formulas in Excel. Excel does not correctly obey the standard order of operations! Recall from basic algebra that an exponent applies only to whatever is to its immediate left. Unfortunately, Excel is programmed to apply negation (the negative sign) before applying exponents, whether parentheses are present or not. So the formula "= –A1^2" will return the value of (-A1)^2, instead of the negative of (A1)^2. Therefore, when using Excel, you should adopt the convention of enclosing the base and the exponent in parentheses. ***Figure 7*** shows the result of three different formulas. If you wish to calculate -3^2= -9, then you should use the formula shown in the last column of the table.

A2	=A2^2	= -A2^2	= -(A2^2)
3	9	9	-9

Figure 7

g. Cell Value

Every cell has a numeric value, even an empty one (which has a default value of 0). If you type a number into a cell, then that will also be the cell's value. If you type a numeric formula into a cell, then that cell's value will be the outcome of the formula. When the cell contains text or has a formula that returns text, then the value of that cell will default to 0. Remember that the value of a cell, what the cell displays, and the cell's content are three different things. As seen in the example illustrated in ***Figure 6,*** the value of the cell is a 0.75, while the cell's content is the formula "=0*3+3/4" and the cell's display is 75%.

h. Formulas

As seen earlier, you can type formulas into a cell. All formulas start with an "=". A formula can have numbers, algebraic operations, functions, and addresses for cells and ranges. Notice that we do not have variables in the form *x, y, .z*... Any time we want to use a value from another cell in a formula, we can instead use the cell's address simply by clicking on it or by typing the cell's address. Let us consider an example.

Suppose you wanted to store in cell D2 the average of three numbers that you have stored in cells B1, B2 and B3. This can be done with a formula. In cell D2, type "=(B1+B2+B3)/3" and press “Enter”. The cell D2 displays the average. If you reselect cell D2, the formula bar will display the formula that the cell contains, while the cell displays its value 2 (***Figure 8***).

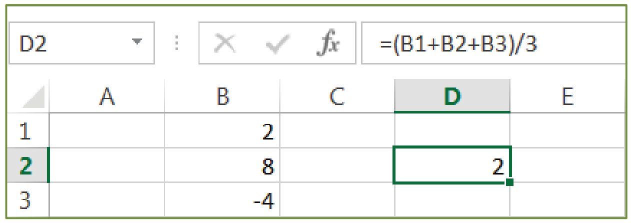


Figure 8

For many common calculations, such as averaging, there are built-in functions. To view a list of these functions, click on a cell and type “=”. The name box changes and becomes a drop-down list of functions you have used recently, but for our purposes the most interesting choice is “More Functions" (***Figure 9***). Through this choice, you can explore and use a whole host of functions; it is certainly worth your while to check this out! Another way of obtaining a function is by clicking on “Formulas” tab and then on f_x in the ribbon. Select your function from the newly appeared pop-up screen. You can also type in functions directly if you know the correct spelling and appropriate arguments. Even if you remember and type the only the first letter of the function name after “=” Excel will help you remember the rest by offering several choices of functions beginning with that letter. In ***Figure 10*** , the function "=AVERAGE(B1:B3)" gives the same result as the formula in ***Figure 8***.

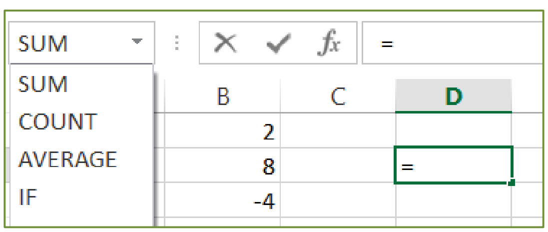


Figure 9

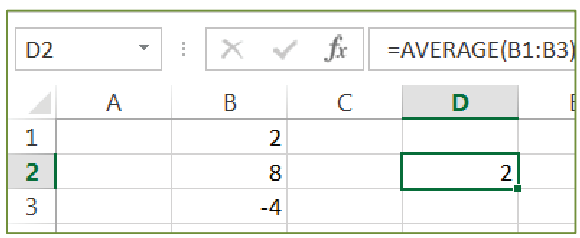
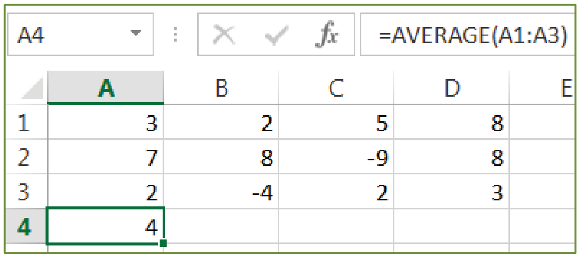


Figure 10

i. Copy, Paste, and Fill

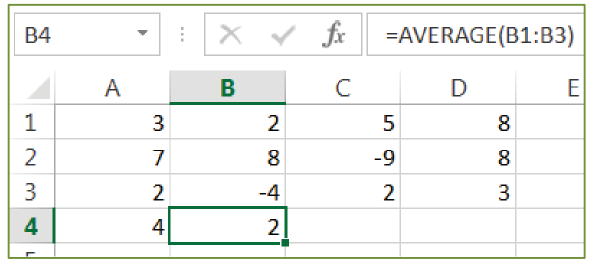
When copying cells, if the cell’s content is anything other than a formula, the contents are identically transcribed. However, when the content of a copied cell is a function, then the addresses within the function are adjusted by the relative distance from the copied cell to the pasted cell.

As an example, consider ***Figure 11***, where you have a table of values of which you want the average of each column. Type in the appropriate formula for averaging the first column in A4, and then copy that cell and paste it into the cell B4 under the next column (***Figure 12***). Notice how the address range, A1:A3, is shifted by exactly one column value to B1:B3. This is precisely the relative change from the copied cell (A4) to the pasted cell (B4). If you pasted from the cell A4 to the cell C9, the relative change would be two to the right and five down: thus, the pasted formula would be "=AVERAGE(C6:C8)".


A4 =AVERAGE(A1:A3)

	A	B	C	D	E
1	3	2	5	8	
2	7	8	-9	8	
3	2	-4	2	3	
4	4				

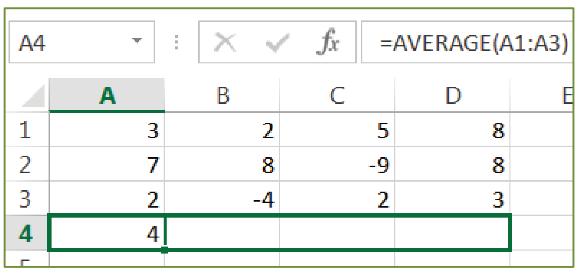
Figure 11 Average function typed


B4 =AVERAGE(B1:B3)

	A	B	C	D	E
1	3	2	5	8	
2	7	8	-9	8	
3	2	-4	2	3	
4	4	2			

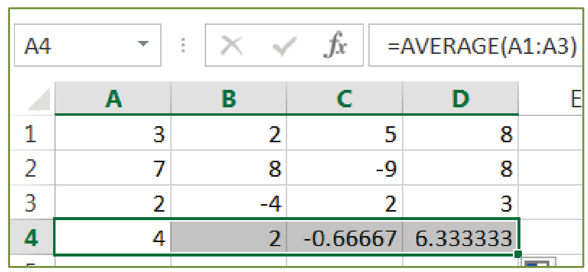
Figure 12 The next is copied from the first

Coming back to our example, there is a faster way of copying one cell to multiple cells. The technique is called filling, and as with most things, there are several ways of accomplishing it. As noted earlier, every highlighted cell has a fill handle in the lower right-hand corner. If you hold-click this handle (grab) and drag in any direction, an automatic copy/paste is performed on all covered cells. In ***Figure 13*** the handle in cell B4 is being grabbed and dragged across cells C4 and D4. When the mouse is released, the average function is pasted into cells C4 and D4 with the appropriate adjustment to the address ranges within the function (***Figure 14***).


A4 =AVERAGE(A1:A3)

	A	B	C	D	E
1	3	2	5	8	
2	7	8	-9	8	
3	2	-4	2	3	
4	4				

Figure 13 The fill handle is dragged


A4 =AVERAGE(A1:A3)

	A	B	C	D	E
1	3	2	5	8	
2	7	8	-9	8	
3	2	-4	2	3	
4	4	2	-0.66667	6.333333	

Figure 14 All averages are filled in

There may be times when you will copy/paste a cell and you want one or more parts of the address ranges in the function's arguments to remain unchanged. The trick is to use a "$" in front of every part of the address you want to freeze. For instance, suppose you are filling from a cell with the formula "=A1+B3-C7". If you do not want the column of the address A1 to change, you would instead use the formula "=$A1+B3-C7". If you do not want the row address of B3 to change, you would use the formula "=A1+B$3-C7". If you do not want the address C7 to change at all, you would use the formula "=A1+B3-$C$7". When an address is completely frozen, e.g. $C$7, this is called an absolute reference. When no part of the cell reference is frozen, e.g. C7, this is called a relative reference. When only part of a cell reference is frozen, e.g. $C7 or C$7, this is called a mixed reference.

j. Charts

A picture is worth a thousand words. Spreadsheets are loaded with graphical tools for displaying sets of data. Creating a chart with sets of data is conveniently simple. Consider the data in ***Figure 15*** of homework and test grades over the first four chapters. Select the data to be visualized (range B2:C5) and engage column charts via the "Insert" tab. There are many choices of possible charts, including bar charts (***Figure 16***), pie charts, and scatter plots.

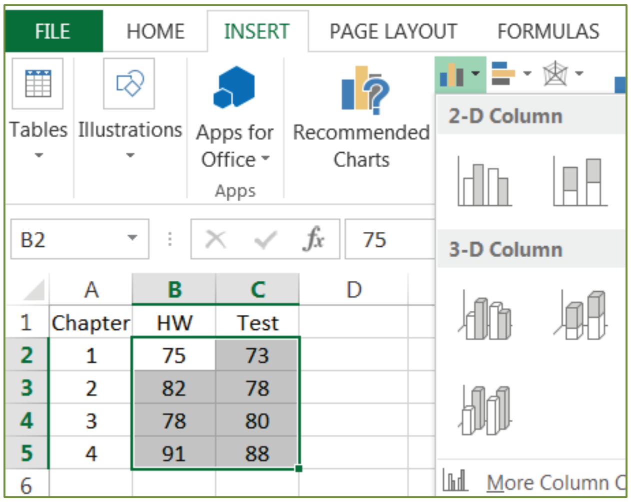


Figure 15 Grades

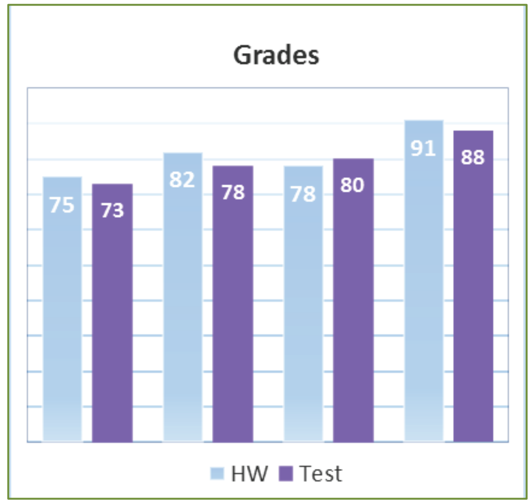


Figure 16 Bar chart

You are given a vast number of choices to fine tune and annotate your graph. After the chart is created, changes can be made by simply right-clicking the part of the chart in which you are interested. If you left-click on the chart a new tab “Chart Tools” will appear, where you’ll find many options under “Format”. If you want to change format of letters in chart title, right-click on the title, and select “Format Chart Title” from the resulting menu. You can also make any other major changes by right-clicking in the white area of the chart. A new tab “CHART TOOLS” will appear at the top. Click on it to obtain ribbon that allows you to change chart type, colors, switch row/columns, etc. Later in this manual you will see that by right-clicking the data graphics, one can even include trendlines.

k. Conclusion

Once you become comfortable with the spreadsheet environment, you should experiment with all the available charts, formulas, and built-in abilities. The possibilities for effectively and efficiently using Excel, or any other spreadsheet program, are practically endless. If you are using Microsoft Word or Microsoft PowerPoint, all tables and charts from Excel can be copied and pasted into a Word document or PowerPoint presentation so that you can create attractive papers and presentations. You can even import data from other programs and databases, as well as from the World Wide Web. Even though this manual only demonstrates how to use Excel for problems that arise in finite mathematics and in applied calculus, Excel also has applicability to other courses, including statistics, accounting, economics, biology, chemistry, and physics, just to name a few.

You may also find spreadsheets useful for keeping track of your college credits, personal finances, or even as an address book for e-mail and mailing addresses and phone numbers. You are limited only by your imagination and patience.

Chapter 1 Solving Equations

a. Using the Quadratic Formula

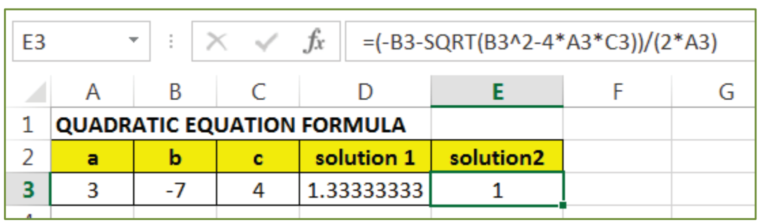
One way to solve the quadratic equation $3x^2 - 7x + 4 = 0$ is to calculate solutions by using the quadratic formula.

If this procedure will not be frequently used, than the simplest is direct calculation. Once "=" is entered in a cell Excel will act as a calculator. Excel notation for square root is "SQRT". Coefficients $a = 3$, $b = -7$ and $c = 4$ entered in the formula as following will result in solutions $x \approx 1.33$ and $x = 1$.

"=(-(-7)+SQRT((-7)^2-4*3*4))/(2*3)" and "=(-(-7)-SQRT((-7)^2-4*3*4))/(2*3)"

For frequent use of the quadratic formula it is recommended that we program cells to calculate quadratic equations solutions and save the file for future use. First step is to enter the coefficients $a = 3$, $b = -7$ and $c = 4$ into cells as shown in ***Figure 17***. Type the formulas bellow in cells D2 and E2 respectively to obtain the final solutions.

"=(-B3+SQRT(B3^2-4*A3*C3))/(2*A3)" and " =(-B3-SQRT(B3^2-4*A3*C3))/(2*A3)"


E3 =(-B3-SQRT(B3^2-4*A3*C3))/(2*A3)

	A	B	C	D	E	F	G
1	QUADRATIC EQUATION FORMULA						
2	a	b	c	solution 1	solution2		
3	3	-7	4	1.33333333	1		

Figure 17

To solve a different quadratic equation simply change the coefficients in the table and the new solutions will automatically be calculated in cells D2 and E2.

b. Using SOLVER

To solve the cubic equation $x^3 - 2.7x^2 - 13.6x + 10.5 = 0$ we actually need to find value(s) of x that will result in $y = 0$ for the function $y = x^3 - 2.7x^2 - 13.6x + 10.5$.

Spreadsheet programs have a built-in add-in called "Solver". To use it in Excel you need to load it first. Click on "File" tab at the top left corner, and then click "Options". Select "Add-ins" and then in the Manage box, select "Excel Add-ins" and click "Go". Check the "Solver" box and click "OK". From now on the "Solver" Add-in will be listed under the "Analysis" group in the "Data" tab ribbon as shown in ***Figure 18***.

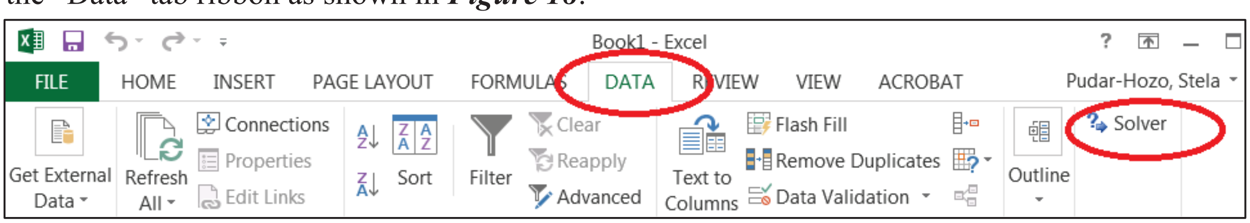


Figure 18

Solver will calculate the objective *y*'s for different values of variable *x* in the formula $y = x^3 - 2.7x^2 - 13.6x + 10.5$ until $y = 0$ is obtained. Assign an initial *x*-value for the adjustable cell B2 as shown in ***Figure 19***. We selected "-7", but any value may be used here. Enter the cubic function "=B2^3-2.7*B2^2-13.6*B2+10.5" in objective cell C2. Select "Solver" and fill the boxes as shown in ***Figure 19***.

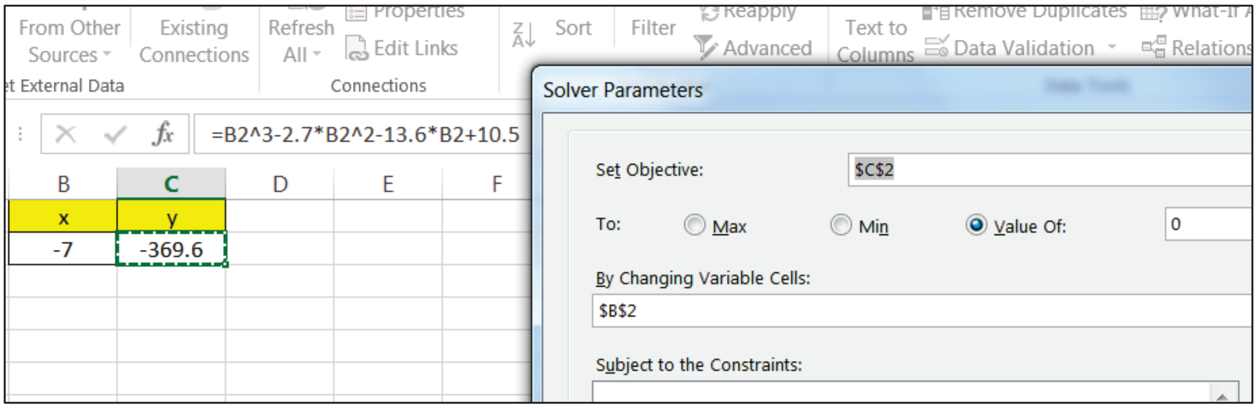


Figure 19

Click on "Solve" and the coordinates of the point where the function is maximized will be determined. In this example, the *x*-value 0.7 will appear in cell B2 and the *y*-value 0 appears in cell C2 as the coordinates of the solution of our cubic equation. To find the second solution pick a different initial *x*-value and use "Solver" again. In our example initial value 8 will produce the second solution $x = 5$.The third solution is 3.

c. Solving Equations Using Graphs

We can read solutions of equation $1.5x^2 - 2.25x - 10.5 = 0$ as *x*-intercepts of the function $y = 1.5x^2 - 2.25x - 10.5$. Excel will graph it, but we need to provide the list of *x*-values we are interested in. For this example, we are interested in values -4, -3.5, -3 etc. One fast way to list the *x* values under observation is to enter numbers -4 and -3.5 in cells A2:A3, then select cells A2:A3 together and then drag the fill handle from the lower right corner down over column A until cells A2:A18 are filled with the desired numbers. A simple way to list the *y* values under observation is to click into B2 and enter the formula " =1.5*A2^2-2.25*A2-10.5 ". Drag the fill handle over column B to copy the formula down. Values of variable *y:* 22.5, 15.75, 9.75, etc. will appear in cells B2:B18. Values of *x* and *y* are now sorted in the table. One of many ways to create a graph of our quadratic function is to first select cells A2:B18 together to indicate *x* and *y* coordinates of the observed points. Select the "Insert" tab and then select the "Scatter" graph to the right. Select the chart sub-type "Scatter with smooth lines" as in ***Figure 20***.

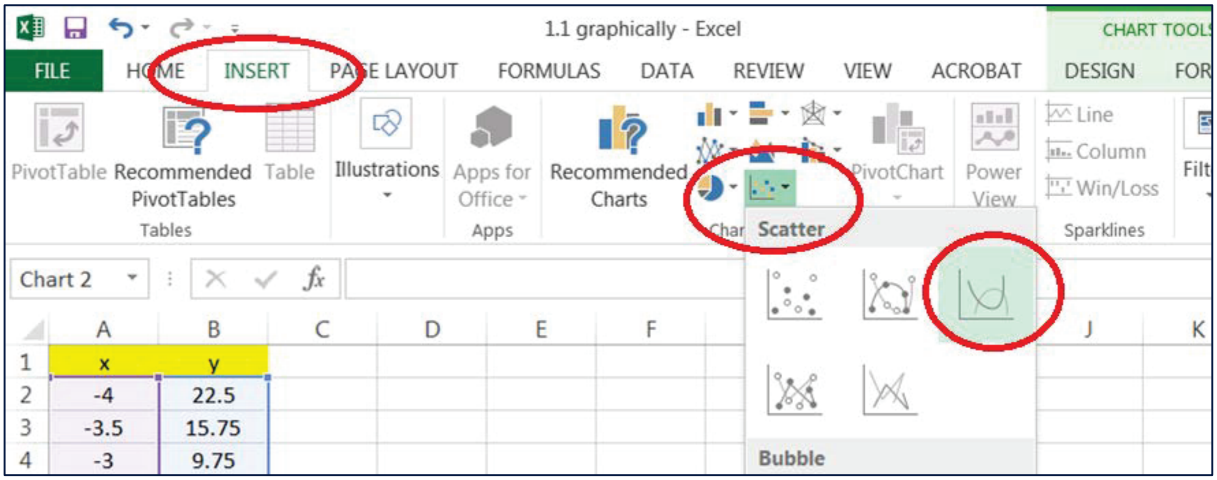


Figure 20

The final look of your graph is a matter of your personal taste. Click on your chart and the "Chart Tools" menu will appear. Use it to upgrade the style. Right-clicking on various sections of the final chart will let you select more options for gridlines, background color, text font, etc. Position your mouse exactly over one of the specially marked data points on your final graph, but do not click. After a short delay, the exact coordinates of that point will appear in a small gray box next to the point, like in ***Figure 21***.

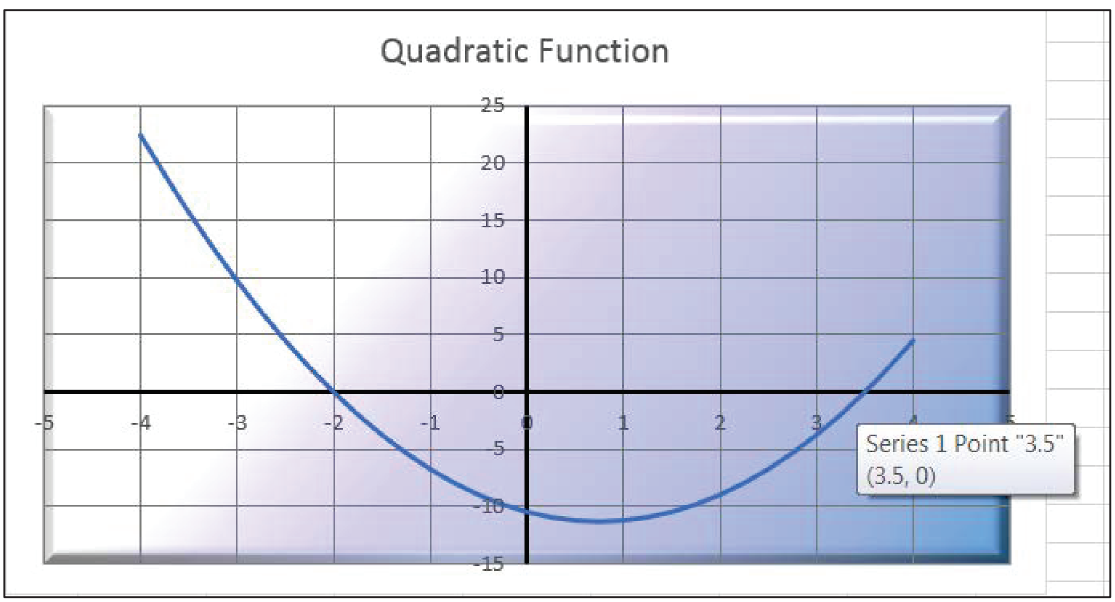


Figure 21

The graph shows that x-intercepts are -2 and 3.5. These values of x are solutions of equation $1.5x^2 - 2.25x - 10.5 = 0$.

Chapter 2 Functions

a. Calculating Numerical Expressions

Spreadsheet can be used as a calculator once we use symbol "=". To evaluate numerical expression "$7.33^3 - 14(12+55.12)$" we type "=7.33^3-14*(12+55.12)" in any cell. The copy of your expression will appear in formula bar, next to f_x. Enter. The result of calculation -545.847 will appear in the cell, as shown in ***Figure 22***.

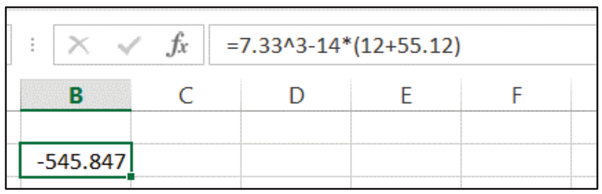


Figure 22

b. Using Function Notation

Excel allows us to create and use our own functions but it also has built-in functions that are performing various operations on data entered in cells.

To calculate value $f(0.9)$ for our own function $f(x) = -1.8x^5 + 3x^3 - 7.3$ we enter the value of *x* in B2 and we enter the function in cell C2 as "=-1.8*B2^5+3*B2^3-7.3". ***Figure 23*** shows the final result $f(0.9) = -6.17588$. In case of long algebraic expressions it is easier to type the function inside the formula bar. Notice that instead of *x* in the formula we are entering the value of cell B2. You can type "B2" or obtain the same by clicking on the cell B2.

Once your function is entered, you can evaluate it for a different value of *x*, say $x = 2.03$ by changing cell B2 value to 2.03. That way *y* value will change automatically to -44.2553.

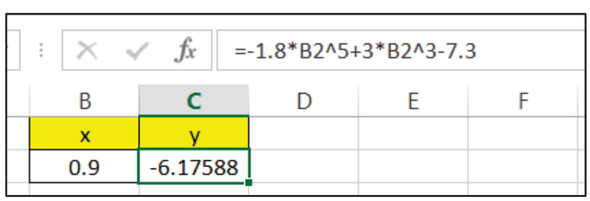


Figure 23

The list of built-in functions is found in the name box, to the left from the formula bar, after you type symbol "=" in any cell. Click on arrow next to the name box to obtain the list, like in ***Figure 24***. Click on "More Functions" to search over 100 built–in functions sorted in categories like "Statistics", "Logical" etc., like in ***Figure 25***.

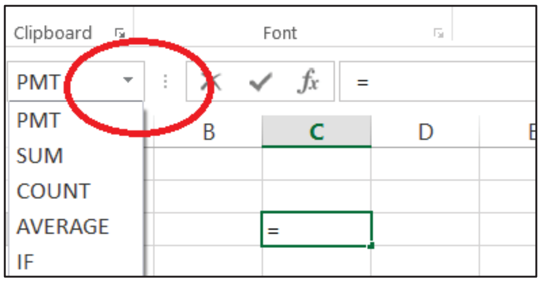


Figure 24

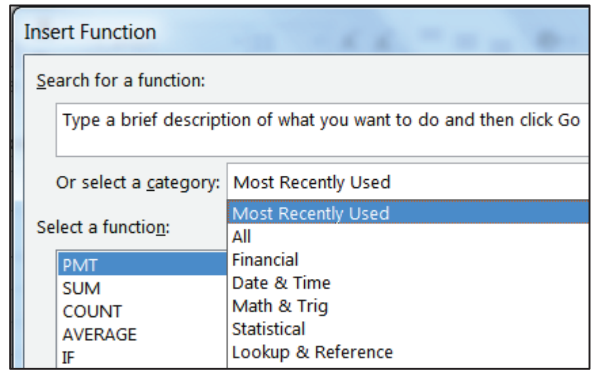


Figure 25

Another way to find and insert a build-in function is by clicking on the tab "Formulas" and selecting a function from the ribbon, like in ***Figure 26***. If you remember only the beginning of the name of the function, type it in a cell after "=". The spreadsheet will help you remember the rest of the name by offering possible choices.

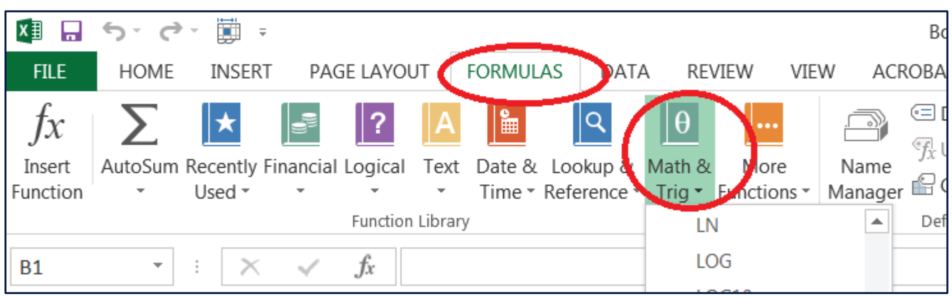


Figure 26

The built-in function "SUM" makes it easy to quickly sum columns, rows, or individual cells of data in a worksheet. To add numbers in cells A1:A4 we'll type "=" and select built-in function SUM as in ***Figure 27***. One easy way to tell Excel that you want the numbers in cells A1:A4 is by clicking on the small matrix to the right from the "Number1" box, and then selecting the array A1:A4.

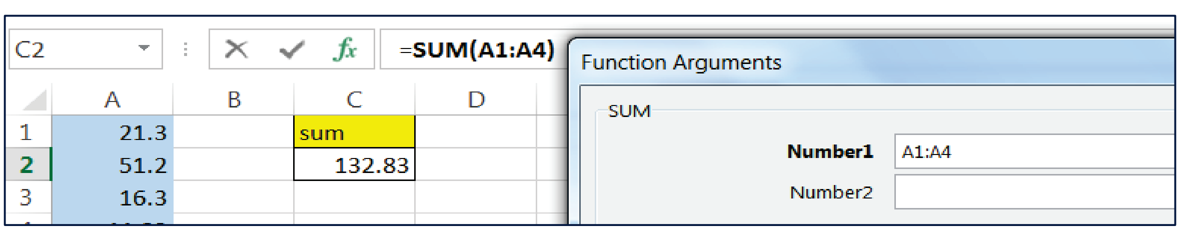


Figure 27

c. Creating Function Tables

Excel can graph the function $y = x^3 - 5x + 1$ after the table of (x, y) values of interest is created. Say, we are interested in y values of the function for $x = -3.0, -2.5, -2.0, \ldots\ldots, 2.5, 3.0$.

Enter the first two values -3 and -2.5 in cells A2 and A3. One fast way to type the remaining *x* values in column A is to select the array A2:A3 and drag the handle from the lower right corner down, over column A, until cells A4:A14 are filled with the desired *x* values, as in ***Figure 28***. Column B is reserved for the *y* values of our cubic function. A simple way to list the *y* values under observation is to click on B2 and enter the formula "=A2^3-5*A2+1". Drag the handle from the lower right corner over column B as in ***Figure 29*** to copy the formula down. Values -11, -2.13, 3, etc. will appear in cells B2:B14.

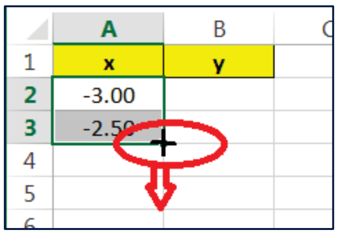


Figure 28

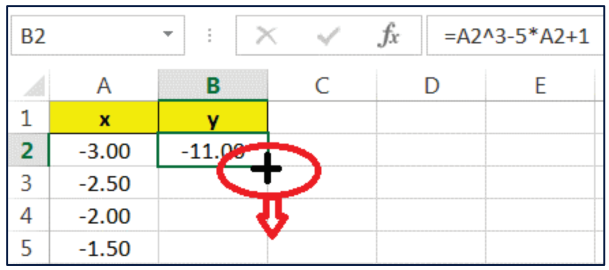


Figure 29

d. Graphing Function

Once we have table of (x, y) values listed in cells A2:B14 we can work on a graph. One of many ways to create a graph of our cubic function $y = x^3 - 5x + 1$ is to first select array A2:B14 to indicate *x* and *y* coordinates of points under observation. Select the "Insert" tab and then in the ribbon select the "Scatter" graph to the right. Select the chart sub-type "Scatter with smooth lines and markers" as shown in ***Figure 30***.

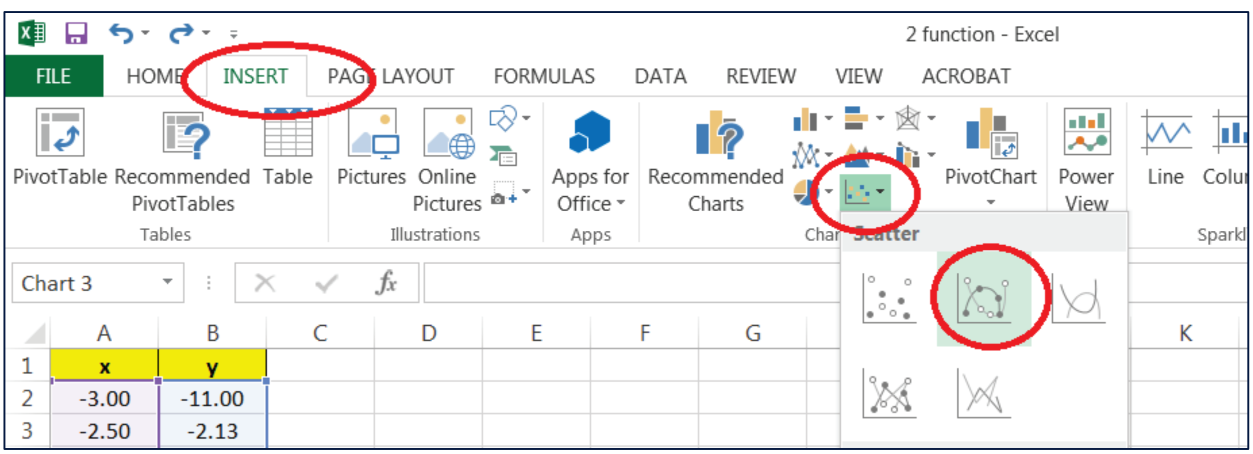


Figure 30

The final look of your graph is a matter of your personal taste. Click on your chart and the "Chart Tools" menu will appear as a new ribbon: use this to upgrade the style. Right-clicking on various sections of the final chart will let you select more options for gridlines, background color, text font, etc. Position your mouse exactly over one of the specially marked data points on your final graph, but do not click. After a short delay, the exact coordinates of that point will appear in a small yellow box next to the point, like in ***Figure 31***.

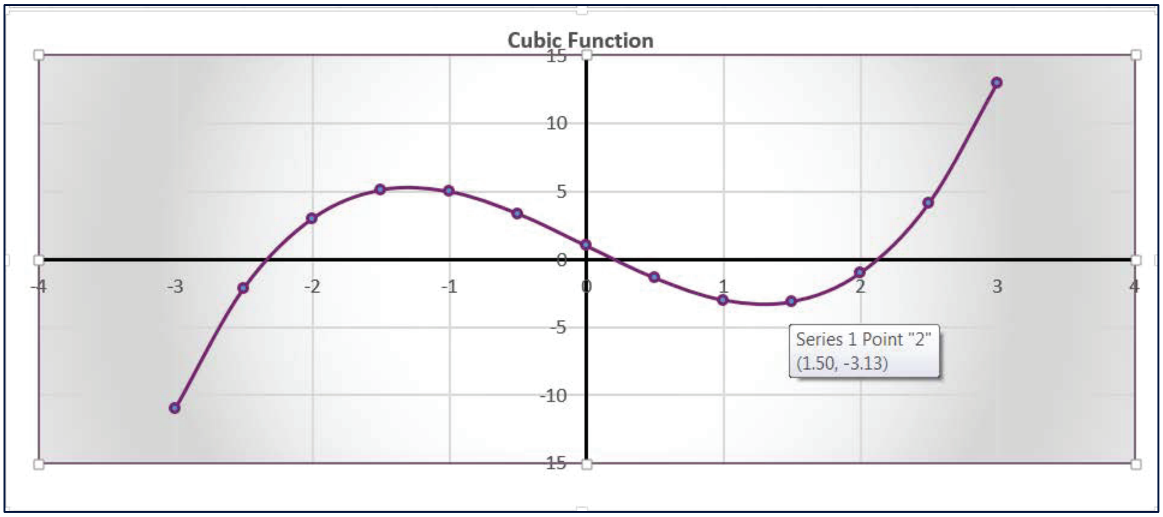


Figure 31

e. Piecewise Functions

To graph a piecewise defined function we'll create table of (*x, y*) values we are interested in, insert graph, and then define the domain of each. As an example we'll use the function:

$$f(x)=\begin{cases} x^2 & \text{if } x<2 \\ 6 & \text{if } x=2 \\ 10-x & \text{if } x>2 \end{cases}$$

Figure 32 shows table of values of all three functions: $y=x^2$, $y=6$, and $y=10-x$ for $x=-4.0,-3.5,-3.0,....$ To graph all three function together on $(-4,6)$ we select A2:D22 and inserted scatter graph with smooth lines.

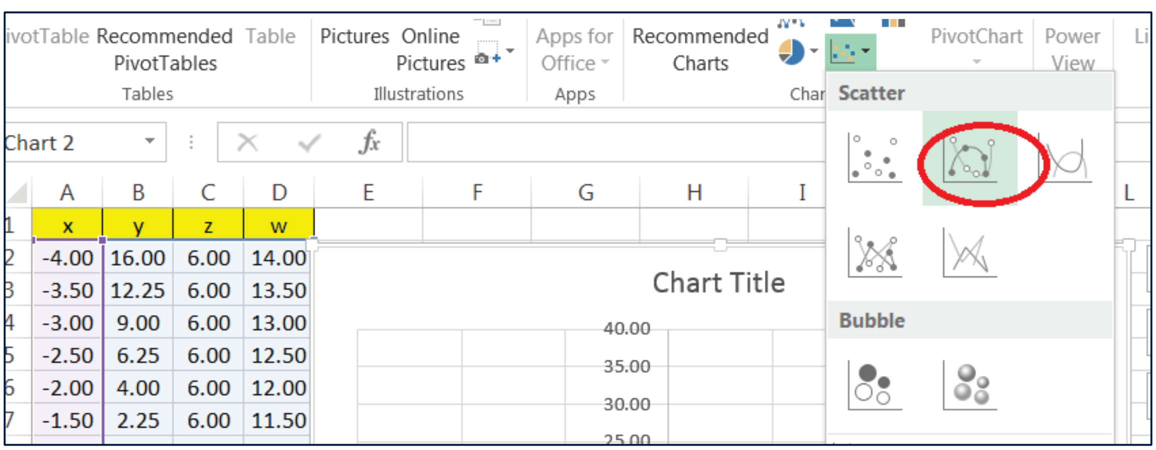


Figure 32

The chart we obtained shows all three functions, each on the entire domain (-4, 4), as shown in ***Figure 33***. We need to adjust the domain of each.

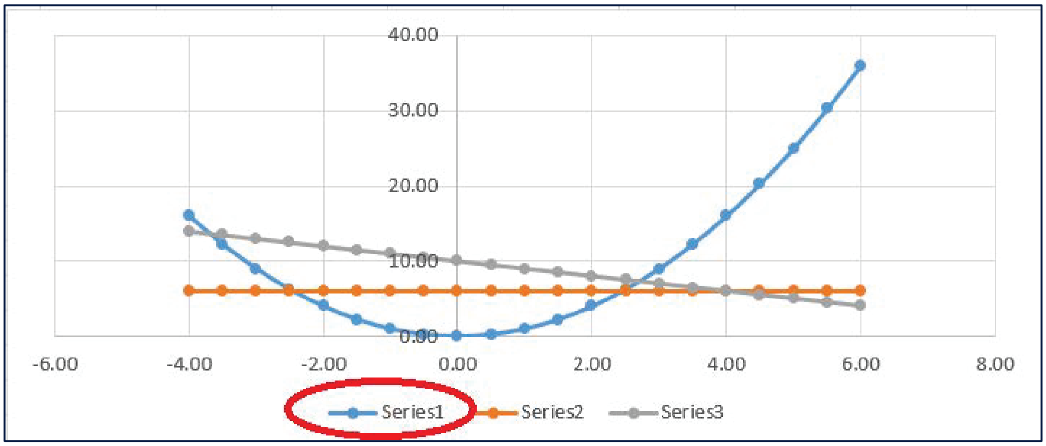


Figure 33

Right-click on "Series1" in legend under the graph and click on "Select Data". The new pop-up screen will help change the visible domain of the first function $y = x^2$ to $(-4, 2)$. Select "Series1" and "Edit" as in ***Figure 34***.

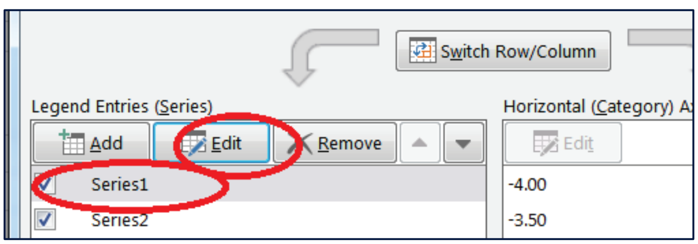


Figure 34

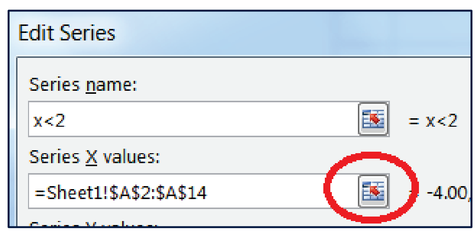


Figure 35

The new pop-up screen allowed us to change "Series1" name to "x<2" in the legend. Click to encircled box matrix in ***Figure 35.*** In the new pop-up screen select the array A2:A14 to indicate that the first function domain is $(-4, 2)$. The final graph is shown in ***Figure 36***.

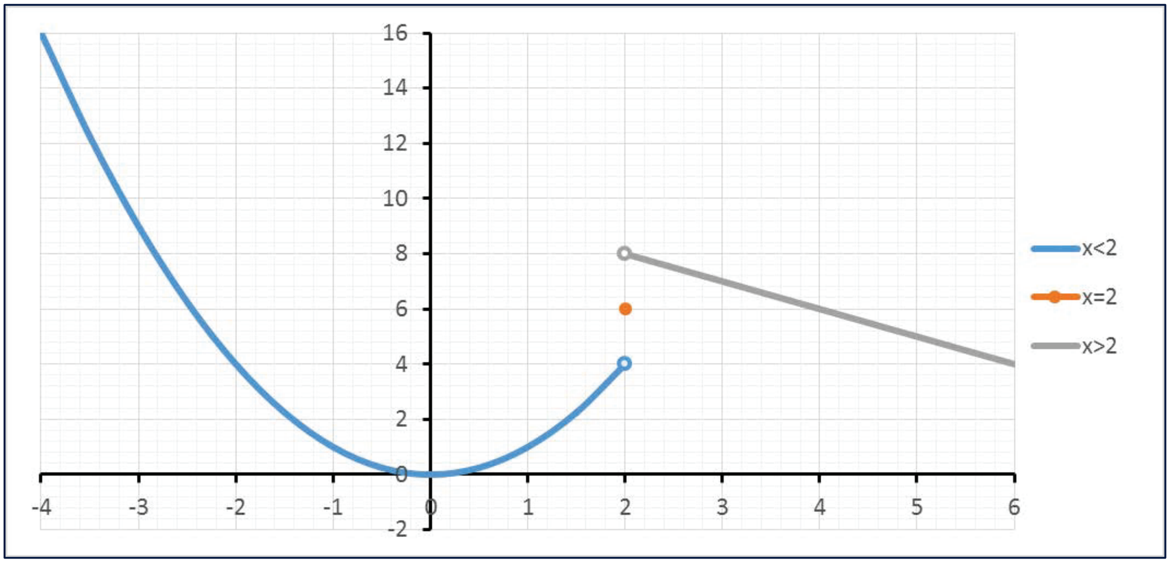


Figure 36

f. Finding Intersection Points

Suppose that the supply function is $p = 0.2q + 51$ and the demand function is $p = 3000/(q+5)$. We'd like to find the exact intersection point of these two functions (the equilibrium point).

One way to do it is to look at the table of values and examine *y* values for supply and demand functions. The first two values of *x* that are 0 and 5 are entered in cells A2 and A3, then the array A2:A3 is copied down to cells A4:A42 by dragging the fill handle from the lower right corner. The functions are entered in cells B2 and C2 as formulas "=0.2*A2+51" and "=3000/(A2+5)", and copied down by dragging the fill handle. ***Figure 37*** shows the equilibrium point in the table.

	A	B	C
1	x	supply	demand
8	30	57.00	85.71
9	35	58.00	75.00
10	40	59.00	66.67
11	45	60.00	60.00
12	50	61.00	54.55
13	55	62.00	50.00
14	60	63.00	46.15

Figure 37

It is even better if we graph the functions together, and find the intersection point on the graph. To obtain the graph select cells array A2:B42 and then "Scatter with Smooth Lines" from the "Insert" tab ribbon. Position the cursor exactly above the intersection point, but do not click. After a brief delay coordinates of the intersection point appear, like in ***Figure 38***.

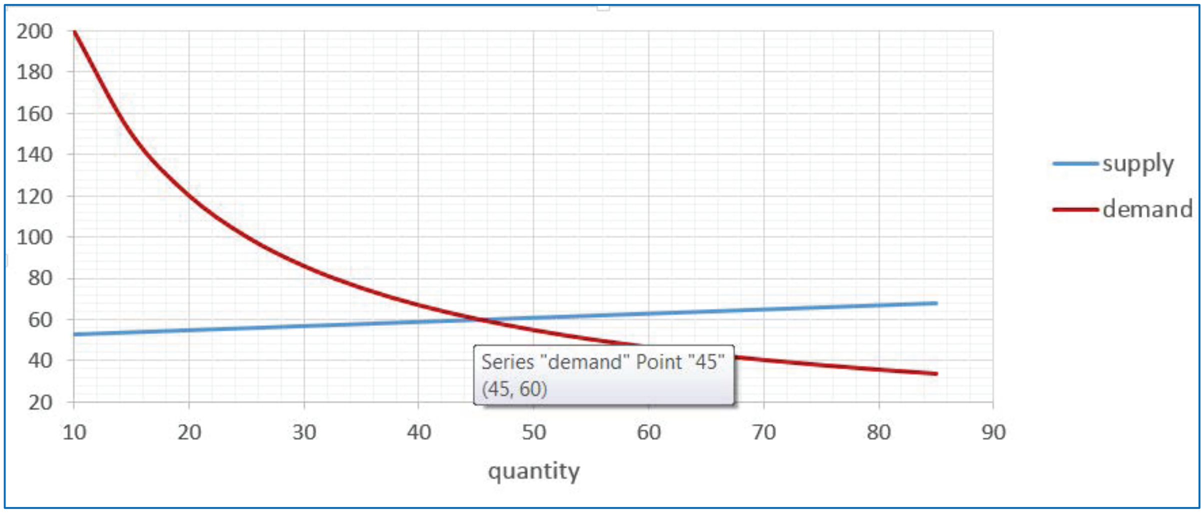


Figure 38

One more way to find the intersection point of functions $p = 0.2q + 51$ and $p = 3000/(q+5)$ is to find zeros of the function $y = (0.2x + 51) - 3000/(x+5)$. Type "0" in cell H2 as shown in ***Figure 39***, and type the formula "=(0.2*H2+51)-3000/(H2+5)" in cell I2. Select "What-If Analysis" in "Data" tab ribbon. Click on "Goal Seek" and fill the obtained pop-up screen as shown in ***Figure 40***. Cell H2 value will change to 45, that is the first coordinate of the intersection point.

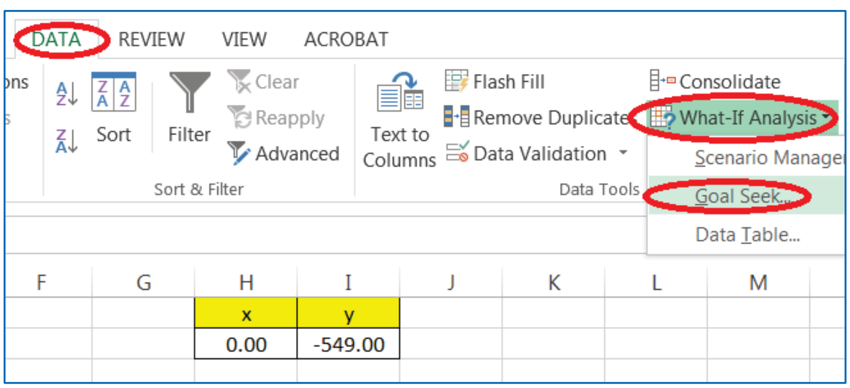


Figure 39

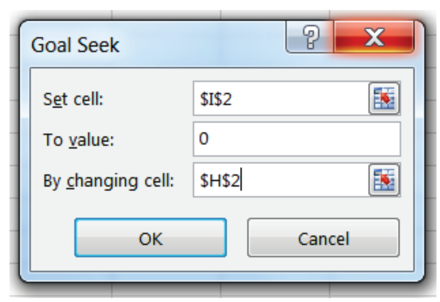


Figure 40

g. Finding Maximum and Minimum Points

We can estimate the maximum of the function $y = -0.0037x^3 - 0.0591x^2 + 2.534x + 38.21$ by creating a spreadsheet table (see chapter 2 c) and examining y values. Once we have the table we might decide to graph the function (see chapter 2 d) and position the mouse over the point. Coordinates (11, 54.0082) will appear as in ***Figure 41***.

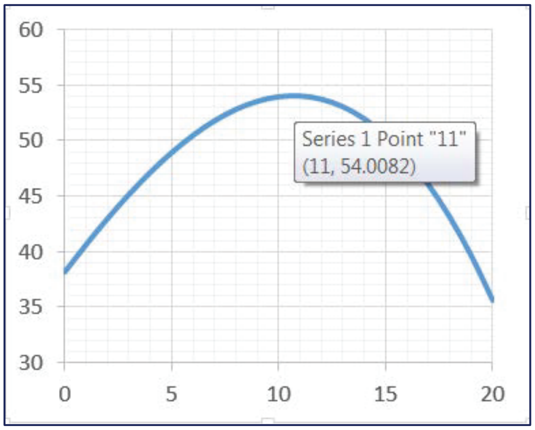


Figure 41

By looking at the y values of marked data points from the graph, we can estimate that the percentage will be maximized where the x -value is between 10 and 11. Assume we'd like to know the exact maximum of our function. Spreadsheet programs have a built-in Add-in called Solver. To use it in Excel you need to load it first. Click on "File" tab, select "Options" and then select 'Add-Ins' the Microsoft Office Button at the top left corner, and then click "Excel Options". Select "Add-ins". After you load "Solver Add-In" to your "Add-Ins", the "Solver", command is available in "Data" tab ribbon as one of "Analysis" tools.

Excel's command "Solver" will calculate y's for different x values in the formula $y = -0.0037x^3 - 0.0591x^2 + 2.534x + 38.21$ until the maximum or minimum value for y is found. Assign an initial x-value for the variable cell F2 as shown in ***Figure 42***. We selected "3", but any value may be used here. Enter the quadratic function spreadsheet formula "=-0.0037*F2^3-0.0591*F2^2+2.534*F2+38.21" in the objective cell G2. Select "Solver" and fill the boxes. Click on "Solve" and the coordinates of the point where the function is maximized will be determined. In this example, the x-value 10.7 will appear in cell F2 and the y-value 54.02 will in cell C2 as the coordinates of the maximum.

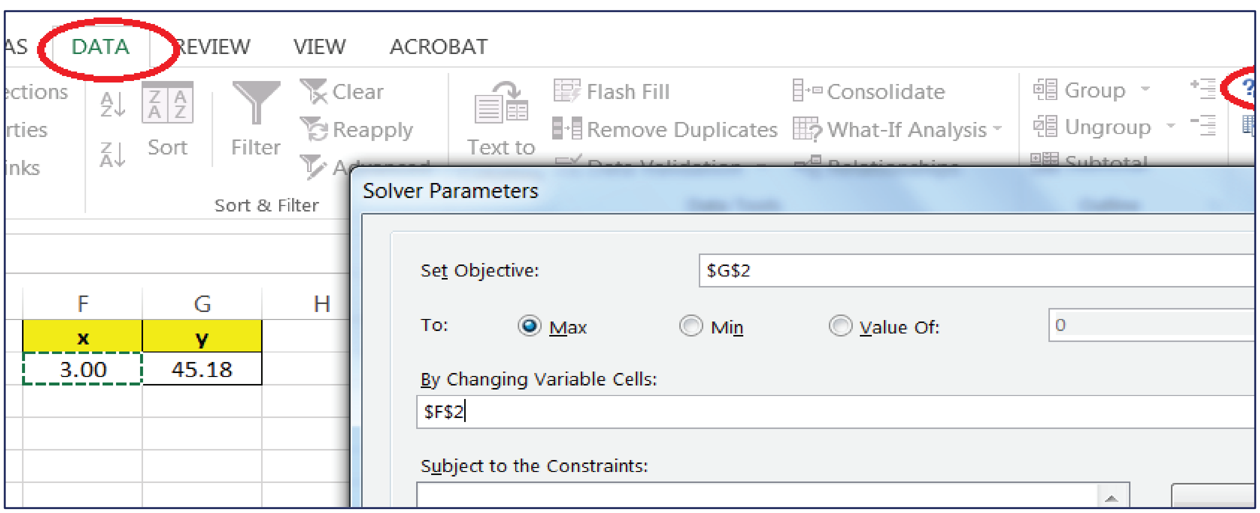


Figure 42

Chapter 3 Exponential and Logarithmic Functions

a. Evaluating Powers of e

Powers of Euler's constant *e* are calculated in Excel using the build-in function "=EXP( )". It is one of many functions that will appear in the name box menu after typing "=" in a cell. This function is also found in "Formulas" tab ribbon as one of "Math & Trig" functions.

Say, a car value, *x* years after the purchase, can be modeled using the function $f(x) = 21679\, e^{-0.1x}$. To calculate the value of this car 5 years after the purchase we enter "=21679*EXP(-0.1*5)" in any cell. The result of this calculation is $13148.98.

b. Evaluating Expressions Involving Logarithms

We can calculate the natural logarithm of 73.5 by typing "=LN(73.5)" in any cell. The result of the calculation 4.297 will appear after entering. This function is also found in "Formulas" tab ribbon as one of "Math & Trig" functions.

The common logarithm (base 10) of number 334 is calculated by typing "=LOG10(334)" in any cell. The result is 2.524. This function is also found in "Formulas" tab ribbon as one of "Math & Trig" functions.

The logarithm of 3000 to base 2 is 11.55. This result is obtained in Excel by typing "=LOG(3000,2)" in any cell. This function is also found in "Formulas" tab ribbon as one of "Math & Trig" functions.

Chapter 4 Regression

a. Linear Regression

Enrollment (in millions) for all US colleges and universities is given in the ***Figure 43***.

Year	2005	2007	2008	2009	2010
Enrollment	17.5	18.2	19.1	20.4	21

Figure 43

Let x=5 correspond to year 2005. We'll use a spreadsheet to find the regression line that models the data. Enter years x=5, 7, 8, 9, 10 in cells B2:B6 and enter the enrollment data y in cells C2:C6. Select cells B2:C6 and click the "Scatter" from the "Insert" tab ribbon, as shown in ***Figure 44***.

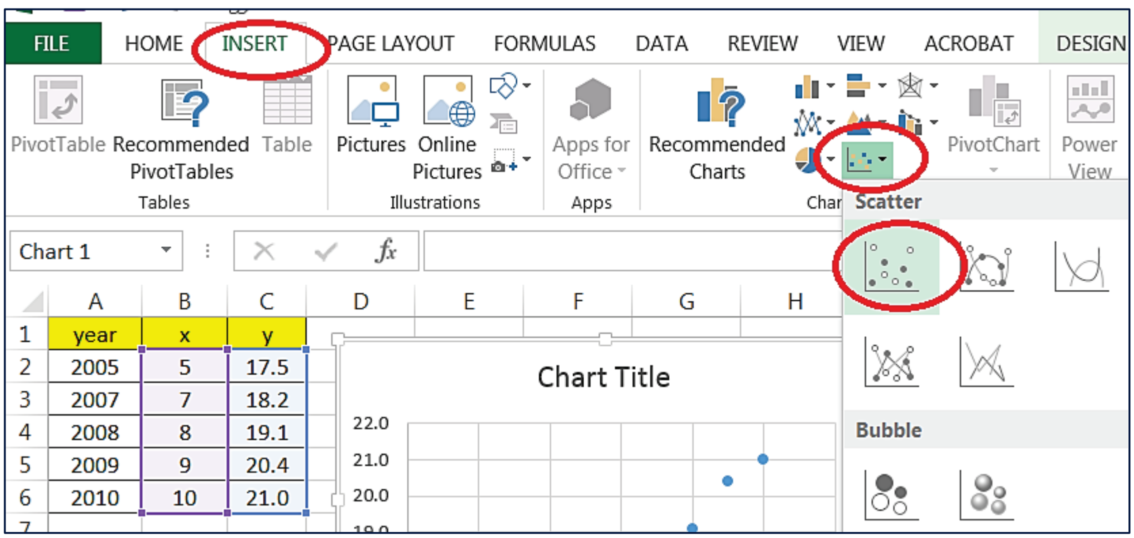


Figure 44

Right-click one of the points from the newly obtained graph and select "Add Trendline", as in ***Figure 45***.

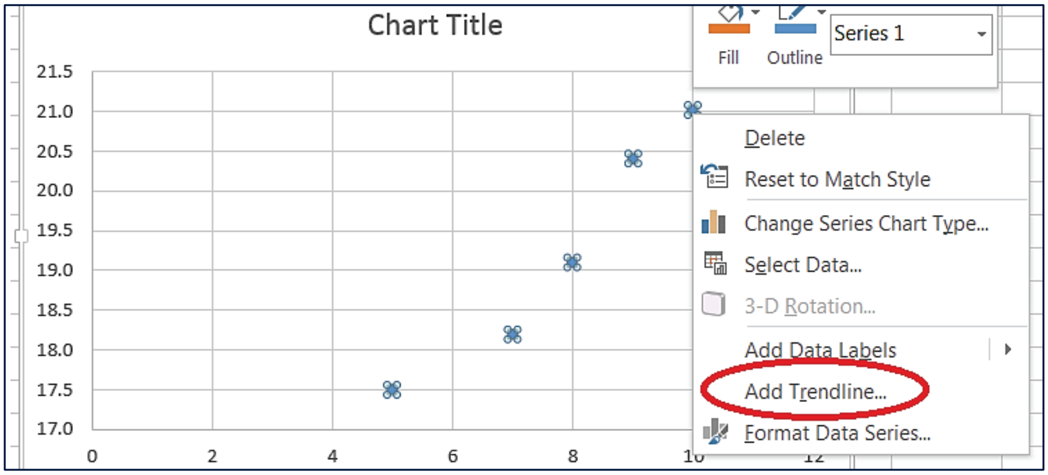


Figure 45

From the new pop-up screen, select the "Linear" graph and also check both "Display Equation on chart" and "Display R-squared value on chart" to obtain the graph shown in ***Figure 46***. The correlation coefficient $r \approx 0.97$ is the square root of the displayed value $R^2 \approx 0.9434$.

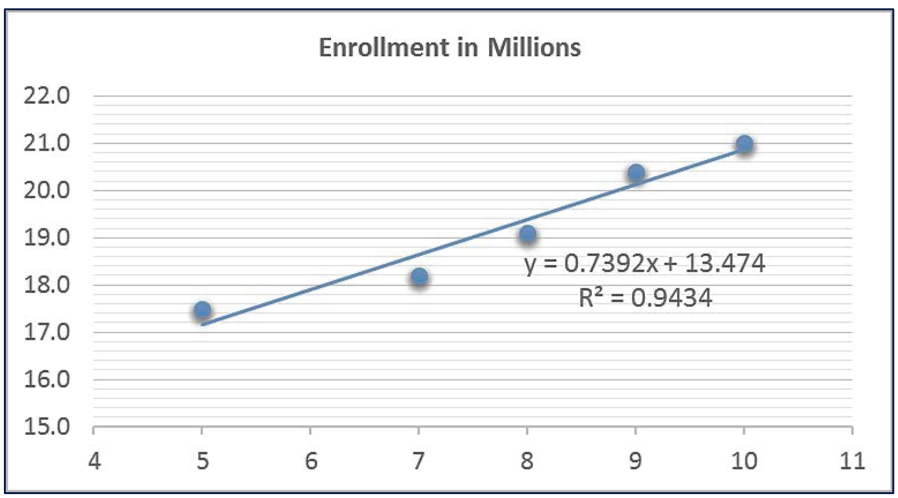


Figure 46

Notice that while adding the trendline, aside from selecting linear, you could select polynomial, exponential, or logarithmic regression. After the regression type has been selected, the Excel procedure for graphing non-linear regression and also displaying both the equation and the R^2 value is the same.

b. Quadratic Regression

The acreage (in millions) consumed by Forest Fires in the United States is given in ***Figure 47***.

Year	1985	1988	1991	1994	1997	2000	2003	2006	2009	2012
Acres	2.9	5	3	4.1	2.9	7.4	4	9.9	5.9	9.2

Figure 47

Let x=5 correspond to the year 1985. We'll find the best-fit quadratic function that models the data using Excel. Enter x values 5, 8, 11,…in cells B2:B11 and y values 2.9, 5.0, 3.0,… in cells C2:C11. Select cells B2:C11 and click the "Scatter" from the "Insert" tab ribbon. Right-click on any point of the graph and select "Add Trendline". Since we want to obtain a quadratic model, select "Polynomial" and select "Order 2" as shown in ***Figure 48***. Scroll down to check "Display Equation" and "display R-squared" as in ***Figure 49***.

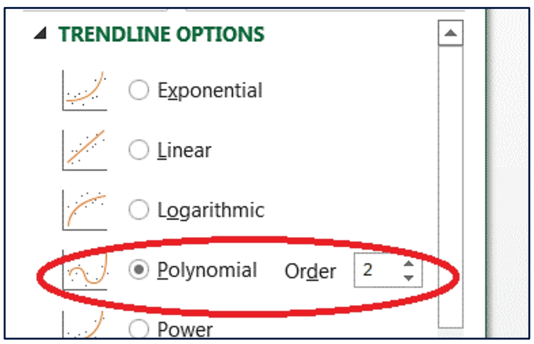


Figure 48

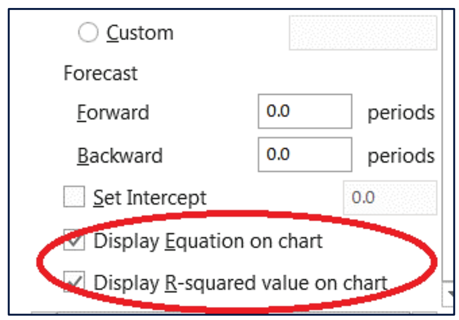


Figure 49

The graph that we obtained (***Figure 50***) shows that the quadratic model is $y = 0.0067x^2 - 0.0437x + 3.4338$, and that $r = \sqrt{0.5456} \approx 0.74$.

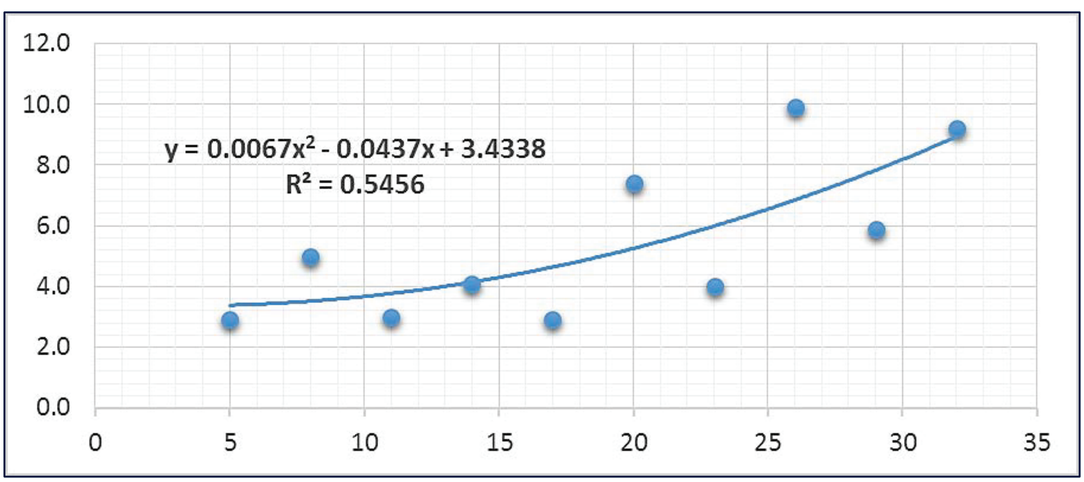


Figure 50

c. Exponential Regression

The table in ***Figure 51*** shows the purchasing power of a dollar in recent years, with the year 2000 being the base year. We'll use Excel to create an exponential regression model.

Year	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Purchasing power of $1	1.00	0.97	0.96	0.94	0.91	0.88	0.85	0.83	0.79	0.80	0.79	0.77	0.75

Figure 51

Enter values x=0, 1, 2,... for years 2000, 2001, 2002, …in cells B2:B1. One way to perform it easier is to enter 0 and 1 in cells B2 and B3 and selecting array B2:B3 and dragging the fill handle from the lower right corner over B4:B14 to copy the pattern. Enter values y=1.00, 0.97, 0.96,… in cells C2:C14. Select cells array B2:C14. Click the "Scatter" from the "Insert" tab ribbon. Right-click on any point of the graph and select "Add Trendline". Since we want to obtain an exponential model, select "Exponential" and scroll down to check "Display Equation" and "Display R-squared". ***Figure 52*** shows the result.

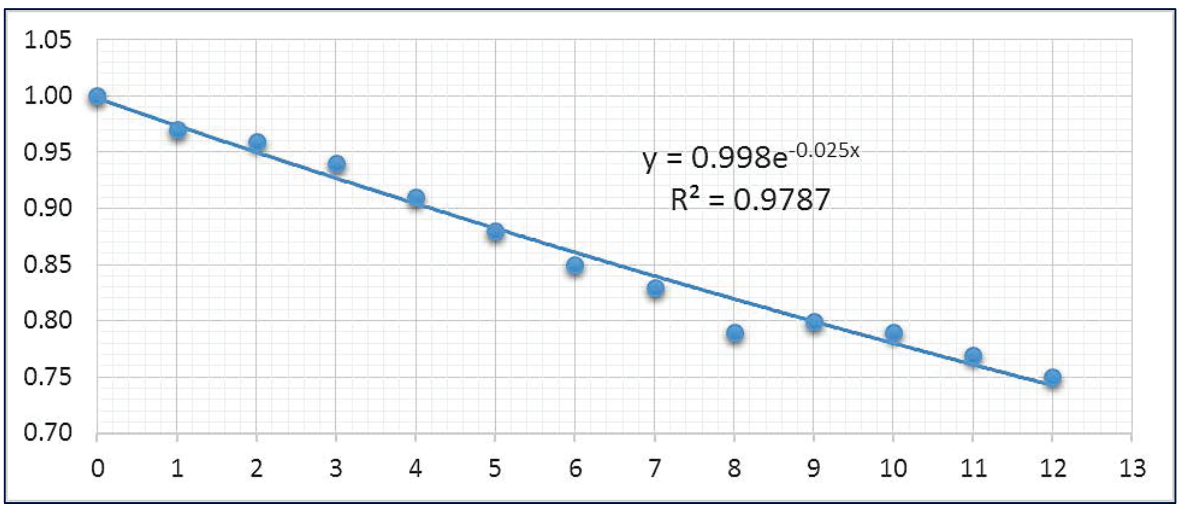


Figure 52

Chapter 5 Systems of Linear Equations and Matrices

a. Matrix Addition and Subtraction

We'll use Excel to perform operation A+B-C on matrices:

$$A=\begin{bmatrix}5.3 & -3 & -4.5\\ 0 & 2 & 0.5\\ 0 & 1 & 7\end{bmatrix}\quad B=\begin{bmatrix}1.3 & -6 & 12.7\\ 0 & 8 & 2\\ 4 & -1 & 2\end{bmatrix}\quad C=\begin{bmatrix}10 & 5 & 0\\ 10 & 5 & 0\\ 10 & 5 & 0\end{bmatrix}$$

Type the matrix entries as shown in ***Figure 53***. The result will be a 3x3 matrix: thus, highlight cells M2:O4 as the place for the future result. Type "=A2:C4+E2:G4-I2:K4". Press "Ctrl+Shift+Enter" at the same time to obtain the final answer, shown in ***Figure 54.***

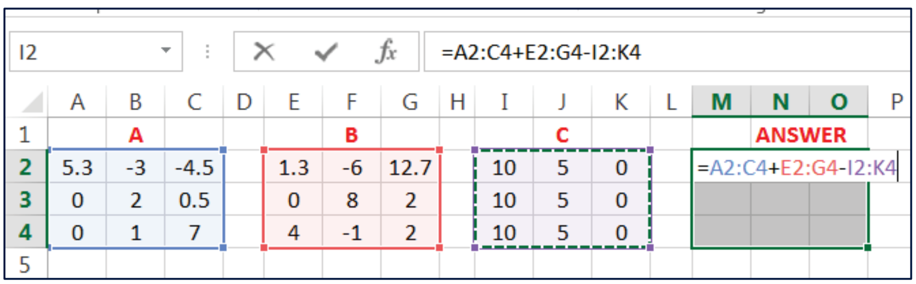


Figure 53

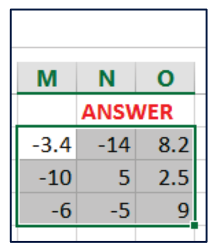


Figure 54

A faster way to type the formula "=A2:C4+E2:G4-I2:K4" is to highlight the cells array A2:C4, then enter "+" then highlight E2:G4, then enter "-"and highlight I2:K4.

b. Scalar and Matrix Multiplication

We'll use Excel to multiply the scalar and the matrix:

$$3\cdot\begin{bmatrix}44 & 2 & -8\\ 0.3 & 0 & 17\end{bmatrix}$$

Type the matrix entries as shown in ***Figure 55***. The result will be a 2x3 matrix, therefore, highlight cells E2:G3 as the place for the future answer. Type "=3*A2:C3". Press "Ctrl+Shift+Enter" at the same time to obtain the final answer, shown in ***Figure 56***.

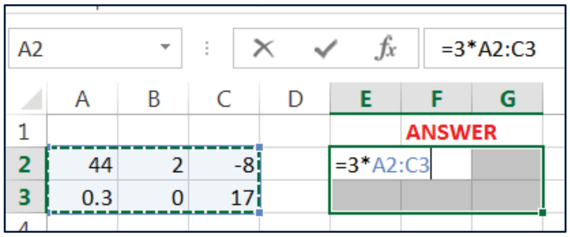


Figure 55

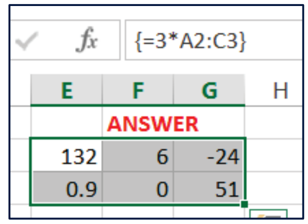


Figure 56

c. Product of Two Matrices

We'll use Excel to find the product AxB for matrices:

$$A = \begin{bmatrix} 5.3 & -3 & -4.5 \\ 0 & 2 & 0.5 \\ 0 & 1 & 7 \\ 6 & 6 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0.2 \\ 8 & 0 & 17 \end{bmatrix}$$

Type the matrix entries as shown in ***Figure 57***. The result will be a 4x3 matrix, therefore, highlight cells J2:L5 as the place for the future answer. Type "=MMULT(A2:C5,E2:G4)". Press "Ctrl+Shift+Enter" at the same time to obtain the final answer, shown in ***Figure 58*** .

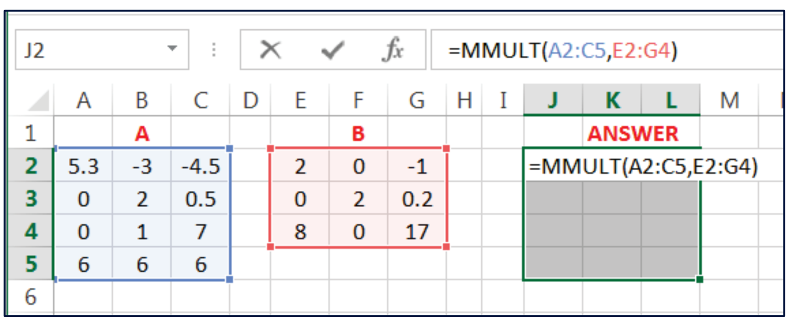


Figure 57

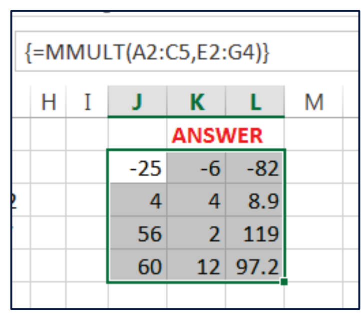


Figure 58

d. Inverse of a Matrix

We'll use Excel to find inverse of matrix A:

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0.2 \\ 8 & 0 & 17 \end{bmatrix}$$

Type the matrix entries as shown in ***Figure 59***. Highlight cells E2:G4 as the place for the future answer. Type "=MINVERSE(A2:C4)". Press "Ctrl+Shift+Enter" at the same time to obtain the final answer, shown in ***Figure 60***.

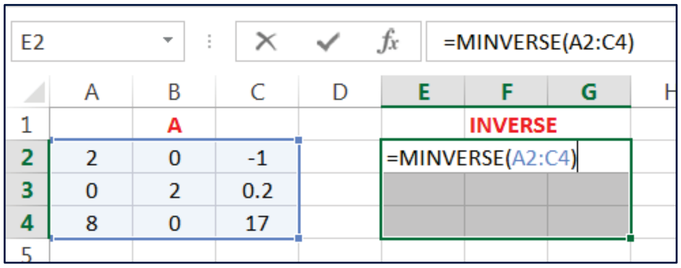


Figure 59

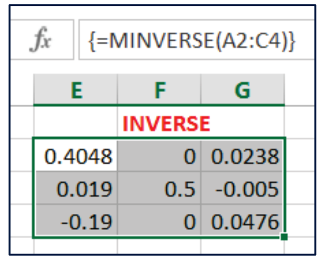


Figure 60

Format cells to fractions if needed, by right-clicking and selecting "Format Cells."

e. Determinant of a Matrix

We'll use Excel to find inverse of the matrix:

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0.2 \\ 8 & 0 & 17 \end{bmatrix}$$

Type the matrix entries as shown in ***Figure 61***. Type "=MDETERM(A2:C4)" in any cell and enter.

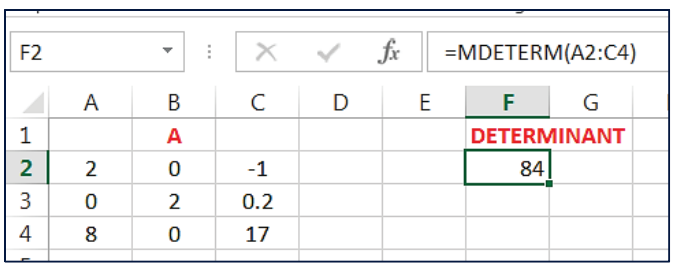


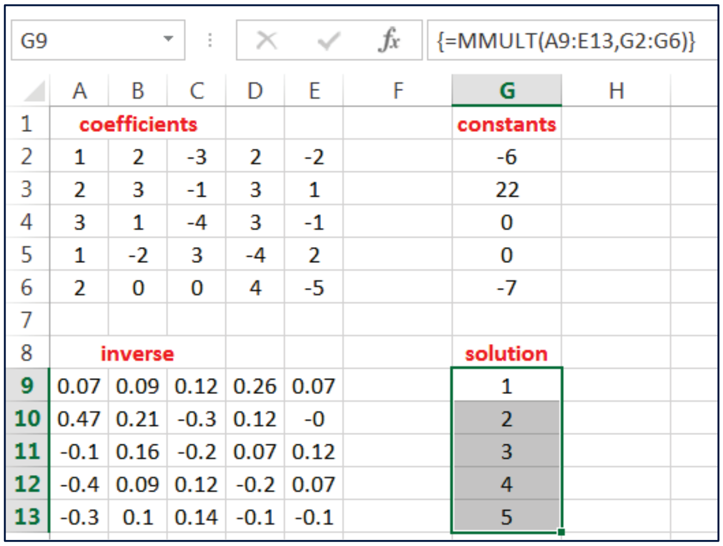
Figure 61

f. Solving System Using Matrices

We'll use Excel to solve the system:

$$\begin{aligned} x_1 + 2x_2 - 3x_3 + 2x_4 - 2x_5 &= -6 \\ 2x_1 + 3x_2 - x_3 + 3x_4 + x_5 &= 22 \\ 3x_1 + x_2 - 4x_3 + 3x_4 - x_5 &= 0 \\ x_1 - 2x_2 + 3x_3 - 4x_4 + 2x_5 &= 0 \\ 2x_1 + 4x_4 - 5x_5 &= -7 \end{aligned}$$

Type the coefficients and constants as shown in ***Figure 62***. To find inverse of the coefficients matrix highlight cells A9:E13 as the place for the future inverse. Type "=MINVERSE(A2:E6)". Press "Ctrl+Shift+Enter" at the same time to obtain the inverse.



G9 | {=MMULT(A9:E13,G2:G6)}

	A	B	C	D	E	F	G	H
1	coefficients						constants	
2	1	2	-3	2	-2		-6	
3	2	3	-1	3	1		22	
4	3	1	-4	3	-1		0	
5	1	-2	3	-4	2		0	
6	2	0	0	4	-5		-7	
7								
8	inverse						solution	
9	0.07	0.09	0.12	0.26	0.07		1	
10	0.47	0.21	-0.3	0.12	-0		2	
11	-0.1	0.16	-0.2	0.07	0.12		3	
12	-0.4	0.09	0.12	-0.2	0.07		4	
13	-0.3	0.1	0.14	-0.1	-0.1		5	

Figure 62

If we multiply the inverse matrix by the constants matrix, the result will be the solutions matrix. Highlight cells G9:G13 as the place for the future solution matrix. Type "=MMULT(A9:E13,G2:G6)", press "Ctrl+Shift+Enter" at the same time and the solution $x_1 = 1,\ x_2 = 2,\ x_3 = 3,\ x_4 = 4,\ x_5 = 5$ will appear in cells G9:G13.

Chapter 6 Linear Programming

a. Graphing an Inequality

To graph the inequality $y \geq 2x - 5$ in Excel we need to plot the line $y = 2x - 5$. Enter x values -5, -4, -3 …in A2:A15. Enter the formula "=2*A2-5" in cell B2 and copy it down to cells B2:B15 to obtain y values. Highlight x-y coordinates in A2:B15. From the "Insert" Tab select "Scatter with Smooth Lines", as shown in ***Figure 63***.

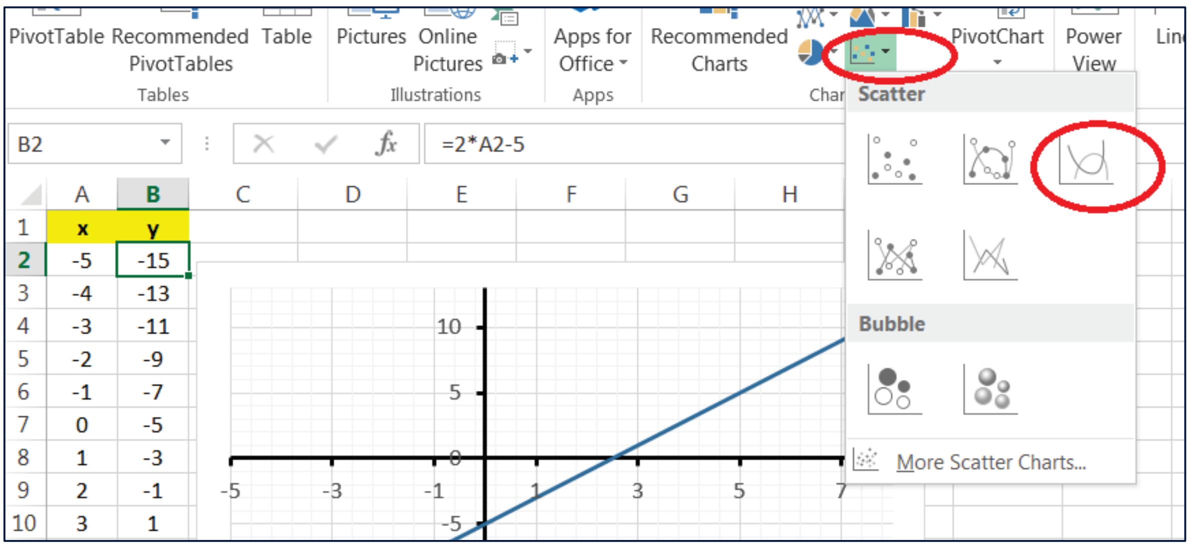


Figure 63

Shading the half plane is done using "Freeform" graphical tool from "Shapes", found as in ***Figure 64.*** Click on the three corners to define the triangular shape of the half plane. To change the transparency of the triangular shade of the half plane, click on it. The "Drawing Tools" tab will appear. Click on "Format Shape" that is a small box in bottom-right corner of "Shape Styles" as in ***Figure 65***. Change the transparency of the shape in the new menu that will appear to the right.

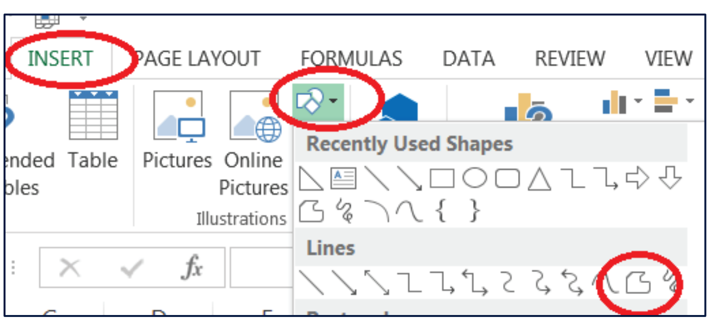


Figure 64

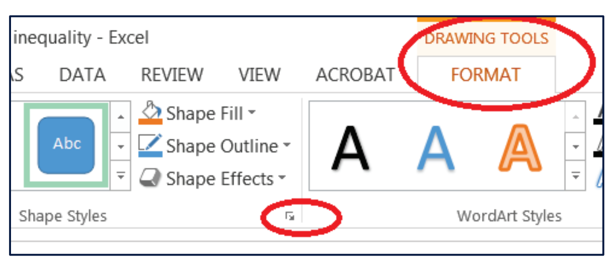


Figure 65

The graph of the inequality $y \ge 2x - 5$ is shown in ***Figure 66.***

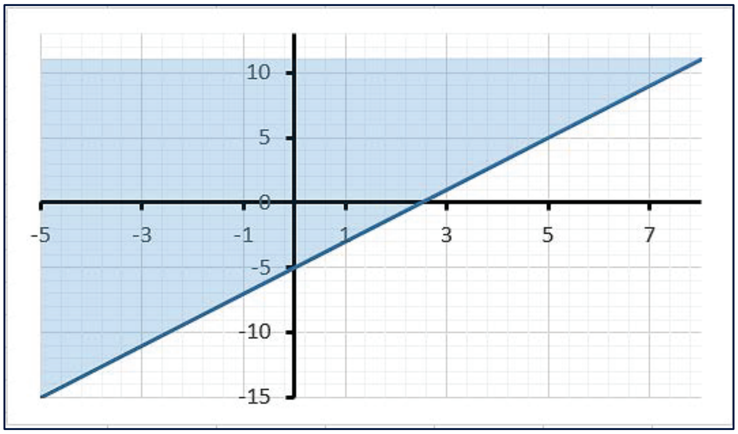


Figure 66

b. Graphing Systems of Inequalities

We'll graph the following system of inequalities

$$\begin{cases} y \ge -2x + 1 \\ y > \dfrac{3}{2}x - 1 \end{cases}$$

To graph the corresponding lines, first enter x values -3, -2, -1 …in cells A2:A8. To calculate y values for the first line points enter the formula "=-2*A2+1" in cell B2, and copy it in cells B3:B8. An easy way to do it is by dragging the fill handle from the lower right corner. To calculate y values for the second line points enter the formula "=(3/2)*A2-1" in cell C2, and copy it in cells C3:C8. Highlight x-y coordinates in cells array A2:C8. From the "Insert" tab ribbon select "Scatter with Smooth Lines". The second line should be dashed. Right-click exactly on the second line and select "Outline" as shown in ***Figure 67*** . One of options that will appear is "Dashes".

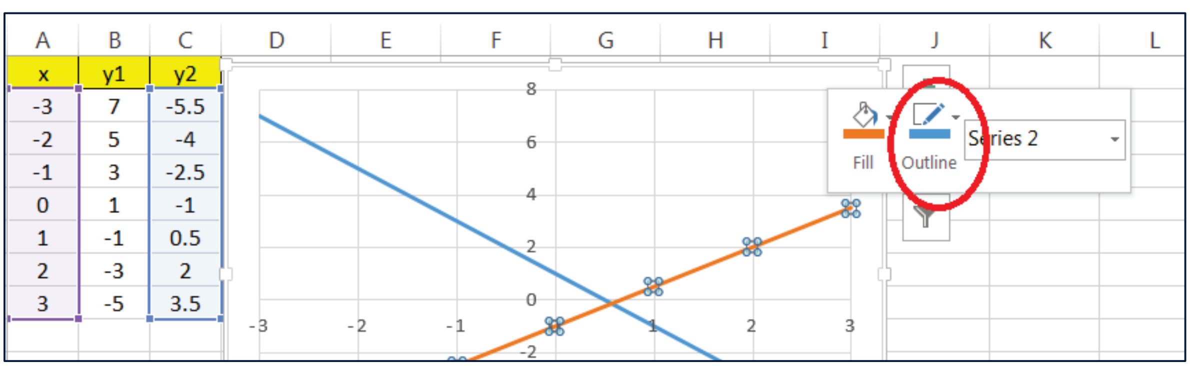


Figure 67

Shading the half plane is done using "Freeform" graphical tool from "Shapes" in "Insert" tab ribbon. Click on the corners to define the shaded shape. To change the transparency of the shaded area of the half plane, click on it. The "Drawing Tools" tab will appear. Click on "Format Shape" to change the transparency of the shape. ***Figure 68*** shows the final result.

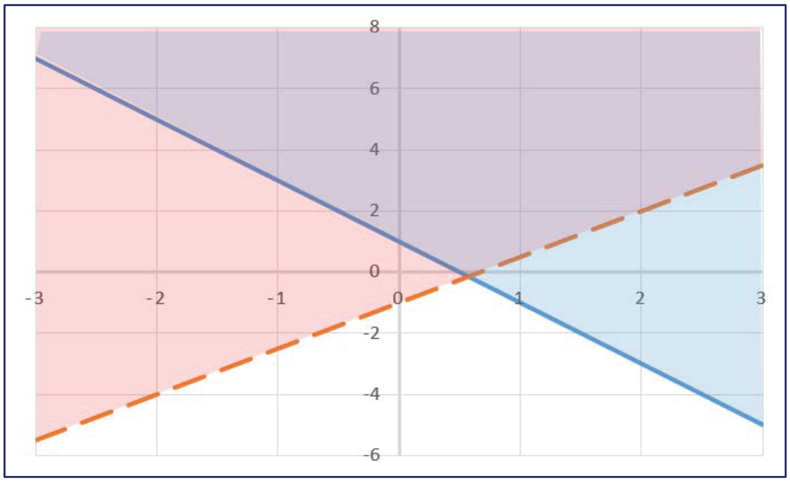


Figure 68

c. Maximization

We'll solve the following linear programming problem:

$$\begin{aligned} \text{Maximize } & z = 8x + 12y \\ \text{subject to } & 40x + 80y \le 560 \\ & 6x + 8y \le 72 \\ & x \ge 0, y \ge 0 \end{aligned}$$

Excel's "Solver", one of add-ins, can quickly find solutions to linear programming problems of all types. Here is how it works: In case that we select x=0 and y=0 the value of objective function is $z = 8 \cdot 0 + 12 \cdot 0 = 0$ and both constraints are satisfied since $40 \cdot 0 + 80 \cdot 0 \le 560$ and $6 \cdot 0 + 8 \cdot 0 \le 72$. Solver will keep changing values of x and y until the maximal value of objective function z is obtained.

	A	B
1		
2	Objective	
3	z	=8*B10+12*B11
4		
5	Constraints	
6	first	=40*B10+80*B11
7	second	=6*B10+8*B11
8		
9	Decision	
10	x=	0
11	y=	0

Figure 69

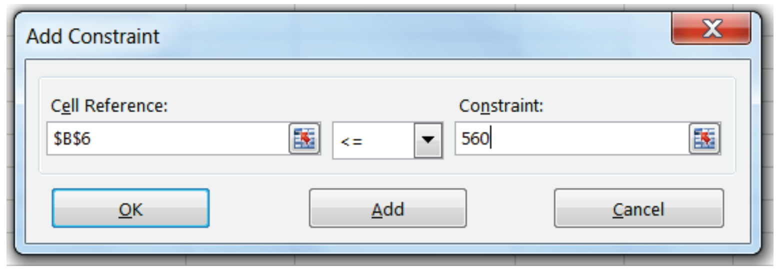


Figure 70

Enter the linear programming problem information as shown in ***Figure 69***. If you entered everything correctly, cells B3, B6 and B7 will be showing zeros.

Select "Solver" from the "Data" tab ribbon "Analysis" tools and fill entries in the following way: first check box 'Make Unconstrained Variables Non-Negative' since our *x* and *y* values cannot be negative. The next step is to define the first constraint. Click on "Add", and a new window will appear with three empty boxes as in ***Figure 70***. Click on the constraint cell B6 to define it as "Cell Reference", then click on arrow in the middle and select "<=" from the drop-down menu, and finally type 560 in the "Constraint" box as it is the maximal value that cell B6 may assume. To define the second constraint, click on "Add", and a new window will appear with three empty boxes. Click on the constraint cell B7 to define it as "Cell Reference", then select "<=" from the drop-down menu, and finally type 72 in the "Constraint" box as it is the maximal value that cell B7 may assume. Click "OK". Set the remaining as shown in ***Figure 71***. The empty "Set Objective" box was filled by clicking on B3. An easy way to fill the empty "By Changing Variable Cells" is to select B10:B11. Click on "Solve" to obtain the final answer *x*=8 and *y*=3 as shown in ***Figure 72.***

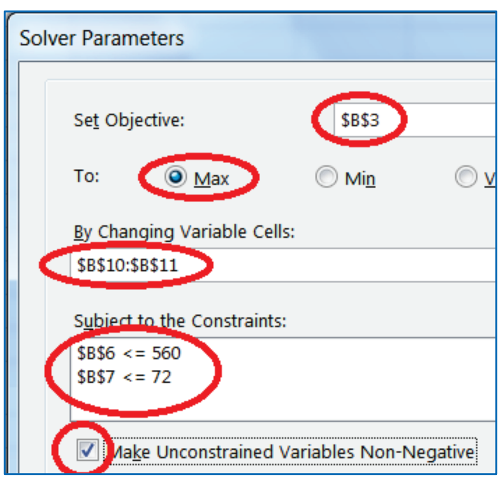


Figure 71

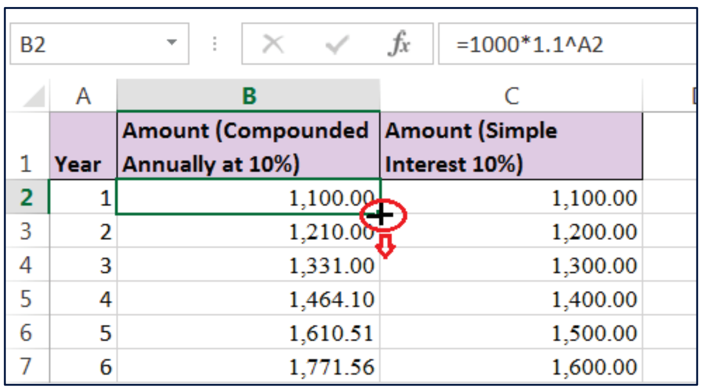
	A	B
1		
2	Objective	
3	z	100
4		
5	Constraints	
6	first	560
7	second	72
8		
9	Decision	
10	x=	8
11	y=	3
12		

Figure 72

Chapter 7 Mathematics of Finance

a. Compound Interest

A spreadsheet can be used to compare two interest schemes. We'll create a table with one column showing the growth of $1,000 investment at an annual rate of 10% and another column showing the maturity value with simple interest. Enter appropriate column headings to represent years, amount compounded annually, and simple interest amount. Fill column A with the numbers 1 through 20 to indicate the years since the principal was invested. An easy way to perform it is to enter "1" and "2" in cells A2 and A2, then select cell array A2:A3 and drag the fill handle from the lower right corner over cells A4:A21.
If $1,000 is invested at a rate of 10% compounded annually, the future value of the money is $A = 1000(1+0.1/1)^{1\cdot t} = 1000\cdot 1.1^{t}$ where t is the number of years. Thus, enter the formula "=1000*1.1^A2" in cell B2. Copy the formula into cells B3:B21. An easy way to do it is by dragging the fill handle from the lower right corner of the cell, shown in ***Figure 73***.



	A	B	C
1	Year	Amount (Compounded Annually at 10%)	Amount (Simple Interest 10%)
2	1	1,100.00	1,100.00
3	2	1,210.00	1,200.00
4	3	1,331.00	1,300.00
5	4	1,464.10	1,400.00
6	5	1,610.51	1,500.00
7	6	1,771.56	1,600.00

Figure 73

If the same amount is invested at a simple interest rate of 10%, then the future value of the money is $A = 1000(1+0.1t) = 1000+100t$. Enter the formula "=1000+100*A2" in cell C2. Copy the formula into cells C3:C21. The first 6 rows of the resulting table are shown.

b. Effective Rate

Excel has many built-in functions for calculating financial values. These functions are located in the "Financial" submenu of the "Function" menu.

Bank A offered its costumers 10% interest compounded annually, bank B offered 9.6% interest compounded monthly, while bank C offered 9.7% interest compounded quarterly. We'll use Excel built-in function "EFFECT(nominal_rate,npery)" to calculate the lowest APY. Type the following in any blank cell in a worksheet:

"=EFFECT(0.1,1)" to obtain APY of 0.1=10% at bank A,
"=EFFECT(0.096,12)" to obtain APY of 0.100339=10.0339% at bank B,
"=EFFECT(0.097,4)" to obtain APY of 0.100586=10.0586% at bank C.

c. Present Value for Compound Interest

The "PV" function calculates the present value of an account, when the nominal interest rate, total number of interest payments, and the future value of the account are known. In any empty cell, type "=", then click on arrow in the Name Box and select the function "PV" from the "Financial" submenu. A pop-up window will appear (see ***Figure 74***). Another way to find it is under "Financial" in "Formulas" tab ribbon.

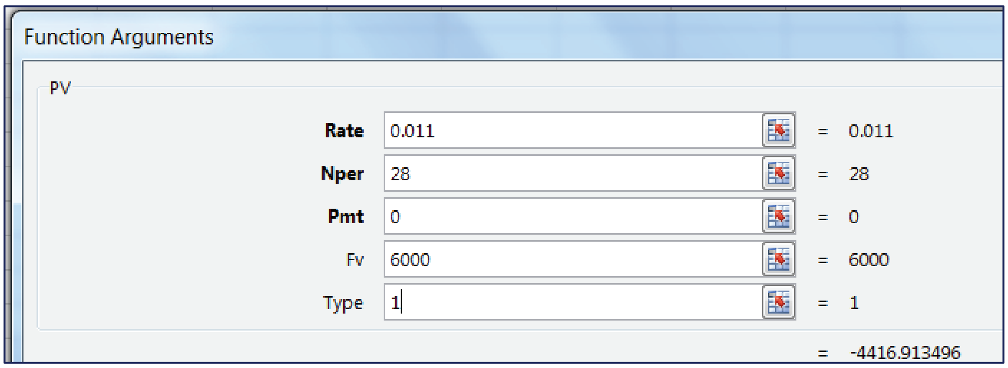


Figure 74

We'll calculate what amount deposited today at 4.4% compounded quarterly will amount to $6,000 in 7 years. Enter the interest rate per compounding period, as a decimal 0.044/4=0.011 (Rate); the total number of compound periods over the life of the account 7x4=28 (Nper); the amount that will be paid into the account by the account holder at the beginning of each period 0 (Pmt); then the desired future value of the account 6,000 (FV). Notice that "1" was entered as the type of the account. This indicates that the account holder will make a payment at the beginning of a period. However, in this example, the account holder will be making only one payment, not periodic payments; this is why "0" was entered for "Pmt". When you click "OK", the resulting present value of $4,416.91 will appear in the selected cell. This value will be shown as a red value, since this amount must be paid out by the account holder.

d. Future Value of the Ordinary Annuity

Excel has a built-in function, "FV", for calculating the future value of an ordinary annuity. This function is also located in the "Financial" collection of functions. From an empty cell, type "=", then click on arrow in the "Name Box" and select the function "FV" from the "Financial" submenu. Another way to find it is under "Financial" in "Formulas" tab ribbon. If you remember the exact name of this function, simply type it in any cell after entering "=".

We'll calculate the future value of the annuity if $1,500 is deposited at the end of each 6-month period for the next 5 years in an account paying 4% interest compounded annually. ***Figure 75*** shows the appropriate values for this example.

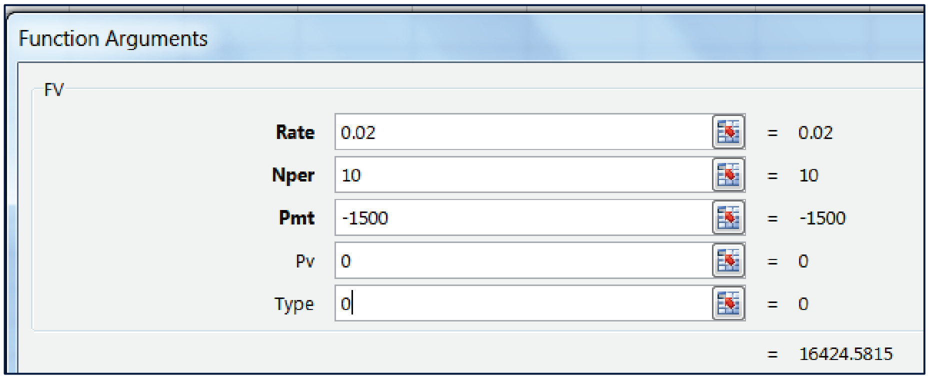


Figure 75

The annual interest rate per compounding period was entered as a decimal 0.04/2=0.01 (Rate), while the total number of compound periods over the life of the account was 5x2=10 (Nper). The payment, "Pmt", is entered as a negative number since this amount will be paid out by the account holder. Also note that for an annuity "0" should be entered as the present value. This time "0" was selected as the "Type", since the money will be deposited at the end of each period. The future value is $16,424.58.

e. Future Value of the Annuity Due

Excel's built-in function, "FV" will calculate the future value in case when $1,500 is deposited at the beginning of each 6-month period for the next 5 years in an account paying 4% interest compounded annually. From an empty cell, type "=", then click on arrow in the "Name Box" and select the function FV from the "Financial" submenu. Another way to find it is under "Financial" in "Formulas" tab ribbon. ***Figure 76*** shows the appropriate values for this example. Note that for an annuity due "1" should be entered as the "Type". The future value is $16,753.07.

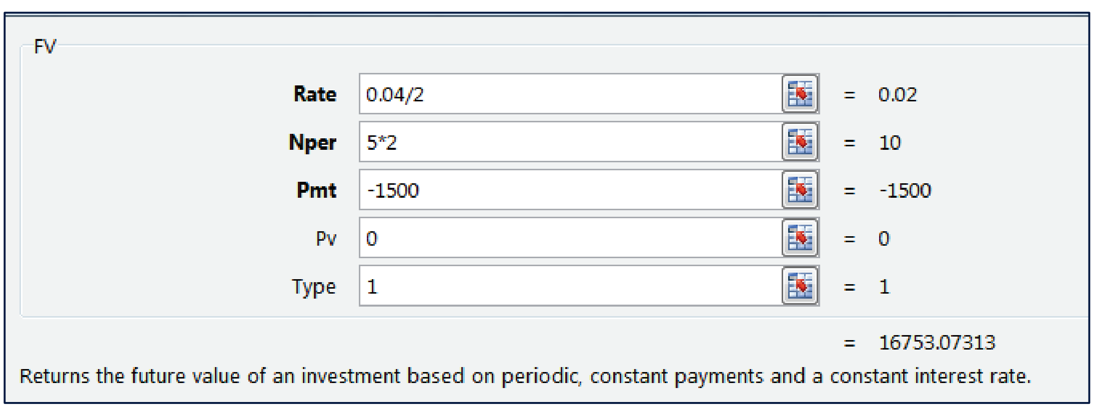


Figure 76

f. Amortization Payments

Excel has a built-in function that will calculate the monthly mortgage payment for the family that purchased a home for $272,900, with the down payment of 20%. They take out a 30-year mortgage at an annual interest rate of 3.43%. From any empty cell, type "=", then click on arrow in the Name Box and select the function "PMT" from the "Financial" submenu. A pop-up window will appear. Fill the entries as indicated in ***Figure 77***. The monthly payment is $971.84.

PMT

Rate	0.0343/12	=	0.002858333
Nper	12*30	=	360
Pv	0.80*272900	=	218320
Fv	0	=	0
Type	0	=	0
		=	-971.8435088

Figure 77

g. Amortization Schedules

To determine the exact remaining balance after each loan payment, financial institutions normally use an amortization schedule, which lists how much of each payment is interest, how much goes to reduce the balance, and how much is still owed after each payment.

We'll create an amortization schedule for Beth Hill, who borrowed $1,000 for one year at 12% annual interest, so her monthly payments are $88.85 each. Enter the appropriate column headings, like in ***Figure 78.*** Fill column A with the numbers 0 through 12, indicating the payments for this example. At this point, you may wish to format the cells in columns B, C, D, and E so that numerical values are displayed as currency. "Chapter 0 Getting Started" of this manual explains the formatting. In cell E2, enter the amount of the loan, $1000. Since monthly payments are $88.85, fill cells B3:B14 with this amount. Since interest is 1% per month (12% compounded monthly), enter the formulas into the cells C3, D3, and E3 as shown in ***Figure 78.***

	A	B	C	D	E
1	Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
2	0				$1,000.00
3	1	$88.85	=0.01*E2	=B3-C3	=E2-D3

Figure 78

Drag the fill handle from the lower-right corner of cell C3 to fill the rest of column C. Do the same for columns D and E. The resulting amortization schedule table is featured in the ***Figure 79.***

	A	B	C	D	E
1	Payment Number	Amount of Payment	Interest for Period	Portion to Principal	Principal at End of Period
2	0				$1,000.00
3	1	$88.85	$10.00	$78.85	$921.15
4	2	$88.85	$9.21	$79.64	$841.51
5	3	$88.85	$8.42	$80.43	$761.08
6	4	$88.85	$7.61	$81.24	$679.84
7	5	$88.85	$6.80	$82.05	$597.79
8	6	$88.85	$5.98	$82.87	$514.91
9	7	$88.85	$5.15	$83.70	$431.21
10	8	$88.85	$4.31	$84.54	$346.67
11	9	$88.85	$3.47	$85.38	$261.29
12	10	$88.85	$2.61	$86.24	$175.05
13	11	$88.85	$1.75	$87.10	$87.96
14	12	$88.85	$0.88	$87.97	($0.02)

Figure 79

Chapter 8 Probability

a. Factorial, Permutations, Combinations

To calculate factorial “9!” enter “=FACT(9)" in any empty cell. The result is 362880. If you cannot remember the name of the function, find it in the “Formulas” tab ribbon, among “Math & Trig” functions, or find it using arrow in the “Name Box” that appears after entering “=” in any cell.

To evaluate the number of permutations “$_8P_3$” type the following in any empty cell "=PERMUT(8,3)". The result is 336. If you cannot remember the name of the function, go to tab “Formulas”, click on “Insert Function” and find it among “Statistical” functions.

To evaluate the number of combinations “$_{15}C_{12}$” type the following in any empty cell "=COMBIN(15,12)". The result is 445. If you cannot remember the name of the function, go to tab “Formulas”, click on “Insert Function” and find it among “Statistical” functions.

b. Expected Value

The table in ***Figure 80*** gives each possible outcome of a raffle drawing and the corresponding probabilities. To calculate the expected value of winning from this example**,** click on an empty cell and type “=”. The arrow in the Name Box will help you locate the function "SUMPRODUCT(Array1,Array2)" from the "Math & Trig" submenu. Select the *x* values from the table B2:E2 for "Array1", and select their corresponding probabilities for "Array2", as shown. Excel will then multiply each entry from Array1 with its corresponding entry from Array2, and subsequently add all such products. A faster way is to type “=SUMPRODUCT(B1:E1,B2:E2)” in any cell. The result of this calculation shows that, on average, one would lose $.74 per ticket.

G1 =SUMPRODUCT(B1:E1,B2:E2)

	A	B	C	D	E	F	G
1	x	$399.00	$79.00	$19.00	-$1.00		-$0.74
2	P(x)	0.0005	0.0005	0.0010	0.9980		

Figure 80

c. Binomial Probability

A 9-person jury acts independently of the other members and makes correct decision with probability 0.65. We’ll find probability that the majority (at least 5) will reach the correct decision.

Excel has a built-in function, "BINOM.DIST" that allows binomial probabilities calculations. From an empty cell insert the function “BINOM.DIST" by clicking “=” and then using arrow in the Name Box to list offered functions. You can also find it in the “Formulas” tab ribbon, among “Math & Trig” functions, or using arrow in the “Name Box” that appears after entering “=” in any cell.

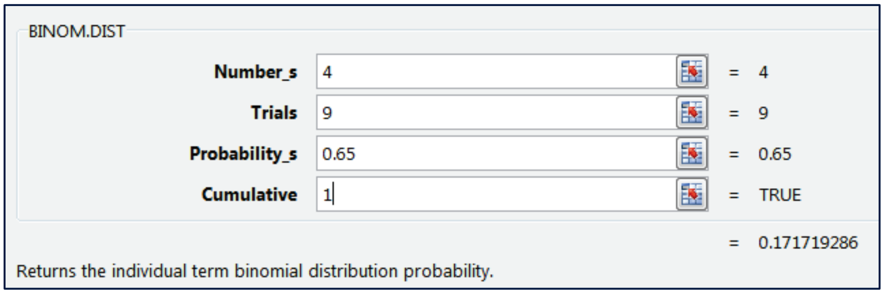


Figure 81

Figure 81 shows the appropriate values for completing this example. The value “1” is used in the last text box to indicate that we are calculating the cumulative probability $P(x \leq 4)$. Click on “OK” and the Excel calculation will show that the probability of 4 or fewer jurors making the correct decision is 0.171719. By subtracting this number from 1, we get 0.828281: this is the probability that at least 5 of the jurors will make the correct decision.

Chapter 9 Statistics

a. Frequency Distributions

The annual tuition in thousands for a random sample of 40 community colleges is:

3.8	1.1	2.5	3.5	4.0	3.8	3.9	4.2	3.1	3.9
5.0	1.1	2.0	3.4	3.5	1.1	2.3	4.8	1.1	3.1
3.5	5.1	3.3	3.2	2.1	5.3	2.6	2.3	2.5	3.6
4.9	3.0	2.4	3.6	3.1	3.7	5.1	3.5	2.0	4.4

To create a frequency histogram with Excel, begin by creating the frequency table. First enter the tuition in cellsA2:A41. Enter "1.4" in cell B2 to indicate that the upper bound of the first frequency interval 1.0-1.4 is 1.4. Enter "1.9" in B3 to indicate that the upper bound of the second frequency interval 1.5-1.9 is 1.9 and so on. Select cells C2:C10 to display the frequency column. Type “=” and insert the function "FREQUENCY" from the functions menu in the Name Box as in ***Figure 82***. Fill the data as shown. Press CONTROL+SHIFT+ENTER simultaneously to finish the frequency table.

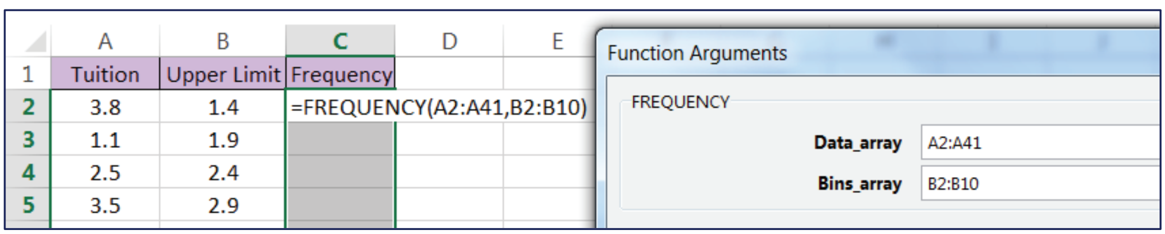


Figure 82

The newly created frequency table in cells C2:C10 will be used to draw histogram. We want to show the frequency intervals on the *x*-axes of our histogram. That is why we’ll enter them in column D.

Select the frequency values in cells C2:C10 from the table and click on the "Insert" tab. Use the "2-D Column" chart as in ***Figure 83*** and your chart will appear.

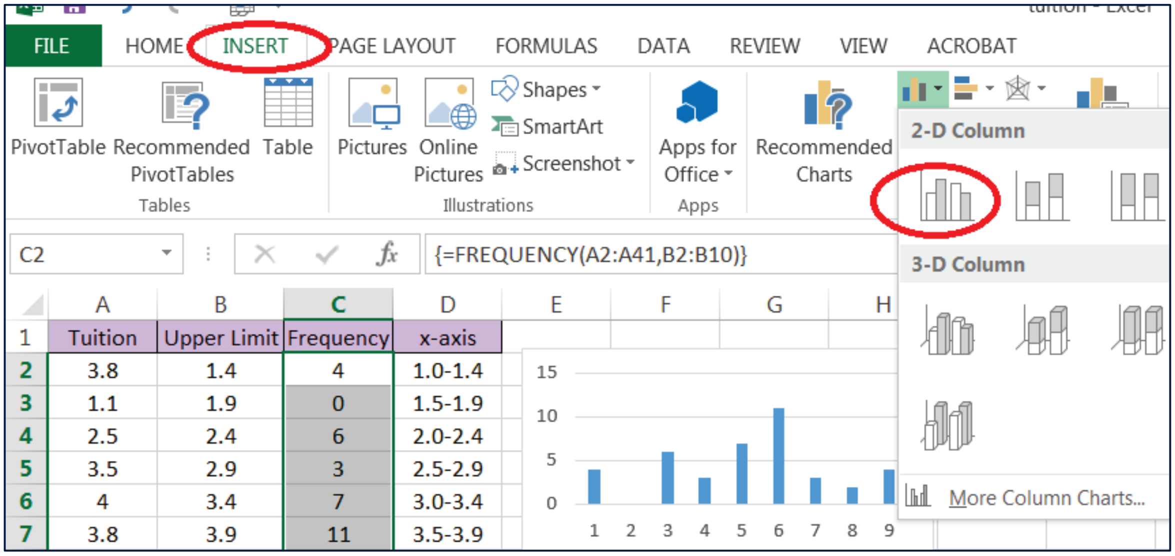


Figure 83

To define what will be seen on the *x*-axis click anywhere on the graph. The Chart Filter will be one of three symbols that will appear to the right from the chart area. Double-click it and then click on "Select Data" at the bottom of the new window. Click on "Edit" shown in ***Figure 84*** and select cells D2:D10 instead as your choice of *x*-axis labels in the new window as in ***Figure 85***.

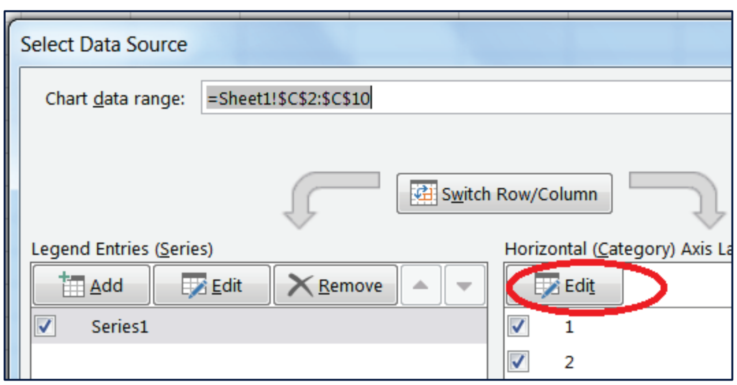


Figure 84

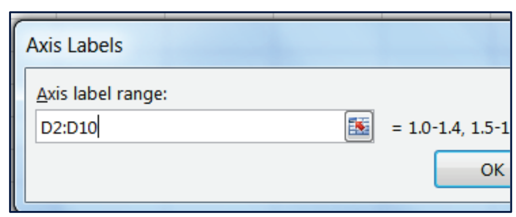


Figure 85

To make your bars wider, click on one of them and right-click. Select "Format Data Series" and change "Gap Width" to a desired "0". ***Figure 86*** shows the histogram.

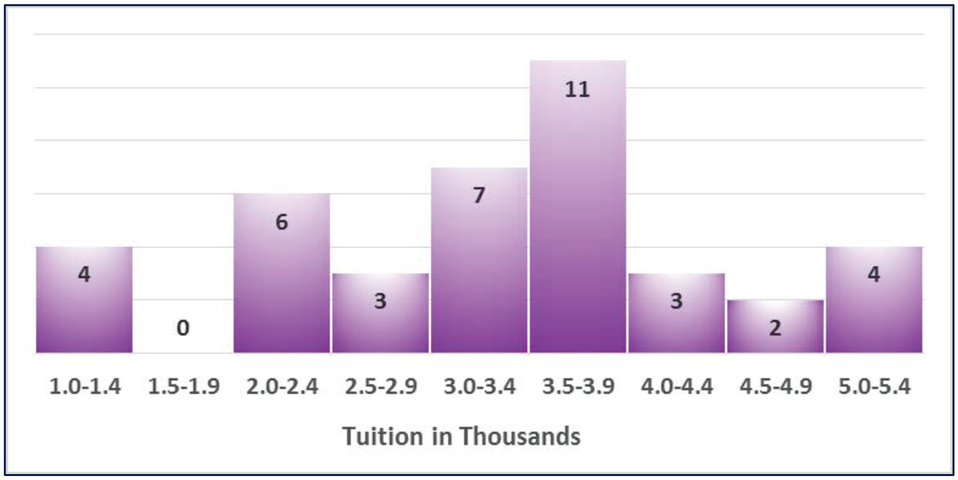


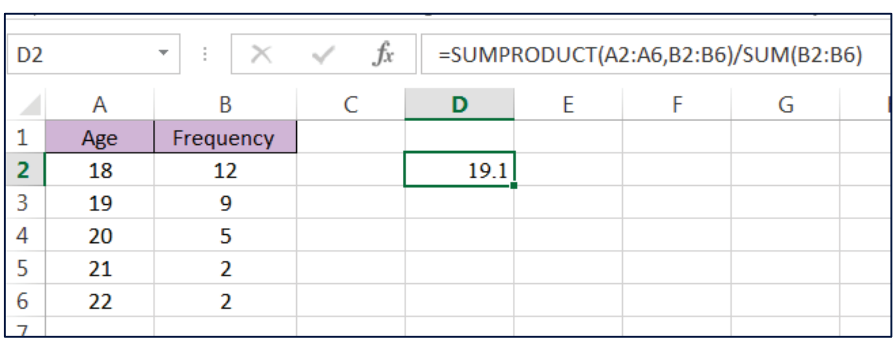
Figure 86

The frequency polygon might be created by following the same routine, except for using "Line Chart" instead of "Column Chart" from the Chart Wizard standard chart types.

b. Mean, Median and Mode

The built-in function, "AVERAGE", will calculate the arithmetic mean. To find the mean of numbers 1248, 1362, 1800, 1943, 1951 type the numbers in cells A1:A5. Type "=AVERAGE(A1:A5)" in any cell. The mean of 1669.8 will appear after entering. This function might be also found in the "Functions" tab ribbon as one of the "Statistical" functions. If you remember the beginning of the name of the function type it in any cell after the symbol "=" and Excel will help you remember the rest by offering you choices.

The mean of data that has been arranged into a frequency distribution table is found in a similar way. An instructor collected data on the age of her finite mathematics class students and the frequency of each age. The age is entered in cells A2:A6 while their frequencies are entered in cells B2:B6. The mean of 19.1 will be obtained by using the formula: "=SUMPRODUCT(A2:A6,B2:B6)/SUM(B2:B6)", as shown in ***Figure 87***.

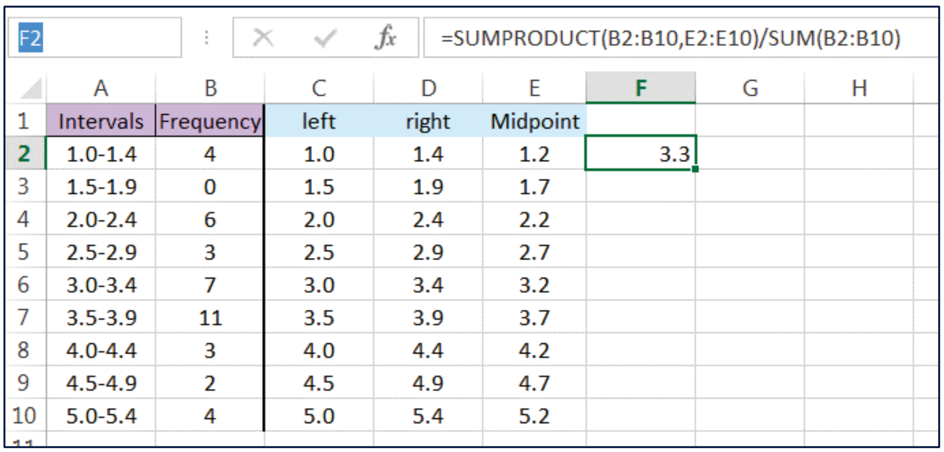

D2 =SUMPRODUCT(A2:A6,B2:B6)/SUM(B2:B6)

	A	B	C	D	E	F	G
1	Age	Frequency					
2	18	12		19.1			
3	19	9					
4	20	5					
5	21	2					
6	22	2					
7							

Figure 87

The mean from the grouped frequency distribution is calculated in a similar way. The annual tuition in thousands for a random sample of 40 community colleges is grouped by tuition intervals and sorted in a frequency table. Excel will calculate midpoints automatically.

To prepare our data first separate endpoints of intervals into two columns, C and D. Type the left endpoints of intervals in cells C2:C10, and type the right endpoints in cells D2:D10 as in ***Figure 88***. Cell E2 will contain the midpoint *x* of the first interval after entering the formula "=(C2+D2)/2". Copy this formula through cells E3:E10 to apply the formula for midpoint to all given intervals. A fast way to do it is by dragging the fill handle in the lower right-hand corner over column D. To find the mean from the grouped frequency distribution it is helpful to use the built-in function, "SUMPRODUCT" from the "Math & Trig" submenu. This function will multiply midpoints E2:E10 with their corresponding frequencies B2:B10 and then add all such products. To obtain the mean from the grouped frequency distribution the result of this calculation should be divided by the sum of all frequencies, found in cells B2:B10. The formula "=SUMPRODUCT(B2:B10,E2:E10)/SUM(B2:B10)" entered in any cell will return 3.3, which is the mean of the grouped frequency distribution.


F2 =SUMPRODUCT(B2:B10,E2:E10)/SUM(B2:B10)

	A	B	C	D	E	F	G	H
1	Intervals	Frequency	left	right	Midpoint			
2	1.0-1.4	4	1.0	1.4	1.2	3.3		
3	1.5-1.9	0	1.5	1.9	1.7			
4	2.0-2.4	6	2.0	2.4	2.2			
5	2.5-2.9	3	2.5	2.9	2.7			
6	3.0-3.4	7	3.0	3.4	3.2			
7	3.5-3.9	11	3.5	3.9	3.7			
8	4.0-4.4	3	4.0	4.4	4.2			
9	4.5-4.9	2	4.5	4.9	4.7			
10	5.0-5.4	4	5.0	5.4	5.2			

Figure 88

To find the median of numbers 12, 7, 10, 20, 22, type the numbers in cells A1:A5, type "=MEDIAN(A1:A5)" in any cell. The result is 12. If you cannot remember the name of the function, go to tab "Formulas", click on "Insert Function" and find it among "Statistical" functions.

To find the mode of numbers 13, 7, 13, 20, 22, type the numbers in cells A1:A5, type "=MODE(A1:A5)" in any cell. The result is 12. If you cannot remember the name of the function, go to tab "Formulas", click on "Insert Function" and find it among "Statistical" functions.

c. Measures of Variation

To find the sample variance of numbers 2, 8, 3, 2, 6, 11, 31, 9, type the numbers in cells A1:A8, and type "=VAR.S(A1:A8)" in any cell. The result is 90.28571. If you cannot remember the name of the function, go to tab "Formulas", click on "Insert Function" and find it among "Statistical" functions.

To find the sample standard deviation of numbers 2, 8, 3, 2, 6, 11, 31, 9, type the numbers in cells A1:A8, then type and enter "=STDEV.S(A1:A8)" in any cell. The result is

9.50188. The tab “Formulas” ribbon is another place where you can find this function.

On the other hand, if you're working with the entire population, you calculate the standard deviation using "=STDEV.P(A1:A8)". The result is 8.889.

The annual tuition in thousands for a random sample of 40 community colleges is grouped by tuition intervals and sorted in a frequency table. We’ll find the standard deviation for the grouped data.

Previously we showed how to use a spreadsheet to calculate midpoints as averages of endpoints. To calculate the mean for the grouped distribution the following formula was used: "=SUMPRODUCT(B2:B10,E2:E10)/SUM(B2:B10)" in cell H2. The resulting mean was $\overline{x} = 3.3$.

The function "=SUM(B2:B10)" entered into H5 will add the frequencies giving us the final result *n*=30. We can calculate fx^2 for each interval by entering the formula "=B2*E2^2" in cell F2 and dragging the fill handle in the lower right-hand corner over column F. The sum of these values will be calculated by using the formula "=SUM(F2:F10)" in cell H8. The value of *s* will be calculated using the formula $\sqrt{\left(\sum fx^2 - n\overline{x}^2\right)/\left(n-1\right)}$. Enter “=SQRT((H8-H5*H2^2)/(H5-1))" in H11 to obtain $s \approx 1.1163$. ***Figure 89*** shows the final table.

H11 | =SQRT((H8-H5*H2^2)/(H5-1))

	A	B	C	D	E	F	G	H
1	Intervals	Frequency	left	right	Midpoint	fx^2		Mean
2	1.0-1.4	4	1.0	1.4	1.2	5.76		3.3
3	1.5-1.9	0	1.5	1.9	1.7	0		
4	2.0-2.4	6	2.0	2.4	2.2	29.04		n
5	2.5-2.9	3	2.5	2.9	2.7	21.87		40
6	3.0-3.4	7	3.0	3.4	3.2	71.68		
7	3.5-3.9	11	3.5	3.9	3.7	150.59		$\sum fx^2$
8	4.0-4.4	3	4.0	4.4	4.2	52.92		484.2
9	4.5-4.9	2	4.5	4.9	4.7	44.18		
10	5.0-5.4	4	5.0	5.4	5.2	108.16		s
11								1.116313

Figure 89

If you only know frequency values *f* in column B and midpoints *x* in column E, and you'd like to calculate the standard deviation for grouped data directly, click on any empty cell and enter the formula “=SQRT(((SUMPRODUCT(B2:B10,E2:E10^2)-((SUMPRODUCT(B2:B10,E2:E10))^2/SUM(B2:B10)))/(SUM(B2:B10)-1)))”.
This is the formula for the standard deviation for a grouped distribution from your text after $n = \sum f$ and $\overline{x} = \sum fx / \sum f$ is used in it, and it can be simplified to:

$$s = \sqrt{\left(fx^2 - \left(\sum fx\right)^2 / \sum f\right) / \left(\sum f - 1\right)}.$$

d. Normal Distributions

Excel has a built-in function that returns cumulative areas under the normal standard curve. We'll use it to find the area between z=0 and z=1.4. Entering "=NORMSDIST(1.4)" in any empty cell will return the answer 0.919243, which is the area shaded in ***Figure 90***. The area between z=0 and z=1.4 is calculated as 0.919243-0.5=0.419243. If you cannot remember the name of the function, go to tab "Formulas", click on "Insert Function" and find it among "Statistical" functions.

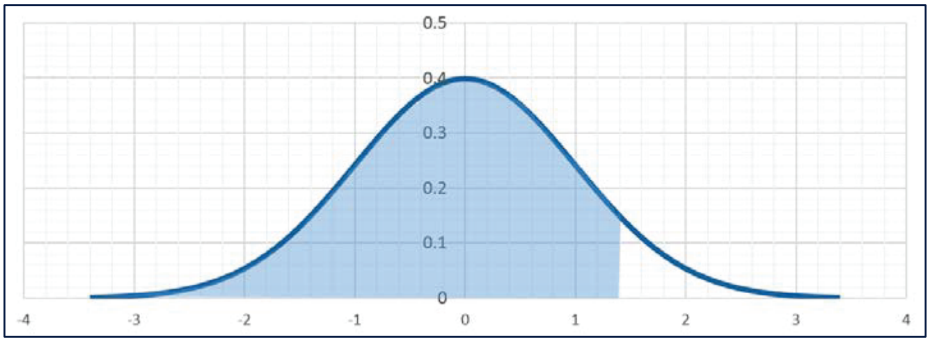


Figure 90

To graph the normal standard curve enter -3.4, -3.2, -3, -2.8… 3.2, 3.4 in cells A1:A35, then insert the formula "=NORM.DIST(A1,0,1,0)" in B1 and copy it in cells B2:B35. Select cells A1:B35 and insert "Scatter with Smooth Lines" graph. One easy way to shade the area under the graph is by using "Freeform" illustrations tool from the "Insert" tab.

For the normal distribution that has mean of 6 and standard deviation of 12, we'll determine what percentage of the distribution occurs for x less than or equal to 9. For that reason we'll use the function"=NORM.DIST(x, mean, standard_dev, cumulative)" from the statistical menu. Entering"=NORM.DIST(9,6,12,1)" in any empty cell will return answer 0.5987, which is the area shaded in ***Figure 91***

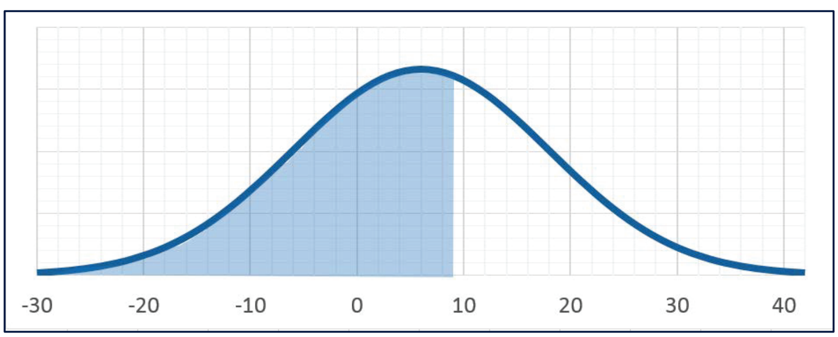


Figure 91

To graph the standard curve with mean of 6 and standard deviation of 12enter -30, -27, -24, -21,…, 45, 48 in cells A1:A27, then insert the formula "=NORM.DIST(A1,6,12,0)" in B1 and copy it in cells B2:B27. Select cells A1:B27 and insert "Scatter with Smooth Lines" graph. One easy way to shade the area under the graph is by using "Freeform" illustrations tool from the "Insert" tab.

e. Boxplots

Excel doesn't have a built-in boxplot graphing tool, but we can still use a modified stacked bars to create such a graph after a careful data preparation. We'll compare the revenues in billions by graphing the data:

Year	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013
Home Depot	64.8	73.1	81.5	90.8	66.2	71.3	66.2	68.0	70.4	74.8
Lowe's	30.8	36.5	43.2	46.9	47.2	48.2	47.2	48.8	50.2	50.5

We'll prepare our data by creating the three tables and entering data and formulas as shown in ***Figure 92***. Copy the summary table cells F2:F6 for Home Depot into G2:G6 cells for Lowe's to apply the same group of formulas. Do the same in "Data for Chart" table.

	A	B	C	D	E	F	G	H	I	J	K
1	Year	Home Depot	Lowe's		Summary	Home Depot	Lowe's		Data for Chart	Home Depot	Lowe's
2	2004	64.8	30.8		Minimum	=MIN(B2:B11)			Series 1	=F2	
3	2005	73.1	36.5		25th Percentile	=PERCENTILE.INC(B2:B11,0.25)			Series 2	=F3-F2	
4	2006	81.5	43.2		50th Percentile	=PERCENTILE.INC(B2:B11,0.5)			Series 3	=F4-F3	
5	2007	90.8	46.9		75th Percentile	=PERCENTILE.INC(B2:B11,0.75)			Series 4	=F5-F4	
6	2008	66.2	47.2		Maximum	=MAX(B2:B11)			Series 5	=F6-F5	
7	2009	71.3	48.2								
8	2010	66.2	47.2								
9	2011	68.0	48.8								
10	2012	70.4	50.2								
11	2013	74.8	50.5								

Figure 92

Select cells I1:K5 and insert the "Stacked Bar" chart.

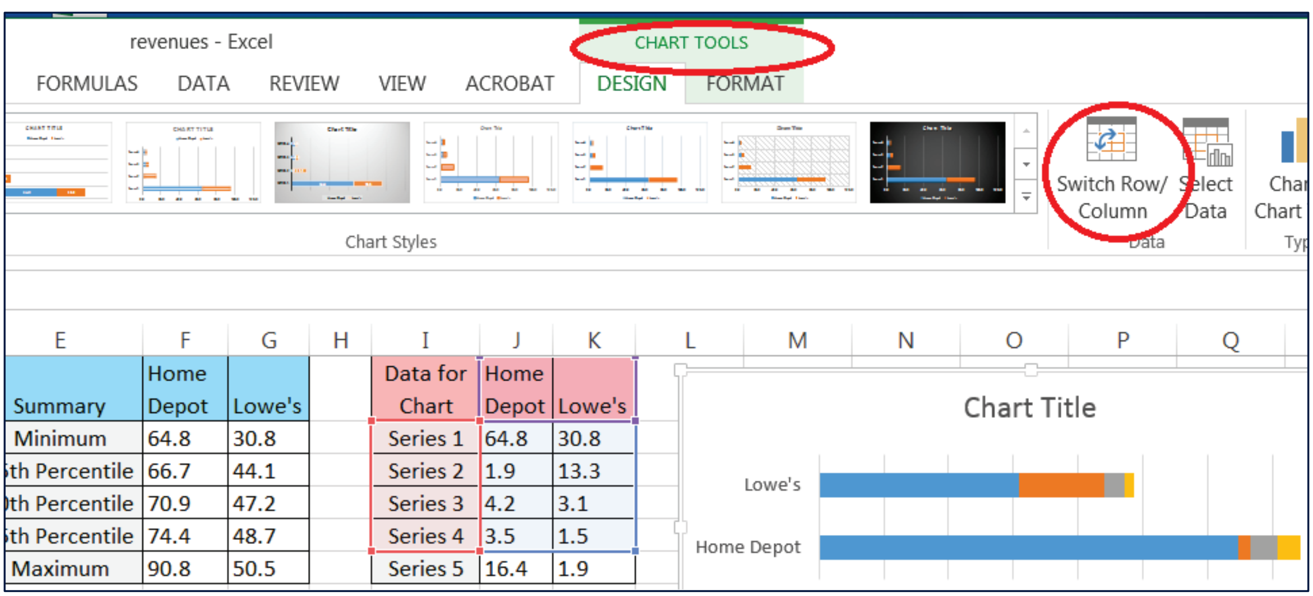


Figure 93

Notice that the last row from the Data Chart was not selected. Click anywhere on the graph and the Chart Tools tab will appear. Click on “Switch Row/Column” to obtain the graph from ***Figure 93*.**

Left-click on the blue area of one of the chart bars, then right click to obtain the “Fill” selection menu. And the “Outline” selection menu. Check boxes “No Fill” and “No Outline” to visually remove the blue part of bars, as in ***Figure 94***. Do the same for the next, brown area, but this time stay in this area since we’ll change it into “Error Bars”. In “Chart Tools” tab select “Add Chart Element” from the ribbon. Pick “Error Bars” from the menu, and then select “More Error Bars Options”. A new menu will appear to the right. Select buttons “Minus” and “Percentage”. Change the percentage to 100. The first set of whiskers will appear, as in ***Figure 95.***

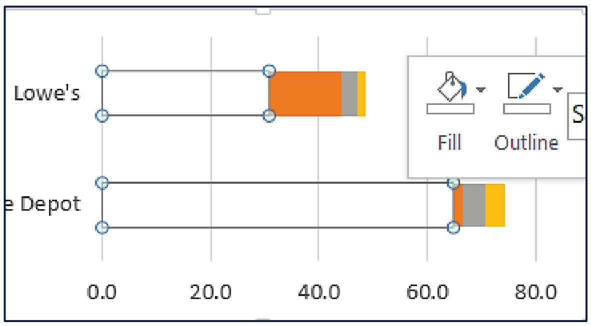


Figure 94

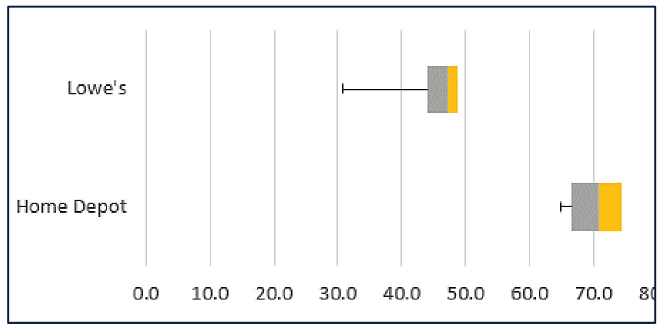


Figure 95

To create the second set of whiskers, first click on the last, orange part of bars. Again go to “More Error Bars Options”, but this time select buttons “Plus” and “Custom”. Click on “Specify the Value”. In the “Positive Error Value” name box use cells J6:K12 since they represent the right hand value of whiskers. The final box plot is shown in ***Figure 96***.

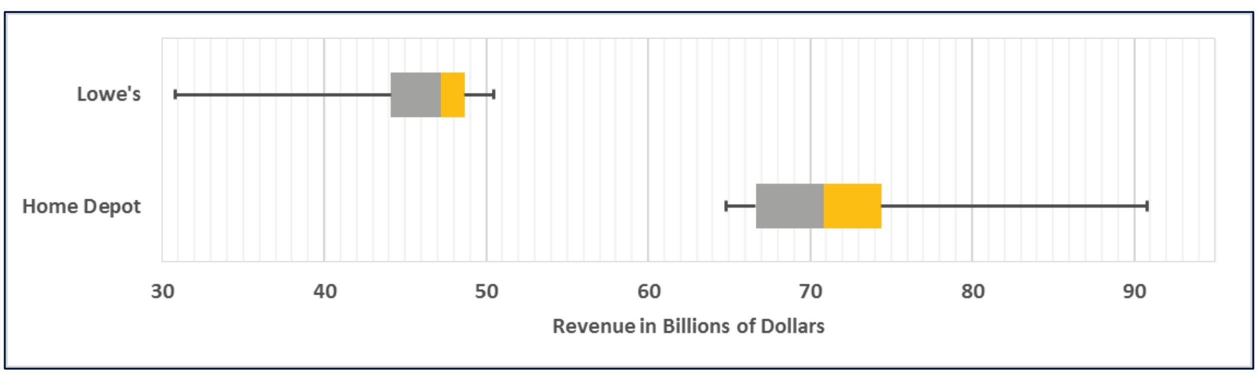


Figure 96

Chapter 10 Differentiation

a. Limits

Excel is an excellent tool for learning explorations of limits. We'll examine the function $f(x) = (5x^3 - 7x)/(2x^3 + 3)$ as x is approaching infinity. For our table we choose x values 5, 50, 500, 5000, 50000, 500000 and enter them in cells A2:A7. Type the formula "=(5*A2^3-7*A2)/(2*A2^3+3)" in cell B2 and copy it in cells B3:B7. The resulting y values are shown in ***Figure 97***.

B2 | fx =(5*A2^3-7*A2)/(2*A2^3+3)

	A	B	C	D	E
1	x	y			
2	5	2.33201581027668			
3	50	2.49857001715979			
4	500	2.49998597000017			
5	5,000	2.49999985997000			
6	50,000	2.49999999859997			
7	500,000	2.49999999998600			

Figure 97

b. Rate of Change

We'll use a spreadsheet to create an interactive table of rates of change for the function $f(x) = 0.5x^2$. Change of x_1 and x_2 values in cells A2 and B2 will result in change of the average rate of change and instantaneous rates of change at x_1 and x_2 in the table. To create such a table, enter any two values in cells A2 and B2. Enter the formulas for our function as shown in the ***Figure 98.***

	A	B	C	D	E
1	x1	x2	AV RATE CHANGE	INST RATE CH at x1	INST RATE CH at x2
2	-0.5	1.8	= (0.5*B2^2-0.5*A2^2)/(B2-A2)	=A2	=B2
3					
4					
5	Any values		$AVE = \frac{0.5(x_2)^2 - 0.5(x_1)^2}{x_2 - x_1}$	$f'(x_1) = x_1$	$f'(x_2) = x_2$
6					
7					

Figure 98

The final table for values x_1=2.3 and x_2=2.5 is shown in ***Figure 99***. To create an interactive table that will help us examine rates of change for a different function, say $f(x) = x^3$, we need to change formulas. In that case C2 cell formula would become "=(B2^3-A2^3)/(B2-A2)", D2 cell formula would be "=3*A2^2", while E2 formula would be "=3*B2^2".

	A	B	C	D	E
1	x1	x2	AV RATE CHANGE	INST RATE CH at x1	INST RATE CH at x2
2	2.3	2.5	2.4	2.3	2.5

Figure 99

A spreadsheet might be used to create an interactive graph of function and its tangent. We'll make it for the function $f(x) = 0.5x^2$, but any function can be used. Change of x_1 value in cell A1 will result in change of the tangent line on the graph. To create such a graph, enter any value in cell A2, then enter the formulas for our function as shown in the ***Figure 100.*** Drag handles down one by one as shown to copy formulas over columns D2:D14, also E2:E17 and F2:F17. Notice that the symbol "$" is used to "freeze" some values. They will remain the same when the formula is copied. Otherwise they'd adjust to the cell position.

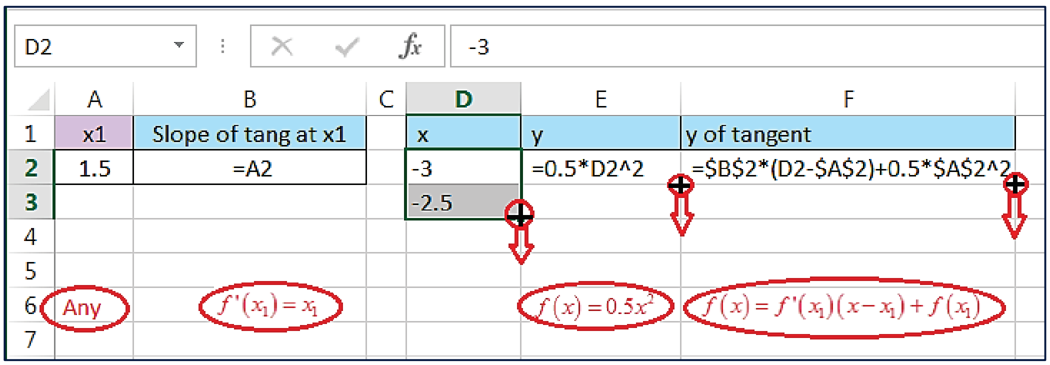


Figure 100

Select cells D2:F14 and insert scatter chart with smooth line. The resulting graph in ***Figure 101*** shows the tangent line of the function $f(x) = 0.5x^2$ at x_1=1.5.

	A	B	C	D	E	F
1	x1	Slope of tang at x1		x	y	y of tangent
2	1.5	1.5		-3	4.5	-5.625
3				-2.5	3.125	-4.875
4				-2	2	-4.125
5				-1.5	1.125	-3.375
6				-1	0.5	-2.625
7				-0.5	0.125	-1.875
8				0	0	-1.125
9				0.5	0.125	-0.375
10				1	0.5	0.375
11				1.5	1.125	1.125
12				2	2	1.875
13				2.5	3.125	2.625
14				3	4.5	3.375

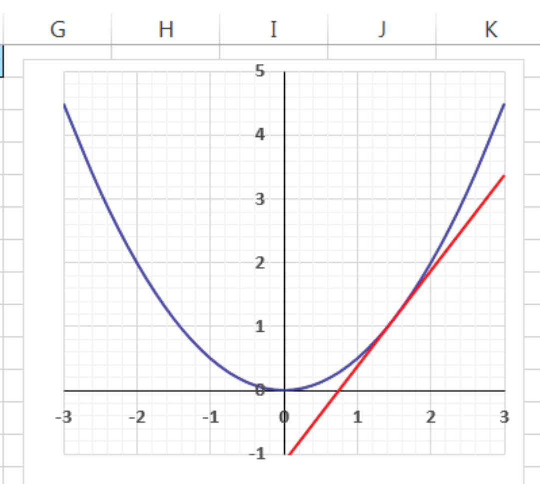


Figure 101

Change the x_1 value in the table to obtain a different tangent. To adjust the view of values on y axis to (-1,5), double-click on any number labeling y axis and a new menu "Format Axis" will appear to the right. Adjust the "Axis Options" (bars icon) bounds minimum and maximum values to -1 and 5.

We can also use a spreadsheet to create an interactive graph showing secant tangents together for the function $f(x) = 3 - 0.4x^2$. Enter values as shown in ***Figure 102*** and drag the corner handles as indicated.

	A	B	C	D	E	F
1	x1	x2	AV RATE CHANGE	INST RATE CH at x1	INST RATE CH at x2	
2	-0.3	1.5	= ((3-0.4*B2^2)-(3-0.4*A2^2))/(B2-A2)	=-0.8*A2	=-0.8*B2	
3						

	G	H	I	J	K
1	x coord	y coordin	y of tang1	y of tang 2	y of secant
2	-3	=3-0.4*G2^2	=$D$2*(G2-$A$2)+3-0.4*$A$2^2	=$E$2*(G2-$B$2)+3-0.4*$B$2^2	=$C$2*(G2-$A$2)+3-0.4*$A$2^2
3	-2.5				
4					
5					

Figure 102

Select cells G2:K14 and insert scatter chart with smooth line. The resulting graph in ***Figure 103*** shows the secant and tangent lines at x_1=-0.3 and x_2=1.5. Change these values in cells A2 and B2 to see secant and tangents move.

	A	B	C	D	E	F	G	H	I	J	K
1	x1	x2	AV RATE CHANGE	INST RATE CH at x1	INST RATE CH at x2		x coord	y coordin	y of tang1	y of tang 2	y of secant
2	-0.3	1.5	-0.48	0.24	-1.2		-3	-0.6	2.316	7.5	4.26
3							-2.5	0.5	2.436	6.9	4.02
4							-2	1.4	2.556	6.3	3.78
5							-1.5	2.1	2.676	5.7	3.54
6							-1	2.6	2.796	5.1	3.3
7							-0.5	2.9	2.916	4.5	3.06
8							0	3	3.036	3.9	2.82
9							0.5	2.9	3.156	3.3	2.58
10							1	2.6	3.276	2.7	2.34
11							1.5	2.1	3.396	2.1	2.1
12							2	1.4	3.516	1.5	1.86
13							2.5	0.5	3.636	0.9	1.62
14							3	-0.6	3.756	0.3	1.38
15											

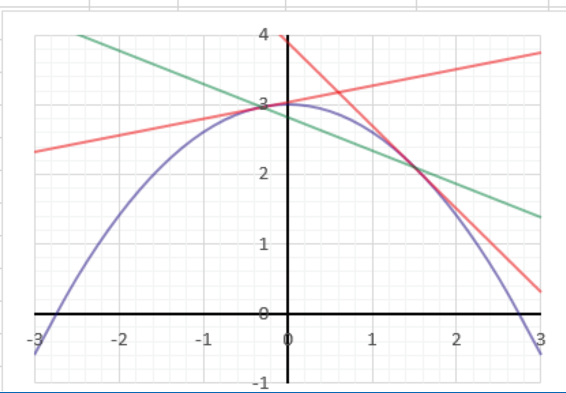


Figure 103

c. Extrema of Functions of Several Variables

We'll use Excel to find the local minimum of the function $f(x, y) = 6x^2 + 6y^2 + 6xy + 36x - 54y - 5$. For that reason we'll use a built-in add-in called "Solver", described in detail in Chapter 1. After installing it, it can be located under the "Analysis" group in the "Data" tab ribbon. Solver will calculate z (the target cell) for different *x* and *y* values (the adjustable cells) until the minimum value for *z* is found.

Assign initial values for adjustable cells *x* and *y*. We selected "1" and "1" for initial values, but any value may be used here except the coordinates of the saddle point, since such selection would block further Excel calculations. Enter the formula "=6*A2^2+6*B2^2+6*A2*B2+36*A2-54*B2-5" in target cell C2. Click on a cell and click on "Solver". Fill the boxes as shown in ***Figure 104*.** Uncheck the box "Make Unconstrained Variables Non-Negative" since we are looking for the solution among negatives also.

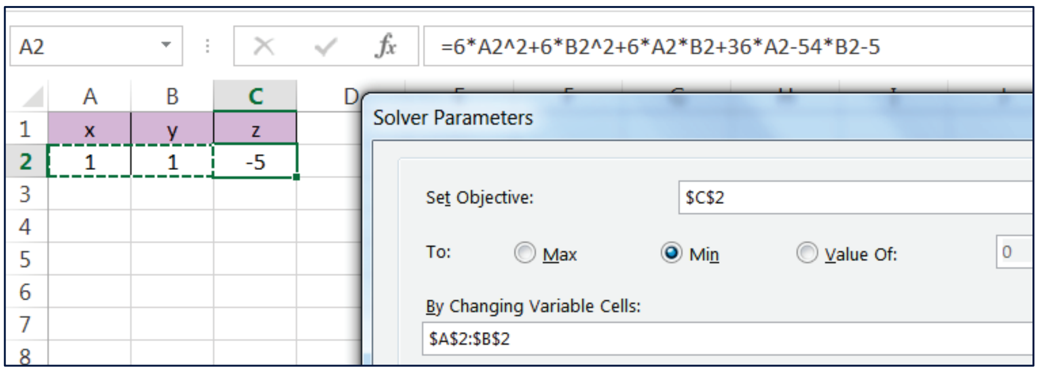


Figure 104

To obtain a solution, click on "Solve" and the optimal values, if they exist, will be determined. In this example, the *x*-value -7 appears in cell A2, the *y*-value 8 appears in cell B2, and the minimum *z*-value -347 appears in cell C2.

d. Lagrange Multipliers

Using a spreadsheet we'll maximize the function $f(x, y, z) = xyz^2$ subject to the constraint $x + y + z = 50$. Assign your choice of initial values for variables *x*, *y*, and *z* and enter them in cells A2, B2, and C2. Notice that some initial values will block Excel from further calculations. If this is the case, change the selection and start again. Enter the formula "=A2*B2*C2^2" in cell D2. Enter the formula "=A2+B2+C2" into cell E2. Click on a cell and select "Solver". Fill the newly appeared pop-up screen as shown in n ***Figure 105***.

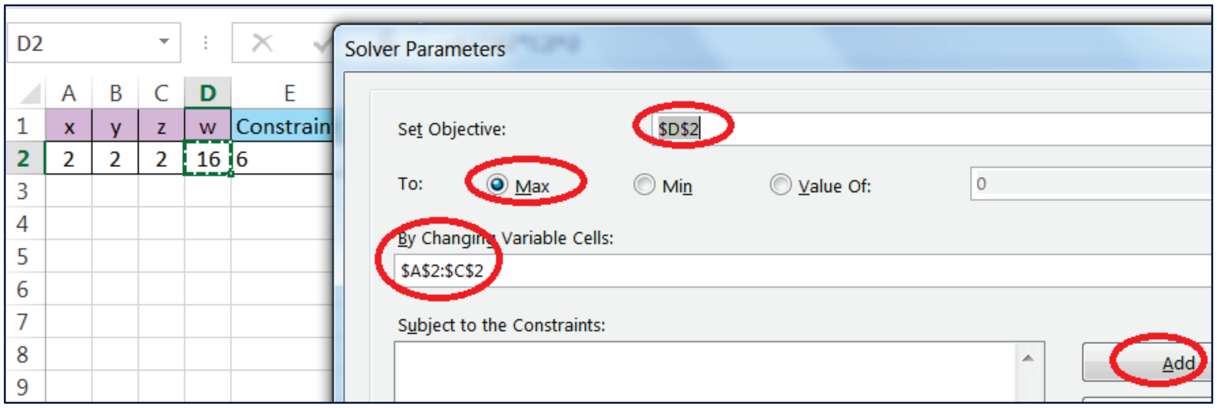


Figure 105

Click on "Add" and a new dialog box will appear where the constraint $x + y + z = 50$ will be entered as shown in ***Figure 106***. Fill the "Cell Reference" box by clicking on E2. Select "=" from the drop bar in the middle. Set the value in cell E2 to 50, as this will be the right hand side of the constraint equation.

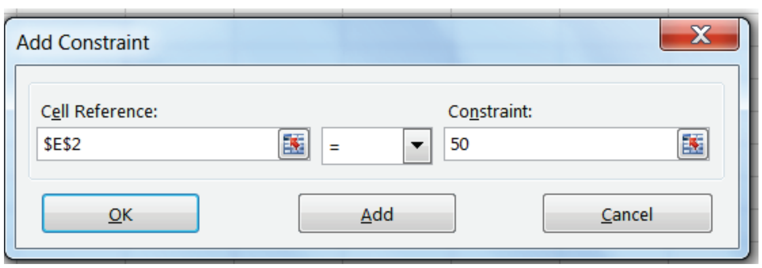


Figure 106

Click "OK" and "Solve" to obtain the final answer x=12.5, y=12.5, z=25 and $f(x, y, z) = 97656$.

Chapter 11 Integrals

a. Numerical Integration

Excel doesn't have a build-in function that calculates integrals, but we can easily approximate a definite integrals by performing numerical integration. To approximate the integral $\int_0^{2.6} x^2 dx$ enter x values 0, 0.1, 0.2, 0.3… 2.5, 2.6 in cells A2:A28. One way to do it fast is to type the first two entries, select cells A2:A3 together and then drag the black corner marker over cells A4:A28 to copy the pattern. The integral will be approximated as the sum $f(x_1)\Delta x + f(x_2)\Delta x + ... + f(x_{27})\Delta x$ where $x_1, x_2, ..., x_{27}$ are midpoints and $\Delta x = 0.1$. For that reason enter the formula "=0.1*(A2+0.05)^2" in cell B2. Copy this formula to cells B3:B27 by dragging the black corner marker as in ***Figure 107***.

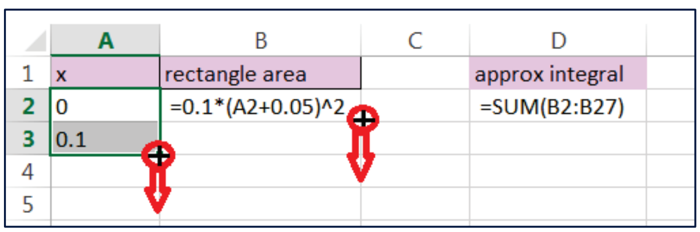


Figure 107

Enter the formula "=SUM(B2:B27)" in any cell to obtain the final answer 5.8565. The exact value of this integral rounded to 4 decimal places is 5.8587.

To estimate an integral of a different function on a different interval change the formula in cell B2 and adjust the formula in cell D2. For example, to approximate $\int_0^{1.7} \sqrt{x}\, dx$ use "=0.1*(A2+0.05)^0.5" and "=SUM(B2:B19)".

To obtain a more accurate approximation of the first integral use x values 0, 0.05, 0.1….and use "=0.05*(A2+0.025)^2" in cell B2.

b. The Definite Integral

We'll create an interactive spreadsheet that will help explore the definition of definite integral. The interactive spreadsheet will calculate the sum of areas of rectangles for a different choice of n. As the number of rectangles n is increasing, the sum of areas of rectangles is becoming more accurate approximation of area under the curve. Our spreadsheet is created to work with the function $f(x) = 5/x$, but other functions might be used after adjustment of cell G2. The choice of the interval endpoints x_1 and x_2 is also interactive

Enter values and formulas as indicated in ***Figure 108***. Drag the corner handle down up to row 17 to copy pattern (column E) or to copy a formula (columns F, G, and H).

	A	B	C	D	E	F	G	H	I
1	x1	x2	n	interval length	rectang number	midpoint x	y	rectangle area	areas sum
2	1	5	10	=(B2-A2)/C2	1	=IF(E2<$C$2+1,$A$2+$D$2*(E2-0.5),0)	=IF(E2<$C$2+1,5/F2,0)	=$D$2*G2	=SUM(H2:H17)
3					2				
4									
5–6	Any			$\Delta x=\frac{x_2-x_1}{n}$		Up to x2: $x_n^*=x_1+\Delta x(n-0.5)$	Up to x2: $f(x_n^*)=5/x_n^*$	$=\Delta x\cdot f(x_n^*)$	$=\sum\Delta x\cdot f(x_n^*)$
7									

Figure 108

If created exactly as in ***Figure 108***, our interactive spreadsheet calculates approximation of $\int_1^5 \frac{5}{x}\,dx$, using n=10 rectangles, each 0.4 wide. The approximation in cell I2 is 8.016. Change the number of rectangles in cell C2 to n=16, and the new integral approximation will be 8.035, as shown in ***Figure 109***. The exact value rounded to 3 decimal places is 8.048.

We can add a graph of rectangles by selecting y-coordinates from column G, and inserting “Clustered Column” graph. Right-click on any number on x-axis and pick “Select Data”. In the newly appeared pop-up screen click on the right-hand “Edit” and select x coordinates from column F. To adjust the appearance of x-axis after change of n, click on graph and then click on the small image of filter that will appear to the right. To make bars wider, right-click on any bar and then click on “Format Data Series”. Change the “Gap Width” is one of “Series Options”.

C	D	E	F	G	H	I
n	interval length	rectangle number	midpoint x	y	rectangle area	areas sum
16	0.25	1	1.125	4.444444	1.111111111	8.034827342
		2	1.375	3.636364	0.909090909	
		3	1.625	3.076923	0.769230769	
		4	1.875	2.666667	0.666666667	
		5	2.125	2.352941	0.588235294	
		6	2.375	2.105263	0.526315789	
		7	2.625	1.904762	0.476190476	
		8	2.875	1.73913	0.434782609	
		9	3.125	1.6	0.4	
		10	3.375	1.481481	0.37037037	
		11	3.625	1.37931	0.344827586	
		12	3.875	1.290323	0.322580645	
		13	4.125	1.212121	0.303030303	
		14	4.375	1.142857	0.285714286	
		15	4.625	1.081081	0.27027027	
		16	4.875	1.025641	0.256410256	

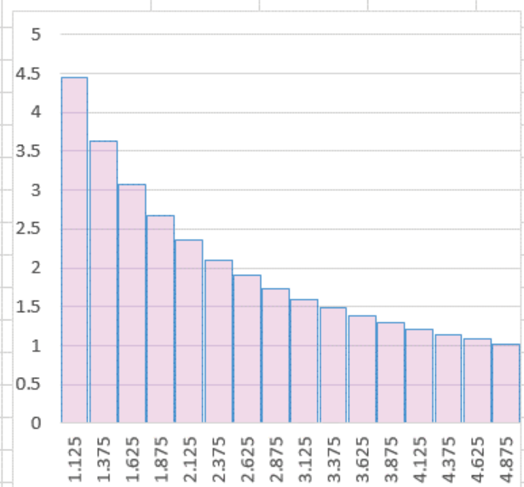


Figure 109

To estimate an integral of a different function change the formula in cell B2 and adjust the formula in cell G2. For example, to approximate $\int_1^5 \sqrt{x}\, dx$ use “=IF(E2<$C$2+1,F2^0.5,0)”.