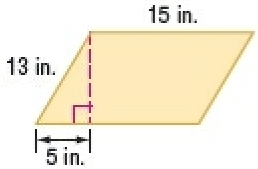
Find the perimeter and area of each parallelogram or triangle. Round to the nearest tenth if necessary.

1. 

SOLUTION:

Use the Pythagorean Theorem to find the height h, of the parallelogram.

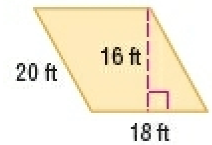
$$a^2 + b^2 = c^2$$
$$5^2 + h^2 = 13^2$$
$$h^2 = 13^2 - 5^2$$
$$h^2 = 169 - 25$$
$$h = \sqrt{144}$$
$$h = 12$$

$$A = bh$$
$$= 15(12)$$
$$= 180$$

$$P = 2(13 + 15)$$
$$= 2(28)$$
$$= 56$$

ANSWER:

56 in., 180 in^2

2. 

SOLUTION:

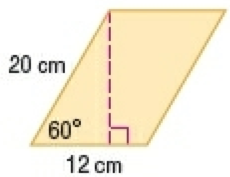
$$A = bh$$
$$= 18(20)$$
$$= 360$$

Each pair of opposite sides of a parallelogram is congruent to each other.

$$P = 2(18 + 20)$$
$$= 2(38)$$
$$= 76$$

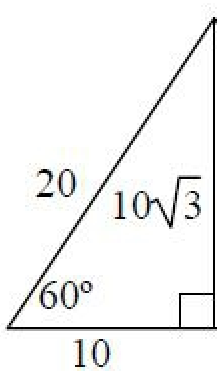
ANSWER:

76 ft, 288 ft^2

3. 

SOLUTION:

Use the 30-60-90 triangle to find the height.



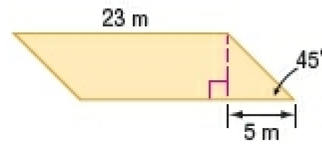
$$\begin{aligned} A &= bh \\ &= 12\left(10\sqrt{3}\right) \\ &\approx 207.85 \end{aligned}$$

Each pair of opposite sides of a parallelogram is congruent to each other.

$$\begin{aligned} P &= 2(12+20) \\ &= 2(32) \\ &= 64 \end{aligned}$$

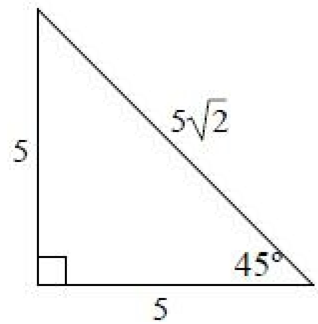
ANSWER:

64 cm, 207.8 cm^2

4. 

SOLUTION:

Use the 45-45-90 triangle to find the height.

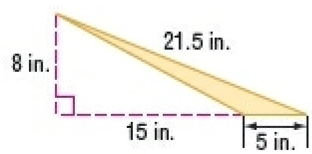


$$\begin{aligned} A &= bh \\ &= 23(5) \\ &= 115 \end{aligned}$$

$$\begin{aligned} P &= 2(23+5\sqrt{2}) \\ &= 46+10\sqrt{2} \\ &\approx 60.1 \end{aligned}$$

ANSWER:

60.1 m, 115 m^2

5. 

SOLUTION:

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(8)(5)$$
$$= 20$$

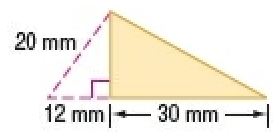
Use the Pythagorean Theorem to find the length of the third side of the triangle.

$$a^2 + b^2 = c^2$$
$$8^2 + 15^2 = c^2$$
$$64 + 225 = c^2$$
$$\sqrt{289} = c$$
$$17 = c$$

Therefore, the perimeter is 17 + 5 + 21.5 = 43.5 in.

ANSWER:

43.5 in., 20 in^2

6. 

SOLUTION:

Use the Pythagorean Theorem to find the height *h*, of the triangle.

$$a^2 + b^2 = c^2$$
$$12^2 + b^2 = 20^2$$
$$b^2 = 20^2 - 12^2$$
$$b^2 = 400 - 144$$
$$b = \sqrt{256}$$
$$b = 16$$

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(30)(16)$$
$$= 240$$

Use the Pythagorean Theorem to find the length of the third side of the triangle.

$$a^2 + b^2 = c^2$$
$$30^2 + 16^2 = c^2$$
$$900 + 256 = c^2$$
$$\sqrt{1156} = c$$
$$34 = c$$

Therefore, the perimeter is 30 + 16 + 34 = 80 mm.

ANSWER:

80 mm, 240 mm^2

7. **CRAFTS** Marquez and Victoria are making pinwheels. Each pinwheel is composed of 4 triangles with the dimensions shown. Find the perimeter and area of one triangle.

SOLUTION:

The perimeter is 9 + 11 + 8.5 or 28.5 in.

Use the Pythagorean Theorem to find the height *h*, of the triangle.

$$a^2 + b^2 = c^2$$
$$a^2 = c^2 - b^2$$
$$a^2 = 8.5^2 - 4^2$$
$$a^2 = 72.25 - 16$$
$$a = 7.5$$

$$\text{Area} = \frac{1}{2}bh$$

$$\text{Area} = \frac{1}{2}(9)(7.5)$$
$$\approx 33.8$$

Use Pythagorean Theorem for the missing side of the triangle.

$$(4+9)^2 + 7.5^2 = c^2$$
$$225.25 = c^2$$
$$15 \approx c$$

□

Perimeter = 15 + 9 + 8.5 = 32.5 in.

ANSWER:

$32.5\text{in.}, 33.8\text{in}^2$

Find *x*.

8. △*JKL* with *J*(5, 6), *K*(–3, –1), and *L*(–2, 6)

SOLUTION:

△*JKL* with *J*(5, 6), *K*(–3, –1), and *L*(–2, 6)

To find the area, let *JL* be the base and the altitude to *JL* be the height.

JL = 5 – (–2) = 7
height = 6 – (–1) = 7

Area = 0.5(7)(7) = 29.5

To find the perimeter, first find the length of the other two sides.

$$JK = \sqrt{(5-(-3))^2 + (6-(-1))^2}$$
$$JK = \sqrt{113}$$

$$LK = \sqrt{(-2-(-3))^2 + (6-(-1))^2}$$
$$LK = \sqrt{50} = 5\sqrt{2}$$

Perimeter = $7 + \sqrt{113} + 5\sqrt{2}$

ANSWER:

Area = 29.5

Perimeter = $7 + \sqrt{113} + 5\sqrt{2}$

9. $\triangle WXY$ with $W(7, 4)$, $X(-1, 5)$, and $Y(7, -1)$

SOLUTION:

To find the area and perimeter of $\triangle WXY$ with $W(7, 4)$, $X(-1, 5)$, and $Y(7, -1)$, first find the length of each side, and the height in relation to one base.

Let the base be WY.

$WY = 4 - (-1) = 5$
height $= 7 - (-1) = 8$

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(5)(8)$$
$$A = 20$$

$$WX = \sqrt{(7-(-1))^2 + (4-5)^2}$$
$$WX = \sqrt{65}$$
$$YX = \sqrt{(7-(-1))^2 + (-1-5)^2}$$
$$YX = \sqrt{100} = 10$$

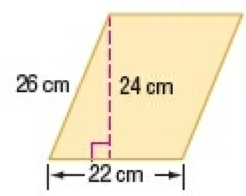
Perimeter $= 5 + 10 + \sqrt{113} = 15 + \sqrt{113}$

ANSWER:

Area = 20

Perimeter $= 15 + \sqrt{113}$

STRUCTURE Find the perimeter and area of each parallelogram or triangle. Round to the nearest tenth if necessary.

10. 

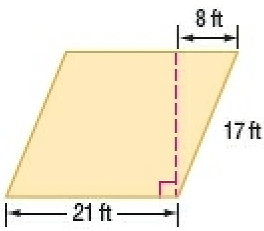
SOLUTION:

$$A = bh$$
$$= 22(24)$$
$$= 528$$

$$P = 2(26 + 22)$$
$$= 2(48)$$
$$= 96$$

ANSWER:

96 cm, 528 cm^2

11. 

SOLUTION:

Use the Pythagorean Theorem to find the height h, of the parallelogram.

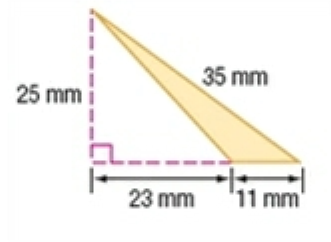
$$a^2 + b^2 = c^2$$
$$8^2 + h^2 = 17^2$$
$$h^2 = 17^2 - 8^2$$
$$h^2 = 289 - 64$$
$$h = \sqrt{225}$$
$$h = 15$$

$$A = bh$$
$$= 21(15)$$
$$= 315$$

$$P = 2(21 + 17)$$
$$= 2(38)$$
$$= 76$$

ANSWER:

76 ft, 315 ft^2

12. 

SOLUTION:

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(11)(25)$$
$$= 137.5$$

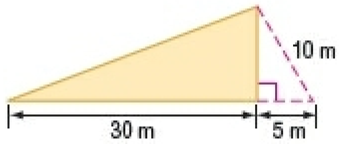
Use the Pythagorean Theorem to find the length of the third side of the triangle.

$$a^2 + b^2 = c^2$$
$$25^2 + 23^2 = c^2$$
$$625 + 529 = c^2$$
$$\sqrt{1154} = c$$
$$34 \approx c$$

The perimeter is about 35 + 11 + 34 or 80 mm.

ANSWER:

80 mm, 137.5 mm^2

13. 

SOLUTION:

Use the Pythagorean Theorem to find the height *h* of the triangle.

$$\begin{aligned} a^2+b^2&=c^2\\ 5^2+h^2&=10^2\\ h^2&=10^2-5^2\\ h^2&=100-25\\ h&=\sqrt{75}\\ h&=5\sqrt{3} \end{aligned}$$

$$\begin{aligned} A&=\tfrac{1}{2}bh\\ &=\tfrac{1}{2}(30)\left(5\sqrt{3}\right)\\ &=75\sqrt{3}\\ &\approx 129.9 \end{aligned}$$

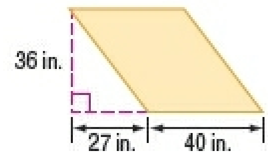
Use the Pythagorean Theorem to find the length of the third side of the triangle.

$$\begin{aligned} a^2+b^2&=c^2\\ 30^2+\left(\sqrt{75}\right)^2&=c^2\\ 900+75&=c^2\\ \sqrt{975}&=c\\ 31.2&\approx c \end{aligned}$$

The perimeter is about 8.7 + 30 + 31.2 = 69.9 m.

ANSWER:

69.9 m, 129.9 m^2

14. 

SOLUTION:

$$\begin{aligned} A&=bh\\ &=40(36)\\ &=1440 \end{aligned}$$

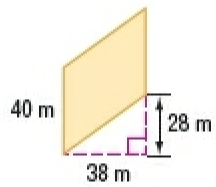
Use the Pythagorean Theorem to find the length of the other pair of opposite sides of the parallelogram.

$$\begin{aligned} a^2+b^2&=c^2\\ 36^2+27^2&=c^2\\ 1296+729&=c^2\\ \sqrt{2025}&=c\\ 45&=c \end{aligned}$$

The perimeter is 2(40 + 45) = 170

ANSWER:

170 in., 1440 in^2

15. 

SOLUTION:

$$A = bh$$
$$= 40(38)$$
$$= 1520$$

Use the Pythagorean Theorem to find the length of the other pair of opposite sides of the parallelogram.

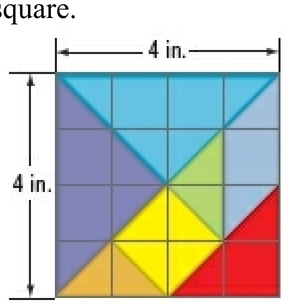
$$a^2 + b^2 = c^2$$
$$38^2 + 28^2 = c^2$$
$$1444 + 784 = c^2$$
$$\sqrt{2228} = c$$
$$47.2 \approx c$$

The perimeter is about 2(40 + 47.2) = 174.4.

ANSWER:

174.4 m, 1520 m^2

16. **TANGRAMS** The tangram shown is a 4-inch square.



a. Find the perimeter and area of the purple triangle. Round to the nearest tenth.
b. Find the perimeter and area of the blue parallelogram. Round to the nearest tenth.

SOLUTION:

a.

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(4)(2)$$
$$= 4$$

Use the Pythagorean Theorem to find the length of the two congruent sides of the triangle. Note that each side is also a hypotenuse for a triangle with sides of length 2.

$$a^2 + b^2 = c^2$$
$$2^2 + 2^2 = c^2$$
$$4 + 4 = c^2$$
$$\sqrt{8} = c$$
$$2.83 \approx c$$

The perimeter is about 2.83 + 2.83 + 4 or 9.7 in.

b.
$$A = bh$$
$$= 2(1)$$
$$= 2$$

Use the Pythagorean Theorem to find the length of the other pair of opposite sides of the parallelogram.

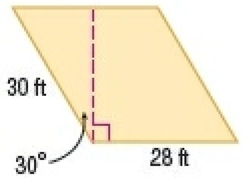
$$a^2 + b^2 = c^2$$
$$1^2 + 1^2 = c^2$$
$$2 = c^2$$
$$\sqrt{2} = c$$
$$1.4 \approx c$$

The perimeter is about 2(2 + 1.4) = 6.8.

ANSWER:

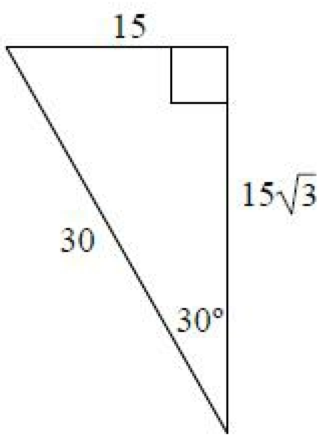
a. 9.7 in.; 4 in^2
b. 6.8 in.; 2 in^2

STRUCTURE Find the area of each parallelogram. Round to the nearest tenth if necessary.

17. 

SOLUTION:

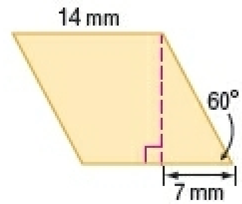
Use the 30-60-90 triangle to find the height.



$$\begin{aligned} A &= bh \\ &= 28\left(15\sqrt{3}\right) \\ &\approx 727.5 \end{aligned}$$

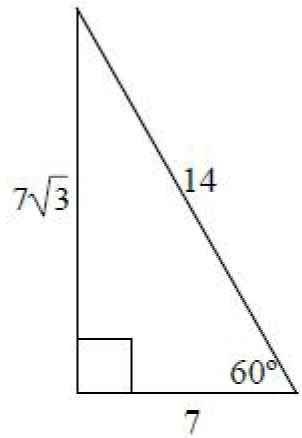
ANSWER:

727.5 ft^2

18. 

SOLUTION:

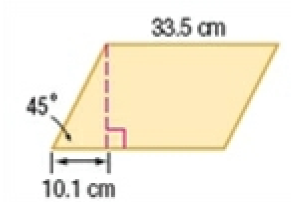
Use the 30-60-90 triangle to find the height.



$$\begin{aligned} A &= bh \\ &= 14\left(7\sqrt{3}\right) \\ &\approx 169.7 \end{aligned}$$

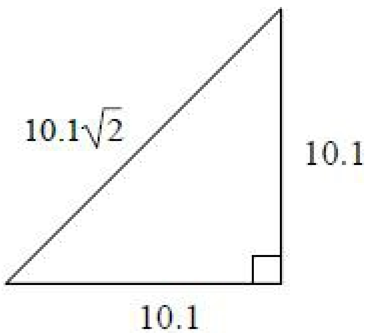
ANSWER:

169.7 mm^2

19. 

SOLUTION:

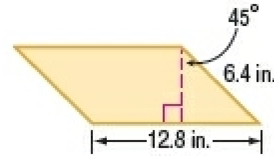
Use the 45-45-90 triangle to find the height.



$$\begin{aligned} A &= bh \\ &= 33.5(10.1) \\ &= 338.4 \end{aligned}$$

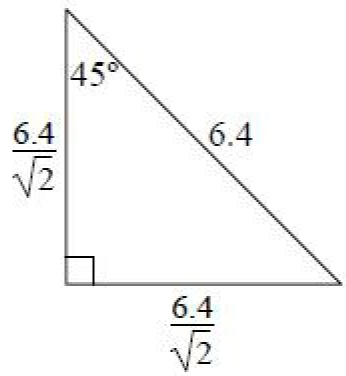
ANSWER:

338.4 cm^2

20. 

SOLUTION:

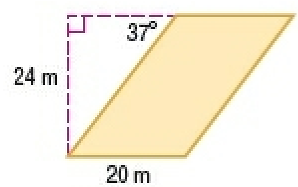
Use the 45-45-90 triangle to find the height.



$$\begin{aligned} A &= bh \\ &= 12.8\left[\frac{6.4}{\sqrt{2}}\right] \\ &\approx 57.9 \end{aligned}$$

ANSWER:

57.9 in^2

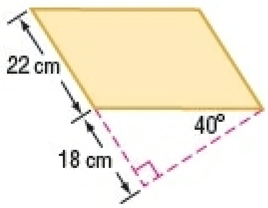
21. 

SOLUTION:

$$\begin{aligned} A &= bh \\ &= 20(24) \\ &= 480 \end{aligned}$$

ANSWER:

480 m^2

22. 

SOLUTION:

Use the tangent ratio of an angle to find the height of the parallelogram.

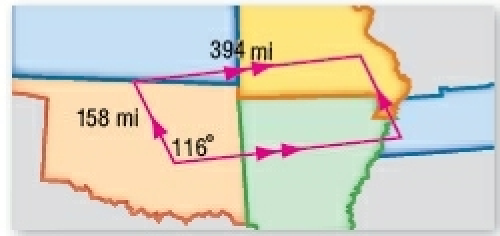
$$\tan x = \frac{\text{opposite}}{\text{adjacent}}$$
$$\tan 40 = \frac{18}{h}$$
$$h \tan 40 = 18$$
$$h = \frac{18}{\tan 40}$$
$$h \approx 21.45$$

$$A = bh$$
$$= 22\left[\frac{18}{\tan 40}\right]$$
$$\approx 471.9$$

ANSWER:

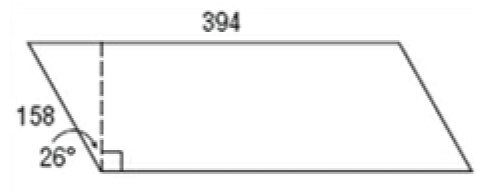
471.9 cm^2

23. **WEATHER** Tornado watch areas are often shown on weather maps using parallelograms. What is the area of the region affected by the tornado watch shown? Round to the nearest square mile.



SOLUTION:

We have a figure as shown.



Use the cosine ratio of an angle to find the height of the parallelogram.

$$\cos 26° = \frac{h}{158}$$
$$158(\cos 26°) = h$$
$$142 \approx h$$

The area of a parallelogram is the product of its base length b and its height h.

$b = 394$ and $h \approx 142$. So, the area of the parallelogram is about 394(142) = 55,948. Therefore, an area of about 55,948 mi^2 is affected by the tornado.

ANSWER:

$55{,}948 \text{ mi}^2$

Find the perimeter and area of each figure.

24. $\triangle DEF$ with $D(4, -4)$, $E(-5, 1)$, and $F(11, -4)$

SOLUTION:

To find the area of $\triangle DEF$ with $D(4, -4)$, $E(-5, 1)$, and $F(11, -4)$, find the length of a base and height.

Let DF be the height.
$DF = 11 - 4 = 7$
height = $1 - (-4) = 5$

Area = $(0.5)(7)(5) = 17.5$

To find perimeter, first find the lengths of the other sides.

$$DE = \sqrt{(4-(-5))^2 + (-4-1)^2}$$
$$DE = \sqrt{106}$$
$$FE = \sqrt{(11-(-5))^2 + (-4-1)^2}$$
$$FE = \sqrt{281}$$

Perimeter = $7 + \sqrt{106} + \sqrt{281}$

ANSWER:

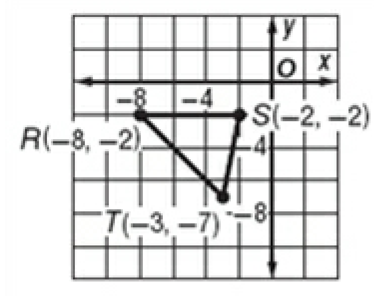
Area = $(0.5)(7)(5) = 17.5$

Perimeter = $7 + \sqrt{106} + \sqrt{281}$

25. ΔRST with $R(-8, -2)$, $S(-2, -2)$, and $T(-3, -7)$

SOLUTION:

Graph the diagram.



The base of the triangle is horizontal and from $x = -2$ to $x = -8$, so it is 6 units long.

The height of the triangle vertical and from $y = -2$ to $y = -7$, so it is 5 units long.

The area is $0.5(6)(5) = 15$ units^2.

To find the perimeter, calculate the lengths of the other two sides.

$$RT = \sqrt{(-8-(-3))^2 + (-2-(-7))^2}$$
$$RT = \sqrt{50} = 5\sqrt{2}$$
$$ST = \sqrt{(-2-(-3))^2 + (-2-(-7))^2}$$
$$ST = \sqrt{26}$$

Perimeter = $6 + 5\sqrt{2} + \sqrt{26}$

ANSWER:

Area = 15

Perimeter = $6 + 5\sqrt{2} + \sqrt{26}$

10-1 Areas of Parallelograms and Triangles

26. $\triangle MNP$ with $M(0, 6)$, $N(–2, 8)$, and $P(–2, –1)$

SOLUTION:

To find the area of $\triangle MNP$ with $M(0, 6)$, $N(–2, 8)$, and $P(–2, –1)$, find its base and height.

Let NP be the base.
$NP = 8 – (–1) = 9$
height $= 0 – (–2) = 2$

Area $= (0.5)(9)(2) = 9$

Then find the length of the other two sides to find the perimeter.

$MN = \sqrt{(0-(-2))^2+(6-8)^2}$

$MN = \sqrt{8} = 2\sqrt{2}$

$MP = \sqrt{(0-(-2))^2+(6-(-1))^2}$

$MP = \sqrt{53}$

Perimeter = $9+2\sqrt{2}+\sqrt{53}$

ANSWER:

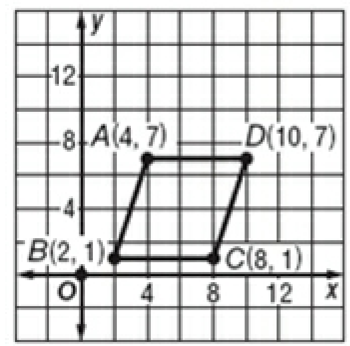
Area = 9

Perimeter = $9+2\sqrt{2}+\sqrt{53}$

27. ▱$ABCD$ with $A(4, 7)$, $B(2, 1)$, $C(8, 1)$, and $D(10, 7)$

SOLUTION:

Graph the diagram.



The base of the parallelogram is horizontal and from $x = 2$ to $x = 8$, so it is 6 units long.

The height of the parallelogram vertical and from $y = 1$ to $y = 7$, so it is 6 units long.

The area is $(6)(6) = 36$ units^2.

To find the perimeter, find DC or AB, which should be equal since the figure is a parallelogram.

$AB = \sqrt{(4-2)^2+(7-1)^2}$

$AB = \sqrt{40} = 2\sqrt{10}$

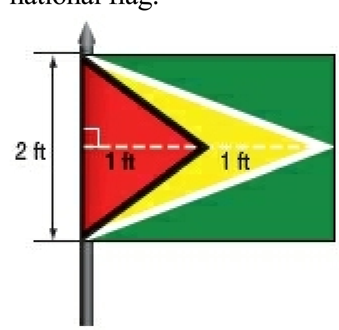
Perimeter = $2(6+2\sqrt{10}) = 12+4\sqrt{10}$

ANSWER:

Area = 36

Perimeter = $12+4\sqrt{10}$

28. **FLAGS** Omar wants to make a replica of Guyana's national flag.



a. What is the area of the piece of fabric he will need for the red region? for the yellow region?

b. If the fabric costs \$3.99 per square yard for each color and he buys exactly the amount of fabric he needs, how much will it cost to make the flag?

SOLUTION:

a. The red region is a triangle with a base of 2 feet and a height of 1 foot. The area of a triangle is half the product of a base b and its corresponding height h. So, the area of the fabric required for the red region is $\frac{1}{2}(2)(1) = 1 \text{ ft}^2$.

The area of the yellow region is the difference between the areas of the triangle of base 2 ft and height 2 ft and the red region. The area of the triangle of base 2 ft and height 2 ft is $\frac{1}{2}(2)(2) = 2 \text{ ft}^2$ and that of the red region is 1 ft^2.

Therefore, the fabric required for the yellow region is $2 - 1 = 1 \text{ ft}^2$.

b. The amount of fabric that is required for the entire flag is 2 ft · 2 ft or 4 ft^2.

$$3.99 \cdot 4 \text{ ft}^2 = (3 \text{ ft})^2 \cdot x$$
$$15.96 = 9x$$
$$1.77 \approx x$$

Therefore, the total cost to make the flag will be about \$1.77.

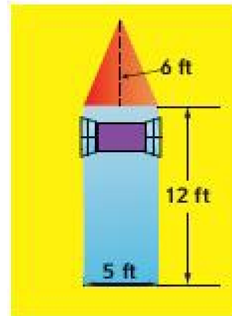
ANSWER:

a. $1 \text{ ft}^2; 1 \text{ ft}^2$

b. \$1.77

29. **MULTI-STEP** Madison is in charge of the set design for the drama club's rendition of *Romeo and Juliet*. The backdrop shown is 12 feet wide and 20 feet tall and needs 3 coats of paint. The window is 4 feet wide and 1 foot high. The paint store has the following available. One quart of paint covers 87.5 square feet.

Size	8 oz	1 qt	1 gal
Cost (\$)	3.75	14	30



a. What should she buy to minimize cost?
b. Explain your solution process.

SOLUTION:

a-b. 1 quart of paint covers 87.5 ft^2. There are 4 quarts in a gallon, so 1 gallon covers 4(87.5) or 350 ft^2. There are 32 oz in a qt, so 1 oz covers $\frac{87.5}{32} \approx 2.734375 \text{ ft}^2$. The paint is sold in 8-oz bottles, so one bottle covers $8(2.734375) = 21.88 \, ft^2$. The purple window has an area of $4 \cdot 1 = 4 \text{ ft}^2$ She should buy one bottle of purple paint at \$3.75. It is more economical for Madison to mix the purple paint with the remaining red and blue paint. She needs enough purple paint to cover $4 \cdot 3 = 12 \text{ ft}^2$, since she needs three coats, or an additional 6 ft^2 with both red and blue paint.

The roof has an area of $\frac{1}{2}(6)(5) = 15 \text{ft}^2$, so she needs enough red paint to cover 15(3) + 6 = 51 ft^2. She could buy 1 quart for \$14 or 3 bottles for \$11.25.
The tower has an area of $5(12) - 4$ or 56 ft^2. She needs enough blue paint to cover 56(3) + 6 or 174 ft^2. She could buy 1 gallon for \$30 or 2 quarts for \$28.

The area of the background is $12(20) - 71$ or 169 ft^2. She needs enough yellowpaint to cover 169(3) = 507 ft^2. She could buy 2 gallons for \$60, 1 gallon and 2 quarts for \$58, or 1 gallon, 1 quart, and 4 8-oz. bottles for \$59.

Therefore, to minimize costs, Madison should buy 1 gal, 1 qt, and four 8-oz bottles of yellow paint; 2

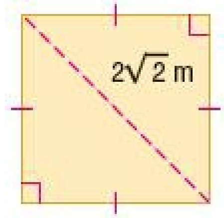
quarts of blue paint; and three 8-oz bottles of red paint: and one 8-oz bottle of purple paint.

ANSWER:

a. yellow: 1 gal, 1 qt, and 3 8-oz bottles; blue: 2 qt; red: 3 8-oz bottles; purple 1 - 8oz bottle

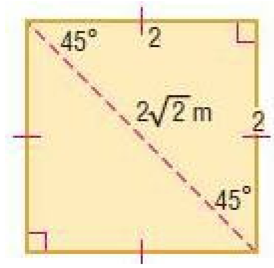
b. 1 qt of paint covers 87.5 ft^2 . There are 4 qts in a gal, so 1 gal covers 4(87.5) or 350 ft^2 . There are 32 oz in a qt, so 1 oz covers 2.73 ft^2 . The paint is sold in 8-oz bottles, so one bottle covers 21.88 ft^2 . The purple window has an area of 4 ft^2 . It is more economical for Madison to mix the purple paint with the remaining red and blue paint. She needs enough purple paint to cover 12 ft^2 , or an additional 6 ft^2 with both red and blue paint. The roof has an area of 15 ft^2 , so she needs enough red paint to cover 15(3) + 6 = 51 ft^2 . She could buy 1 qt for \$14 or 3 bottles for \$11.25. The tower has an area of $5(12) - 4$ or 56 ft^2 . She needs enough blue paint to cover 56(3) + 6 or 174 ft^2 . She could buy 1 gal for \$30 or 2 qts for \$28. The area of the background is $12(20) - 71$ or 169 ft^2 . She needs enough yellow paint to cover 169(3) = 507 ft^2 . She could buy 2 gal for \$60, 1 gal and 2 qts for \$58, or 1 gal, 1 qt, and 4 bottles for \$59.

Find the perimeter and area of each figure. Round to the nearest hundredth, if necessary.

30. 

SOLUTION:

Use the 45°-45°-90° triangle to find the lengths of the sides.

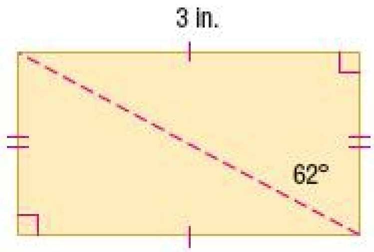


The perimeter is 4 • 2 = 8

The area is 2^2 or 4 m^2.

ANSWER:

8 m; 4 m^2

31. 

SOLUTION:

Use trigonometry to find the width of the rectangle.

$$\tan x = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 62 = \frac{3}{w}$$

$$w\tan 62 = 3$$

$$w = \frac{3}{\tan 62}$$

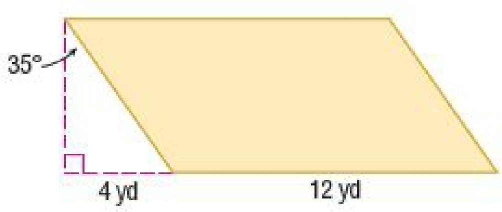
$$w \approx 1.6$$

$$\begin{aligned} P &= 2\left(3 + \frac{3}{\tan 62}\right) \\ &= 6 + \frac{6}{\tan 62} \\ &\approx 9.19 \end{aligned}$$

$$\begin{aligned} A &= \ell w \\ &= 3 \cdot \frac{3}{\tan 62} \\ &\approx 4.79 \end{aligned}$$

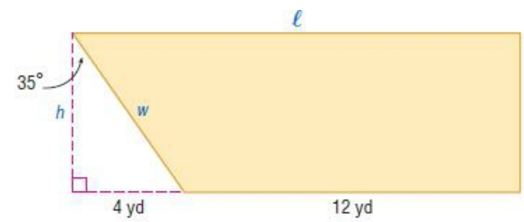
ANSWER:

9.19 in.; 4.79 in^2

32. 

SOLUTION:

Use trigonometry to find the side length of the parallelogram.



$$\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin 35 = \frac{4}{w}$$

$$w\sin 35 = 4$$

$$w = \frac{4}{\sin 35}$$

$$w \approx 7.0$$

$$\begin{aligned} P &= 2\left(12 + \frac{4}{\sin 35}\right) \\ &= 24 + \frac{8}{\sin 35} \\ &\approx 37.95 \end{aligned}$$

Use trigonometry to find h.

$$\tan x = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 35 = \frac{4}{h}$$

$$h\tan 35 = 4$$

$$h = \frac{4}{\tan 35}$$

$$h \approx 5.7$$

$$\begin{aligned} A &= \ell h \\ &= 12\left(\frac{4}{\tan 35}\right) \\ &\approx 68.55 \end{aligned}$$

ANSWER:

37.95 yd; 68.55 yd^2

33. **ALGEBRA** The base of a triangle is twice its height. If the area of the triangle is 49 square feet, find its base and height.

SOLUTION:
Use the formula for the area of a triangle.

$A = 0.5bh$
$49 = 0.5(2h)(h)$
$49 = h^2$
$7 = h$

$b = 2h$
$b = 2(7) = 14$

ANSWER:
$h = 7$ ft
$b = 14$ ft

34. **ALGEBRA** The height of a triangle is 3 meters less than its base. If the area of the triangle is 44 square meters, find its base and height.

SOLUTION:
Use the formula for the area of a triangle.

$A = \frac{1}{2}bh$

$44 = \frac{1}{2}b(b - 3)$

$88 = b^2 - 3b$

$0 = b^2 - 3b - 88$

$0 = (b - 11)(b + 8)$

$b = 11, b = -8$

Only a positive number makes sense here, so the base is 11 m, and the height is 8 m.

ANSWER:
$b = 11$ m, $h = 8$ m

35. **HERON'S FORMULA** Heron's Formula relates the lengths of the sides of a triangle to the area of the triangle. The formula is $A = \sqrt{s(s-a)(s-b)(s-c)}$, where s is the *semiperimeter*, or one half the perimeter, of the triangle and a, b, and c are the side lengths.
a. Use Heron's Formula to find the area of a triangle with side lengths 7, 10, and 4.
b. Show that the areas found for a 5–12–13 right triangle are the same using Heron's Formula and using the triangle area formula you learned earlier in this lesson.

SOLUTION:
a. Substitute $a = 7$, $b = 10$ and $c = 4$ in the formula.

$$s = \frac{a+b+c}{2} = \frac{7+10+4}{2} = 10.5$$
$$A = \sqrt{10.5(10.5-7)(10.5-10)(10.5-4)}$$
$$= \sqrt{119.4375}$$
$$\approx 10.9 \text{ unit}^2$$

b.
$$\sqrt{s(s-a)(s-b)(s-c)} \stackrel{?}{=} \frac{1}{2}bh$$
$$\sqrt{15(15-5)(15-12)(15-13)} \stackrel{?}{=} \frac{1}{2}(5)(12)$$
$$\sqrt{15(10)(3)(2)} \stackrel{?}{=} 30$$
$$\sqrt{900} \stackrel{?}{=} 30$$
$$30 = 30 \checkmark$$

ANSWER:
a. 10.9 units^2
b.
$$\sqrt{s(s-a)(s-b)(s-c)} \stackrel{?}{=} \frac{1}{2}bh$$
$$\sqrt{15(15-5)(15-12)(15-13)} \stackrel{?}{=} \frac{1}{2}(5)(12)$$
$$\sqrt{15(10)(3)(2)} \stackrel{?}{=} 30$$
$$\sqrt{900} \stackrel{?}{=} 30$$
$$30 = 30$$

36. **MULTIPLE REPRESENTATIONS** In this problem, you will investigate the relationship between the area and perimeter of a rectangle.

a. ALGEBRAIC A rectangle has a perimeter of 12 units. If the length of the rectangle is x and the width of the rectangle is y, write equations for the perimeter and area of the rectangle.

b. TABULAR Tabulate all possible whole-number values for the length and width of the rectangle, and find the area for each pair.

c. GRAPHICAL Graph the area of the rectangle with respect to its length.

d. VERBAL Describe how the area of the rectangle changes as its length changes.

e. ANALYTICAL For what whole-number values of length and width will the area be greatest? least? Explain your reasoning.

SOLUTION:

a. $P = 2x + 2y$; $A = xy$

b. Solve for y.

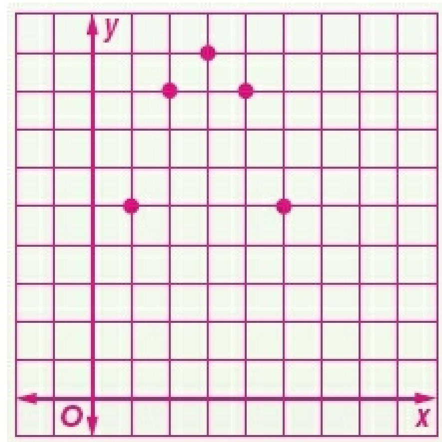
$$\begin{aligned} 12 &= 2x + 2y \\ 12 - 2x &= 2y \\ 6 - x &= y \end{aligned}$$

Now input values for x to get corresponding values for y.

Find xy to get the area.

Length, x	Width, y	Area
1	5	5
2	4	8
3	3	9
4	2	8
5	1	5

c. Length is x and is inputted as the x-coordinates. Area is inputted as the y-coordinates.



d. Sample answer: The plots (and the y-coordinates) go up as the x-coordinates move from 1 to 3, and then move down as the x-coordinates move from 3 to 5.

The area increases as the length increases from 1 to 3, is highest at 3, then decreases as the length increases to 5.

e. Sample answer: The graph reaches its highest point when $x = 3$, so the area of the rectangle will be greatest when the length is 3. The graph reaches its lowest points when $x = 1$ and 5, so the area of the rectangle will be the smallest when the length is 1 or 5, assuming the lengths are always whole numbers.

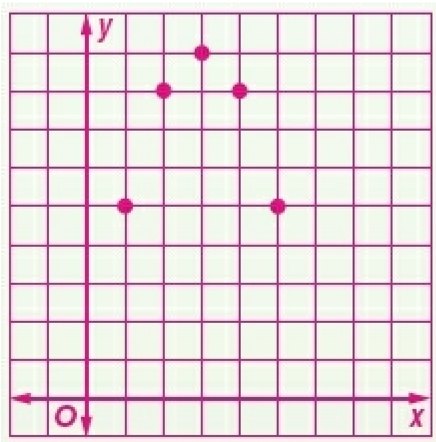
ANSWER:

a. $P = 2x + 2y$; $A = xy$

b.

Length, x	Width, y	Area
1	5	5
2	4	8
3	3	9
4	2	8
5	1	5

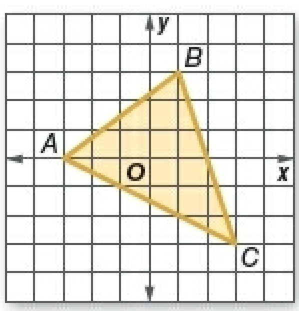
c.



d. Sample answer: The area increases as the length increases from 1 to 3, is highest at 3, then decreases as the length increases to 5.

e. Sample answer: The graph reaches its highest point when $x = 3$, so the area of the rectangle will be greatest when the length is 3. The graph reaches its lowest points when $x = 1$ and 5, so the area of the rectangle will be the smallest when the length is 1 or 5.

37. **CHALLENGE** Find the area of ΔABC. Explain your method.

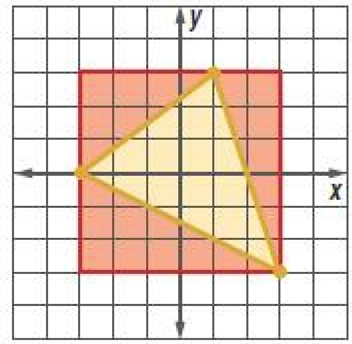


SOLUTION:

One method of solving would be to find the length of one of the bases and then calculate the corresponding height. The length of the sides can be found by using the distance formula; for example by finding the distance between *A* and *C* to calculate the length of *AC*. The height will be more challenging, because we will need to determine the perpendicular from point *B* to line *AC*.

We could also use Heron's formula, which is described in problem #35 in this lesson.

A third method will be to inscribe the triangle in a 6 by 6 square. When we do this, we form three new triangles as shown.



The area of the square is 36 units^2. The three new triangles are all right triangles and the lengths of their sides can be found by subtracting the coordinates.

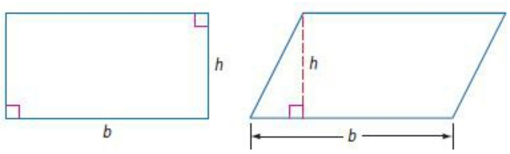
The areas of the three triangles are 6 unit^2, 6 unit^2, and 9 unit^2 respectively. Therefore, the area of the main triangle is the difference or 15 unit^2.

ANSWER:

15 units^2 ; Sample answer: I inscribed the triangle in a 6-by-6 square. I found the area of the square and subtracted the areas of the three right triangles inside the square that were positioned around the given triangle. The area of the given triangle is the difference, or 15 units^2 .

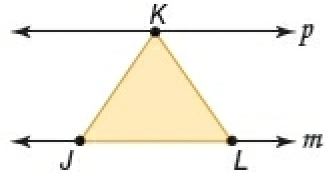
38. **CONSTRUCT ARGUMENTS** Will the perimeter of a nonrectangular parallelogram *sometimes*, *always*, or *never* be greater than the perimeter of a rectangle with the same area and the same height? Explain.

SOLUTION:
The perimeter of a nonrectangular parallelogram *always* is greater than the perimeter of a rectangle with the same area and the same height. If the areas are equal, the perimeter of the nonrectangular parallelogram will always be greater because the length of the side that is not perpendicular to the height forms a right triangle with the height. The height is a leg of the triangle and the side of the parallelogram is the hypotenuse of the triangle. Since the hypotenuse is always the longest side of a right triangle, the non-perpendicular side of the parallelogram is always greater than the height. The bases of the quadrilaterals have to be the same because the areas and the heights are the same. Since the bases are the same and the height of the rectangle is also the length of a side, the perimeter of the parallelogram will always be greater.

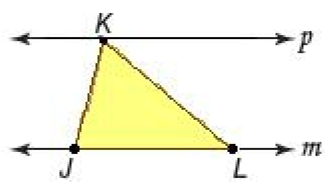


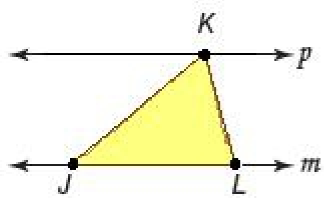
ANSWER:
Always; sample answer: If the areas are equal, the perimeter of the nonrectangular parallelogram will always be greater because the length of the side that is not perpendicular to the height forms a right triangle with the height. The height is a leg of the triangle and the side of the parallelogram is the hypotenuse of the triangle. Since the hypotenuse is always the longest side of a right triangle, the non-perpendicular side of the parallelogram is always greater than the height. The bases of the quadrilaterals have to be the same because the areas and the heights are the same. Since the bases are the same and the height of the rectangle is also the length of a side, the perimeter of the parallelogram will always be greater.

39. **WRITING IN MATH** Points J and L lie on line m. Point K lies on line p. If lines m and p are parallel, describe how the area of $\triangle JKL$ will change as K moves along line p.



SOLUTION:
Sample answer: The area will not change as K moves along line p. Since lines m and p are parallel, the perpendicular distance between them is constant. That means that no matter where K is on line p, the perpendicular distance to line p, or the height of the triangle, is always the same.





Since point J and L are not moving, the distance between them, or the length of the base, is constant. Since the height of the triangle and the base of the triangle are both constant, the area will always be the same.

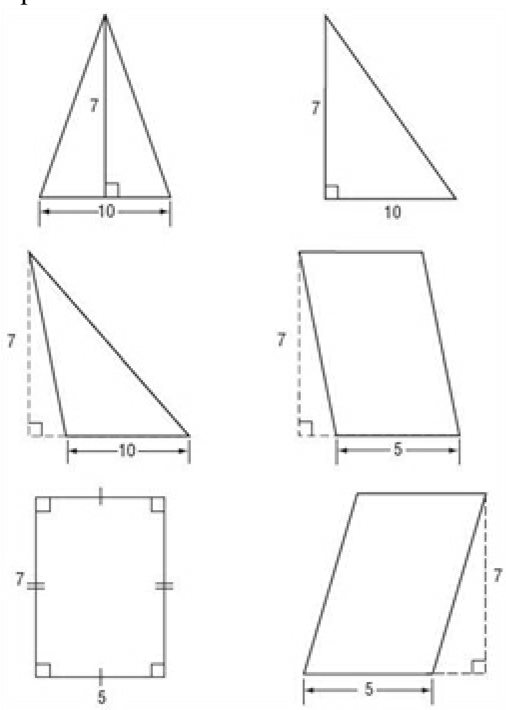
ANSWER:
Sample answer: The area will not change as K moves along line p. Since lines m and p are parallel, the perpendicular distance between them is constant. That means that no matter where K is on line p, the perpendicular distance to line p, or the height of the triangle, is always the same. Since point J and L are not moving, the distance between them, or the length of the base, is constant. Since the height of the triangle and the base of the triangle are both constant, the area will always be the same.

40. **OPEN-ENDED** The area of a polygon is 35 square units. The height is 7 units. Draw three different triangles and three different parallelograms that meet these requirements. Label the base and height on each.

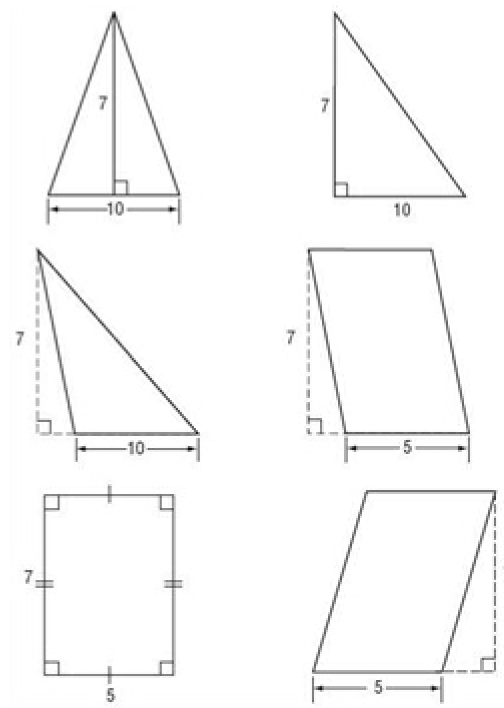
SOLUTION:

For each triangle, maintain a height of 7 and a base of 10, and the area will not change from 35 square units.

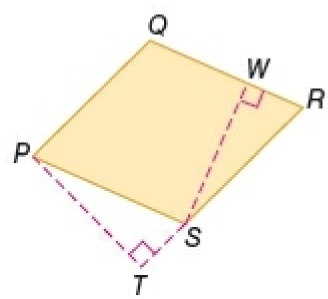
For each parallelogram, maintain a height of 7 and a base of 5, and the area will not change from 35 square units.



ANSWER:



41. **WRITING IN MATH** Describe two different ways you could use measurement to find the area of parallelogram *PQRS*.

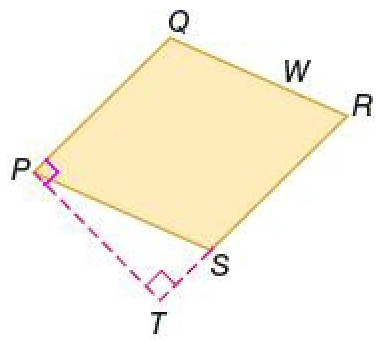


SOLUTION:

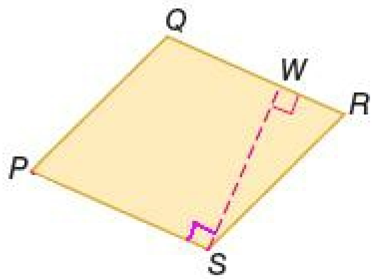
Sample answer:

The area of a parallelogram is the product of the base and the height, where the height is perpendicular to the base. Opposite sides of a parallelogram are parallel so either base can be multiplied by the height to find the area.

To find the area of the parallelogram, you can measure the height $\overline{PT}$ and then measure one of the bases $\overline{PQ}$ or $\overline{SR}$ and multiply the height by the base to get the area.



You can also measure the height $\overline{SW}$ and measure one of the bases $\overline{QR}$ or $\overline{PS}$ and then multiply the height by the base to get the area.

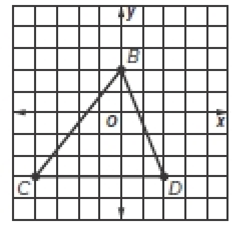


It doesn't matter which side you choose to use as the base, as long as you use the height that is perpendicular to that base to calculate the area.

ANSWER:

Sample answer: To find the area of the parallelogram, you can measure the height $\overline{PT}$ and then measure one of the bases $\overline{PQ}$ or $\overline{SR}$ and multiply the height by the base to get the area. You can also measure the height $\overline{SW}$ and measure one of the bases $\overline{QR}$ or $\overline{PS}$ and then multiply the height by the base to get the area. It doesn't matter which side you choose to use as the base, as long as you use the height that is perpendicular to that base to calculate the area.

Consider $\triangle BCD$.



42. Find the perimeter of $\triangle BCD$. If necessary, round to the nearest tenth.

A 12.4 units
B 17.8 units
C 29.0 units
D 35.6 units

SOLUTION:

To find the perimeter, find the length of each side of the triangle.

$CD = 2 - (-4) = 6$

$BC = \sqrt{(0-(-4))^2 + (2-(-3))^2}$

$BC = \sqrt{41}$

$BD = \sqrt{(0-2)^2 + (2-(-3))^2}$

$BD = \sqrt{29}$

Perimeter = $6 + \sqrt{41} + \sqrt{29}$

Perimeter ≈ 17.79

The correct answer is choice C, 17.8 units.

ANSWER:

C

43. Find the area of $\triangle BCD$. If necessary, round to the nearest tenth.

A 15 units^2
B 16.2 units^2
C 17.2 units^2
D 19.2 units^2

SOLUTION:
To find the area, first find the base and the height.

Let the base be *CD*.

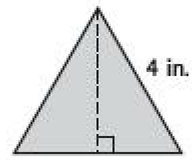
CD = 2 – (–4) = 6
height = 2 – (–3) = 5

Area = (0.5)(6)(5) = 15 square units.

The correct answer is choice A.

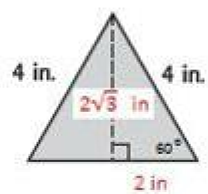
ANSWER:
A

44. Katie makes coasters by cutting pieces of cardboard into equilateral triangles with the dimensions shown.



What is the area of each coaster?
A $2\sqrt{3}$ in^2
B 4 in^2
C $4\sqrt{2}$ in^2
D $4\sqrt{3}$ in^2
E $8\sqrt{3}$ in^2

SOLUTION:
The area of an equilateral triangle can be found by finding its height and base. Since the altitude of an equilateral triangle divides the triangle into two congruent 30-60-90 special right triangles, the height of the equilateral triangle is $2\sqrt{3}$ inches.



$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2}(4)(2\sqrt{3})$$
$$A = 4\sqrt{3}\text{ in}^2$$

The correct choice is D.

ANSWER:
D

45. A parallelogram is in the shape of a rectangle. The parallelogram has a length of 7.5 meters and a width of 6.5 meters. What is the perimeter and area of the parallelogram?

A 19 meters and 36 square meters
B 21 meters and 40 square meters
C 25 meters and 50 square meters
D 28 meters and 48.75 square meters

SOLUTION:
To find the perimeter and area of a rectangle, use the formulas for each.

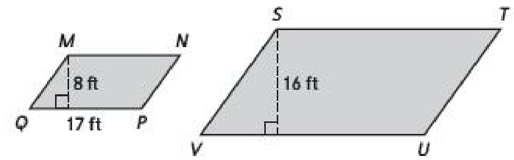
length = 7.5 meters, width = 6.5 meters

Area = (7.5)(6.5) = 48.75 square meters
Perimeter = 2(7.5 + 6.5) = 28 meters

The correct answer is choice D.

ANSWER:
D

46. The area of parallelogram *STUV* is 4 times the area of parallelogram *MNPQ*.



What is the length of $\overline{VU}$?
A 68 ft
B 34 ft
C 32 ft
D 25 ft

SOLUTION:
First, find the area of parallelogram *MNPQ*:
$A = bh$
$A = 17 \cdot 8$
$A = 136\,\text{ft}^2$

Therefore, the area of parallelogram of *STUV* is 4(136) or 544 ft^2. Use this area, along with the given height of 16 ft, to find the length of base $\overline{VU}$:

$A = bh$
$544 = (VU)(16)$
$34 = VU$ The correct choice is B.

ANSWER:
B

47. Ben uses software to draw an isosceles right triangle with a hypotenuse that has endpoints at (6, 0) and (0, 6). What is the area of the triangle in square units?

SOLUTION:
An isosceles right triangle with endpoints of the hypotenuse at (0, 6) and (6, 0) will have its other vertex at either (0, 0) or (6, 6).

The legs of this triangle will be 6 units long each. The area of this triangle is
$A = \frac{1}{2}(6)(6) = 18 \text{ square units}$.

ANSWER:
18

48. A raised garden is shaped like a parallelogram with two sides that meet at a 45° angle. The garden has an area of $84\sqrt{2}$ square feet and a base of 14 feet. What is the perimeter of the garden in feet?

SOLUTION:

A raised garden is shaped like a parallelogram with two sides that meet at a 45° angle. The garden has an area of $84\sqrt{2}$ square feet and a base of 14 feet. What is the perimeter of the garden in feet?

Area = (base)(height)

$$84\sqrt{2} = 14h$$
$$6\sqrt{2} = h$$

The other side of the parallelogram is the hypotenuse of a 45°-45°-90° triangle, so its length is $s = h\sqrt{2} = 6\left(\sqrt{2}\right)^2 = 12$.

The perimeter is 2(12 + 14) = 52 feet.

ANSWER:

52

49. **MULTI-STEP** A parallelogram has a base that is six times the length of the height.

a. What is an expression for the area of the parallelogram in terms of the height, *h*?

b. A triangle has the same area and height *h* as the parallelogram. What is an expression for the base of the triangle?

c. Find the area of the parallelogram if the height is 6.

d. Find the length of the base of a triangle that has the same area and a height of 6.

SOLUTION:

a. The parallelogram has a base that is six times its height.

Area = bh

$$A = 6h(h) = 6h^2$$

b. A triangle has the same area and height *h* as the parallelogram.

$$A = \frac{1}{2}bh$$
$$6h^2 = \frac{1}{2}bh$$
$$12h = b$$

c. Substitute to find the area of the parallelogram if the height is 6.

$$A = 6h^2$$
$$A = 6(6)^2$$
$$A = 216$$

d. Substitute to find the length of the base of a triangle that has the same area and a height of 6.

$$A = \frac{1}{2}bh$$
$$216 = \frac{1}{2}b(6)$$
$$72 = b$$

ANSWER:

a. $6h^2$

b. $12h$

c. 216

d. 72