Lesson 17: The Area of a Circle

Student Outcomes

- Students give an informal derivation of the relationship between the circumference and area of a circle.
- Students know the formula for the area of a circle and use it to solve problems.

Lesson Notes

- Remind students of the definitions for circle and circumference from the previous lesson. The Opening Exercise is a lead-in to the derivation of the formula for the area of a circle.
- Not only do students need to know and be able to apply the formula for the area of a circle, it is critical for them to also be able to draw the diagram associated with each problem in order to solve it successfully.
- Students must be able to translate words into mathematical expressions and equations and be able to determine which parts of the problem are known and which are unknown or missing.

Classwork

Exercises 1–3 (4 minutes)

Exercises 1–3

Solve the problem below individually. Explain your solution.

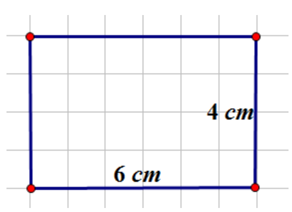
1. **Find the radius of the following circle if the circumference is 37.68 inches. Use $\pi \approx 3.14$.**

   If $C = 2\pi r$, then $37.68 = 2\pi r$. Solving the equation for r:

$$37.68 = 2\pi r$$
$$\left(\frac{1}{2\pi}\right)37.68 = \left(\frac{1}{2\pi}\right)2\pi r$$
$$\frac{1}{6.28}(37.68) \approx r$$
$$6 \approx r$$

   The radius of the circle is approximately 6 in.

2. **Determine the area of the rectangle below. Name two ways that can be used to find the area of the rectangle.**



   The area of the rectangle is $24\ \text{cm}^2$. The area can be found by counting the square units inside the rectangle or by multiplying the length (6 cm) by the width (4 cm).

3. Find the length of a rectangle if the area is 27 cm^2 and the width is $3\ cm$.

If the area of the rectangle is $\text{Area} = \text{length} \cdot \text{width}$, *then* $27\text{ cm}^2 = \text{l} \cdot 3\text{ cm}$.

$$\frac{1}{3} \cdot 27\text{ cm}^2 = \frac{1}{3} \cdot \text{l} \cdot 3\text{ cm}$$

$$9\text{ cm} = \text{l}$$

Exploratory Challenge (10 minutes)

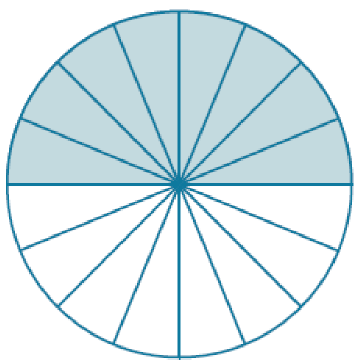
Complete the exercise below.

Scaffolding:

Provide a circle divided into 16 equal sections for students to cut out and re-assemble as a rectangle.

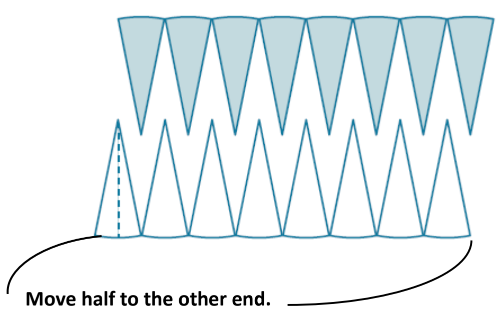
Exploratory Challenge

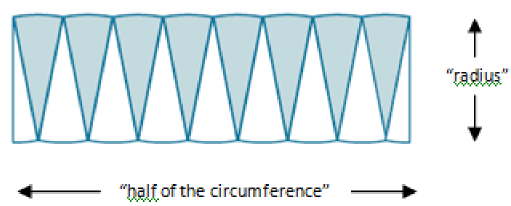
To find the formula for the area of a circle, cut a circle into 16 equal pieces.



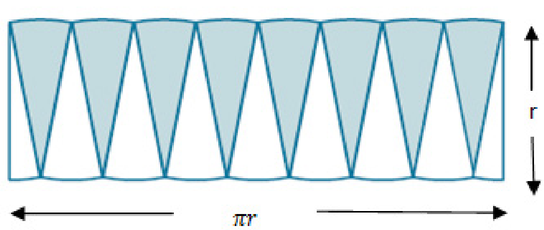
MP.7

Arrange the triangular wedges by alternating the "triangle" directions and sliding them together to make a "parallelogram." Cut the triangle on the left side in half on the given line, and slide the outside half of the triangle to the other end of the parallelogram in order to create an approximate "rectangle."





The circumference is $2\pi r$, where the radius is "r." Therefore, half of the circumference is πr.



What is the area of the "rectangle" using the side lengths above?

The area of the "rectangle" is base times height, and, in this case, $A = \pi r \cdot r$.

Are the areas of the rectangle and the circle the same?

Yes, since we just rearranged pieces of the circle to make the "rectangle," the area of the "rectangle" and the area of the circle are approximately equal. Note that the more sections we cut the circle into, the closer the approximation.

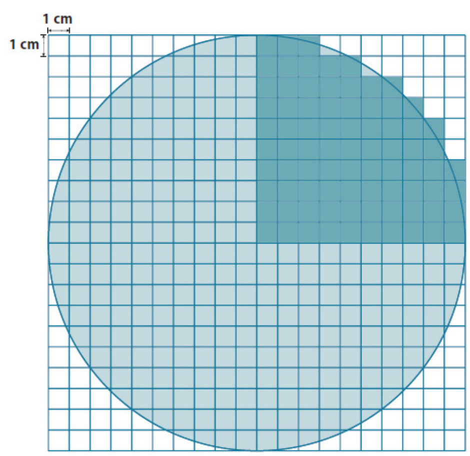
If the area of the rectangular shape and the circle are the same, what is the area of the circle?

The area of a circle is written as $A = \pi r \cdot r$, *or* $A = \pi r^2$.

Example 1 (4 minutes)

Example 1

Use the shaded square centimeter units to approximate the area of the circle.



What is the radius of the circle?

10 cm

What would be a quicker method for determining the area of the circle other than counting all of the squares in the entire circle?

Count $\frac{1}{4}$ *of the squares needed; then, multiply that by four in order to determine the area of the entire circle.*

Using the diagram, how many squares were used to cover one-fourth of the circle?

The area of one-fourth of the circle is approx. 79 cm^2.

What is the area of the entire circle?

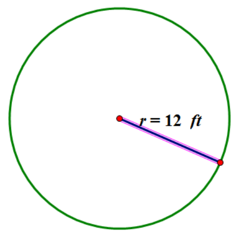
$$A \approx 4 \cdot 79\text{ cm}^2$$
$$A \approx 316\text{ cm}^2$$

Example 2 (4 minutes)

Example 2

A sprinkler rotates in a circular pattern and sprays water over a distance of 12 feet. What is the area of the circular region covered by the sprinkler? Express your answer to the nearest square foot.

Draw a diagram to assist you in solving the problem. What does the distance of 12 feet represent in this problem?



The radius is 12 feet.

What information is needed to solve the problem?

The formula to find the area of a circle is $A = \pi r^2$. If the radius is 12 ft., then $A = \pi \cdot (12 \text{ ft.})^2 = 144\pi \text{ ft}^2$, or approximately 452 ft^2.

Make a point of telling students that an answer in exact form is in terms of π, not substituting an approximation of pi.

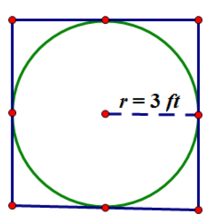
Example 3 (4 minutes)

Example 3

Suzanne is making a circular table out of a square piece of wood. The radius of the circle that she is cutting is 3 feet. How much waste will she have for this project? Express your answer to the nearest square foot.

Draw a diagram to assist you in solving the problem. What does the distance of 3 feet represent in this problem?

The radius of the circle is 3 feet.



What information is needed to solve the problem?

The area of the circle and the area of the square are needed so that we can subtract the area of the square from the area of the circle to determine the amount of waste.

What information do we need to determine the area of the square and the circle?

Circle: just radius because $A = \pi r^2$. Square: one side length.

How will we determine the waste?

The waste will be the area left over from the square after cutting out the circular region. The area of the circle is $A = \pi \cdot (3\text{ ft.})^2 = 9\pi\text{ ft}^2 \approx 28.26\text{ ft}^2$. *The area of the square is found by first finding the diameter of the circle, which is the same as the side of the square. The diameter is* $d = 2r$*; so,* $d = 2 \cdot 3\text{ ft.}$ *or* 6 ft. *The area of a square is found by multiplying the length and width; so,* $A = 6\text{ ft.} \cdot 6\text{ ft.} = 36\text{ ft}^2$. *The solution will be the difference between the area of the square and the area of the circle; so,* $36\text{ ft}^2 - 28.26\text{ ft}^2 \approx 7.74\text{ ft}^2$.

Does your solution answer the problem as stated?

Yes, the amount of waste is 7.74 ft^2.

Exercises 4–6 (11 minutes)

Solve in cooperative groups of two or three.

Exercises 4–6

4. A circle has a radius of 2 cm.

a. Find the exact area of the circular region.

$A = \pi \cdot (2\text{ cm})^2 = 4\pi\text{ cm}^2$

b. Find the approximate area using 3.14 to approximate π.

$A = 4 \cdot \pi\text{ cm}^2 \approx 4\text{ cm}^2 \cdot 3.14 \approx 12.56\text{ cm}^2$

5. A circle has a radius of 7 cm.

a. Find the exact area of the circular region.

$A = \pi \cdot (7\text{ cm})^2 = 49\pi\text{ cm}^2$

b. Find the approximate area using $\frac{22}{7}$ to approximate π.

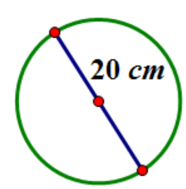
$A = 49 \cdot \pi\text{ cm}^2 \approx \left(49 \cdot \frac{22}{7}\right)\text{cm}^2 \approx 154\text{ cm}^2$

c. What is the circumference of the circle?

$C = 2\,\pi \cdot 7\text{ cm} = 14\pi\text{ cm} \approx 43.96\text{ cm}$

6. Joan determined that the area of the circle below is $400\pi\text{ cm}^2$. Melinda says that Joan's solution is incorrect; she believes that the area is $100\pi\text{ cm}^2$. Who is correct and why?

Melinda is correct. Joan found the area by multiplying π *by the square of* 20 cm *(which is the diameter) to get a result of* $400\pi\text{ cm}^2$*, which is incorrect. Melinda found that the radius was* 10 cm *(half of the diameter). Melinda multiplied* π *by the square of the radius to get a result of* $100\pi\text{ cm}^2$.

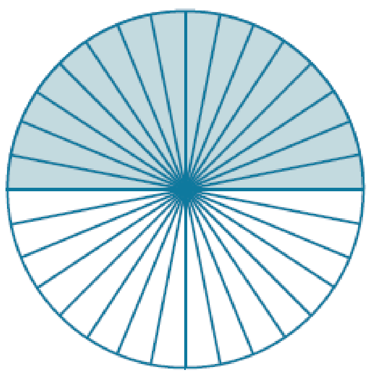


Closing (3 minutes)

- Strategies for problem solving include drawing a diagram to represent the problem and identifying the given information and needed information to solve the problem.
- Using the original circle in this lesson, cut it into 64 equal slices. Reassemble the figure. What do you notice?
  - *It looks more like a rectangle.*

Ask students to imagine repeating the slicing into even thinner slices (infinitely thin). Then, ask the next two questions.

- What does the length of the rectangle become?
  - *An approximation of half of the circumference of the circle.*
- What does the width of the rectangle become?
  - *An approximation of the radius.*
- Thus, we conclude that the area of the circle is $A = \frac{1}{2}Cr$.
  - If $A = \frac{1}{2}Cr$, then $A = \frac{1}{2} \cdot 2\pi r \cdot r$ or $A = \pi r^2$.
  - Also see video link: http://www.youtube.com/watch?v=YokKp3pwVFc



> **Relevant Vocabulary**
>
> **CIRCULAR REGION (OR DISK): Given a point C in the plane and a number $r > 0$, the *circular region (or disk) with center C and radius r* is the set of all points in the plane whose distance from the point C is less than or equal to r.**
>
> **The boundary of a disk is a circle. The "area of a circle" refers to the area of the disk defined by the circle.**

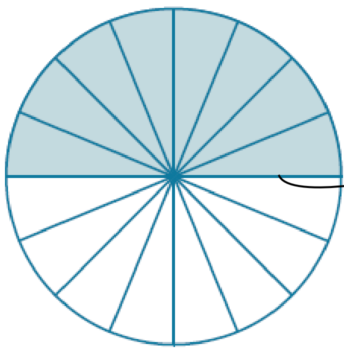
Exit Ticket (4 minutes)

Name ____ Date____________________

Lesson 17: The Area of a Circle

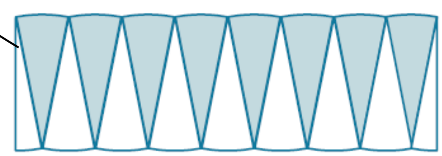
Exit Ticket

Complete each statement using the words or algebraic expressions listed in the word bank below.



1. The length of the ___________________ of the rectangular region approximates the length of the ___________________ of the circle.

2. The ___________________ of the rectangle approximates the length as one-half of the circumference of the circle.



3. The circumference of the circle is _________________.

4. The _________________ of the ___________________ is $2r$.

5. The ratio of the circumference to the diameter is ______.

6. Area (circle) = Area of (_____________) = $\frac{1}{2} \cdot \text{circumference} \cdot r = \frac{1}{2}(2\pi r) \cdot r = \pi \cdot r \cdot r =$ _____________.

Word bank				
Radius	**Height**	**Base**	$2\pi r$	
Diameter	**Circle**	**Rectangle**	πr^2	π

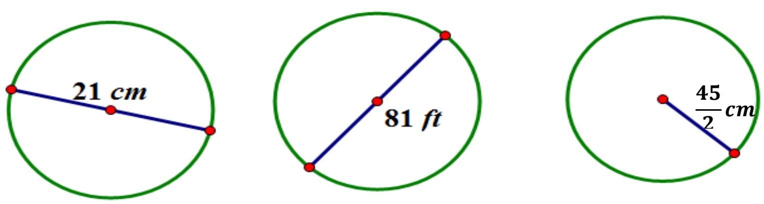
Exit Ticket Sample Solutions

Complete each statement using the words or algebraic expressions listed in the word bank below.

1. The length of the ***height*** of the rectangular region approximates the length of the ***radius*** of the circle.
2. The ***base*** of the rectangle approximates the length as ***one-half of the circumference*** of the circle.
3. The ***circumference*** of the circle is $2\pi r$.
4. The ***diameter*** of the circle is $2r$.
5. The ratio of the circumference to the diameter is π.
6. Area (circle) = Area of (***rectangle***) $= \frac{1}{2} \cdot \text{circumference} \cdot r = \frac{1}{2}(2\pi r) \cdot r = \pi \cdot r \cdot r = \pi r^2$.

Problem Set Sample Solutions

1. The following circles are not drawn to scale. Find the area of each circle. (Use $\frac{22}{7}$ as an approximation for π.)



346.5 cm^2 $\quad$ $5{,}155.1\text{ ft}^2$ $\quad$ $1{,}591.1\text{ cm}^2$

2. A circle has a diameter of 20 inches.

   a. Find the exact area and find an approximate area using $\pi \approx 3.14$.

   If the diameter is 20 in., then the radius is 10 in. If $A = \pi r^2$, then $A = \pi \cdot (10\text{ in.})^2$ or $100\pi\text{ in}^2$.
   $A \approx (100 \cdot 3.14)\text{ in}^2 \approx 314\text{ in}^2$.

   b. What is the circumference of the circle using $\pi \approx 3.14$?

   If the diameter is 20 in., then the circumference is $C = \pi d$ or $C \approx 3.14 \cdot 20\text{ in.} \approx 62.8\text{ in.}$

3. A circle has a diameter of 11 inches.

   a. Find the exact area and an approximate area using $\pi \approx 3.14$.

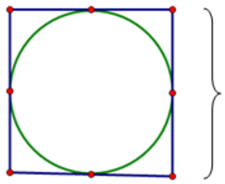
   If the diameter is 11 in., then the radius is $\frac{11}{2}$ in. If $A = \pi r^2$, then $A = \pi \cdot \left(\frac{11}{2}\text{ in.}\right)^2$ or $\frac{121}{4}\pi\text{ in}^2$.

   $$A \approx \left(\frac{121}{4} \cdot 3.14\right)\text{in}^2 \approx 94.985\text{ in}^2$$

   b. What is the circumference of the circle using $\pi \approx 3.14$?

   If the diameter is 11 inches, then the circumference is $C = \pi d$ or $C \approx 3.14 \cdot 11\text{ in.} \approx 34.54\text{ in.}$

4. **Using the figure below, find the area of the circle.**



In this circle, the diameter is the same as the length of the side of the square. The diameter is 10 cm; so, the radius is 5 cm. $A = \pi r^2$, so $A = \pi(5\text{ cm})^2 = 25\pi\text{ cm}^2$.

5. **A path bounds a circular lawn at a park. If the inner edge of the path is 132 ft. around, approximate the amount of area of the lawn inside the circular path. Use $\pi \approx \frac{22}{7}$.**

The length of the path is the same as the circumference. Find the radius from the circumference; then, find the area.

$$C = 2\pi r$$
$$132\text{ ft.} \approx 2 \cdot \frac{22}{7} \cdot r$$
$$132\text{ ft.} \approx \frac{44}{7} r$$
$$\frac{7}{44} \cdot 132\text{ ft.} \approx \frac{7}{44} \cdot \frac{44}{7} r$$
$$21\text{ ft.} \approx r$$
$$A \approx \frac{22}{7} \cdot (21\text{ ft.})^2$$
$$A \approx 1386\text{ ft}^2$$

6. **The area of a circle is $36\pi\text{ cm}^2$. Find its circumference.**

Find the radius from the area of the circle; then, use it to find the circumference.

$$A = \pi r^2$$
$$36\pi\text{ cm}^2 = \pi r^2$$
$$\frac{1}{\pi} \cdot 36\pi\text{ cm}^2 = \frac{1}{\pi} \cdot \pi r^2$$
$$36\text{ cm}^2 = r^2$$
$$6\text{ cm} = r$$
$$C = 2\pi r$$
$$C = 2\pi \cdot 6\text{ cm}$$
$$C = 12\pi\text{ cm}$$

7. **Find the ratio of the area of two circles with radii 3 cm and 4 cm.**

The area of the circle with radius 3 cm is $9\pi\text{ cm}^2$. The area of the circle with the radius 4 cm is $16\pi\text{ cm}^2$. The ratio of the area of the two circles is $\frac{9\pi\text{ cm}^2}{16\pi\text{ cm}^2}$ or $\frac{9}{16}$.

8. **If one circle has a diameter of 10 cm and a second circle has a diameter of 20 cm, what is the ratio of the area of the larger circle to the area of the smaller circle?**

The area of the circle with the diameter of 10 cm will have a radius of 5 cm. The area of the circle with the diameter of 10 cm is $\pi \cdot (5\text{ cm})^2$ or $25\pi\text{ cm}^2$. The area of the circle with the diameter of 20 cm will have a radius of 10 cm. The area of the circle with the diameter of 20 cm is $\pi \cdot (10\text{ cm})^2$ or $100\pi\text{ cm}^2$. The ratio of the diameters is 20 to 10 or $2:1$, while the ratio of the areas is $100\pi\text{ cm}^2$ to $25\pi\text{ cm}^2$ or $4:1$.

9. Describe a rectangle whose perimeter is 132 ft. and whose area is less than 1 ft^2. Is it possible to find a circle whose circumference is 132 ft. and whose area is less than 1 ft^2? If not, provide an example or write a sentence explaining why no such circle exists.

A rectangle that has a perimeter of 132 ft. can have a length of 65.995 ft. and a width of 0.005 ft. The area of such a rectangle is 0.329975 ft^2, which is less than 1 ft^2. No, because a circle that has a circumference of 132 ft. will have a radius of approximately 21 ft.

$$A = \pi r^2 = \pi(21)^2 = 1387.96 \neq 1$$

10. If the diameter of a circle is double the diameter of a second circle, what is the ratio of the area of the first circle to the area of the second?

If I choose a diameter of 24 cm for the first circle, then the diameter of the second circle is 12 cm. The first circle has a radius of 12 cm and an area of $144\pi\text{ cm}^2$. The second circle has a radius of 6 cm and an area of $36\pi\text{ cm}^2$. The ratio of the area of the first circle to the second is $144\pi\text{ cm}^2$ to $36\pi\text{ cm}^2$, which is a 4 to 1 ratio. The ratio of the diameters is 2, while the ratio of the areas is the square of 2, or 4.

Lesson 18: More Problems on Area and Circumference

Student Outcomes

- Students examine the meaning of *quarter circle* and *semicircle*.
- Students solve area and perimeter problems for regions made out of rectangles, quarter circles, semicircles, and circles, including solving for unknown lengths when the area or perimeter is given.

Classwork

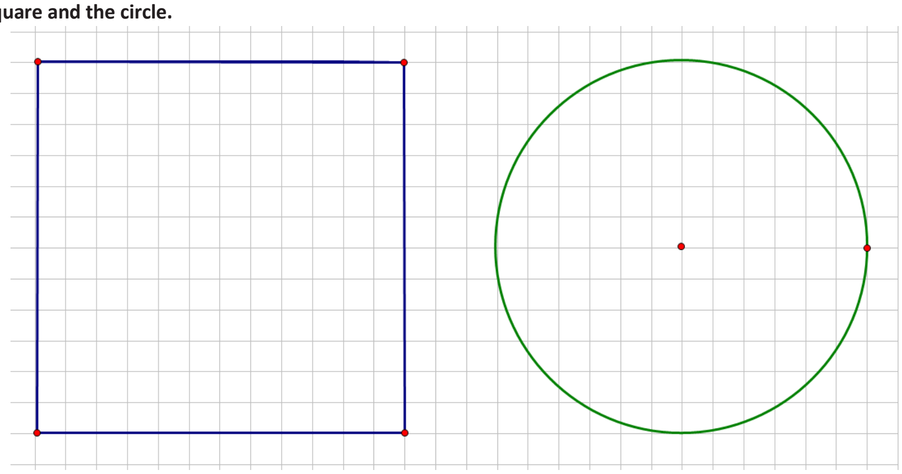
Opening Exercise (5 minutes)

Students use prior knowledge to find the area of circles, semicircles, and quarter circles and compare their areas to areas of squares and rectangles.

MP.1

Opening Exercise

Draw a circle with a diameter of 12 cm and a square with a side length of 12 cm on grid paper. Determine the area of the square and the circle.



Area of square: $A = (12\text{ cm})^2 = 144\text{ cm}^2$*; Area of circle:* $A = \pi \cdot (6\text{ cm})^2 = 36\pi\text{ cm}^2$

Brainstorm some methods for finding half the area of the square and half the area of the circle.

Some methods include folding in half and counting the grid squares, cutting each in half and counting the squares, etc.

Find the area of half of the square and half of the circle, and explain to a partner how you arrived at the area.

The area of half of the square is 72 cm^2*. The area of half of the circle is* $18\pi\text{ cm}^2$*. Some students may count the squares; others may realize that half of the square is a rectangle with side lengths of* 12 cm *and* 6 cm *and use* $A = l \cdot w$ *to determine the area. Some students may fold the square vertically, and some may fold it horizontally. Some students will try to count the grid squares in the semicircle and find that it is easiest to take half of the area of the circle.*

What is the ratio of the new area to the original area for the square and for the circle?

The ratio of the areas of the rectangle (half of the square) to the square is $\frac{72\text{ cm}^2}{144\text{ cm}^2}$ *or* $\frac{1}{2}$*. The ratio for the areas of the circles is* $\frac{18\pi\text{ cm}^2}{36\pi\text{ cm}^2}$ *or* $\frac{1}{2}$*.*

Find the area of one-fourth of the square and one-fourth of the circle, first by folding and then by another method. What is the ratio of the new area to the original area for the square and for the circle?

Folding the square in half and then in half again will result in one-fourth of the original square. The resulting shape is a square with a side length of 6 cm *and an area of* 36 cm^2*. Repeating the same process for the circle will result in an area of* $9\pi\text{ cm}^2$*. The ratio for the areas of the squares is* $\frac{36\text{ cm}^2}{72\text{ cm}^2}$ *or* $\frac{1}{4}$*. The ratio for the areas of the circles is* $\frac{9\pi\text{ cm}^2}{36\pi\text{ cm}^2}$ *or* $\frac{1}{4}$*.*

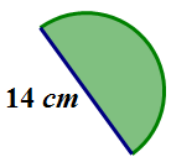
Write an algebraic expression that will express the area of a semicircle and the area of a quarter circle.

Semicircle: $A = \frac{1}{2}\pi r^2$*; Quarter circle:* $A = \frac{1}{4}\pi r^2$

Example 1 (8 minutes)

Example 1

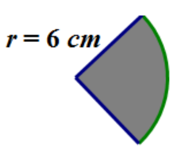
Find the area of the following semicircle. Use $\pi \approx \frac{22}{7}$.



If the diameter of the circle is 14 cm, then the radius is 7 cm. The area of the semicircle is half of the area of the circular region.

$$A \approx \frac{1}{2}\cdot\frac{22}{7}\cdot(7\text{ cm})^2$$
$$A \approx \frac{1}{2}\cdot\frac{22}{7}\cdot 49\text{ cm}^2$$
$$A \approx 77\text{ cm}^2$$

What is the area of the quarter circle? Use $\pi \approx \frac{22}{7}$.

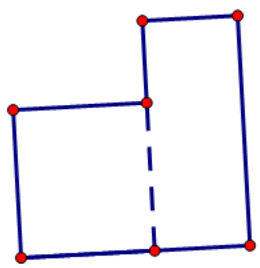


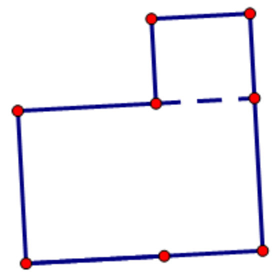
$$A \approx \frac{1}{4}\cdot\frac{22}{7}(6\text{ cm})^2$$
$$A \approx \frac{1}{4}\cdot\frac{22}{7}\cdot 36\text{ cm}^2$$
$$A \approx \frac{198}{7}\text{ cm}^2$$

Let students reason out and vocalize that the area of a quarter circle must be one-fourth of the area of an entire circle.

Discussion

Students should recognize that composition area problems involve the decomposition of the shapes that make up the entire region. It is also very important for students to understand that there are several perspectives in decomposing each shape and that there is not just one correct method. There is often more than one "correct" method; therefore, a student may feel that his/her solution (which looks different than the one other students present) is incorrect. Alleviate that anxiety by showing multiple correct solutions. For example, cut an irregular shape into squares and rectangles as seen below.



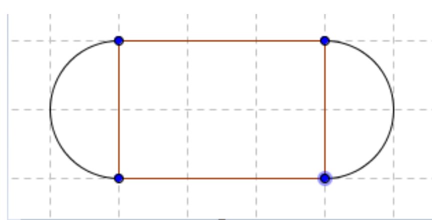


Example 2 (8 minutes)

Example 2

Marjorie is designing a new set of placemats for her dining room table. She sketched a drawing of the placement on graph paper. The diagram represents the area of the placemat consisting of a rectangle and two semicircles at either end. Each square on the grid measures 4 inches in length.

Find the area of the entire placemat. Explain your thinking regarding the solution to this problem.



The length of one side of the rectangular section is 12 inches in length, while the width is 8 inches. The radius of the semicircular region is 4 inches. The area of the rectangular part is $(8\text{ in.})\cdot(12\text{ in.}) = 96\text{ in}^2$. The total area must include the two semicircles on either end of the placemat. The area of the two semi-circular regions is the same as the area of one circle with the same radius. The area of the circular region is $A = \pi\cdot(4\text{ in.})^2 = 16\pi\text{ in}^2$. In this problem, using $\pi \approx 3.14$ will make more sense because there are no fractions in the problem. The area of the semicircular regions is approximately 50.24 in^2. The total area for the placemat is the sum of the areas of the rectangular region and the two semicircular regions, which is approximately $(96 + 50.24)\text{ in}^2 = 146.24\text{ in}^2$.

Common Mistake: Ask students to determine how to solve this problem and arrive at an incorrect solution of 196.48 in^2. A student would arrive at this answer by including the area of the circle twice instead of once $(50.24 + 50.24 + 96)$.

If Marjorie wants to make six placemats, how many square inches of fabric will she need? Assume there is no waste.

There are 6 placemats that are each 146.24 in^2, so the fabric needed for all is $6 \cdot 146.24\text{ in}^2 = 877.44\text{ in}^2$.

Marjorie decides that she wants to sew on a contrasting band of material around the edge of the placemats. How much band material will Marjorie need?

The length of the band material needed will be the sum of the lengths of the two sides of the rectangular region and the circumference of the two semicircles (which is the same as the circumference of one circle with the same radius).

$$P = (l + l + 2\pi r)\text{ in.}$$

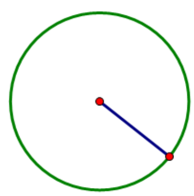
$$P = (12 + 12 + 2 \cdot \pi \cdot 4)\text{ in.} = 49.12\text{ in.}$$

Example 3 (4 minutes)

Example 3

The circumference of a circle is 24π cm. What is the exact area of the circle?

Draw a diagram to assist you in solving the problem.



What information is needed to solve the problem?

The radius is needed to find the area of the circle. Let the radius be r cm. Find the radius by using the circumference formula.

$C = 2\pi r$

$24\pi = 2\pi r$

If $24\pi = 2\pi r$, then $\left(\frac{1}{2\pi}\right)24\pi\ cm = \left(\frac{1}{2\pi}\right)2\pi r$.

This yields $r = 12$ cm.

Next, find the area.

$A = \pi r^2$

$A = \pi(12)^2 = 144\pi$

The exact area of the circle is 144π cm².

Exercises (10 minutes)

Students should solve these problems individually at first and then share with their cooperative groups after every other problem.

Exercises

1. **Find the area of a circle with a diameter of 42 cm. Use $\pi \approx \frac{22}{7}$.**

   If the diameter of the circle is 42 cm, then the radius is 21 cm.

   $A = \pi r^2$
   $A \approx \frac{22}{7}(21\ cm)^2$
   $A \approx 1386\ cm^2$

2. **The circumference of a circle is 9π cm.**

   a. **What is the diameter?**

   If $C = \pi d$, then 9π cm $= \pi d$.

   Solving the equation for the diameter, d, $\frac{1}{\pi} \cdot 9\pi$ cm $= \frac{1}{\pi}\pi \cdot d$.

   So, 9 cm $= d$.

   b. **What is the radius?**

   If the diameter is 9 cm, then the radius is half of that or $\frac{9}{2}$ cm.

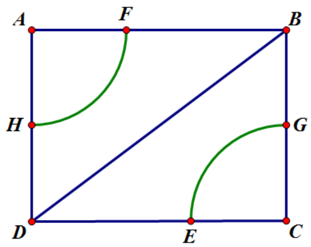
   c. **What is the area?**

   The area of the circle is $A = \pi \cdot \left(\frac{9}{2} cm\right)^2$, so $= \frac{81}{4}\pi$ cm².

3. **If students only know the radius of a circle, what other measures could they determine? Explain how students would use the radius to find the other parts.**

   If students know the radius, then they can find the diameter. The diameter is twice as long as the radius. The circumference can be found by doubling the radius and multiplying the result by π. The area can be found by multiplying the radius times itself and then multiplying that product by π.

4. **Find the area in the rectangle between the two quarter circles if $AF = 7$ ft., $FB = 9$ ft., and $HD = 7$ ft. Use $\pi \approx \frac{22}{7}$. Each quarter circle in the top-left and lower-right corners have the same radius.**



The area between the quarter circles can be found by subtracting the area of the two quarter circles from the area of the rectangle. The area of the rectangle is the product of the length and the width. Side AB has a length of 16 ft. and Side AD has a length of 14 ft. The area of the rectangle is $A = 16\text{ ft.} \cdot 14\text{ ft.} = 224\text{ ft}^2$. The area of the two quarter circles is the same as the area of a semicircle, which is half the area of a circle. $A = \frac{1}{2}\pi r^2$.

$$A \approx \frac{1}{2} \cdot \frac{22}{7} \cdot (7\text{ ft.})^2$$
$$A \approx \frac{1}{2} \cdot \frac{22}{7} \cdot 49\text{ft}^2$$
$$A \approx 77\text{ ft}^2$$

The area between the two quarter circles is $224\text{ ft}^2 - 77\text{ ft}^2 = 147\text{ ft}^2$.

Closing (5 minutes)

- The area of a semicircular region is $\frac{1}{2}$ of the area of a circle with the same radius.
- The area of a quarter of a circular region is $\frac{1}{4}$ of the area of a circle with the same radius.
- If a problem asks you to use $\frac{22}{7}$ for π, look for ways to use fraction arithmetic to simplify your computations in the problem.
- Problems that involve the composition of several shapes may be decomposed in more than one way.

Exit Ticket (5 minutes)

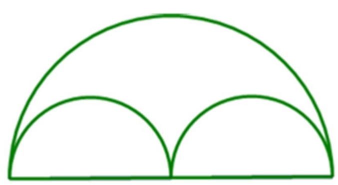
Name ____ Date____________________

Lesson 18: More Problems on Area and Circumference

Exit Ticket

1. Ken's landscape gardening business creates odd-shaped lawns that include semicircles. Find the area of this semicircular section of the lawn in this design. Use $\frac{22}{7}$ for π.

2. In the figure below, Ken's company has placed sprinkler heads at the center of the two small semicircles. The radius of the sprinklers is 5 ft. If the area in the larger semicircular area is the shape of the entire lawn, how much of the lawn will not be watered? Give your answer in terms of π and to the nearest tenth. Explain your thinking.



Exit Ticket Sample Solutions

1. Ken's landscape gardening business creates odd-shaped lawns that include semicircles. Find the area of this semicircular section of the lawn in this design. Use $\frac{22}{7}$ for π.

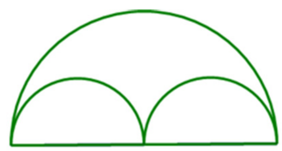
If the diameter is 5 m, then the radius is $\frac{5}{2}\text{ m}$. Using the formula for area of a semicircle,

$A = \frac{1}{2}\pi r^2$, $A \approx \frac{1}{2} \cdot \frac{22}{7} \cdot \left(\frac{5}{2}\text{cm}\right)^2$. *Using the order of operations,*

$A \approx \frac{1}{2} \cdot \frac{22}{7} \cdot \frac{25}{4}\text{cm}^2 \approx \frac{550}{56} \approx 9.8\text{ m}^2$.

2. In the figure below, Ken's company has placed sprinkler heads at the center of the two small semi-circles. The radius of the sprinklers is 5 ft. If the area in the larger semicircular area is the shape of the entire lawn, how much of the lawn will not be watered? Give your answer in terms of π and to the nearest tenth. Explain your thinking.

The area not covered by the sprinklers would be the area between the larger semicircle and the two smaller ones. The area for the two semicircles is the same as the area of one circle with the same radius of 5 ft. The area not covered by the sprinklers can be found by subtracting the area of the two smaller semicircles from the area of the large semicircle.



$$\text{Area Not Covered} = \text{Area of large semicircle} - \text{Area of two smaller semicircles}$$

$$A = \frac{1}{2}\pi \cdot (10\text{ ft.})^2 - \left(2 \cdot \left(\frac{1}{2}(\pi \cdot (5\text{ ft.})^2)\right)\right)$$

$$A = \frac{1}{2}\pi \cdot 100\text{ ft}^2 - \pi \cdot 25\text{ ft}^2$$

$$A = 50\pi\text{ ft}^2 - 25\pi\text{ ft}^2 = 25\pi\text{ ft}^2$$

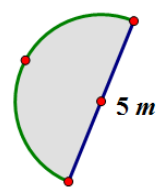
Let $\pi \approx 3.14$

$A \approx 78.5\text{ ft}^2$

The sprinklers will not cover $25\pi\text{ ft}^2$ *or* 78.5 ft^2 *of the lawn.*

Problem Set Sample Solutions

1. Mark created a flowerbed that is semicircular in shape. The diameter of the flower bed is 5 m.



   a. What is the perimeter of the flower bed? (Approximate π to be 3.14.)

   The perimeter of this flower bed is the sum of the diameter and one-half the circumference of a circle with the same diameter.

   $P = \text{diameter} + \frac{1}{2}\pi \cdot \text{diameter}$

   $P \approx 5\text{ m} + \frac{1}{2} \cdot 3.14 \cdot 5\text{ m}$

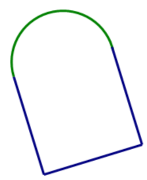
   $P \approx 12.85\text{ m}$

b. What is the area of the flowerbed? (Approximate π to be 3.14.)

$$A = \frac{1}{2}\pi\,(2.5\text{ m})^2$$
$$A = \frac{1}{2}\pi\,(6.25\text{ m}^2)$$
$$A \approx 0.5 \cdot 3.14 \cdot 6.25\text{ m}^2$$
$$A \approx 9.8\text{ m}^2$$

2. A landscape designer wants to include a semicircular patio at the end of a square sandbox. She knows that the area of the semicircular patio is 25.12 cm^2.

a. Draw a picture to represent this situation.



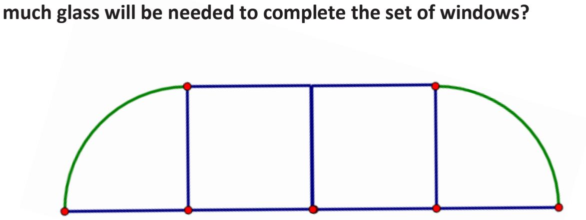
b. What is the length of the side of the square?

If the area of the patio is 25.12 cm^2, then we can find the radius by solving the equation $A = \frac{1}{2}\pi r^2$ and substituting the information that we know. If we approximate π to be 3.14 and solve for the radius, r, then $25.12\text{ cm}^2 \approx \frac{1}{2}\pi r^2$.

$$\frac{2}{1} \cdot 25.12\text{ cm}^2 \approx \frac{2}{1} \cdot \frac{1}{2}\pi r^2$$
$$50.24\text{ cm}^2 \approx 3.14r^2$$
$$\frac{1}{3.14} \cdot 50.24\text{ cm}^2 \approx \frac{1}{3.14} \cdot 3.14r^2$$
$$16\text{ cm}^2 \approx r^2$$
$$4\text{ cm} \approx r$$

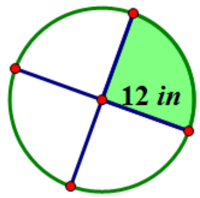
The length of the diameter is 8 cm; therefore, the length of the side of the square is 8 cm.

3. A window manufacturer designed a set of windows for the top of a two-story wall. If the window is comprised of 2 squares and 2 quarter circles on each end, and if the length of the span of windows across the bottom is 12 feet, approximately how much glass will be needed to complete the set of windows?



The area of the windows is the sum of the areas of the two quarter circles and the two squares that make up the bank of windows. If the span of windows is 12 feet across the bottom, then each window is 3 feet wide on the bottom. The radius of the quarter circles is 3 feet, so the area for one quarter circle window is $A = \frac{1}{4}\pi \cdot (3\text{ ft.})^2$ or $A \approx 7.065\text{ ft}^2$. The area of one square window is $A = (3\text{ ft.})^2$ or 9 ft^2. The total area is $2(\text{area of quarter circle}) + 2(\text{area of square})$, or $A \approx (2 \cdot 7.065\text{ ft}^2) + (2 \cdot 9\text{ ft}^2) \approx 32.13\text{ ft}^2$.

4. Find the area of the shaded region. (Approximate π to be $\frac{22}{7}$.)



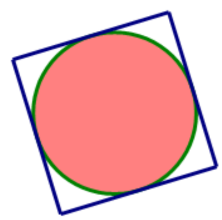
$A = \frac{1}{4}\pi(12 \text{ in})^2$

$A = \frac{1}{4}\pi \cdot 144 \text{ in}^2$

$A \approx \frac{1}{4} \cdot \frac{22}{7} \cdot 144 \text{ in}^2$

$A \approx \frac{792}{7} \text{in}^2$ *or* 113.1 in.

5. The figure below shows a circle inside of a square. If the radius of the circle is 8 cm, find the following and explain your solution.



a. The circumference of the circle

$C = 2\pi \cdot 8 \text{ cm}$

$C = 16\pi \text{ cm}$

b. The area of the circle

$A = \pi \cdot (8 \text{ cm})^2$

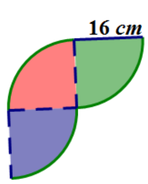
$A = 64\,\pi \text{ cm}^2$

c. The area of the square

$A = 16 \text{ cm} \cdot 16 \text{ cm}$

$A = 256 \text{ cm}^2$

6. Michael wants to create a tile pattern out of three quarter circles for his kitchen backsplash. He will repeat the three quarter circles throughout the pattern. Find the area of the tile pattern that Michael will use. Approximate π as 3.14.



There are three quarter circles in the tile design. The area of one quarter circle multiplied by 3 will result in the total area.

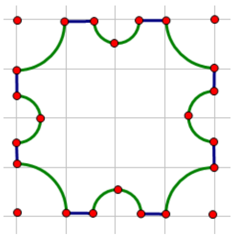
$A = \frac{1}{4}\pi \cdot (16 \text{ cm})^2$

$A \approx \frac{1}{4} \cdot 3.14 \cdot 256 \text{ cm}^2$

$A \approx 200.96 \text{ cm}^2$

The area of the tile pattern is $A \approx 3 \cdot 200.96 = 602.88 \text{ cm}^2$.

7. A machine shop has a square metal plate with sides that measure 4 cm each. A machinist must cut four semicircles with a radius of $\frac{1}{2}$ cm and four quarter circles with a radius of 1 cm from its sides and corners. What is the area of the plate formed? Use $\frac{22}{7}$ to approximate π.



The area of the metal plate is determined by subtracting the four quarter circles (corners) and the four half-circles (on each side) from the area of the square. Area of the square: $A = (4 \text{ cm})^2 = 16 \text{ cm}^2$.

The area of four quarter circles is the same as the area of a circle with a radius of 1 cm*:* $A \approx \frac{22}{7}(1 \text{ cm})^2 \approx \frac{22}{7} \text{ cm}^2$.

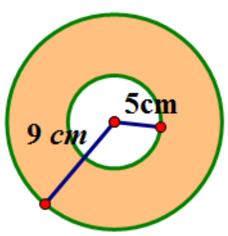
The area of the four semicircles with radius $\frac{1}{2}$ cm *is*

$$A \approx 4 \cdot \frac{1}{2} \cdot \frac{22}{7} \cdot \left(\frac{1}{2} \text{ cm}\right)^2$$
$$A \approx 4 \cdot \frac{1}{2} \cdot \frac{22}{7} \cdot \frac{1}{4} \text{ cm}^2 \approx \frac{11}{7} \text{cm}^2.$$

Area of metal plate:

$$A \approx 16 \text{ cm}^2 - \frac{22}{7} \text{ cm}^2 - \frac{11}{7} \text{ cm}^2 \approx \frac{79}{7} \text{ cm}^2$$

8. A graphic artist is designing a company logo with two concentric circles (two circles that share the same center but have different radii). The artist needs to know the area of the shaded band between the two concentric circles. Explain to the artist how he would go about finding the area of the shaded region.



The artist should find the areas of both the larger and smaller circles. Then, the artist should subtract the area of the smaller circle from the area of the larger circle to find the area between the two circles. The area of the larger circle is

$A = \pi \cdot (9 \text{ cm})^2$ *or* $81\pi \text{ cm}^2$.

The area of the smaller circle is

$A = \pi(5 \text{ cm})^2$ *or* $25\pi \text{ cm}^2$.

The area of the region between the circles is $81\pi \text{ cm}^2 - 25\pi \text{ cm}^2 = 56\pi \text{ cm}^2$. *If we approximate* π *to be* 3.14*, the then* $A \approx 175.84 \text{ cm}^2$.

9. Create your own shape made up of rectangles, squares, circles, or semicircles and determine the area and perimeter.

Student answers may vary.