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# MESH FREE METHODS

## Moving beyond the Finite Element Method

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# Dedication

*To Zuona*

*Yun, Kun, Run,*

*and my family*

*for the time and support they gave to me*

# Preface

Topics related to modeling and simulation play an increasingly important role in building an advanced engineering system in rapid and cost-effective ways. For centuries, people have been using the finite difference method (FDM) to perform the task of modeling and simulation of engineering systems, in particular to solve partial differential equation systems. It works very well for problems of simple geometry. For decades, we have used techniques of finite element methods (FEM) to perform more-challenging tasks arising from increasing demands on flexibility, effectiveness, and accuracy in challenging problems with complex geometry. I still remember, during my university years, doing a homework assignment using FDM to calculate the temperature distribution in a rectangular plate. This simple problem demonstrated the power of numerical methods. About a year later, I created an FEM program to solve a nonlinear mechanics problem for a frame structural system, as my final year project. Since then, FEM has been one of my major tools in dealing with many engineering and academic problems. In the past two decades I have participated in and directed many engineering problems of very large scale with millions of degrees of freedom (DOFs). I thought, and many of my colleagues agreed, that with the advances of FEM and the computer, there were very few problems left to solve. Soon, I realized that I was wrong and for very simple reasons. When a class of problems is solved, people simply move on to solve a class of problems that are more complex and to demand results that are more accurate. In reality, problems can be as complex as we want them to be; hence, we can never claim that problems are solved. We solve problems that are idealized and simplified by us. Once the simplification is relaxed, new challenges arise. The older methods often cannot meet the demands of new problems of increasing complexity, and newer and more advanced methods are constantly born.

I heard about meshless methods in about 1993, while I was working at Northwestern University, but somehow I was reluctant to move into this new research area probably because I was quite happy with what I was doing using techniques of FEM. It was also partially because I was concentrating on the development of my strip element method (see the monograph by G. R. Liu and Xi, 2001). During 1995–1996, I handled a number of practical engineering problems for the defense industry using FEM packages, and encountered difficulties in solving mesh distortion-related problems. I struggled to use re-meshing techniques, but the solution was far from satisfactory. I then began to look for methods that can solve the mesh distortion problems encountered in my industrial research work. I immediately started to learn more about meshless methods.

I worked alone for about a year feeling as if I was walking in a maze of this new research area. I wished that I had a book on mesh free methods to guide me. I was excited for a time about the small progress I made, which motivated me to work day and night to write a proposal for a research grant from NSTB (a research funding agency of the Singapore government). I was lucky enough to secure the grant, which quickly enabled me to form a research team at the Centre for Advanced Computations in Engineering Science (ACES) working on element free methods. The research team at ACES is still working very hard in the area of mesh free methods. This book will cover many of the research outcomes from this research group.

This book provides systematic steps that lead the reader to understand mesh free methods; how they work; how to use and develop a mesh free method, as well as the problems associated with the element free methods. I experienced difficulties in the process of learning mesh free methods, because no single book was thus far available dedicated to the topic. I therefore hope my effort in writing this monograph can help researchers, engineers, and students who are interested in exploring mesh free methods.

My work in the area of mesh free methods has been profoundly influenced by the works of Professors Belytschko, Atluri, W. K. Liu, and many others working in this area. Without their significant contributions to this area, this book would not exist.

In preparing this book, a number of my colleagues and students have supported and contributed to its writing. I express my sincere thanks to all of them. Special thanks to Y. T. Gu, X. L. Chen, L. Liu, V. Tan, L. Yan, K. Y. Yang, M. B. Liu, Y. L. Wu, Z. H. Tu, J. G. Wang, X. M. Huang, Y. G. Wu, Z. P. Wu, K. Y. Dai, and X. Han. Many of these individuals have contributed examples to this book in addition to their hard work in carrying out a number of projects related to the mesh free methods covered in this book.

G. R. Liu

# *The Author*

**G. R. Liu** received his Ph.D. from Tohoku University, Japan in 1991. He was a postdoctoral fellow at Northwestern University, U.S.A. He is currently the Director of the Centre for Advanced Computations in Engineering Science (ACES), National University of Singapore. He is also an associate professor at the Department of Mechanical Engineering, National University of Singapore. He has authored more than 250 technical publications including four books and 150 international journal papers. He is the recipient of the Outstanding University Researchers Awards (1998), for his development of the strip element method. He is also a recipient of the Defence Technology Prize (national award, 1999) for his contribution to development of underwater shock technology at Singapore. He won the Silver Award at CrayQuest 2000 (nationwide competition in 2000) for his development of mesh free methods. His research interests include computational mechanics, element free methods, nano-scale computation, vibration and wave propagation in composites, mechanics of composites and smart materials, inverse problems, and numerical analysis.

# Contents