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# Dispersion Properties of Co-Existing Low Frequency Modes in Quantum Plasmas

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## 1. Introduction

The underlying physics of nonconventional quantum plasmas has been introduced long ago. Analytical investigations of collective interactions between an ensemble of degenerate electrons in a dense quantum plasma dates back to early fifties. The general kinetic equations for quantum plasmas were derived and the dispersion properties of plasma waves were studied (Klimotovich & Silin, 1952). It was thought that the quantum mechanical behaviour of electrons, in the presence of heavier species modifies the well known properties of plasma. The dynamics of quantum plasmas got particular attention in the framework of relationship between individual particle and collective behavior. Emphasizing the excitation spectrum of quantum plasmas, theoretical investigations describe the dispersion properties of electron plasma oscillations involving the electron tunneling (Bohm & Pines, 1953; Pines, 1961). A general theory of electromagnetic properties of electron gas in a quantizing magnetic field and many particle kinetic model of non-thermal plasmas was also developed treating the electrons quantum mechanically (Zyrianov et al., 1969; Bezzerides & DuBois, 1972). Since the pioneering work of these authors which laid foundations of quantum plasmas, many theoretical studies have been done in the subsequent years. The rapidly growing interest in quantum plasmas in the recent years has several different origins but is mainly motivated by its potential applications in modern science and technology (e.g. metallic and semiconductor micro and nanostructures, nanoscale plasmonic devices, nanotubes and nanoclusters, spintronics, nano-optics, etc.). Furthermore, quantum plasmas are ubiquitous in planetary interiors and in compact astrophysical objects (e.g., the interior of white dwarfs, neutron stars, magnetars, etc.) as well as in the next generation intense laser-solid density plasma interaction experiments. Such plasmas also provide promises of important futuristic developments in ultrashort pulsed lasers and ultrafast nonequilibrium phenomena (Bonitz, 1898; Lai, 2001; Shukla & Eliasson, 2009).

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Contrary to classical plasmas, the number density of degenerate electrons, positrons/holes in quantum plasmas is extremely high and they obey Fermi-Dirac statistics whereas the temperature is very low. Plasma and quantum mechanical effects co-exist in such systems and many unusual effects like tunneling of electrons, quantum destabilization, pressure ionization, Bose-Einstein condensation and crystallization etc. may be equally important (Bonitz et al., 2009). Their properties in equilibrium and nonequilibrium are governed by many-body effects (collective and correlation effects) which require quantum statistical theories and versatile computational techniques. The average inter-particle distance  $n^{-1/3}$  (where  $n$  is the particle density) is comparable with electron thermal de Broglie wavelength  $\lambda_{Be}$  ( $= \hbar/mv_{te}$ , where  $\hbar$  is Planck's constant divided by  $2\pi$ ,  $m$  is the electronic mass and  $v_{te}$  is thermal speed of electron). The overlapping of wave functions associated with electrons or positrons take place which leads to novel quantum effects.

It was recognized long ago that the governing quantum-like equations describing collective behavior of dense plasmas can be transformed in the form of hydrodynamic (or fluid) equations which deals with macroscopic variables only (Madelung, 1926). Here, the main line of reasoning starts from Schrodinger description of electron. The N-body wave function of the system can be factored out in N one-body wave functions neglecting two-body and higher order correlations. This is justified by weak coupling of fermions at high densities. The coupling parameter of quantum plasmas decreases with increase in particle number density. For hydrodynamic representation, the electron wave function is written as  $\psi = \sqrt{n} \exp(iS/\hbar)$  where  $n$  is amplitude and  $S$  is phase of the wave function. Such a decomposition of  $\psi$  was first presented by Bohm and de Broglie in order to understand the dynamics of electron wave packet in terms of classical variables. It introduces the Bohm-de Broglie potential in equation of motion giving rise to dispersion-like terms. In the recent years, a vibrant interest is seen in investigating new aspects of quantum plasmas by developing non-relativistic quantum fluid equations for plasmas starting either from real space Schrodinger equation or phase space Wigner (quasi-) distribution function. (Haas et al., 2003, Manfredi & Haas, 2001; Manfredi, 2005). Such approaches take into account the quantum statistical pressure of fermions and quantum diffraction effects involving tunneling of degenerate electrons through Bohm-de Broglie potential. The hydrodynamic theory is also extended to spin plasmas starting from non-relativistic Pauli equation for spin- $\frac{1}{2}$  particles (Brodin & Marklund, 2007; Marklund & Brodin, 2007). Generally, the hydrodynamic approach is applicable to unmagnetized or magnetized plasmas over the distances larger than electron Fermi screening length  $\lambda_{Fe}$  ( $= v_{Fe}/\omega_{pe}$ , where  $v_{Fe}$  is the electron Fermi velocity and  $\omega_{pe}$  is the electron plasma frequency). It shows that the plasma effects at high densities are very short scaled.

The present chapter takes into account the dispersive properties of low frequency electrostatic and electromagnetic waves in dense electron-ion quantum plasma for the cases of dynamic as well as static ions. Electrons are fermions (spins=1/2) obeying Pauli's exclusion principle. Each quantum state is occupied only by single electron. When electrons are added, the Fermi energy of electrons  $\epsilon_{Fe}$  increases even when interactions are neglected ( $\epsilon_{Fe} \propto n^{2/3}$ ). This is because each electron sits on different step of the ladder according to Pauli's principle which in turn increases the statistical (Fermi) pressure of electrons. The de Broglie wavelength associated with ion as well as its Fermi energy is much smaller as compared to electron due to its large mass. Hence the ion dynamics is classical. Quantum

diffraction effects (quantum pressure) of electrons are significant only at very short length scales because the average interparticle distance is very small. This modifies the collective modes significantly and new features of purely quantum origin appear. The quantum ion-acoustic type waves in such system couples with shear Alfvén waves. The wave dispersion due to gradient of Bohm-de Broglie potential is weaker in comparison with the electrons statistical/Fermi pressure. The statistical pressure is negligible only for wavelengths smaller than the electron Fermi length. For plasmas with density greater than the metallic densities, the statistical pressure plays a dominant role in dispersion.

## 2. Basic description

The coupling parameter for a traditional classical plasma is defined as

$$\Gamma^C = \frac{\langle U \rangle}{\langle K \rangle} \propto \frac{\sqrt[3]{n}}{T}, \quad (1)$$

where  $\langle U \rangle = \frac{1}{2} \sum_{i \neq j} \frac{e_i e_j}{\bar{r}}$  is the two-particle Coulomb interaction (potential) energy,  $\langle K \rangle = \frac{3}{2} k_B T$  is the average kinetic energy,  $k_B$  is the Boltzmann constant and  $T$  is the system's temperature. The average interparticle distance  $\bar{r}$  is given by

$$\bar{r} = \langle r_i - r_j \rangle \propto \frac{1}{\sqrt[3]{n}}. \quad (2)$$

The parameter  $\Gamma^C$  may also be written in the form  $\left( \frac{1}{n \lambda_D^3} \right)^{2/3}$  where  $\lambda_D = \left( \frac{k_B T}{4 \pi n e^2} \right)^{1/2}$  is the Debye screening length. Such a plasma obeys Boltzmann distribution function in which the ordering  $\Gamma^C \ll 1$  corresponds to collisionless and  $\Gamma^C \simeq 1$  to collisional regime. So, a classical plasma can be said collisionless (ideal) when long-range self-consistent interactions (described by the Poisson equation) dominate over short-range two-particle interactions (collisions).

When the density is very high,  $\bar{r}$  become comparable to thermal de Broglie wavelength of charged particles defined by

$$\lambda_B = \frac{h}{\sqrt{2 \pi m k_B T}}, \quad (3)$$

where  $h$  is the Planck's constant. Here, the degeneracy effects cannot be neglected i.e.,  $1 \lesssim n \lambda_B^3$  and the quantum mechanical effects along with collective (plasma) effects become important at the same time. Such plasmas are also referred to as quantum plasmas. Some common examples are electron gas in an ordinary metal, high-density degenerate plasmas in white dwarfs and neutron stars, and so on. From quantum mechanical point of view, the state of a quantum particle is characterized by the wave function associated with the particle instead of its trajectory in phase space. The Heisenberg uncertainty principle leads to fundamental modifications of classical statistical mechanics in this case. The de Broglie wavelength has no role in classical plasmas because it is too small compared to the average

interparticle distances. There is no overlapping of the wave functions and consequently no quantum effects. So the plasma particles are considered to be point like and treated classically.

However, in quantum plasmas, the overlapping of wave functions takes place which introduces novel quantum effects. It is clear from (3) that the de Broglie wavelength depends upon mass of the particle and its thermal energy. That is why, the quantum effects associated with electrons are more important than the ions due to smaller mass of electron which qualifies electron as a true quantum particle. The behavior of such many-particle system is now essentially determined by statistical laws. The plasma particles with symmetric wave functions are termed as Bose particles and those with antisymmetric wave function are called Fermi particles. We can subdivide plasmas into (i) quantum (degenerate) plasmas if  $1 < n\lambda_B^3$  and (ii) classical (nondegenerate) plasmas if  $n\lambda_B^3 < 1$ . The border between the degenerate and the non-degenerate plasmas is roughly given by

$$n\lambda_B^3 = n \left( \frac{h}{\sqrt{2\pi mk_B T}} \right)^3 = 1. \quad (4)$$

For quantum plasmas, the Boltzmann distribution function is strongly modified to Fermi-Dirac or Bose-Einstein distribution functions in a well known manner, i.e.,

$$f(\epsilon) = \frac{1}{\left[ e^{\beta(\epsilon-\mu)} \pm 1 \right]}, \quad (5)$$

where  $\beta = 1/k_B T$ ;  $\epsilon$  is the particle energy and  $\mu$  is the chemical potential. The '+' sign corresponds to Fermi-Dirac distribution function (for fermions with spin  $1/2, 3/2, 5/2, \dots$ ) and '-' sign to Bose-Einstein distribution function (for bosons with spin  $0, 1, 2, 3, \dots$ ). The different signs in the denominators of (5) are of particular importance at low temperatures. For fermions, this leads to the existence of Fermi energy (Pauli principle), and for bosons, to the possibility of macroscopic occupation of the same quantum state which is the well known phenomenon of Bose-Einstein condensation.

Let us consider a degenerate Fermi gas of electrons at absolute zero temperature. The electrons will be distributed among the various quantum states so that the total energy of the gas has its least possible value. Since each state can be occupied by not more than one electron, the electrons occupy all the available quantum states with energies from zero (least value) to some largest possible value which depends upon the number of electrons present in the gas. The corresponding momenta also starts from zero to some limiting value (Landau & Lifshitz, 1980). This limiting momentum is called the Fermi momentum  $p_F$  given by

$$p_F = \hbar (3\pi^2 n)^{1/3}. \quad (6)$$

Similarly, the limiting energy is called the Fermi energy  $\epsilon_F$  which is

$$\epsilon_F = \frac{p_F^2}{2m} = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3}. \quad (7)$$

The Fermi-Dirac distribution function becomes a unit step function in the limit  $T \rightarrow 0$ . It is zero for  $\mu < \epsilon$  and unity for  $\epsilon < \mu$ . Thus the chemical potential of the Fermi gas at  $T = 0$  is same as the limiting energy of the fermions ( $\mu = \epsilon_F$ ). The statistical distribution of plasma particles changes from Maxwell-Boltzmann  $\propto \exp(-\epsilon/k_B T)$  to Fermi-Dirac statistics  $\propto \exp[-(\beta(\epsilon - \epsilon_F) + 1)]^{-1}$  whenever  $T$  approaches the so-called Fermi temperature  $T_F$ , given by

$$k_B T_F \equiv \epsilon_F = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3}. \quad (8)$$

Then the ratio  $\chi = T_F / T$  can be related to the degeneracy parameter  $n\lambda_B^3$  as,

$$\chi = T_F / T = \frac{1}{2} (3\pi^2)^{2/3} (n\lambda_B^3)^{2/3} \quad (9)$$

It means that the quantum effects are important when  $1 \lesssim T_F / T$ . In dense plasmas, the plasma frequency  $\omega_p = (4\pi n e^2 / m)^{1/2}$  becomes sufficiently high due to very large equilibrium particle number density. Consequently, the typical time scale for collective phenomena  $(\omega_p)^{-1}$  becomes very short. The thermal speed  $v_T = (k_B T / m)^{1/2}$  is sufficiently smaller than the Fermi speed given by

$$v_F = \left( \frac{2\epsilon_F}{m} \right)^{1/2} = \frac{\hbar^2}{m} (3\pi^2 n)^{1/3}. \quad (10)$$

With the help of plasma frequency and Fermi speed, we can define a length scale for electrostatic screening in quantum plasma i.e., the Fermi screening length  $\lambda_F (= v_{Fe} / \omega_{pe})$  which is also known as the quantum-mechanical analogue of the electron Debye length  $\lambda_{De}$ . The useful choice for equation of state for such dense ultracold plasmas is of the form (Manfredi, 2005)

$$P = P_0 \left( \frac{n}{n_0} \right)^\gamma, \quad (11)$$

where the exponent  $\gamma = (d + 2) / d$  with  $d = 1, 2, 3$  denoting the dimensionality of the system, and  $P_0$  is the equilibrium pressure. In three dimensions,  $\gamma = 5/3$  and  $P_0 = (2/5) n_0 \epsilon_F$  which leads to

$$P = \left( \hbar^2 / 5m \right) (3\pi^2)^{2/3} n^{5/3}. \quad (12)$$

For one dimensional case,  $\gamma = 3$  and  $P = \left( m v_F^2 / 3 n_0^2 \right) n_e^3$ . It shows that the electrons obeying Fermi-Dirac statistics introduce a new pressure at zero temperature called the Fermi pressure, which is significant in dense low temperature plasmas. The Fermi pressure increases with increase in number density and is different from thermal pressure.

Like classical plasmas, a coupling parameter can be defined in a quantum plasma. For strongly degenerate plasmas, the interaction energy may still be given by  $\langle U \rangle$ , but the kinetic energy is now replaced by the Fermi energy. This leads to the quantum coupling parameter

$$\Gamma^Q = \frac{\langle U \rangle}{\epsilon_F} \propto \frac{\sqrt[3]{n}}{T_F}, \quad (13)$$

which shows that  $\Gamma^Q \propto n^{-1/3}$ . So the peculiar property of quantum plasma is that it increasingly approaches the more collective (ideal) behavior as its density increases. Quantum plasma is assumed to be collisionless when  $\Gamma^Q \ll 1$  because the two body correlation can be neglected in this case. This condition is fulfilled in high density plasmas since  $\epsilon_F = \epsilon_F(n)$ . In the opposite limit of high temperature and low density, we have  $1 \gg n\lambda_B^3$  and the system behaves as a classical ideal gas of free charge carries. Another useful form of  $\Gamma^Q$  is as follows

$$\Gamma^Q \approx \left( \frac{1}{n\lambda_F^3} \right)^{2/3} = \left( \frac{\hbar\omega_p}{\epsilon_F} \right)^{2/3}, \quad (14)$$

which shows the resemblance with classical coupling parameter which may be recovered in the limit  $\lambda_F \rightarrow \lambda_D$ .

### 3. Governing equations

Suppose that the N-particle wave function of the system can be factorized into N one-particle wave functions as  $\Psi(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N) = \psi_1(\mathbf{x}_1)\psi_2(\mathbf{x}_2) \dots \psi_N(\mathbf{x}_N)$ . Then, the system is described by statistical mixture of N states  $\psi_\alpha$ ,  $\alpha = 1, 2, \dots, N$  where the index  $\alpha$  sums over all particles independent of species. We then take each  $\psi_\alpha$  to satisfy single particle Schrodinger equation where the potentials ( $\mathbf{A}$ ,  $\phi$ ) is due to the collective charge and current densities. For each  $\psi_\alpha$ , we have corresponding probability  $p_\alpha$  such that  $\sum_{\alpha=1}^N p_\alpha = 1$  and all types of entanglements are neglected in the weak coupling limit. To derive the quantum fluid description, we define  $\psi_\alpha = \sqrt{n_\alpha} \exp(iS_\alpha / \hbar)$  where  $n_\alpha$  and  $S_\alpha$  are real and the velocity of  $\alpha$ th particle is  $v_\alpha = \nabla S_\alpha / m_\alpha - (q_\alpha / m_\alpha c) \mathbf{A}$ . Next, defining the global density  $n = \sum_{\alpha=1}^N p_\alpha n_\alpha$ , the global velocity  $\mathbf{v} = \sum_{\alpha=1}^N p_\alpha \frac{n_\alpha}{n} \mathbf{v}_\alpha \equiv \langle \mathbf{v}_\alpha \rangle$ , and separating the real and imaginary parts in Schrodinger equation, the resulting continuity and momentum balance equation take the form

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0, \quad (15)$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = \frac{q}{m} \left( \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) + \frac{\hbar^2}{2m^2} \nabla \left( \frac{\nabla^2 \sqrt{n}}{\sqrt{n}} \right) - \frac{1}{mn} \nabla P. \quad (16)$$

The last term in (16) is the statistical pressure term. For high temperature plasmas, it is simply thermal pressure. However, in low temperature and high density regime, the Fermi pressure is significant which corresponds to fermionic nature of electrons and  $P$  is given by equation (12). In the model (15)-(16), it is assumed that pressure  $P = P(n)$  which leads to the appropriate

equation of state to obtain the closed system of equations. For typical length scales larger than  $\lambda_{Fe}$ , we have approximated the  $\hbar$ -term as  $\sum_{\alpha=1}^N p_{\alpha} (\nabla^2 \sqrt{n_{\alpha}} / \sqrt{n_{\alpha}}) \approx (\nabla^2 \sqrt{n} / \sqrt{n})$ . This term is gradient of the so called Bohm-de Broglie potential. The equations (15)-(16) are commonly known as the quantum fluid equations which are coupled to the Poisson's equation and Ampere's law to study the self-consistent dynamics of quantum plasmas (Manfredi 2005; Brodin & Marklund, 2007). This model has obtained considerable attention of researchers in the recent years to study the behaviour of collisionless plasmas when quantum effects are not negligible. Starting from simple cases of electrostatic linear and nonlinear modes in two component and multicomponent plasmas, e.g., linear and nonlinear quantum ion waves in electron-ion (Haas et al., 2003, Khan et al., 2009), electron-positron-ion (Khan et al., 2008, 2009) and dust contaminated quantum plasmas (Khan et al., 2009), the studies are extended to electromagnetic waves and instabilities (Ali, 2008). Some particular developments have also appeared in spin-1/2 plasmas (Marklund & Brodin, 2007; Brodin & Marklund, 2007; Shukla, 2009), quantum electrodynamic effects (Lundin et al., 2007) and quantum plasmadynamics (Melrose, 2006). It is to mention here that the inclusion of simple collisional terms in such model is much harder and the exclusion of interaction terms is justified by small value of  $\Gamma^Q$ .

#### 4. Fermionic pressure and quantum pressure

For dense electron gas in metals with equilibrium density  $n_{e0} \simeq 10^{23} \text{ cm}^{-3}$ , the typical value of Fermi screening length is of the order of Angstrom while the plasma oscillation time period ( $\omega_{pe}^{-1}$ ) is of the order of femtosecond. The electron-electron collisions can be ignored for such short time scales. The Fermi temperature of electrons is very large in such situations i.e.,  $T_{Fe} \simeq 9 \times 10^4 \text{ K}$  which shows that the electrons are degenerate almost always (Manfredi & Haas, 2001). The Fermi energy, which increases with the plasma density, becomes the kinetic energy scale. The quantum criterion of ideality has the form

$$\Gamma^Q \approx \frac{e^2 \sqrt[3]{n}}{\epsilon_{Fe}} \ll 1. \quad (17)$$

The parameter  $\Gamma^Q$  decreases with increasing electron density, therefore, a degenerate electron plasma becomes even more ideal with compression. So, even in the fluid approximation, it is reasonable to compare the statistical pressure term arising due to the fermionic character of electrons and the quantum Bohm-de Broglie potential term in the ultracold plasma.

Let us consider two-component dense homogenous plasma consisting of electrons and ions. The plasma is embedded in a very strong uniform magnetic field  $B_0 \hat{z}$ ; where  $B_0$  is the strength of magnetic field and  $\hat{z}$  is the unit vector in z-axis direction. However, plasma anisotropies, collisions and the spin effects are not considered in the model for simplicity. The low frequency (in comparison with the ion cyclotron frequency  $\Omega_{ci} = eB_0/m_i c$ , where  $e$ ,  $m_i$  and  $c$  are the magnitude of electron charge, ion mass and speed of light in vacuum, respectively) electric and magnetic field perturbations are defined as  $\mathbf{E} = -\nabla\phi - c^{-1}(\partial A_z / \partial t)\hat{z}$  and  $\mathbf{B}_{\perp} = \nabla_{\perp} A_z \times \hat{z}$ , respectively, where  $\phi$  is the electrostatics wave

potential and  $A_z$  is the component of vector potential along z-axis. For very low temperature plasma by assuming that the ions behave classically in the limit  $T_{Fi} \ll T_{Fe}$  (where  $T_{Fi}$  is the ion Fermi temperature), the pressure effects of quantum electrons are relevant only. In this situation the Fermi pressure which is contribution of the electrons obeying the Fermi-Dirac equilibrium is of most significance. In linearized form, the gradient of Fermi pressure for spin- $\frac{1}{2}$  electrons from (12) leads to

$$\nabla P_{Fe1} \approx \frac{\hbar^2}{3m_e} (3\pi^2 n_{e0})^{2/3} \nabla n_{e1}, \quad (18)$$

where the perturbation is assumed to be proportional to  $\exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$ . The index 0 and 1 is used to denote the equilibrium, and perturbation, respectively. The  $\hbar$ -term in expression (16) i.e., the gradient of the Bohm-de Broglie potential in the linear limit may be written as

$$\nabla P_{q1} \approx \frac{\hbar^2 k^2}{4m_e} \nabla n_{e1}, \quad (19)$$

where  $P_q$  has the dimensions of pressure. Notice that  $(3\pi^2)^{2/3} \approx 9.6$  and  $n_{e0}^{2/3} = \frac{1}{\bar{r}^2}$  where  $\bar{r}$  is the average interparticle distance. If  $k \sim 10^6 \text{cm}^{-1}$  is assumed, then near metallic electron densities i.e.,  $n_{e0} \sim 10^{23} \text{cm}^{-3}$ , we have,  $\bar{r} \sim 10^{-8} \text{cm}$ ; which shows that

$$k^2 \ll \frac{1}{\bar{r}^2}. \quad (20)$$

The inequality (20) shows that the variation of quantities should be on length scales that are larger than  $\lambda_{Fe}$  and the fluid model is useful on such scales (Khan & Saleem, 2009).

## 5. Dynamics of ions and electrons

Starting from the fluid equations (15)-(16), it is assumed that the quantum effects of ions are neglected due to their larger mass in the limit  $T_{Fi} \ll T_{Fe}$ . The equation of motion for  $j$ th species may be written as,

$$m_j n_j \left( \frac{\partial}{\partial t} + \mathbf{v}_j \cdot \nabla \right) \mathbf{v}_j = q_j n_j (\mathbf{E} + \frac{1}{c} \mathbf{v}_j \times \mathbf{B}) - \nabla P_j + \frac{\hbar^2 n_j}{2m_j} \nabla \left( \frac{\nabla^2 \sqrt{n_j}}{\sqrt{n_j}} \right), \quad (21)$$

where  $j = e, i$  for electron, and ion. Furthermore, the ions are assumed to be singly charged i.e.,  $q_j = e$  ( $-e$ ) for electrons (ions) and the steady state is defined as  $n_{e0} = n_{i0}$ . The linearized ion velocity components in the perpendicular and parallel directions are

$$\mathbf{v}_{i\perp 1} \approx \frac{c}{B_0} \left( \mathbf{z} \times \nabla_{\perp} \phi_1 - \frac{1}{\Omega_{ci}} \frac{\partial \nabla_{\perp} \phi_1}{\partial t} \right) \approx \mathbf{v}_E + \mathbf{v}_{Pi}, \quad (22)$$

$$\partial_t v_{iz1} \approx -\frac{e}{m_i} \left( \frac{\partial \phi_1}{\partial z} + \frac{1}{c} \frac{\partial A_{z1}}{\partial t} \right), \quad (23)$$

with  $v_E$  and  $v_{pi}$  being the electric and polarization drifts, respectively. Similarly, the components of electron velocities in perpendicular and parallel directions can be written, respectively, as,

$$\mathbf{v}_{e\perp 1} \approx \frac{c}{B_0} \hat{\mathbf{z}} \times \nabla_{\perp} \left( \phi_1 + \frac{\hbar^2}{4m_e n_0 e} \nabla^2 n_{e1} - \frac{2k_B T_{Fe}}{3n_0 e} n_{e1} \right), \quad (24)$$

$$\approx \mathbf{v}_E + \mathbf{v}_{qe} + \mathbf{v}_{De},$$

$$\partial_t v_{ez1} \approx \frac{e}{m_e} \frac{\partial}{\partial z} \left( \phi_1 + \frac{\hbar^2}{4m_e n_0 e} \nabla^2 n_{e1} - \frac{2k_B T_{Fe}}{3n_0 e} n_{e1} \right) + \frac{e}{m_e c} \frac{\partial A_{z1}}{\partial t}, \quad (25)$$

where  $v_{qe}$  and  $v_{De}$  are defined as the quantum and diamagnetic type drifts, respectively,  $|\partial_t| \ll \omega_{pe}$ ,  $ck$ , and  $n_{e0} = n_{i0} = n_0$ . The continuity equation for  $j$ th species can be expressed as

$$\partial_t n_{j1} + n_0 (\nabla \cdot \mathbf{v}_{E1} + \nabla \cdot \mathbf{v}_{pj} + \partial_z v_{jz1}) = 0. \quad (26)$$

The Poisson's equation is

$$\nabla^2 \phi_1 = 4\pi e (n_{e1} - n_{i1}), \quad (27)$$

and the Ampere's law can be written as,

$$\nabla_{\perp}^2 A_{z1} = \frac{4\pi n_0 e}{c} (v_{ez1} - v_{iz1}). \quad (28)$$

### 5.1 Mobile ions

Let us consider that the ions as well as electrons are mobile. The electron and ion continuity equations lead to

$$\frac{\partial}{\partial t} (n_{e1} - n_{i1}) - n_0 \nabla \cdot \mathbf{v}_{pi} + n_0 \frac{\partial}{\partial z} (v_{ez1} - v_{iz1}) = 0. \quad (29)$$

Using expressions (22), (27) and (28) in the above equation, we obtain

$$\frac{\partial}{\partial t} \left( \nabla^2 + \frac{c^2}{v_A^2} \nabla_{\perp}^2 \right) \phi_1 + \frac{c \partial \nabla_{\perp}^2 A_{z1}}{\partial z} = 0, \quad (30)$$

where  $v_A = B_0 / \sqrt{4\pi n_0 m_i}$  is the speed of Alfvén wave, and we have defined the current as  $J_{z1} \approx en_0 (v_{iz1} - v_{ez1})$ . Ion continuity equation along with (22) and (23) yields,

$$\frac{\partial^2 n_{i1}}{\partial t^2} - \frac{n_0 c}{B_0 \Omega_{ci}} \frac{\partial^2 \nabla_{\perp}^2 \phi_1}{\partial t^2} - \frac{n_0 e}{m_i} \left( \frac{\partial^2 \phi_1}{\partial z^2} + \frac{1}{c} \frac{\partial}{\partial z} \frac{\partial A_{z1}}{\partial t} \right) = 0. \quad (31)$$

Eliminating  $A_{z1}$  from (30) and (31) and Fourier analyzing, we obtain,

$$\frac{n_{i1}}{n_0} = \frac{1}{\omega^2} \left[ -\rho_q^2 k_{\perp}^2 \omega^2 + \frac{c_q^2 k_z^2}{\omega_A^2} \left\{ \omega_A^2 - \omega^2 \left( 1 + \frac{v_A k^2}{c^2 k_{\perp}^2} \right) \right\} \right] \Phi_1, \quad (32)$$

where we have defined the quantum ion-acoustic type speed as  $c_q = \sqrt{T_q / m_i}$ , the ion Larmor radius at effective electron temperature as  $\rho_q = c_q / \Omega_{ci}$ , the Alfvén wave frequency as  $\omega_A = k_z v_A$  and  $\Phi_1 = e \phi_1 = T_q$ . The effective temperature of electrons (in energy units) is defined as  $T_q = (\hbar^2 k^2 / 4m_e + 2k_B T_{Fe} / 3)$ , which is a pure quantum mechanical effect. The first term in  $T_q$  corresponds to quantum Bohm-de Broglie potential, and the second term represents the electron Fermi energy. So the parameters  $c_q$  and  $\rho_q$  contain the contribution of both the terms. The Poisson's equation (27) gives

$$\frac{n_{e1}}{n_0} = \frac{n_{i1}}{n_0} - \frac{c_q^2 k^2}{\omega_{pi}^2} \Phi_1, \quad (33)$$

with  $\omega_{pi} (= \sqrt{4\pi n_0 e^2 / m_i})$  being the ion plasma frequency. Using (32) and (33), we obtain

$$\frac{n_{e1}}{n_0} = \frac{1}{\omega^2} \left[ -\rho_q^2 k_{\perp}^2 \omega^2 + \frac{c_q^2 k_z^2}{\omega_A^2} (\omega_A^2 - \omega^2) - \frac{c_q^2 k^2}{\omega_{pi}^2} \omega^2 \right] \Phi_1. \quad (34)$$

The electron parallel equation of motion leads to,

$$\frac{\partial v_{ez1}}{\partial t} = \frac{e}{m_e} \left( \frac{\partial \phi_1}{\partial z} + \frac{1}{c} \frac{\partial A_{z1}}{\partial t} \right) - \frac{T_q}{m_e n_0} \frac{\partial n_{e1}}{\partial z}. \quad (35)$$

From Ampère's law, we find  $v_{ez1} = \frac{c}{4\pi n_0 e} \nabla_{\perp}^2 A_{z1} + v_{iz1}$ . Equation (35) along with (30) leads to

$$\frac{n_{e1}}{n_0} = \frac{1}{\omega_A^2} \left[ \omega_A^2 - (1 + \lambda_e^2 k_{\perp}^2) \left( 1 + \frac{v_A^2 k^2}{c^2 k_{\perp}^2} \right) \omega^2 + \frac{m_e}{m_i} \left\{ \omega_A^2 - \left( 1 + \frac{v_A^2 k^2}{c^2 k_{\perp}^2} \right) \omega^2 \right\} \right] \Phi_1. \quad (36)$$

where  $\lambda_e = c / \omega_{pe}$  is the electron collisionless skin depth and the small term in the curly brackets appears from ion parallel velocity component. From (34) and (36), we obtain the linear dispersion relation of low frequency coupled electrostatic and electromagnetic modes in dense cold magnetoplasma as,

$$\left[ \left( 1 + \frac{v_A^2 k^2}{c^2 k_{\perp}^2} \right) \omega^2 + \frac{m_e}{m_i (1 + \lambda_e^2 k_{\perp}^2)} \left\{ \left( 1 + \frac{v_A^2 k^2}{c^2 k_{\perp}^2} \right) \omega^2 - \omega_A^2 \right\} - \frac{c_q^2 k_z^2}{(1 + \lambda_e^2 k_{\perp}^2)} \left( 1 + \frac{v_A^2 k^2}{c^2 k_{\perp}^2} \right) \right] \omega^2 - \frac{\omega_A^2}{(1 + \lambda_e^2 k_{\perp}^2)} \left[ \left( 1 + \frac{c_q^2 k^2}{\omega_{pi}^2} \right) \omega^2 - c_q^2 k_z^2 \right] = \frac{\rho_q^2 k_{\perp}^2 \omega_A^2}{(1 + \lambda_e^2 k_{\perp}^2)} \omega^2. \quad (37)$$

Since  $m_e / m_i \ll 1$ , therefore relation (37) reduces to

$$\left[ \left( 1 + \frac{v_A^2 k^2}{c^2 k_\perp^2} \right) \omega^2 - \frac{c_q^2 k_z^2}{(1 + \lambda_e^2 k_\perp^2)} \left( 1 + \frac{v_A^2 k^2}{c^2 k_\perp^2} \right) \right] \omega^2 - \frac{\omega_A^2}{(1 + \lambda_e^2 k_\perp^2)} \left[ \left( 1 + \frac{c_q^2 k^2}{\omega_{pi}^2} \right) \omega^2 - c_q^2 k_z^2 \right] = \frac{\rho_q^2 k_\perp^2 \omega_A^2}{(1 + \lambda_e^2 k_\perp^2)} \omega^2, \quad (38)$$

In the limit  $\lambda_e^2 k_\perp^2 \ll 1$ , (38) can be written as

$$\left[ \left( 1 + \frac{v_A^2 k^2}{c^2 k_\perp^2} \right) \omega^2 - c_q^2 k_z^2 \left( 1 + \frac{v_A^2 k^2}{c^2 k_\perp^2} \right) \right] \omega^2 - \omega_A^2 \left[ \left( 1 + \frac{c_q^2 k^2}{\omega_{pi}^2} \right) \omega^2 - c_q^2 k_z^2 \right] = \rho_q^2 k_\perp^2 \omega_A^2 \omega^2. \quad (39)$$

If we assume  $v_A \ll ck_\perp / k$  in a quasi-neutral limit  $n_{e1} \approx n_{e1}$ , (39) leads to

$$(\omega^2 - c_q^2 k_z^2)(\omega^2 - \omega_A^2) = \rho_q^2 k_\perp^2 \omega_A^2 \omega^2. \quad (40)$$

The above relation can be found in the recent paper of Saleem et al. (Saleem et al., 2008) if the density inhomogeneity is neglected there and we assume  $k_B T_{Fe} n_{e1} \ll (\hbar^2 / 4m_e) \nabla^2 n_{e1}$ . However, the pressure effects of dense ultracold electron plasma are negligible only for wavelengths smaller than the electron Fermi wavelength (Manfredi & Haas, 2001). The plasmas found in the compact astrophysical objects (e.g., white dwarfs and neutron stars) have very high densities and correspondingly, very small interparticle distances. Then the electron statistical pressure effects are significant over the length scales larger than  $\lambda_{Be}$  and it plays a dominant role in wave dispersion.

In case of small parallel ion current, equation (38) yields,

$$\omega^2 = \frac{k_z^2 v_A^2}{(1 + \lambda_e^2 k_\perp^2)} \left( \frac{1}{\left( 1 + \frac{v_A^2 k^2}{c^2 k_\perp^2} \right)} + \rho_q^2 k_\perp^2 \right). \quad (41)$$

Expression (41) shows the effects of electron inertia on shear Alfvén wave frequency at quantum scale lengths of electrons in a dense ultracold magnetoplasma. If the electron inertia is neglected, (41) may be written as

$$\omega^2 = k_z^2 v_A^2 \left( \frac{1}{\left( 1 + \frac{v_A^2 k^2}{c^2 k_\perp^2} \right)} + \rho_q^2 k_\perp^2 \right). \quad (42)$$

For  $v_A \ll \frac{ck_\perp}{k}$ , we have

$$\omega^2 = k_z^2 v_A^2 \left(1 + \rho_q^2 k_\perp^2\right), \quad (43)$$

which shows the dispersion of shear Alfvén wave frequency due to quantum effects associated with electrons in a dense quantum magnetoplasma.

## 5.2 Immobile ions

Now we assume that the ions are immobile. Then Ampère's law leads to

$$v_{ez1} = \frac{c}{4\pi n_0 e} \nabla_\perp^2 A_{z1}. \quad (44)$$

The perpendicular component of electron fluid velocity from (21) becomes

$$\begin{aligned} \mathbf{v}_{e\perp 1} \approx & \frac{c}{B_0} \hat{\mathbf{z}} \times \nabla_\perp \left( \phi_1 + \frac{\hbar^2}{4m_e n_0 e} \nabla_\perp^2 n_{e1} - \frac{2k_B T_{Fe}}{3n_0 e} n_{e1} \right) + \\ & \frac{c \partial \nabla_\perp}{B_0 \Omega_{ce} \partial t} \left( \phi_1 + \frac{\hbar^2}{4m_e n_0 e} \nabla_\perp^2 n_{e1} - \frac{2k_B T_{Fe}}{3n_0 e} n_{e1} \right). \end{aligned} \quad (45)$$

Using (27), (28) and (45) in the electron continuity equation, we obtain

$$\frac{\partial}{\partial t} \left[ \nabla_\perp^2 \phi_1 + \frac{\omega_{pe}^2}{\Omega_{ce}} \nabla_\perp^2 \left( \phi_1 + \frac{\hbar^2}{4m_e n_0 e} \nabla_\perp^2 n_{e1} - \frac{2k_B T_{Fe}}{3n_0 e} n_{e1} \right) \right] + c \frac{\partial}{\partial z} \nabla_\perp^2 A_{z1} = 0, \quad (46)$$

where  $\Omega_{ce} = eB_0/m_e c$  is the electron cyclotron frequency. Then using (25), (44) and (46) along with electron continuity equation, we have

$$\begin{aligned} (1 - \lambda_e^2 \nabla_\perp^2) \frac{\partial^2}{\partial t^2} \left[ \nabla_\perp^2 \phi_1 + \frac{\omega_{pe}^2}{\Omega_{ce}} \nabla_\perp^2 \left\{ \phi_1 + \frac{1}{n_0 e} \left( \frac{\hbar^2 \nabla_\perp^2}{4m_e} - \frac{2T_{Fe}}{3} \right) n_{e1} \right\} \right] - \\ c^2 \nabla_\perp^2 \frac{\partial^2}{\partial z^2} \left\{ \phi_1 + \frac{1}{n_0 e} \left( \frac{\hbar^2 \nabla_\perp^2}{4m_e} - \frac{2T_{Fe}}{3} \right) n_{e1} \right\} = 0. \end{aligned} \quad (47)$$

Using (47) with Poisson's equation, and Fourier transforming the resulting expression, we obtain the linear dispersion relation as follows

$$\omega^2 = \frac{v_{Ae}^2 k_z^2 (1 + \Lambda_q^2 k^2)}{(1 + \lambda_e^2 k_\perp^2) \left[ 1 + \Lambda_q^2 k^2 + \frac{v_{Ae}^2 k^2}{c^2 k_\perp^2} \right]}, \quad (48)$$

where  $v_{Ae} = B_0 / \sqrt{4\pi n_0 m_e}$ ,  $\Lambda_q = \frac{v_q}{\omega_{pe}}$  and  $v_q = \sqrt{T_q / m_e}$ . The above equation shows that the wave frequency strongly depends on the quantum nature of electrons which gives rise to the dispersion due to the fermionic pressure and diffraction effects. The last term in square brackets in denominator is negligibly small in general. In an earlier paper, such a relation has been derived in the absence of electron statistical effects (Shukla & Stenflo, 2007).

However, it is seen that the contribution of Fermi pressure is dominant in the wave dispersion as compared with the quantum pressure arising due to Bohm-de Broglie potential.

## 6. Parametric analysis

Plasmas found in the interior of Jovian planets (Jupiter, Saturn), in compact astrophysical objects e.g., white dwarf stars and neutron stars (Lai, 2001; Bonitz, et al., 2009) as well as dense electron Fermi gas near metallic densities (Manfredi 2005) are typical examples of what is known as quantum plasmas. Here, we numerically analyze the quantum effects on such dense plasmas arising due to Fermi pressure and Bohm-de Broglie potential of electron using typical parameters. The hydrodynamic model is useful for understanding the properties of long wavelength perturbations ( $> \lambda_{Fe}$ ) in such systems. The density in the interior of white dwarf stars is of the order of  $10^{26}\text{cm}^{-3}$ . For such densities, we have  $v_{Fe} \cong 10^8\text{cm/s}$ ,  $\lambda_{Fe} \cong 4 \times 10^{-9}\text{cm}$  and  $\bar{r} \cong 2 \times 10^{-9}\text{cm}$ . For  $v_{te} < v_{Fe}$ , we have  $\lambda_{Be} > \lambda_{Fe}$  and  $n_{e0}\lambda_{Be}^3 > 1$ . The choice of  $k_{\perp} \sim 10^6\text{cm}^{-1}$  shows that the wavelength of perturbation is much larger than  $\lambda_{Be}$  and  $\bar{r}$ . Since we have assumed  $k_z \ll k_{\perp}$  in deriving the dispersion relation, therefore we take  $k_z/k_{\perp} \sim 0.002$ . These parameters are used to numerically analyze the dispersion relation (38). Figs. (1) and (2) shows the dispersive contribution of electrons quantum effects on shear Alfvén waves and electrostatic waves, respectively. We use the high magnetic field of the order  $10^8\text{G}$  which is within the limits of dense astrophysical plasmas (Lai, 2001). It leads to  $\lambda_e^2 k_{\perp}^2 \cong 0.003$ ,  $\rho_q \cong 0.3 \times 10^{-5}\text{cm}$ ,  $v_A \cong 2 \times 10^6\text{cm/s}$  and  $ck_{\perp}/k \gg v_A$ . The overall contribution of the quantum effects in wave dispersion is weak but the effect of electron Fermi pressure is more important as compared with corresponding quantum diffraction term. The dispersion is predominantly due to the Fermi pressure of electrons. It may be mentioned here that  $T_{Fe}$  is a function of density and assumes very large values. The dispersion relation (48) is plotted in Fig. (3) for relatively less dense plasmas with  $n_{e0} \cong 10^{24}\text{cm}^{-3}$ . Such densities are relevant to

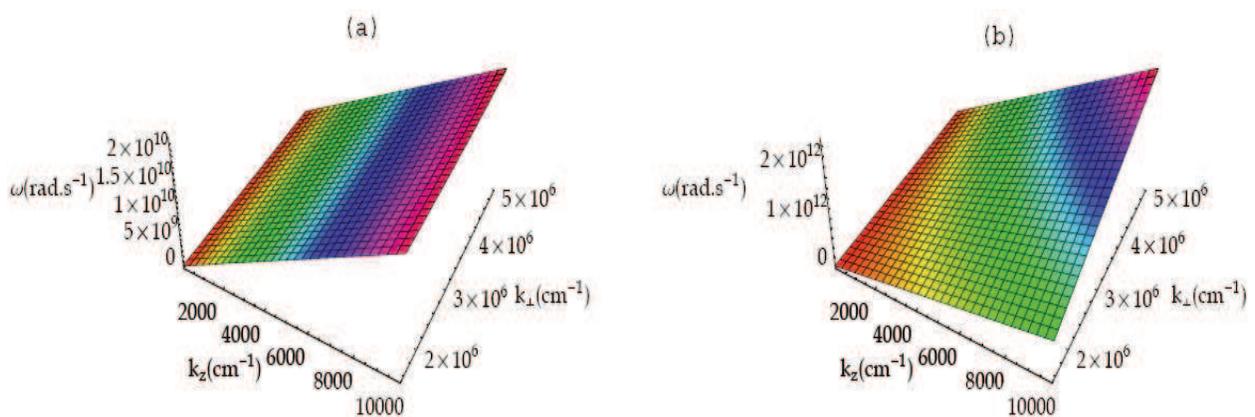


Fig. 1. (Color online). The Alfvén wave frequency  $\omega$  is plotted vs wave numbers  $k_z$  and  $k_{\perp}$  for mobile ions using typical parameters of white dwarf stars i.e.,  $n_{e0} \cong 10^{26}\text{cm}^{-3}$  and  $B_0 \cong 10^8\text{G}$ . Case (a) corresponds to the frequency when the effect of Fermionic pressure is not considered and case (b) when considered. It is evident from (b) that the wave frequency is increased by two orders of magnitude when electron fermionic pressure is taken into account.

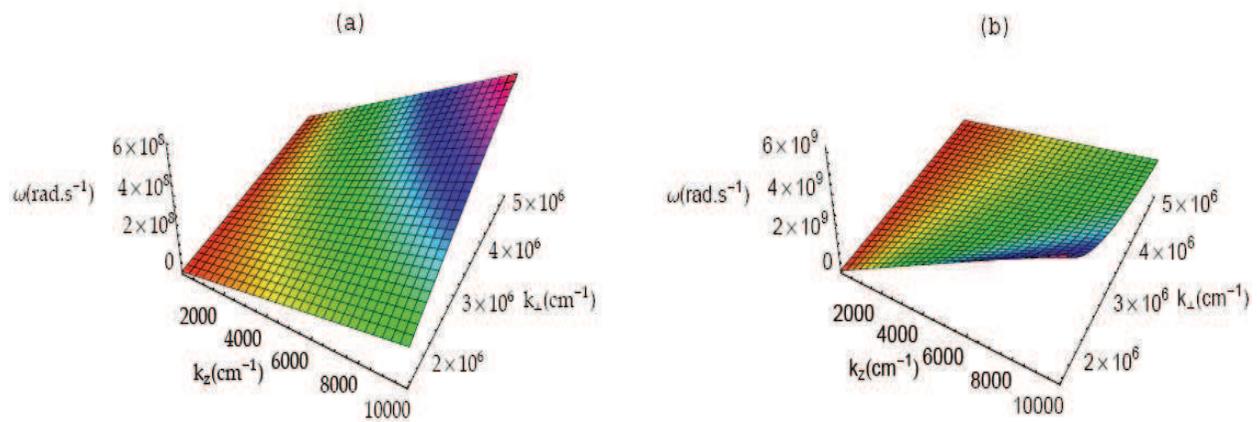


Fig. 2. (Color online). The quantum ion-acoustic wave frequency  $\omega$  is plotted from (38) against the wave numbers  $k_z$  and  $k_\perp$  using the same parameters as in Fig. 1. Case (a) refers to the wave frequency when the effect of Fermi pressure is not included, whereas case (b) when included.

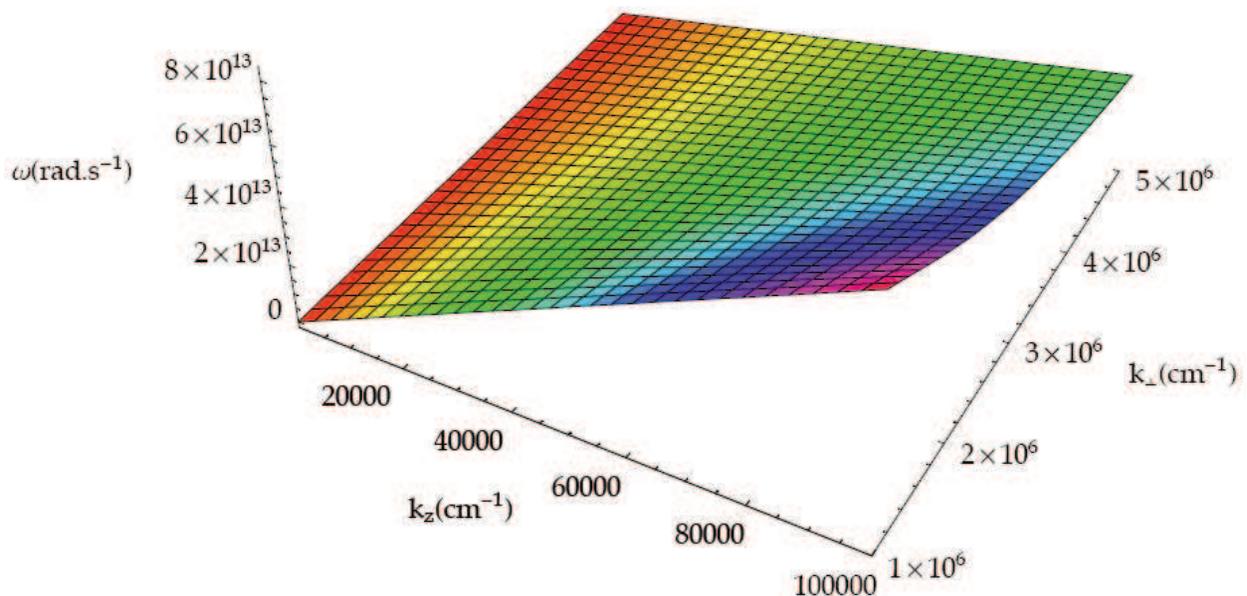


Fig. 3. (Color online). The linear dispersion relation (48) for immobile ions is plotted with  $n_{e0} \cong 1 \times 10^{24} \text{cm}^{-3}$  and  $B_0 \cong 1 \times 10^8 \text{G}$ . The wave frequency is much higher in the present case because of the dynamics of electrons with the background of stationary ions.

the dense Fermi gas of electrons with the background of stationary positive ions (Manfredi 2005) as well as dense plasmas in the outer layers of white dwarfs. For  $\omega \ll \Omega_{ce}$ , and  $k_z \sim 0.002k_\perp$ , it is found that  $\Lambda_q^2 k_\perp^2 \cong 0.03$  which is due to the dominant contribution of electron Fermi pressure since the dispersive effects due to quantum diffraction term are negligibly small. The approximations and the assumptions made in deriving the dispersion relation (48) are satisfied in the parametric range used for numerical work. The fluid model may be used for physical understanding of the waves in dense plasmas even if  $\rho_q^2 k_\perp^2 > 1$  since we have  $T \ll T_{Fe}$ . The results also indicate that the well known electron and ion plasma wave spectra in dense plasmas is significantly modified by quantum nature of electrons.

## 7. Concluding remarks

The self-consistent dynamics of low frequency linear modes in a dense homogenous electron ion quantum magnetoplasma has been investigated. Using quantum hydrodynamic approach, the generalized dispersion relation is obtained to investigate the properties of electrostatic and electromagnetic waves. The dispersion relation for coupled electrostatic and shear Alfvén waves reveals the dispersive effects associated with Fermi pressure of electrons and quantum Bohm-de Broglie potential. For illustration, we have applied our results to the dense magnetoplasma with  $n_0 \sim 10^{26} \text{cm}^{-3}$  which seems to be possible locally in the dense astrophysical objects e.g., white dwarf stars (Lai, 2001; Bonitz et al., 2009). The dispersion relation has also been obtained for dense plasma with the background of stationary ions in the presence of very high magnetic field. In the case of immobile ions, only electromagnetic part survives and the dispersion enters through the electron quantum effects. This case may be of interest for the self-consistent dynamics of electrons in dense plasma systems with background of immobile ions. The analysis of the waves in different limiting cases shows that the contribution of electron Fermi pressure is dominant over the quantum pressure due to Bohm-de Broglie potential for mobile as well as immobile ions. It is also shown that the quantum behavior of electrons in high density low temperature plasmas modifies wave frequency at short length scales.

The field of quantum plasmas is extremely rich and vibrant today. The investigation of quantum plasma oscillations in unmagnetized and magnetized charged particle systems of practical importance has been a subject of interest in the recent years. In dense astrophysical plasmas such as in the atmosphere of neutron stars or the interior of massive white dwarfs, the magnetic field and density varies over a wide range of values. For instance, the magnetic field is estimated to be varying from kilogauss to gigagauss (petagauss) range in white dwarfs (neutron stars) and the density  $n_0$  lies in the range  $10^{23} - 10^{28} \text{cm}^{-3}$ . The quantum corrections to magnetohydrodynamics can be experimentally important in such systems (Haas, 2005). Similarly, in magnetars, and in the next generation intense laser-solid density plasma interaction experiments, one would certainly have degenerate positrons along with degenerate electrons. Apart from theoretical perspective, such plasmas also holds promises of providing future technologies.

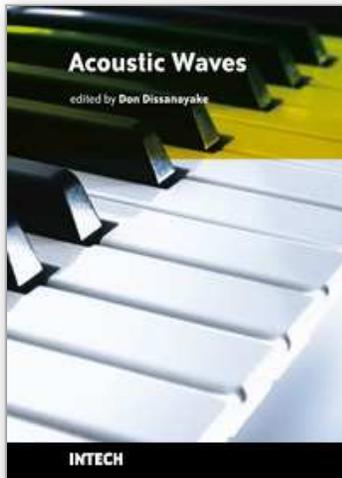
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## **Acoustic Waves**

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SAW devices are widely used in multitude of device concepts mainly in MEMS and communication electronics. As such, SAW based micro sensors, actuators and communication electronic devices are well known applications of SAW technology. For example, SAW based passive micro sensors are capable of measuring physical properties such as temperature, pressure, variation in chemical properties, and SAW based communication devices perform a range of signal processing functions, such as delay lines, filters, resonators, pulse compressors, and convolvers. In recent decades, SAW based low-powered actuators and microfluidic devices have significantly added a new dimension to SAW technology. This book consists of 20 exciting chapters composed by researchers and engineers active in the field of SAW technology, biomedical and other related engineering disciplines. The topics range from basic SAW theory, materials and phenomena to advanced applications such as sensors actuators, and communication systems. As such, in addition to theoretical analysis and numerical modelling such as Finite Element Modelling (FEM) and Finite Difference Methods (FDM) of SAW devices, SAW based actuators and micro motors, and SAW based micro sensors are some of the exciting applications presented in this book. This collection of up-to-date information and research outcomes on SAW technology will be of great interest, not only to all those working in SAW based technology, but also to many more who stand to benefit from an insight into the rich opportunities that this technology has to offer, especially to develop advanced, low-powered biomedical implants and passive communication devices.

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