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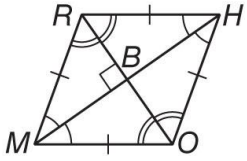
6-5 Study Guide and Intervention

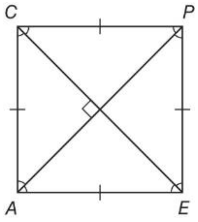
Rhombi and Squares

Properties of Rhombi and Squares A **rhombus** is a quadrilateral with four congruent sides. Opposite sides are congruent, so a rhombus is also a parallelogram and has all of the properties of a parallelogram. Rhombi also have the following properties.

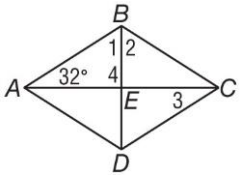


The diagonals are perpendicular.	$\overline{MH} \perp \overline{RO}$
Each diagonal bisects a pair of opposite angles.	$\overline{MH}$ bisects $\angle RMO$ and $\angle RHO$. $\overline{RO}$ bisects $\angle MRH$ and $\angle MOH$.

A **square** is a parallelogram with four congruent sides and four congruent angles. A square is both a rectangle and a rhombus; therefore, all properties of parallelograms, rectangles, and rhombi apply to squares.



Example: In rhombus *ABCD*, $m\angle BAC = 32$. Find the measure of each numbered angle.

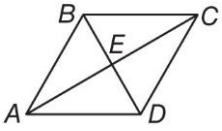


ABCD is a rhombus, so the diagonals are perpendicular and $\triangle ABE$ is a right triangle. Thus $m\angle 4 = 90$ and $m\angle 1 = 90 - 32$ or 58. The diagonals in a rhombus bisect the vertex angles, so $m\angle 1 = m\angle 2$. Thus, $m\angle 2 = 58$.

A rhombus is a parallelogram, so the opposite sides are parallel. $\angle BAC$ and $\angle 3$ are alternate interior angles for parallel lines, so $m\angle 3 = 32$.

Exercises

Quadrilateral *ABCD* is a rhombus. Find each value or measure.



1. If $m\angle ABD = 60$, find $m\angle BDC$. 60

2. If $AE = 8$, find AC. 16

3. If $AB = 26$ and $BD = 20$, find AE. 24

4. Find $m\angle CEB$. 90

5. If $m\angle CBD = 58$, find $m\angle ACB$. 32

6. If $AE = 3x - 1$ and $AC = 16$, find x. 3

7. If $m\angle CDB = 6y$ and $m\angle ACB = 2y + 10$, find y. 10

8. If $AD = 2x + 4$ and $CD = 4x - 4$, find x. 4

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6-5 Study Guide and Intervention *(continued)*

Rhombi and Squares

Conditions for Rhombi and Squares The theorems below can help you prove that a parallelogram is a rectangle, rhombus, or square.

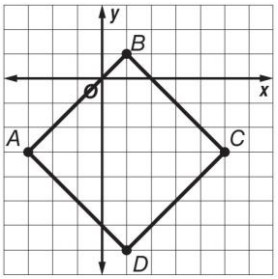
If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus.

If one pair of consecutive sides of a parallelogram are congruent, the parallelogram is a rhombus.

If a quadrilateral is both a rectangle and a rhombus, then it is a square.

Example: Determine whether parallelogram *ABCD* with vertices *A*(–3, –3), *B*(1, 1), *C*(5, –3), *D*(1, –7) is a *rhombus*, a *rectangle*, or a *square*.



$AC = \sqrt{(-3-5)^2 + ((-3-(-3))^2} = \sqrt{64} = 8$

$BD = \sqrt{(1-1)^2 + (-7-1)^2} = \sqrt{64} = 8$

The diagonals are the same length; the figure is a rectangle.

Slope of $\overline{AC} = \frac{-3-(-3)}{-3-5} = \frac{0}{-8} = 0$ The line is horizontal.

Slope of $\overline{BD} = \frac{1-(-7)}{1-1} = \frac{8}{0} =$ *undefined* The line is vertical.

Since a horizontal and vertical line are perpendicular, the diagonals are perpendicular. Parallelogram *ABCD* is a square which is also a rhombus and a rectangle.

Exercises

Given each set of vertices, determine whether $\square ABCD$ is a *rhombus*, *rectangle*, or *square*. List all that apply. Explain.

1. *A*(0, 2), *B*(2, 4), *C*(4, 2), *D*(2, 0)

Rectangle, rhombus, square; the four sides are ≅ and consecutive sides are ⊥.

2. *A*(–2, 1), *B*(–1, 3), *C*(3, 1), *D*(2, –1)

Rectangle; consecutive sides are ⊥.

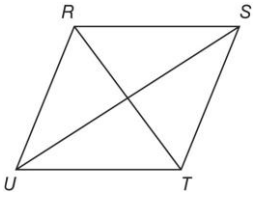
3. *A*(–2, –1), *B*(0, 2), *C*(2, –1), *D*(0, –4)

Rhombus; the four sides are ≅ and consecutive sides are not ⊥.

4. *A*(–3, 0), *B*(–1, 3), *C*(5, –1), *D*(3, –4)

Rectangle; consecutive sides are ⊥.

5. PROOF Write a two-column proof.
Given: Parallelogram *RSTU*. $\overline{RS} \cong \overline{ST}$
Prove: *RSTU* is a rhombus.



Statements	Reasons
1. *RSTU* is a parallelogram $\overline{RS} \cong \overline{ST}$	1. Given
2. $\overline{RS} \cong \overline{UT}$, $\overline{RU} \cong \overline{ST}$	2. Definition of a parallelogram
3. $\overline{UT} \cong \overline{RS} \cong \overline{ST} \cong \overline{RU}$	3. Substitution
4. *RSTU* is a rhombus	4. Definition of a rhombus