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of the American Mathematical Society

May 2009 Volume 56, Number 5

Climate Change and the Mathematics of Transport in Sea Ice

page 562

Mathematics and the Internet: A Source of Enormous Confusion and Great Potential

page 586

Real analysis in polar coordinates (see page 613)

Notices
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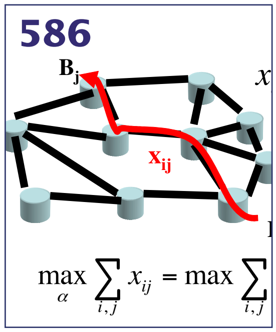

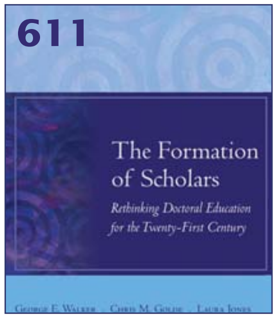

Photograph courtesy of Adam Gully.

Communications

600 WHAT IS...Measure Rigidity?
Manfred Einsiedler

614 Homage to Henri Cartan (1904–2008)
Michèle Audin

616 Encounters with Mischa Cotlar
John Horváth

622 Nebraska Department Wins Exemplary Program Award
Allyn Jackson

627 What Is New in LaTeX? II. TeX Implementations, Evolution or Revolution?
G. Grätzer

630 2009 JPBM Communications Award

632 2009 Award for an Exemplary Program or Achievement in a Mathematics Department

633 MAA Prizes Presented in Washington, DC

634 AWM Awards Given in Washington, DC

Commentary

559 Opinion: K–12 Calculator Woes
Frank Quinn

560 Letters to the Editor

602 *The Calculus Wars*—A Book Review
Reviewed by Brian E. Blank

611 *The Formation of Scholars*—A Book Review
Reviewed by David Manderscheid

Features

562 Climate Change and the Mathematics of Transport in Sea Ice

Kenneth M. Golden

Mathematics Awareness Month 2009 has as its theme "Mathematics and Climate Change". Polar sea ice both regulates climate and serves as an indicator of climate change. Brine flow through sea ice figures in the regulation, and its measurement helps in calculating climate change. The author explains mathematical modeling of brine flow and reports on Antarctic measurements checking the models.

586 Mathematics and the Internet: A Source of Enormous Confusion and Great Potential

Walter Willinger, David Alderson, and John C. Doyle

Graph theory models the Internet mathematically, and a number of plausible mathematically intersecting network models for the Internet have been developed and studied. Simultaneously, Internet researchers have developed methodology to use real data to validate, or invalidate, proposed Internet models. The authors look at these parallel developments, particularly as they apply to scale-free network models of the preferential attachment type.

Notices
of the American Mathematical Society

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Departments

About the Cover 613

Mathematics People 637

Wood and Wickelgren Awarded AIM Five-Year Fellowships, Sarkar Selected as Clay Research Fellow, Gordina Awarded Michler Prize, Bozeman Receives AAAS Mentor Award, ASL Karp and Sacks Prizes Awarded, Heineman Prize Awarded, National Academy of Engineering Elections.

Mathematics Opportunities 639

NSF Postdoctoral Research Fellowships, International Mathematics Competition for University Students.

Inside the AMS 640

From the AMS Public Awareness Office, Epsilon Awards for 2009.

Reference and Book List 641

American Mathematical Society—Contributions 645

Mathematics Calendar 652

New Publications Offered by the AMS 661

Classified Advertisements 668

Meetings and Conferences of the AMS 671

Meetings and Conferences Table of Contents 680

From the AMS Secretary

Officers of the Society 2008 and 2009 Updates 651

K–12 Calculator Woes

In the third grade my daughter complained that she wasn't learning to read. She switched schools, was classified as Learning Disabled, and with special instruction quickly caught up. The problem was that her first teacher used a visual word recognition approach to reading, but my daughter has a strong verbal orientation. The method did not connect with her strongest learning channel and her visual channel could not compensate. The LD teacher recognized this and changed to a phonics approach.

My daughter was not alone. So many children had trouble that verbal methods are now widely used and companies make money offering phonics instruction to students in visual programs.

The concern here is with serious learning deficits associated with calculator use in K–12 math. Calculators may not be making contact with important learning channels. Are they the latest analog of visual reading?

For brevity, connections are presented as "deductions" (this about calculators causes that in learning). However the deficits described are direct observations from many hundreds of hours of one-on-one work with students in elementary university courses.[1] The connections are after-the-fact speculations. If the explanations are off-base, the problems remain and need some other explanation.

Disconnect from mathematical structure. Calculators lead students to think in terms of algorithms rather than expressions. Adding a bunch of numbers is "enter 12, press +, enter 24, press +,…", and they do not see this either figuratively or literally as a single expression "12+24+…". Algorithms are less flexible than expressions: harder to manipulate, generalize, or abstract; and structural commonalities are hidden by implementation differences.[2] The algorithmic mindset has to be overcome before students can progress much beyond primitive numerical calculation.

Disconnect from visual and symbolic thinking. Calculator keystroke sequences are strongly kinetic. But this sort of kinetic learning is disconnected from other channels: touch typists, for instance, often have trouble visually locating keys. Many students can do impressive multi-step numerical calculations but are unable to either write or verbally describe the expressions they are evaluating. Their expertise is not transferred to domains where it can be generalized.

Even among high achievers calculators leave an imprint in things like parenthesis errors. The expression for an average such as $(a + b + c)/3$ requires parentheses. The keystroke sequence does not: the sum is encapsulated by being evaluated before the division is done. Traditional programs also encourage parenthesis problems,[3] but they seem more common among calculator-oriented students.

Lack of kinetic reinforcement. It is ironic that calculators might be too kinetic in one way and not enough in another, but this seems to be the case with graphing. In some K–12 curricula, graphing is now almost entirely visual: students push keys to see a picture on their graphing calculators and are tested by hand math actually connected with ways our brains learn, and the way calculators are used to bypass drudgery has weakened these connections and undercut learning.

If the explanations offered are correct, then there are several further conclusions. First, the learning connections in traditional courses are largely accidental, and a more conscious approach should significantly improve learning. Second, calculators are not actually evil, but we must be much more sophisticated in how such things are designed and used.[4] But most of all, learning must now be the focus in education. Not technology, not teaching, not learning in traditional classrooms, but unfamiliar interactions between odd and variable features of human brains and a complex new environment.

—*Frank Quinn*
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Author's Note: What do you think? Enter opinions and comments at https://survey.vt.edu/survey/entry.jsp?id=1236216488429.

[1] *At the Math Emporium at Virginia Tech,* http://www.emporium.vt.edu.

[2] *For further analysis see "Beneficial high-stakes math tests: An example" at* http://www.math.vt.edu/people/quinn/education.

[3] *See the Teaching Note on Parentheses at* http://amstechnicalcareers.wikidot.org.

[4] *See "Student computing in math: Interface design" at the site in footnote 2 for an attempt.*

Letters to the Editor

Preserve the Old Media Also

I read with some concern Michael Doob's Opinion essay "Preserving our history" in the January 2009 *Notices*. In it Doob discusses the ever-increasing capacity of hardware and software to create new documents and to bring old work online using these technologies. He also suggests in both the title of the essay and in the statement "…the hardest part of preserving your mathematical history may be taking the staples out of those old pages" that use of these technologies effectively preserves the documents. I would like to make a distinction here between preserving your mathematical history and providing access to it; there is, I believe, a common but important misconception about these two distinct activities.

As the archivist for the Archives of American Mathematics at the University of Texas at Austin, I deal every day with the preservation of both paper and digital files. While we in the archives profession have figured out how to preserve the former, we are in the early stages of grappling with the complexities and fragility of the latter.

Archives now regularly receive digital files from donors, and they digitize collections to create online exhibits or to provide online access to documents that would otherwise be available only to researchers who can visit the archives. However, archivists consider digitization to be no more than a method of providing access; if we think of it as preservation at all, it is only as a means of limiting unnecessary handling of the paper originals.

Preserving digital files in the long term is a tricky matter. It depends on many factors, such as ongoing software and hardware compatibility, stability, and obsolescence; maintenance of the metadata associated with the digital file; and the relative fragility of storage media, such as CDs, DVDs, and external hard drives (CDs, for instance, deteriorate faster than previously thought and are otherwise prone to mishandling, while data on an external hard drive can be wiped out in a second).

I am not suggesting a return to paper or other analog formats—digital is here to stay. And the possibilities for wide access to photographs, sound and video recordings, and all manner of textual documents are huge. But I do want people to realize before they digitize everything and discard the paper documents (or audio or video cassettes, etc.) that the more sophisticated the technology, the more unstable the output. Will any of our masses of digital data be readable in even 10, 20, or 50 years? Archivists (and others) are working on that question, but in the meantime, hold onto your analog copies (or consider, perhaps, donating some of those analog and digital documents to an archives, where they really will be preserved).

—*Carol Mead, archivist*
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(Received January 21, 2009)

Reply to Carol Mead

It is undeniable that access and preservation are distinct activities. In addition, it is also true that large amounts of digitally stored data can be destroyed in an instant. Still, preservation without widespread access limits the utility of that which is being stored. For the type of project I described, the originals would not be destroyed, so the digitization being described could only be helpful for both access and preservation.

It is also undeniable and important to avoid using, as much as possible, hardware or software likely to become obsolete. That is why scans must be kept in a publicly known format. I suggested TIFF since it is standard, but others are possible.

The question of the reliability of hardware can be addressed by a good backup strategy. One would not want a repetition of the fire at the library of Alexandria. The question of obsolete hardware is more difficult; I have some eight-inch floppies in my office and no way to read them. It's clear that hardware such as floppy disks is rapidly disappearing. The hard disk may be replaced by solid-state memory soon. Leaving (at least one copy of) the files with competent and aware personnel (such as archivists) certainly has to be part of the picture.

I must say that looking at digitization as a different form of access is really wrong. This is like saying Google is just a backup copy of a library. The ability to scan images for further information and to produce metadata that may be searched is at least as valuable as the images themselves.

The effectiveness of digital media for preserving our mathematical heritage is a wonderful topic with much more to it than can be discussed in a one-page opinion piece. We should continue the conversation.

—*Michael Doob*
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(Received February 11, 2009)

In Defense of Teaching Postdocs

After reading the column by "Professor Nescio" about postdoc positions [*Notices*, February 2009], I wanted to defend the teaching postdoc position that I think the good professor unfairly criticized. I recently finished a three-year teaching postdoc position at the University of Arizona, and I think Professor Nescio somewhat mischaracterized my experience.

Yes, I did on occasion teach three classes a semester, but this was by design. Not every new Ph.D. is going to get a tenure-track job at a major research institution, and having a wide teaching experience is unquestionably a benefit in the job market. There were plenty of opportunities to reduce this to two classes a semester, and I frequently took advantage of that. Since the tradeoff was to supervise the Undergraduate Teaching Assistant program, which was also a plus on my resume, it was a win/win situation for everyone involved. I had the chance to teach a wide range of classes, from low level to very advanced; never got stuck teaching a large lecture (not that I would have minded); always had the opportunity for a grader; and always had time for research.

Sure, I probably would have written more papers if I had taken a more traditional research postdoc position coming out of grad school, but it's not like research opportunities were lacking in the teaching postdoc position. I worked with other postdocs and regular faculty, and significantly strengthened my research breadth and depth. Funding was always available for conferences, and I even helped start a new annual conference in group theory hosted by the University of Arizona.

Finally, to Professor Nescio's claim that after taking a teaching postdoc position, "you may look less attractive to employers": Coming out of grad school, my "success ratio" (interviews/applications) was less than seven percent. After three years as a teaching postdoc, it was just under fifty percent, and I landed the top tenure-track job on my list. And for what it's worth, my new position has a significantly lower teaching load than my postdoc position, a claim that I don't think a lot of research postdocs can make.

—*J. P. Cossey*
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(Received January 24, 2009)

TeX Vector Graphics

Regarding the article by Jim Hefferon and Karl Berry in the March 2009 issue: Your readers may benefit from learning about the vector graphics editor Inkscape (`http://www.inkscape.org/`) and the extension called Textext (`http://www.elisanet.fi/ptvirtan/software/textext/`) written by Pauli Virtanen. Both are free and, in conjunction, can produce excellent graphics that contain embedded and editable LaTeX-generated text objects.

Also of note is the Asymptote vector graphics language (`http://asymptote.sourceforge.net/`), which uses LaTeX for typesetting of labels in graphics.

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(Received March 3, 2009)

Submitting Letters to the Editor

The *Notices* invites readers to submit letters and opinion pieces on topics related to mathematics. Electronic submissions are preferred (`notices-letters@ams.org`); see the masthead for postal mail addresses. Opinion pieces are usually one printed page in length (about 800 words). Letters are normally less than one page long, and shorter letters are preferred.

Identifications

Affiliations of authors of "Letters to the Editor" are provided for identification purposes only. Opinions expressed in letters are those of the authors and do not necessarily reflect those of their employers or, in the case of American Mathematical Society officers or committee members, policies of the Society. Committee reports to the Council of the Society and official communications of officers of the Society, when published in the *Notices*, appear in the section of the *Notices* "From the AMS Secretary".

Climate Change and the Mathematics of Transport in Sea Ice

Kenneth M. Golden

As the boundary layer between the ocean and atmosphere in the polar regions, sea ice is a critical component of the global climate system. As temperatures on Earth have warmed, the Arctic sea ice pack in particular has exhibited a dramatic decline in its summer extent. Indeed, the polar sea ice packs are harbingers of climate change. Predicting what may happen over the next ten, fifty, or one hundred years requires extensive modeling of critical sea ice processes and the role that sea ice plays in global climate. Currently, large-scale climate models in general do not realistically treat a number of sea ice processes that can significantly affect predictions. Moreover, monitoring the state of Earth's sea ice packs, in particular their thickness distributions, provides key data on the impact of global warming. Mathematics is currently playing an important role in addressing these fundamental issues and will likely play an even greater role in the future. Here we give a brief bullet-by-bullet summary of the contents of this article.

Question 1. What is the role of polar sea ice in global climate?

- Sea ice forms the boundary between the polar oceans and the atmosphere, and mediates the exchange of heat, gases, and momentum between them. The polar sea ice packs help regulate Earth's climate and are acute indicators of climate change.
- While sea water absorbs most incident solar radiation, sea ice reflects most of it. Earth's sea ice packs act as solar heat reflectors, as well as ocean insulators, keeping significant heat from escaping to the atmosphere.
- Sea ice is a porous composite of pure ice with liquid brine inclusions. The flow of fluids through sea ice mediates processes important to climate, from melt pond evolution, which controls the reflectance of the sea ice packs, to upward percolation of sea water, which floods the surface and then freezes, forming *snow-ice*, an increasingly important component in the sea ice system. Brine flow though sea ice can also enhance heat exchange.
- Sea ice hosts algal and bacterial communities which support life in the polar oceans. Nutrient replenishment processes are controlled by fluid transport through the microstructure.
- The state of the polar sea ice packs—their extent and thickness—provides important information in monitoring climate change.

Question 2. What is the role of mathematics in modeling transport in sea ice?

- Various techniques within the field of **homogenization** are used to derive macroscopic information about transport in sea ice from partial information about its microstructure.
- Variational formulations for the trapping constant and mean survival time for **diffusion processes** which interact with the pore boundaries are used to obtain rigorous bounds on the **fluid permeability tensor** for sea ice, based on general microstructural information.

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- Fluid transport in sea ice exhibits a permeable/impermeable transition at a critical brine volume fraction of about 5%, which controls geophysical and biological processes. **X-ray computed tomography** and mapping of the pore microstructure onto **random graphs** are used to demonstrate that the brine phase of sea ice undergoes a transition in connectedness at this brine volume fraction. **Percolation theory** is used to theoretically predict the transition and to mathematically characterize the thermal evolution of the fluid pores and their connectedness.
- **Lattice and continuum percolation models** are used to predict critical behavior of the fluid permeability in sea ice, with a universal exponent describing the behavior above the percolation threshold, and **critical path analysis** yields the scaling factor.
- **Hierarchical models** developed for porous rocks are used to predict the fluid permeability of sea ice over the entire range of brine porosities.
- A **random pipe network**, which is equivalent to a random resistor network and is solved using fast **multigrid methods**, is used to numerically simulate fluid flow through sea ice.
- Methods of **complex analysis** and **functional analysis** are used to obtain rigorous bounds on the **effective complex permittivity** of sea ice, the key parameter characterizing its electromagnetic behavior and the response of sea ice in remote sensing applications, such as monitoring sea ice thickness.
- **Inverse methods** yield microstructural information from complex permittivity data, paving the way for electromagnetic monitoring of fluid and thermal transport in sea ice.
- We have made **measurements of fluid and electrical transport** in sea ice on a 2007 Antarctic expedition in order to validate our models and investigate new phenomena.
- Large-scale sea ice and climate models currently do not generally incorporate the key processes our work on transport describes. These global models have grid sizes on the order of many kilometers. **Future mathematical challenges** include quantification of how local transport properties on the scale of individual floes influence pack behavior on much larger scales relevant to global models.

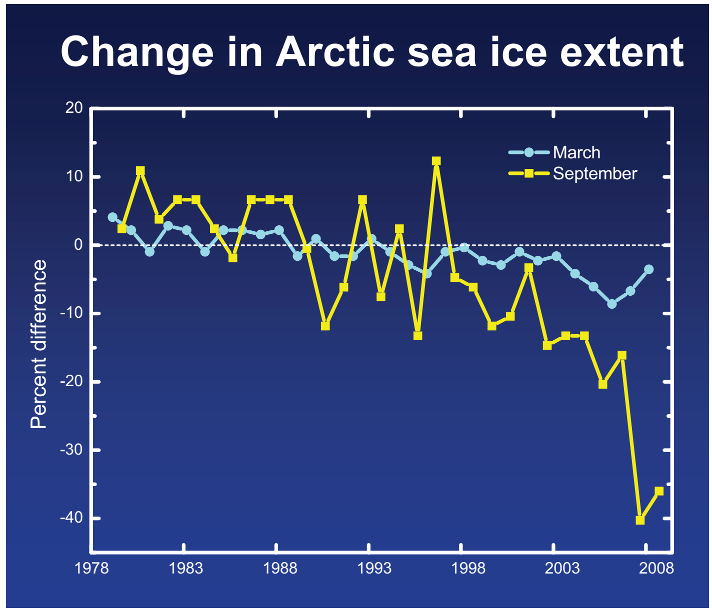


Figure 1. While winter Arctic sea ice extent (blue) has decreased modestly over the past thirty years, summer pack ice (yellow) has experienced a dramatic decline, particularly in the last couple of years. The dotted base line represents the average extent during the period 1979–2000. Some climate scientists wonder if this critical behavior of the system means we have passed through a so-called *tipping point* into a new regime.

This paper is divided into two main parts. First we discuss the role of sea ice in the climate system and how transport mediates key processes important to issues of climate change. Then we outline how mathematics is being used to model fluid and electromagnetic transport in sea ice. Only a small number of key references and starting points for further reading are given here. Many of the references for this work can be found in [8, 9, 10], and a Web link to an extensive bibliography is given at the end.

Sea Ice and Its Role in the Global Climate System

Decline of the summer Arctic sea ice pack

Up until about ten years ago, each summer most of the Arctic Ocean was covered with sea ice of areal extent roughly the same as the continental United States. By 2005, this thick, perennial ice pack had lost the areal equivalent of the U.S. east of the Mississippi River. By the end of the 2007 summer melt season, an area comparable to most of the Midwestern U.S. had been lost as well, and the Arctic ice pack had shrunk by nearly 40 percent from its 1979–2000 average extent. The fabled Northwest Passage was finally open to seafarers

Figure 2. In 1980 the areal extent of the summer Arctic sea ice pack corresponded roughly to the continental U.S. By 2005 the area shown in red was gone, and by 2007 the additional area in purple was gone as well.

for the first time in human memory. These stunning losses in the extent of summer Arctic sea ice represent perhaps the most dramatic, large-scale indicator of global warming on the surface of the Earth so far.

The sea ice pack surrounding the continent of Antarctica is mostly seasonal, meaning it melts in the summer before returning in the cold, dark winter. Will the Arctic become mostly ice-free in summer like the Antarctic? If so, how long will it take? The answers to such questions are critical for global climate. The summer Arctic sea ice pack serves as a major part of Earth's polar refrigerator, cooling it and protecting the Arctic Ocean from absorbing too much solar heat. One of the central questions of climate science today is to understand why the Arctic ice pack has declined so precipitously and to predict how this complex system will evolve in the coming years. In order to see how mathematics can play a role in addressing such fundamental issues, let's first discuss how sea ice forms, explore its material structure, and then outline its role in global climate and polar ecosystems.

Formation and structure of sea ice

Sea ice is frozen ocean water, which freezes at a temperature of about −1.8°C, or 28.8°F. Initially, tiny platelike crystals called *frazil* form near the sea surface. Often they aggregate into slick patches called *grease ice*. When sea ice forms in calm conditions, thin sheets of translucent ice called *nilas* are seen first, which eventually thicken through congelation into gray ice and then white ice in large, undeformed floes. Currents or light winds often push the nilas around so that pieces slide over each other, a process known as *rafting*. Rafting nilas is shown in Figure 3 on top. In turbulent conditions typical of the open ocean, the "greasy" frazil suspension undergoes cyclic compression in the wave field, and during compression the crystals can freeze together to form small cakes of slush. They grow larger by accretion and more solid through continued freezing between the crystals and form plates with raised rims known as *pancake ice* [22]. Patches of new grease ice are shown on the bottom in Figure 3. Pancake ice is shown in the bottom row of Figure 8. The pancakes in the lower right of Figure 8 are "glued" together in the process of forming larger floes. Sea ice that melts after one season is called *seasonal ice*, and sea ice that survives a summer season is called *multiyear ice*.

During the austral winter the continent of Antarctica, sitting atop the South Pole, is ringed mostly by seasonal ice on the Southern Ocean, with maximal ice extent in September. The Arctic is almost the geographical opposite, with an ocean at the North Pole, surrounded by continents, and covered with both seasonal and multiyear sea ice, of maximal extent in March. Sea ice should be contrasted with icebergs, glaciers, ice sheets, and ice shelves, which all originate on land. The great ice sheets covering Antarctica and Greenland are up to two miles thick and are composed of glacial ice formed from snow that has been compressed over thousands or millions of years into ice which flows under great pressure.

As a material, sea ice is quite different from glacial ice. When salt water freezes, the result is a polycrystalline composite of pure ice with inclusions of liquid brine, air pockets, and solid salts. As the temperature of sea ice increases, the porosity or relative volume fraction ϕ of brine increases. Brine inclusions on the submillimeter scale are shown in the upper left panel of Figure 8, and its centimeter scale polycrystalline structure is shown in the upper right panel.

Sea ice exhibits a very interesting and important critical phenomenon [5]. For brine volume fractions ϕ below about 5%, sea ice is effectively impermeable to fluid flow, while for ϕ above 5%, it is increasingly permeable with ϕ. This critical brine volume fraction $\phi_c \approx 5\%$ corresponds to a critical temperature $T_c \approx -5^\circ$C for a typical bulk sea ice salinity of 5 parts per thousand, termed the *rule of fives*. When fluid flows through sea ice, transport is facilitated by *brine channels*—connected brine structures ranging in scale from a few centimeters for horizontal slices to a meter or more in the vertical direction, as shown in the middle panel of Figure 8.

Ice-ocean-atmosphere interactions and global climate

As the boundary layer between the ocean and atmosphere in the polar regions, sea ice is a key player in global climate and the world's ocean system and is a leading indicator of climate change [23, 12, 19, 17]. The sea ice pack mediates the exchange of heat, moisture, and momentum between the ocean and atmosphere, the two prinicpal geophysical fluids on Earth. For example, as winds push sea ice around on the ocean surface, momentum is transferred to the water below; the rougher the ice, the more efficient the transfer. This process can cause large-scale overturning in the upper ocean, bringing warmer water to the surface. As another example, when sea ice freezes, it rejects cold, dense brine, forming descending plumes in the polar oceans. In the Southern Ocean, this process leads to the formation of Antarctic *bottom water*, which then flows like a complex river through the world's oceans. Another important process in atmosphere-ocean interactions is the transfer of heat between them. In winter when the ocean is warmer, heat flows to the atmosphere through the sea ice itself, which forms an insulating blanket over the ocean, as well as through *leads* or openings in the pack. The thermal conductivity of sea ice is thus an important transport coefficient helping to quantify atmosphere-ocean interactions in coupled climate models.

Roughly speaking, most of the solar radiation which is incident on snow-covered sea ice is reflected, while most of the solar radiation which is incident on darker sea water is absorbed. Sea ice is both ocean sunscreen and blanket, preventing solar rays from warming the waters beneath and thwarting ocean heat from escaping to warm the air above. The ratio of reflected sunlight to incident sunlight is called *albedo*. While the albedo of snow-covered ice is close to 1 (larger than 0.8), the albedo of sea water is close to zero (less than 0.1). If warming temperatures melt sea ice over time, fewer bright surfaces are available to reflect sunlight, more heat escapes from the ocean to warm the atmosphere, and the ice melts further. As more ice is melted, the albedo of the polar oceans decreases, leading to more solar absorption and warming, which in turn leads to more melting, in a positive feedback loop. It is believed that this so-called *ice-albedo feedback* has played an important role in the marked decrease in Arctic sea ice extent in summer [16]. Thus, even a small increase in temperature can lead to greater warming over time, making the polar regions the most sensitive areas to climate change on Earth. Global warming is *amplified* in the polar regions. Indeed, global climate models consistently show amplified warming in the high latitude Arctic, although the magnitude varies considerably across different models. For example, the average surface air temperature at the North Pole by the end of the twenty-first century is predicted to rise by a factor of about 1.5 to 4 times the predicted increase in global average surface air temperature.

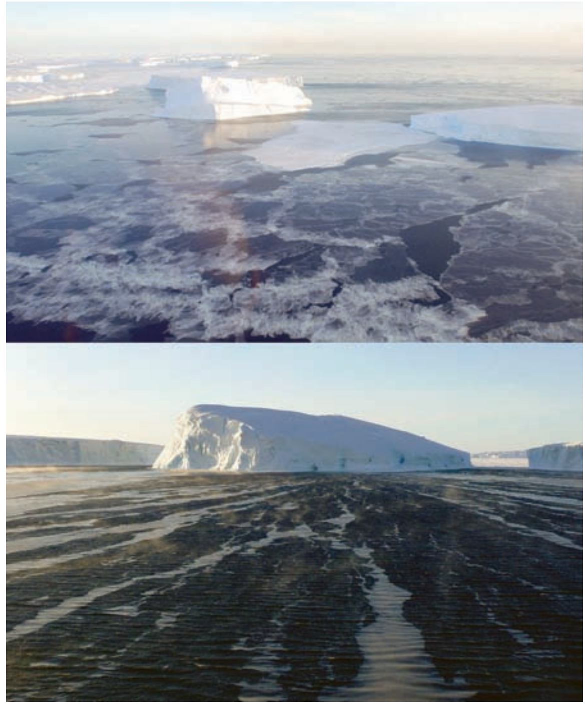


Figure 3. Sea ice forming in Antarctica, with nilas on top and grease ice on the bottom.

In the summer of 2007, the extent of the summer Arctic ice pack reached a record minimum. Ice mass balance observations show that there was an extraordinarily large amount of melting on the bottom of the ice in the Beaufort Sea during that summer [16]. Calculations indicate that solar heating of the upper ocean was the primary source for the observed melting. An increase in the open water fraction resulted in a 500% positive anomaly in solar heat input to the upper ocean, triggering an ice-albedo feedback, accelerating the pack ice retreat. Also of concern is that there has apparently been a significant reduction in the relative amount of thicker, multiyear ice in the Arctic basin. Many authors have recently acknowledged that a relatively younger,

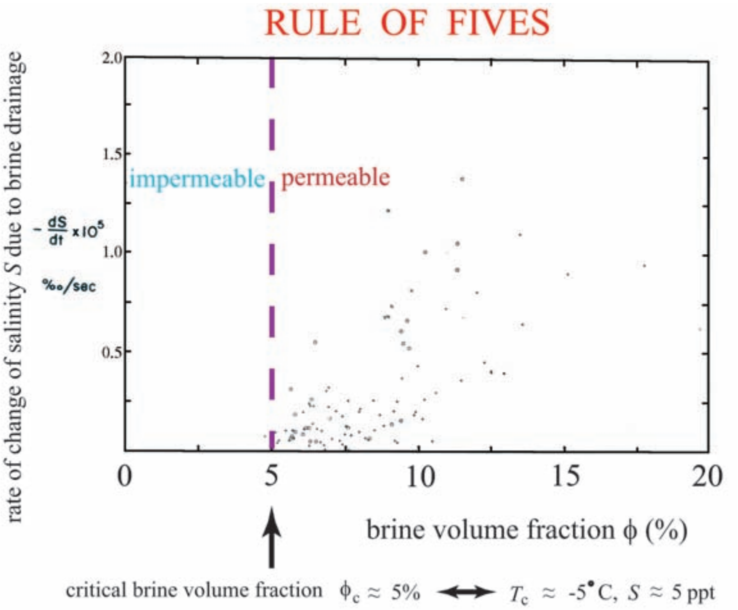


Figure 4. Sea ice was observed in 1975 by Cox and Weeks (see [5]) to be effectively impermeable to brine drainage for ϕ below about 5% and increasingly permeable with ϕ above 5%.

thinner ice cover is more susceptible to the effects of atmospheric and oceanic forcing [17]. In the face of predictions for continued warming, the persistence of recent atmospheric and oceanic circulation patterns, and the amplification of these effects through the ice-albedo feedback mechanism, it is becoming increasingly likely that the Arctic Ocean will eventually change from perennially ice-covered to ice-free in the summer.

As evidenced from the previous discussion, a key determining factor in predicting the future trajectory of Arctic sea ice is understanding how ice pack albedo evolves. Melt ponds which form on the surface of the ice, as shown on the right in Figure 6, determine its albedo. Whether or not these ponds spread, deepen, or drain is controlled largely by the fluid permeability of the sea ice, which is one of the main motivations for our studies of fluid transport in sea ice discussed below.

Another motivation for studying fluid flow in sea ice is the importance of sea ice production from freezing of flooded ice surfaces, particularly in the Antarctic, where it is ubiquitous throughout the pack. As snow loading increases during a storm, a pressure head develops. Sea water is forced upward through the sea ice, as constrained by its permeability. Net flux to the surface can occur only if the temperature throughout the entire sea ice layer has exceeded the critical temperature T_c for fluid flow. Once the surface of the ice is flooded, the slushy mix of sea water and snow freezes into *snow-ice*, which is a dominant form of ice production in the Antarctic. In fact, witnessing this process during a powerful winter storm in the Eastern Weddell Sea while on the Antarctic Zone Flux Experiment (ANZFLUX) in 1994 and watching sea water flood the ice surface in a rather dramatic fashion were what motivated the author to begin studying fluid transport in sea ice. Recent models have even suggested that with increased polar precipitation accompanying global warming, the Antarctic winter sea ice pack could thicken and that snow-ice production could become more important in the Arctic. Fluid flow through sea ice also facilitates convection-enhanced thermal transport and the input of brine and fresh water into the upper ocean from freezing, melting, and drainage processes [23, 19].

Sea ice ecology and fluid transport processes

The brine inclusions in sea ice host extensive algal and bacterial communities [14, 19] that are essential for supporting life in the polar oceans, such as krill, which themselves support fishes, penguins, seals, and Minke whales, and on up the food chain to top predators like killer whales, leopard seals, and polar bears. Nutrient replenishment processes for sea ice microbes are facilitated by fluid flow through the porous microstructure. For example, an algal bloom during the Antarctic fall in the Weddell Sea was fueled by nutrient-laden sea water replacing cold dense brine draining from freezing slush at the surface in a convective overturning process. The bloom effectively shut down, however, once the critical isotherm—above which the ice was impermeable and below which the ice was permeable—passed through the algae layer during the fall freeze-up [5].

Multiscale structure of sea ice over ten orders of magnitude

One of the fascinating yet challenging aspects of modeling sea ice and its role in global climate is the sheer range of relevant length scales of structure, over ten orders of magnitude from the submillimeter scale to hundreds of kilometers. In Figures 8 and 9, we show principal examples of sea ice structure illustrating such a range of scales. Modeling sea ice on a large scale depends on understanding the physical properties of sea ice at the scale of individual floes and smaller. Today's climate models challenge even the most powerful supercomputers to their fullest capacity. However, even the largest computers still limit the horizontal resolution to tens of kilometers and require clever approximations to model the basic physics of sea ice. One of the central themes is how to use information at a smaller scale to predict behavior at a larger scale, such as in classical statistical mechanics. In the sections to follow we focus on the microphysics of sea ice. That is, we address the fundamental issues of characterizing the brine microstructure and its thermal evolution and how to use that knowledge to derive information about the bulk transport properties of sea ice relevant to climate models, as well as monitoring climate change.

Bounds on the Fluid Permeability of Sea Ice

Despite the importance of fluid flow through sea ice in considerations of climate change and polar ecosystems, little attention had been paid to this problem, particularly from the theoretical standpoint, prior to our work [5, 9, 8, 26] (with [3] a notable exception). As a first step toward analyzing the vertical fluid permeability k and its microstructural dependence, we obtained rigorous upper bounds on $k(\phi)$ [9, 8], which we briefly review.

Consider low Reynolds number flow of a fluid through a porous random medium, occupying a region $\Omega \subset \mathbb{R}^3$. Brine of viscosity μ occupies the brine pore space $\Omega_b \subset \Omega$, having a relative volume fraction ϕ, which is determined by the sea ice temperature T and bulk salinity S [23, 19]. The solid ice phase occupies the ice grain space $\Omega_i \subset \Omega$, having a relative volume fraction $1-\phi$. Let $\chi(x)$ be the indicator function of the brine pore space, with $\chi(x) = 1$ if $x \in \Omega_b$ and $\chi(x) = 0$ if $x \in \Omega_i$. The porous microstructure is assumed to have translation invariant statistics for given T and S so that an *ergodic hypothesis* is satisfied, with equivalent ensemble and volume averaging (in the infinite volume limit), denoted by $\langle\cdot\rangle$ [20].

As with many other porous media [20], there is a microscopic length scale ℓ associated with the medium. For example, the scale over which the two-point correlation function obtained from $\chi(x)$ varies is a good measure of this length, which is small compared to a typical macroscopic length scale L, such as a sample size or thickness of a statistically homogeneous layer. Then the parameter $\epsilon = \ell/L$ is small, and one is interested in obtaining the effective fluid transport behavior in the limit as $\epsilon \to 0$. To obtain such information, the method of *two-scale homogenization* or *two-scale convergence* [20, 11] has been developed in various forms, based on the identification of two scales: a slow scale x and a fast scale $y = x/\epsilon$. The velocity and pressure fields in the brine, $v^\epsilon(x)$ and $p^\epsilon(x)$ with $x \in \Omega_b$, are assumed to depend on these two scales x and y. The idea is to *average*, or *homogenize*, over the fast scale using expansion in powers of ϵ, which leads to a simpler equation describing the overall behavior of the flow.

The velocity and pressure in the brine phase satisfy the Stokes equations

$$(1) \qquad \nabla p^\epsilon = \mu \Delta v^\epsilon, \quad \nabla \cdot v^\epsilon = 0, \qquad x \in \Omega_b,$$

Figure 5. The sea ice pack forms the boundary between the ocean and atmosphere in the polar regions. Here sea smoke forms in the track of the icebreaker *Aurora Australis* as heat flows from the warmer ocean to the colder atmosphere. Rougher ice on the right has a larger drag coefficient than smooth ice and transfers more momentum from the winds to the ocean below.

with the no-slip boundary condition on the pore surface $v^\epsilon(x) = 0$, $x \in \partial\Omega_b$. The first equation in (1) is the steady state fluid momentum equation in the zero Reynolds number limit. The macroscopic equations can be derived through a two-scale expansion, and by averaging the leading order term v_0 of the velocity over y, one obtains

$$(2) \qquad v(x) = -\frac{1}{\mu}\,\mathbf{k}\,\nabla p(x), \quad \nabla \cdot v(x) = 0,$$

where $p(x) = p_0(x)$, the leading order term in the expansion of $p^\epsilon(x)$, and $\mathbf{k}$ is the effective permeability tensor. The first equation in (2) is known as Darcy's law and the second is the macroscopic incompressibility condition [11]. We shall be interested in the permeability in the vertical direction $k_{zz} = k$, in units of m^2.

To motivate our results, consider the effective conductivity tensor $\boldsymbol{\sigma}^*$ of a two-phase composite with electrical conductivities σ_1 and σ_2 in the volume fractions ϕ and $1-\phi$. The defining equations here are $J = \sigma E$, where J is the current density, E is the electric field, and $\sigma(x)$ is the local conductivity taking the values σ_1 and σ_2. The local fields satisfy

$$(3) \qquad \nabla \times E = 0, \quad \nabla \cdot J = 0,$$

and the averaged equation $\langle J\rangle = \boldsymbol{\sigma}^*\langle E\rangle$ defines $\boldsymbol{\sigma}^*$, analogous to Darcy's law. Optimal upper and lower bounds on a diagonal component σ^* of $\boldsymbol{\sigma}^*$, such as in the vertical direction, known as the classical arithmetic and harmonic mean bounds

Figure 6. White, ice-covered polar regions are important for reflecting much of the incoming solar radiation during summer, whereas sea water absorbs most of the incident radiation. The reflectance, or albedo, of the sea ice pack is largely determined by melt ponds on the surface, like those shown on the right in the Arctic in June 2007. Melt pond evolution itself is largely determined by the fluid permeability of the sea ice below, as well as by the Sun's rays.

Figure 7. The rich Antarctic marine ecosystem is supported by photosynthetic algae (d), which give sea ice a brownish or yellowish color (e). Nutrient replenishment for these microbes is controlled by fluid flow through the porous brine microstructure. Krill (c) feed on algae, Emperor penguins (a) feed on krill, and killer whales (b) spy-hop looking for penguins to eat.

(or elementary bounds), were obtained almost one hundred years ago by Wiener:

$$\frac{1}{\frac{\phi}{\sigma_1}+\frac{1-\phi}{\sigma_2}} \leq \sigma^* \leq \phi\sigma_1+(1-\phi)\sigma_2. \tag{4}$$

The bounds are optimal, since laminates parallel to the field realize the upper bound, as shown in Figure 10(a), while laminates perpendicular to the field realize the lower bound.

For the fluid problem, we ask what configuration of the pores maximizes k. Intuitively, the best arrangement to maximize k is in vertical pipes of appropriate radii, as shown in Figure 10(b), where in the fluid problem a length scale not present for electrical conductivity arises. This

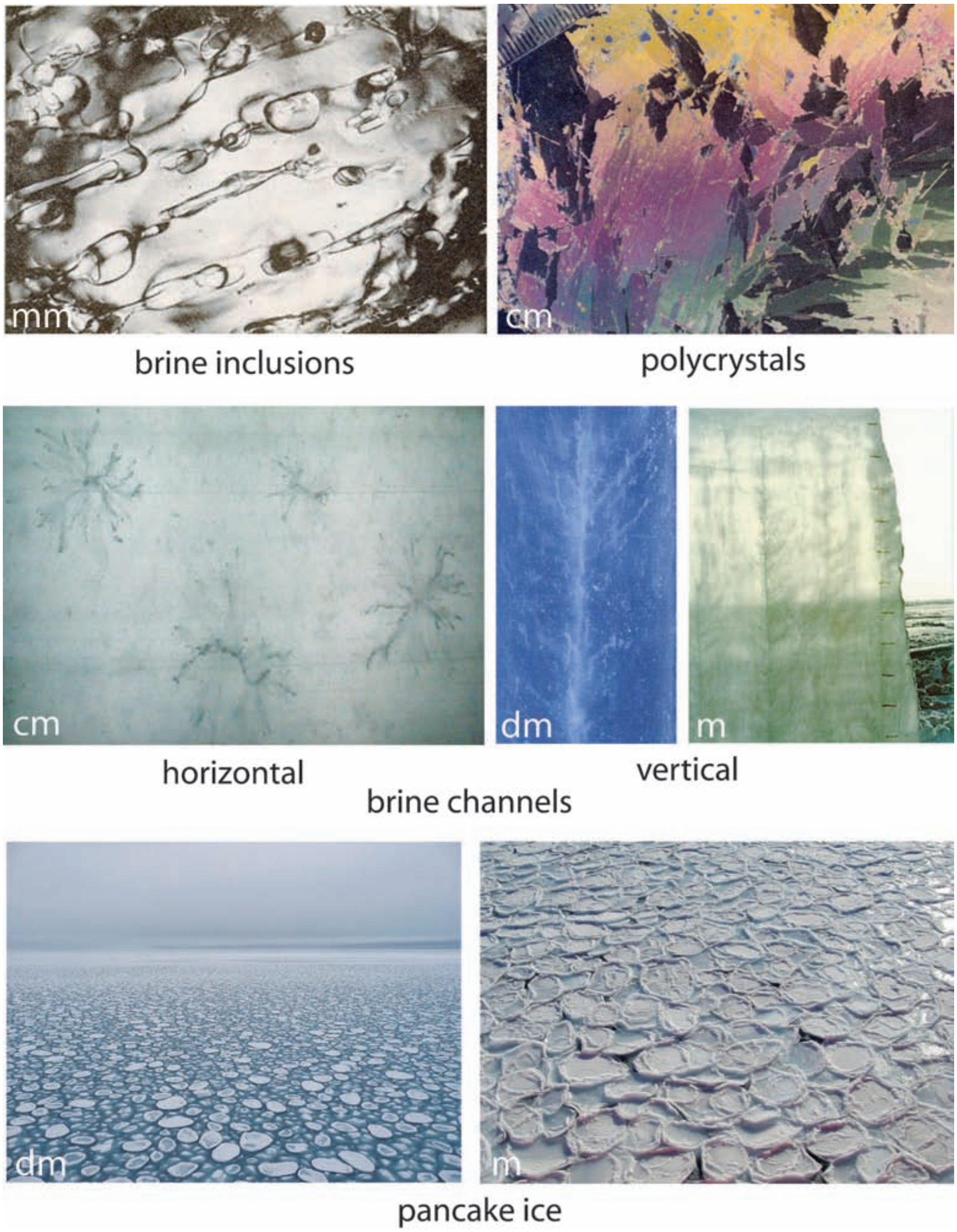


Figure 8. Multiscale nature of sea ice. Sea ice is structured on many length scales, ranging from tenths of a millimeter to hundreds of kilometers.

difference can be seen from the observation that the electrical conductance g_e of a cylinder of unit length and radius r is $g_e = \pi r^2 \sigma$, where σ is the conductivity of the material, while the fluid conductance g_f of a pipe of unit length and inner radius r is $g_f = \pi r^4/8\mu$. In [9] we obtained a *pipe bound*, $k \leq \phi[a(\phi)]^2/8$, with mean brine inclusion horizontal cross-sectional radius $a(\phi) = 7 \times 10^{-5} + (1.6 \times 10^{-4})\phi$ meters. The simple pipe bound, based on uniformly sized pipes whose radii increase with ϕ or T, captures lab data [9]. However, when considering Arctic field data where a broader range of inclusion length scales is relevant, some of the data lie outside the bound. We must consider generalizations which allow for a distribution of radii. The derivation of such bounds [21, 9, 8] employs diffusion processes in the pore space.

Consider the problem of a tracer diffusing in the fluid phase $\Omega_b \subset \Omega \subset \mathbb{R}^3$ of a porous medium. The tracer reacts with partially or completely absorbing *traps* on the boundary $\partial\Omega_b$ of the pore space.

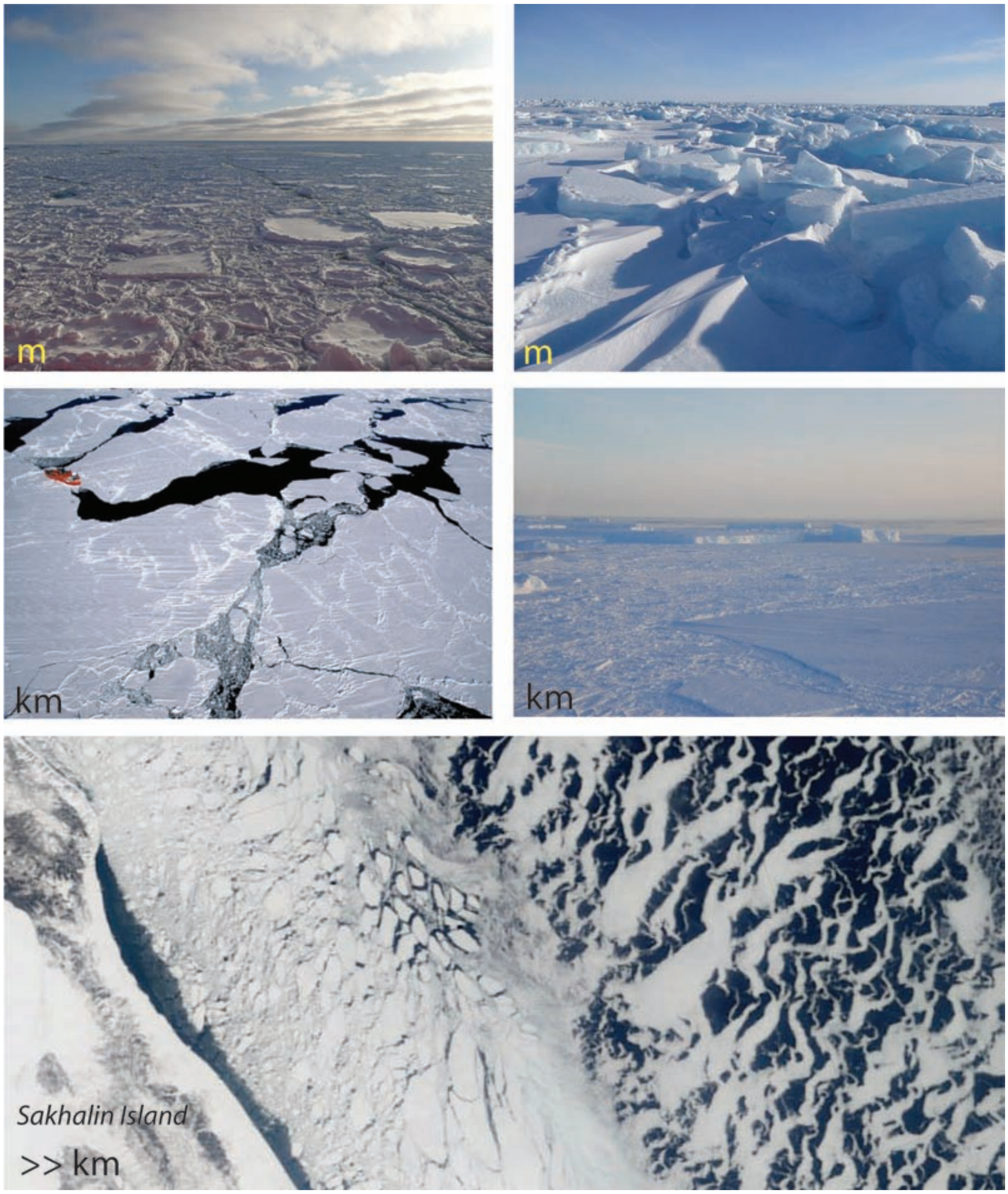


Figure 9. Multiscale nature of sea ice, continued. Pack ice viewed from ice level (top), from a helicopter (middle), and from a satellite (bottom).

Let $c(x,t)$ with $x \in \Omega_b$ be the time-dependent concentration of the reactant governed by the diffusion equation and boundary condition

$$(5)\quad \frac{\partial c}{\partial t} = D\Delta c + G, x \in \Omega_b,\ D\frac{\partial c}{\partial n} + \eta c = 0, x \in \partial\Omega_b,$$

where D is the diffusion coefficient of the reactant in the fluid, η is a positive surface reaction rate constant, G is a generation rate of reactant per unit trap-free volume, and n is the unit outward normal from the pore space. In the diffusion-controlled limit [21], where a reactant will typically diffuse in the pore space for much longer than the characteristic time associated with the surface reaction, we consider the steady state problem where the rate of removal of the reactant by absorbing boundaries is exactly compensated by the production rate per unit volume G of the reactant.

The parameter of interest is the *trapping constant*,

$$(6)\quad \gamma^{-1} = \langle u\chi \rangle,$$

where u is a scaled concentration field satisfying $\Delta u = -1$ in Ω_b and $u = 0$ on the pore boundaries, obtained from a two-scale expansion of c. The trapping constant is inversely related to the *mean survival time* τ for diffusion in the pore space, $\tau = 1/\gamma\phi D$. Using a variational definition for the trapping constant with an appropriately constructed trial field yields a general two-point *void lower bound* [21]

$$\gamma \geq \frac{(1-\phi)^2}{\ell_P^2}, \tag{7}$$

where ℓ_P is a length scale defined in terms of the two-point correlation function of the pores.

In the theory of composite materials, the Hashin-Shtrikman bounds [20, 9, 10] are important in providing optimal estimates on effective transport properties using the volume fraction information as well as isotropy of the material. These bounds are optimal in that they can be attained by actual geometries, namely, hierarchical arrays of coated spheres in $d = 3$ or coated cylinders in $d = 2$, filling all space. The two-point void lower bound in (7) can be evaluated [21] for Hashin-Shtrikman coated cylinder geometries, i.e., brine-filled vertical ice pipes for the sea ice problem, with appropriate distributions of inclusion radii R_I, as shown in Figure 11(a). Using an inequality relating the permeability tensor to the trapping constant [20] yields an optimal, rigorous upper bound,

$$k \leq \frac{\phi \langle R_I^4 \rangle}{8 \langle R_I^2 \rangle}. \tag{8}$$

Measured brine inclusion cross-sectional areas A have been found to obey a lognormal distribution [8]. Then $z = \ln A$ has a normal probability density with mean α and variance ρ^2,

$$P(z) = \frac{1}{\sqrt{2\pi\rho^2}} \, e^{-(z-\alpha)^2/2\rho^2}. \tag{9}$$

We evaluated [8] the moments in (8) using the lognormal distribution, yielding

$$k(\phi) \leq \frac{\phi}{8\pi} \, \langle A(\phi) \rangle \, e^{\rho^2}. \tag{10}$$

With variance $\rho^2 \approx 1$ and $\langle A(\phi) \rangle = \pi a^2(\phi)$ as above, the *lognormal pipe bound* in (10) captures our Arctic field data, as well as the lab data, as shown in Figure 11(b). The field data were taken near Barrow, Alaska, in a hydrological bail test, where a cylindrical hole is drilled, a tube is inserted to seal off horizontal flow, and time series measurements of the water level are used to estimate the permeability of the ice under the hole.

Predictions for the Fluid Permeability of Sea Ice

Percolation theory for sea ice

Percolation theory [18, 20, 11] has been used to model disordered materials where the connectedness of one phase dominates effective behavior. Examples range from porous rocks and bubbly glacial ice to semiconductors and carbon nanotube composites. Consider the square ($d = 2$) or cubic ($d = 3$) network of bonds joining nearest neighbor sites on the integer lattice $\mathbb{Z}^d$. We will initially consider the problem of electrical transport through the network and then fluid transport. The bonds are assigned electrical conductivities $\sigma_0 > 0$ (open) or 0 (closed) with probabilities p and $1 - p$. Groups of connected open bonds are called open clusters, and the average cluster size grows as p increases. The striking feature of this model is that there is a critical probability p_c, $0 < p_c < 1$, called the *percolation threshold*, where an infinite cluster of open bonds first appears. In $d = 2$, $p_c = \frac{1}{2}$, and in $d = 3$, $p_c \approx 0.25$.

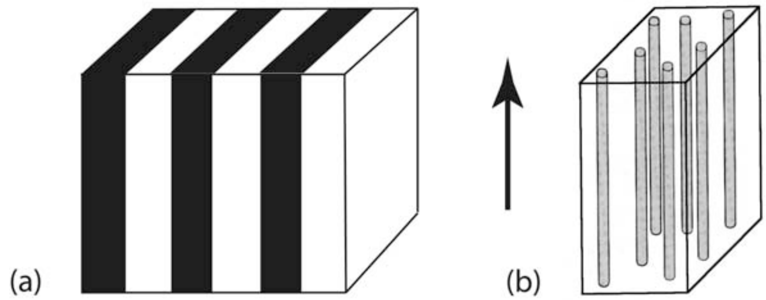


Figure 10. (a) The conductivity of a two-phase composite in a vertical electrical field is maximized by laminate structures parallel to the field. (b) The vertical fluid permeability of a porous medium is maximized by vertical pipes with appropriate radii.

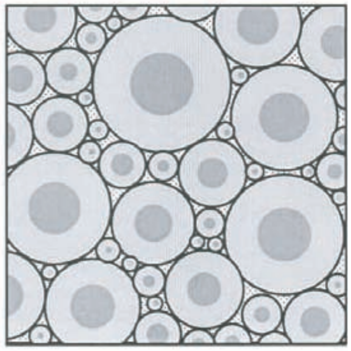
The infinite cluster density $P_\infty(p)$ is defined as the probability that the origin (or any point, by translation invariance) is contained in the infinite cluster, whose graph is shown in Figure 13(b). The two-point function $\tau(x, y)$ measures the probability that two points x and y belong to the same open cluster. As $|x - y| \to \infty$, $\tau(x, y) \sim exp(-|x - y|/\xi)$, where ξ is called the correlation length, which is a measure of the scale of connectedness. The correlation length diverges as p increases toward p_c and the infinite cluster forms, as shown in Figure 13(a). In the neighborhood of p_c, $P_\infty(p)$ and $\xi(p)$ exhibit power laws, with

$$\begin{aligned} P_\infty(p) &\sim (p - p_c)^\beta, \quad p \to p_c^+, \\ \xi(p) &\sim (p_c - p)^{-\nu}, \quad p \to p_c^-, \end{aligned} \tag{11}$$

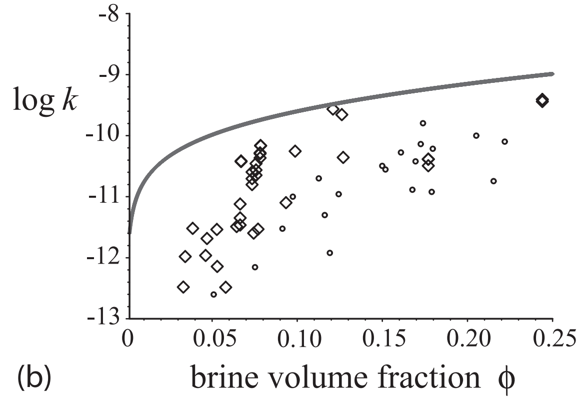
where β and ν are *universal* critical exponents that are believed to depend only on dimension and not on the details of the lattice. In $d = 3$, $\beta \approx 0.41$ and $\nu \approx 0.88$ [18].

Let $\sigma^*(p)$ be the effective conductivity of this random resistor network in the vertical direction, as defined by Kirchoff's laws in the infinite volume limit [4]. For $p < p_c$, $\sigma^*(p) = 0$, as shown in Figure 13 (c). For $p > p_c$ near the threshold, $\sigma^*(p)$ exhibits power law behavior,

$$\sigma^*(p) \sim \sigma_0 (p - p_c)^t, \quad p \to p_c^+, \tag{12}$$



(a)



(b)

Figure 11. (a) Hashin-Shtrikman coated cylinders provide an optimal geometry which attains the two-point void upper bound for the vertical fluid permeability. (b) The lognormal pipe bounds capture all our lab and field data on the vertical permeability of sea ice.

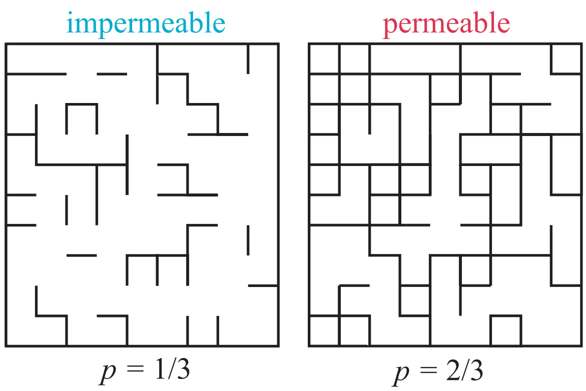


Figure 12. The two-dimensional square bond lattice below its percolation threshold $p_c = 1/2$ in (a), and above its threshold in (b).

where t is the conductivity critical exponent. For lattices, t is believed to be universal, depending only on d. The corresponding fluid permeability $\kappa^*(p)$ of the lattice, where the bonds are open or closed pipes of fluid conductivity $\kappa_0/\mu = r_0^2/8\mu$, behaves like

$$\kappa^*(p) \sim \kappa_0(p - p_c)^e\ , \quad p \to p_c^+, \tag{13}$$

where e is the fluid permeability exponent. For lattices, it is believed that $e = t$ [20]. In $d = 2$, $t \approx 1.3$, and in $d = 3$ [18, 20], $t \approx 2.0$. There is also a rigorous bound [4] that $t \leq 2$.

In view of the apparent critical behavior of brine drainage in columnar sea ice depending on whether ϕ is above or below about 0.05 [23], it was reasonable to try and find a percolation theoretic explanation. However, with $p_c \approx 0.25$ for the $d = 3$ cubic bond lattice, it was apparent that key features of the geometry of the brine microstructure in sea ice were being missed by lattices. In [5] $\phi_c \approx 0.05$ was identified with the critical probability in a continuum percolation model for compressed powders [13] that exhibit microstructural characteristics similar to sea ice. The identification explained the rule of fives, as well as data on algal growth and snow-ice production. The compressed powders, shown in Figure 14, were used in the development of so-called *stealthy* or radar-absorbing composites.

When we applied the compressed powder model to sea ice about a decade ago, we had no *direct* evidence that the brine microstructure undergoes a transition in connectedness at a critical brine volume fraction. This lack of evidence was due partly to the difficulty of imaging and quantitatively characterizing the brine inclusions in three dimensions, particularly the thermal evolution of their connectivity. Through X-ray computed tomography and pore structure analysis [8] we have now analyzed critical behavior of the thermal evolution of brine connectedness in sea ice single crystals, over a temperature range from −18°C to −3°C. We have mapped three-dimensional images of the brine phase onto graphs and analyzed their connectivity as functions of temperature and sample size. Realistic network models of brine inclusions can be derived from porous media analysis of 3-D microtomography images. Approaches include the well-established 3DMA package (3-Dimensional Medial Axis; see [8] for example), and the developmental tight dual model of Glantz and Hilpert. In both cases, a 3-D graph of nodes and edges is derived from the pores and throats. In general terms, the steps are: (1) represent the pore space by a polyhedral complex, (2) decompose this digital pore space into pores and throats and compute individual characteristic radii, (3) map these to nodes and edges tagged with their respective radii. The resulting graphs enable imbibition and drainage characterization, connectivity, and critical path analyses (see below).

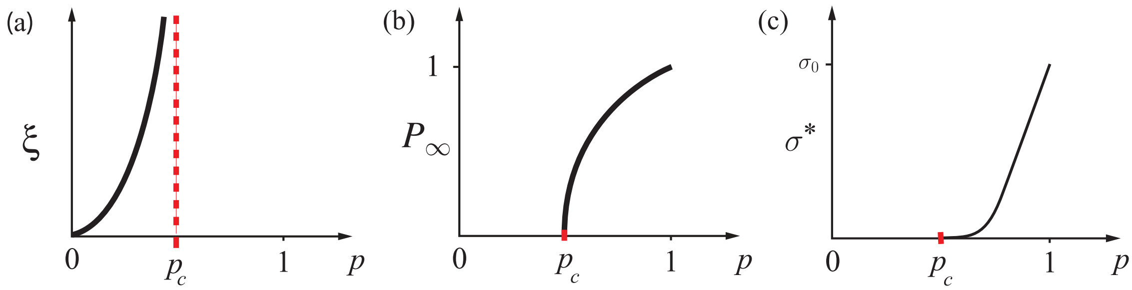


Figure 13. (a) The correlation length diverges as $p \to p_c^-$. (b) The infinite cluster density $P_\infty(p)$ vanishes below p_c and takes off with power law behavior above p_c. (c) Below p_c there is no bulk transport, and above p_c the transport coefficient takes off with a different power law.

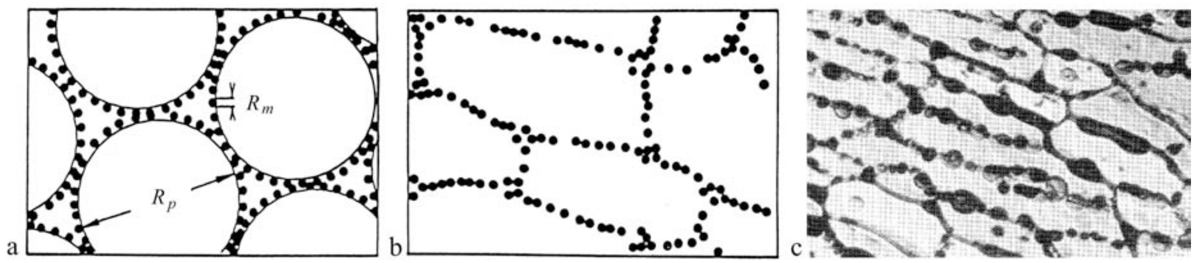


Figure 14. (a) A powder of large polymer spheres mixed with smaller metal spheres [13]. **(b) When the powder is compressed, its microstructure is similar to that of sea ice in (c).**

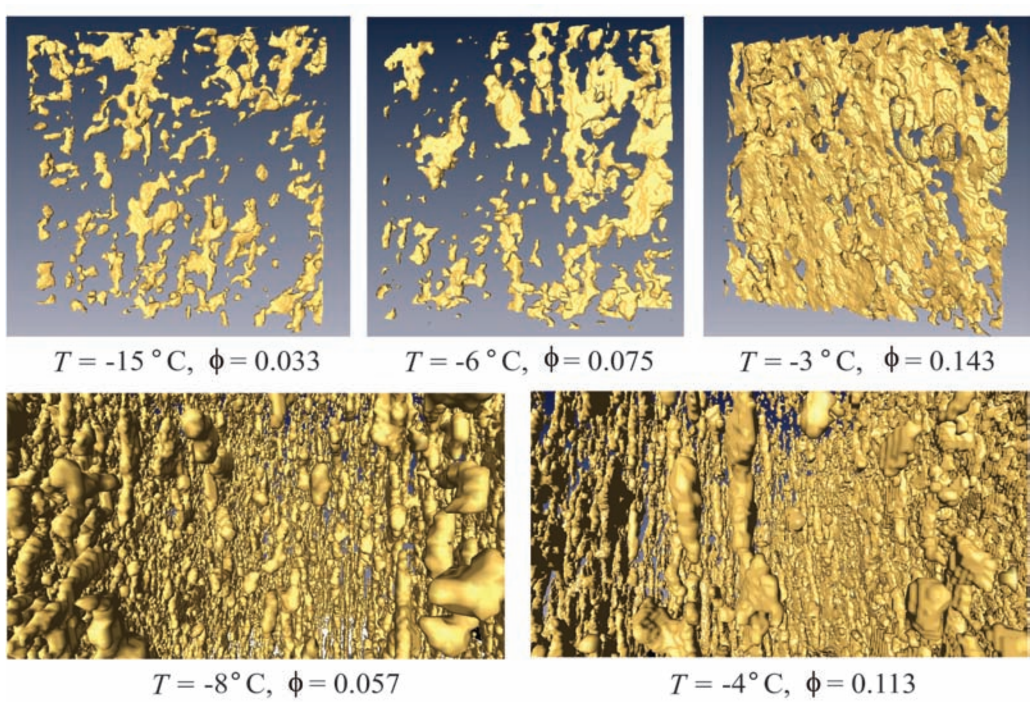


Figure 15. X-ray CT volume rendering of brine layers within a lab-grown sea-ice single crystal with S = 9.3 ppt. The (non-collocated) 8 × 8 × 2 mm sub-volumes (top row) illustrate a pronounced change in the micro-scale morphology and connectivity of the brine inclusions during warming. Images from a cylinder of height 8 mm and diameter 21 mm are shown in the bottom row.

An example of how brine morphology and connectivity evolve with temperature is shown in Figure 15. X-ray CT has provided new insight into small-scale, intracrystalline brine layer connectivity. Our work provides the first direct observation of an onset in connectivity over this scale ($\approx$ 1 cm). Notably, the thermal evolution of pore morphology that we observe is more complex than the classic model [19, 23], a transition from brine sheets to cylindrical to spheroidal inclusions, which underlies a range of porosity models [19, 23].

While percolation theory has been applied to disordered composites for fifty years, key functions such as the percolation probability (which equals 0 for $p < p_c$ and equals 1 for $p > p_c$), the infinite cluster density, the correlation length, and their critical exponents have been computed only for idealized models, like the square or cubic lattice, and *not* for real materials. In general, it is difficult not only to image the microstructure of complex disordered media but to conveniently vary the volume fraction of the connected phase in which one is interested, such as the pore space in rocks or the doping in semiconductors. However, for sea ice, by changing the temperature of a single sample we can significantly vary the brine volume fraction. We have computed these functions from our three-dimensional imaging of the pore-space evolution (in a submitted paper by Pringle, Miner, Eicken, and Golden). For example, from analysis of the two-point connectivity function, we have generated a graph illustrating the divergence of the correlation length quite similar to Figure 13(a).

We have also used our data to find anisotropic percolation thresholds using *finite-size scaling.* Rather than attempting to compute very sensitive parameters such as the percolation threshold or critical exponents from real data and its variation in volume fraction, finite-size scaling is a technique from physics where one exploits the dependence of functions of interest on sample size to obtain such results. In our case, a key relation is

$$P_\infty(p_c, L) \sim L^{-\beta/\nu}, \quad L \to \infty. \tag{14}$$

In single crystals of sea ice the brine microstructure is highly anisotropic, with inclusions usually arranged in layers. Using three different approaches, we have employed finite-size scaling to obtain anisotropic critical porosities of $4.6 \pm 0.7\%$ in the vertical direction and, laterally, $9 \pm 2\%$ parallel to the layers and $14 \pm 4\%$ perpendicular to them. The value of about 4.6% in the vertical direction for single crystals should be smaller than the threshold of about 5% for bulk connectivity, which it is.

Now we consider the application of percolation theory to understanding the fluid permeability of sea ice. In the continuum [18, 20], the permeability and conductivity exponents e and t can take nonuniversal values and need not be equal, such as for the three-dimensional Swiss cheese model. Continuum models have been studied by mapping to a lattice with a probability density $\psi(g)$ of bond conductances g. Nonuniversal behavior can be obtained when $\psi(g)$ is singular as $g \to 0^+$. However, for a lognormal conductance distribution arising from intersections of lognormally distributed inclusions, as in sea ice, the behavior is *universal* [8]. Thus $e \approx 2$ for sea ice.

The permeability scaling factor k_0 for sea ice, analogous to κ_0 in (13), is estimated using critical path analysis (see the article by the author in [11]). For media with g in a wide range, the overall behavior is dominated by a critical, *bottleneck* conductance g_c, the smallest conductance such that the critical path $\{g : g \geq g_c\}$ spans the sample. With most brine channel diameters [23] between 1.0 mm and 1.0 cm, spanning fluid paths have a smallest characteristic radius $r_c \approx 0.5$ mm, and we estimate k_0 by the pipe-flow result $r_c{}^2/8$. Thus

$$k(\phi) \sim 3\,(\phi - \phi_c)^2 \times 10^{-8}\ \text{m}^2, \quad \phi \to \phi_c^+. \tag{15}$$

In Figure 16(c), field data with ϕ in $[0.055, 0.15]$, just above $\phi_c \approx 0.05$ [5], are compared with (15), showing close agreement. A best fit for e of 1.94 was obtained from other data. The striking result that for sea ice $e \approx 2$, the universal *lattice* value in three dimensions [20, 4], is due to the general lognormal structure of the brine inclusion distribution function. The general nature of our results suggests that similar types of porous media, such as saline ice on extraterrestrial bodies, may also exhibit universal critical behavior.

Hierarchical models

To model $k(\phi)$ over all porosities, not just near the percolation threshold, we consider features of the brine phase present over the full range—some degree of connectivity, particularly on small scales, and self-similarity. The basic unit of sea ice microstructure is an ice grain surrounded by brine and ice, having substructure itself on smaller scales. Hierarchical, self-similar models of spheres or other grains surrounded by smaller spheres, and so on, with brine of conductivity σ_w in the pore spaces, have been used to describe the transport properties of sedimentary rocks [24]. Such microstructures are similar to those shown in Figure 11(a) except in this case the basic elements are brine-coated spheres of ice. Effective medium theory gives Archie's law $\sigma = \sigma_w \phi^m$ as the conductivity of the model with exact results for the exponent m, such as $m = 3/2$ for spherical grains. The corresponding permeability exponent [24] is $2m$, so that with k_0 as above and $m = 3/2$, the simplest hierarchical model yields

$$k(\phi) = 3\,\phi^3 \times 10^{-8}\ \text{m}^2. \tag{16}$$

In Figure 17(a) the full *in situ* data set is compared to this theoretical prediction, with close

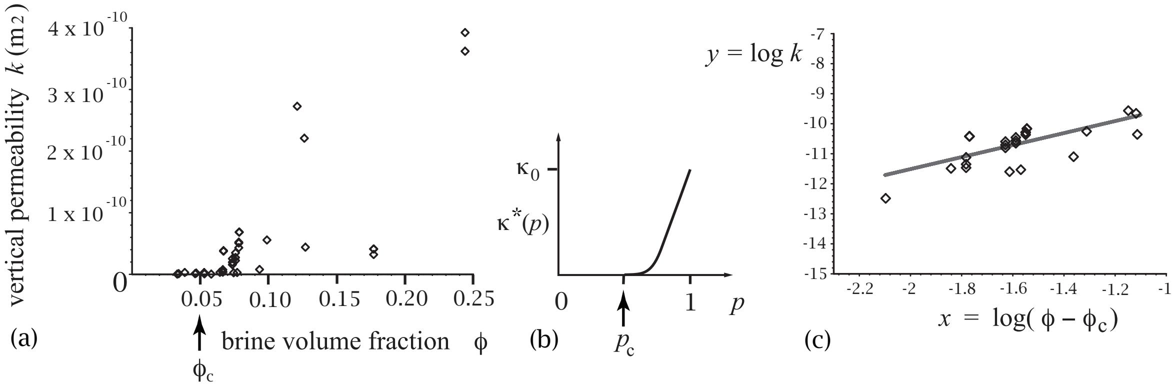


Figure 16. (a) Data for k taken *in situ* on Arctic sea ice, displayed on a linear scale. (b) The effective permeability $\kappa^*(p)$ in a lattice percolation model, displayed on a linear scale. (c) Comparison of Arctic permeability data in the critical regime (25 data points) with percolation theory in (15). In logarithmic variables the predicted line has the equation $y = 2\,x - 7.5$, while a best fit of the data yields $y = 2.07\,x - 7.45$, assuming [5] $\phi_c = 0.05$.

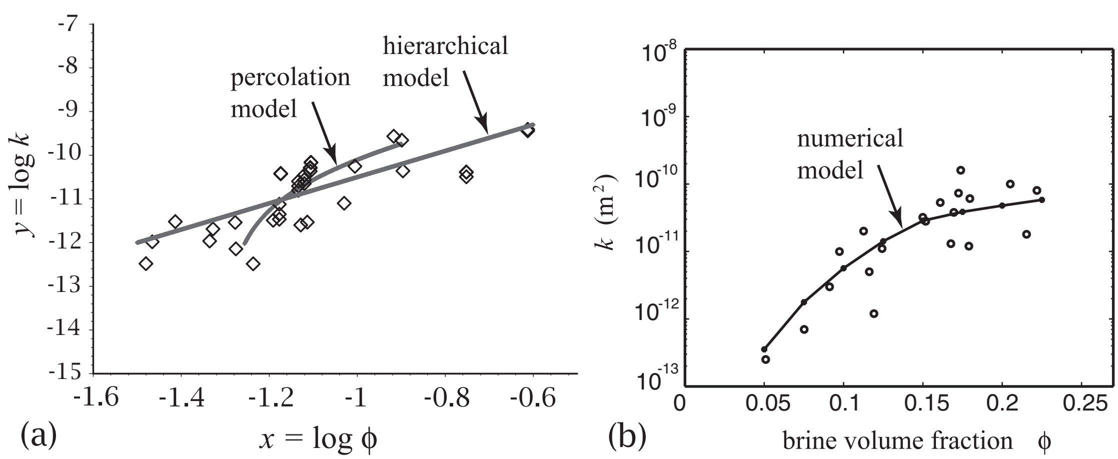


Figure 17. (a) Comparison of Arctic permeability data (37 data points) with our theoretical prediction in (16). In logarithmic variables the predicted line has the equation $y = 3x - 7.5$, while a best fit of the data yields $y = 3.05x - 7.50$. Lab data on artificially grown sea ice [3] give a best fit of $y = 3.1x - 7.7$ [8]. (b) Comparison of lab data with numerical simulation.

agreement. From photomicrographs of marine sand microstructures, compacted platey sand appears the most similar to the cellular substructure of sea ice. In this case, $2m \approx 3.04$, which is closer to the best fit of 3.05 from our field data. For calcite aggregates and Fontainebleau sandstone, a cubic power law like (16) agrees closely with permeability data for porosities above about 10%, with an exponent $2m \approx 3.05$ for Fontainebleau sandstone.

Our results show that the parameters characterizing fluid transport in sea ice are almost the same as in some crustal, sedimentary rocks. Both types of media exhibit critical behavior near a percolation threshold, with $\phi_c \approx 5\%$ for sea ice, and $\phi_c \approx 4\%$ for calcite aggregates and Fontainebleau sandstone. Moreover, even though the porous microstructures in these rocks are quite different from sea ice, the permeability critical exponent in both cases falls into the lattice universality class. The permeability exponent of 3 for the hierarchical model is also about the same in both cases. It is interesting to note that a variation of four orders of magnitude in the permeability of calcite aggregates is achieved with temperatures ranging from 633 K to 833 K, and with confining pressures from 200 MPa to 300 MPa. For sea ice, a similar variation in permeability occurs over a temperature range of about 15 K.

Numerical model for fluid flow through sea ice

In [26] we developed a two-dimensional random pipe network to simulate fluid flow through a porous medium, as illustrated in Figure 18. For

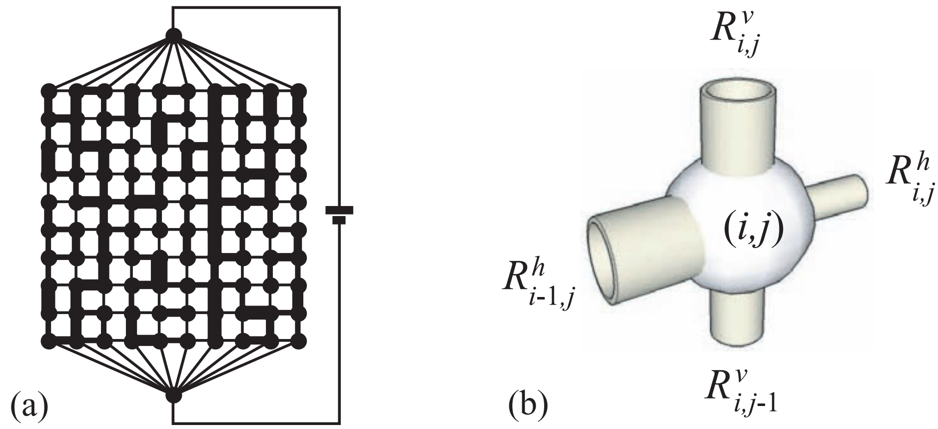


Figure 18. (a) Random pipe or resistor network. (b) Close-up of a node and adjoining ducts.

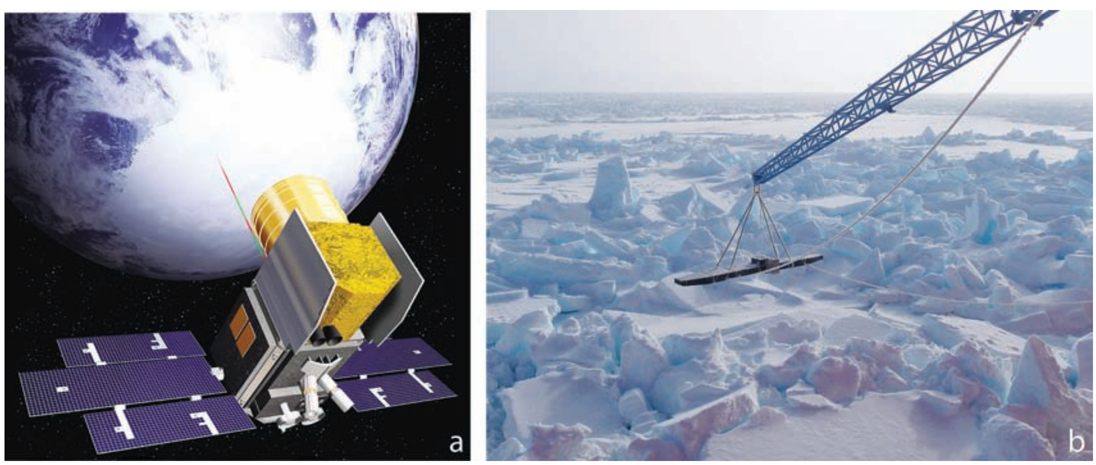


Figure 19. (a) NASA's Ice, Cloud and Land Elevation Satellite (ICESat) over Antarctica. It bounces laser pulses off the surfaces of ice sheets and sea ice to measure changes in thickness and monitor climate change. (b) An electromagnetic induction device for measuring sea ice thickness, called the "Worbot" after Australian sea ice scientist Tony Worby. It is mounted here on the icebreaker *Aurora Australis* in the Antarctic sea ice pack, September 2007.

sea ice the cross-sectional areas A of the pipes are chosen from the lognormal distribution in (9), with $\langle A(\phi)\rangle$ as before and $\rho^2 = 1$. The system is equivalent to a resistor network, which was solved using a fast multigrid method. While the model compared well with lab data for $\phi > 0.15$, it overestimated $k(\phi)$ for $\phi < 0.15$, where percolation effects, which were ignored, become significant. In accordance with our X-ray CT data, we introduced random *disconnections* so that the network falls apart as ϕ decreases. The results then agree well with the lab data, as shown in Figure 17(b).

As discussed above, heat is exchanged between the ocean and atmosphere in the polar regions through sea ice. The thermal conductivity of sea ice is thus an important transport coefficient entering into climate models. We will not discuss our work in this area, but simply mention that the methods for studying transport we have outlined here can be applied to bounding and numerically calculating this important sea ice parameter. One particularly interesting aspect of the thermal transport problem is that when brine flows through sea ice, such as for temperatures above the percolation threshold, there is an advective term in the heat equation. A velocity field helping to move heat around can enhance measured thermal conductivity, which has been observed in Antarctic sea ice. In conjunction with our fluid permeability work, we have been theoretically and numerically modeling such processes.

Sea Ice Remote Sensing: Monitoring the Impact of Climate Change

Because of the global nature of monitoring Earth's sea ice packs, information is usually obtained via remote sensing from platforms on satellites, aircraft, and ships [15, 2, 7, 6]. The physics underlying the problem of remotely sensing sea ice concerns the interaction of electromagnetic waves with a polycrystalline composite of four components: pure ice, brine inclusions, air pockets, and solid salt deposits [23, 19]. Here we will focus on the two main constituents in sea ice, namely, pure ice with brine inclusions. One of the grand challenges of sea ice remote sensing is to accurately recover the thickness distribution of the pack. Assessing the impact of global warming on the polar regions involves monitoring not only the ice extent but the ice volume, which requires knowledge of ice thickness. While microwaves in the GHz frequency range have been commonly used in many applications, due to the absorbing nature of the brine phase and the short wavelengths involved, they typically can penetrate only into the upper portions of a sea ice floe. Lasers, such as on ICESat, illustrated in Figure 19(a), have been used to bounce off the ice or snow surface, and when compared with returns from the sea surface in open leads, they can provide thickness estimates. Changes in ice sheet elevation in Antarctica and Greenland can also be similarly obtained.

Recently there has been increasing interest in using low-frequency electromagnetic induction devices to estimate sea ice thickness [25], as shown in Figure 19(b). Electromagnetic fields in the MHz range, with wavelengths on the scale of meters, are used to probe the air-ice-ocean interface and estimate the thickness of the ice layer. The technique relies on a time-varying primary magnetic field (generated by a transmitter coil) inducing eddy currents in the seawater beneath the comparatively resistive ice. The secondary magnetic field produced is sensed by a receiver coil, determining an apparent conductivity that results essentially from an integration over the vertical distance between the instrument and induced currents. Accurate interpretation of the data relies on understanding how the conductivity of the sea ice, or more generally its complex permittivity, depends on the microstructural characteristics. For example, current methods of obtaining thickness estimates assume a single value for the conductivity. However, this key parameter can vary by over two orders of magnitude with depth throughout a given floe. Since typical sizes of the brine inclusions are on the submillimeter scale, the electromagnetic behavior of the sea ice can be treated using the quasi-static approximation, where the wavelength is much longer than the microstructural scale. Then the electromagnetic response of sea ice can be characterized by its effective complex permittivity tensor $\boldsymbol{\epsilon}^*$. For a given diagonal component in a particular direction, the real part corresponds to the polarizability, or dielectric constant, in that direction, and the imaginary part corresponds to the conductivity in that direction. For example, the reflection coefficient of a wave incident on a flat sea ice surface is determined by the mismatch in complex permittivity between the sea ice and the air above.

There has been considerable work in the past on estimating and bounding $\boldsymbol{\epsilon}^*$ for sea ice, particularly in the microwave region [2, 7]. We will discuss here some of our work on bounding $\boldsymbol{\epsilon}^*$, particularly in this low-frequency range, although the mathematics is quite general. We will also discuss how we've been inverting these bounds and other information to recover microstructural characteristics, which is important in monitoring fluid and thermal transport in sea ice. In this connection, there are new methods, such as cross-borehole tomography developed by M. Ingham and colleagues, which show promise in reconstructing the permittivity profile of sea ice and, in turn, the state of the ice itself.

Here we review our work on theoretical bounds on the principal components of $\boldsymbol{\epsilon}^*$ with extensive outdoor tank and *in situ* data on $\boldsymbol{\epsilon}^*$ at 50 MHz, consisting of thousands of measurements taken with a capacitance probe [10]. In particular, we compare the low-frequency measurements with the Bergman-Milton bounds [7, 10] for the complex permittivity of a two-phase composite with known constituent permittivities ϵ_1 and ϵ_2 and volume fractions p_1 and p_2. There are two types of these bounds which we employ. The first type assumes only the permittivity and volume fraction information and is known as the complex elementary or arithmetic and harmonic mean bounds. The second type further assumes statistical isotropy in the composite microstructure and is known as the complex Hashin-Shtrikman bounds.

All the data considered are situated well inside the complex elementary bounds. However, there are naturally occurring anisotropies in the brine microstructure, such as its preferred vertical orientation [23, 19], which become more pronounced above the critical brine volume fraction of about 5%, where the brine phase percolates [5]. Also, preferred azimuthal alignment of the c-axes of individual crystals within the horizontal plane has long been observed in the Arctic and was present in the Arctic sea ice whose permittivity was measured in one of the data sets considered here. Preferred c-axis alignment is attended by anistropy in the brine microstructure and anisotropic behavior of the complex permittivity within the horizontal plane. Comparing the different data sets with the isotropic bounds yields interesting and useful insights about sea ice microstructure, its evolution with temperature, and its electromagnetic properties. We also invert the complex bounds to obtain rigorous estimates of the brine volume fraction from the permittivity data [6].

Bounds on the complex permittivity of sea ice

Let us briefly describe the analytic continuation method for studying the effective properties of composite materials [7, 10]. Let the local complex permittivity $\epsilon(x)$ be a spatially stationary random field in $x \in \mathbb{R}^d$ for an appropriate probability space representing the set of realizations of the random medium. We assume $\epsilon(x)$ for 50 MHz takes the values $\epsilon_1 = 63.3 + i1930$ in brine and $\epsilon_2 = 3.06$ in ice, and write $\epsilon = \epsilon_1\chi + \epsilon_2(1-\chi)$, where $\chi(x)$ is again the characteristic function of the brine phase. Let $E(x)$ and $D(x)$ be the stationary random electric and displacement fields satisfying the constitutive law $D(x) = \epsilon(x)E(x)$ and the equations

$$(17) \qquad \nabla \times E(x) = 0, \qquad \nabla \cdot D(x) = 0,$$

with $\langle E(x)\rangle = e_k$, where e_k is a unit vector in the k^{th} direction for some $k = 1, \ldots, d$, and $\langle\cdot\rangle$ means an ensemble average or spatial average over all of $\mathbb{R}^d$.

The effective complex permittivity tensor $\boldsymbol{\epsilon}^*$ is defined by

$$(18) \qquad \langle D\rangle = \boldsymbol{\epsilon}^*\langle E\rangle.$$

For simplicity, we focus on one diagonal coefficient, $\epsilon^* = \epsilon^*_{kk}$. Due to the homogeneity of effective parameters, $\epsilon^*(\lambda\epsilon_1, \lambda\epsilon_2) = \lambda\epsilon^*(\epsilon_1, \epsilon_2)$, ϵ^* depends only on the ratio $h = \epsilon_1/\epsilon_2$, and we define $m(h) = \epsilon^*/\epsilon_2$. The two main properties of $m(h)$ are that it is analytic off $(-\infty, 0]$ in the h-plane and that it maps the upper half-plane to the upper half-plane, so that it is an example of a Herglotz, or Stieltjes, function. The key step in the analytic continuation method is obtaining an integral representation for ϵ^*.

It is more convenient to work with the function $F(s) = 1 - m(h)$, where $s = 1/(1-h)$, which is analytic off $[0,1]$ in the s-plane. It was proven that $F(s)$ has the powerful representation

$$(19) \qquad F(s) = \int_0^1 \frac{d\mu(z)}{s-z}, \qquad s \notin [0,1],$$

where μ is a positive measure on $[0,1]$ (not to be confused with the viscosity from before). Formula (19) separates the parameter information in s from information about the mixture geometry contained in μ, a spectral measure of the operator $\Gamma\chi$, where $\Gamma = \nabla(-\Delta)^{-1}\nabla\cdot$. Statistical assumptions about the geometry are incorporated into μ via its moments $\mu_n = \int_0^1 z^n d\mu(z)$, which can be calculated from the correlation functions of the random medium, with $\mu_n = (-1)^n\langle\chi[(\Gamma\chi)^n e_k]\cdot e_k\rangle$.

Bounds on ϵ^*, or $F(s)$, are obtained by fixing s in (19), varying over admissible measures μ (or admissible geometries), such as those that satisfy only $\mu_0 = p_1$, and finding the corresponding range of values of $F(s)$ in the complex plane. Two types of bounds on ϵ^* are obtained. The first bound R_1 assumes only that the relative volume fractions p_1 and $p_2 = 1 - p_1$ of the brine and ice are known, with $\mu_0 = p_1$ satisfied. In this case, the admissible set of measures forms a compact, convex set $\mathcal{M}_0$. Since (19) is a linear functional of μ, the extreme values of F are attained by extreme points of $\mathcal{M}_0$, which are the Dirac point measures $p_1\delta_z$. The values of F lie inside the region R_1 bounded by circular arcs, called the complex elementary bounds. The formulas can be found in [7, 10]. This region collapses to the interval $(p_1/\epsilon_1 + p_2/\epsilon_2)^{-1} \le \epsilon^* \le p_1\epsilon_1 + p_2\epsilon_2$ when ϵ_1 and ϵ_2 are real, which are the arithmetic (upper) and harmonic (lower) mean bounds. The complex elementary bounds are optimal and can be attained by coated ellipsoidal geometries as the aspect ratio varies. If the material is further assumed to be statistically isotropic, i.e., $\epsilon^*_{ik} = \epsilon^*\delta_{ik}$, then $\mu_1 = p_1p_2/d$ must be satisfied as well. A smaller bounding region R_2 is then obtained, which forms the complex Hashin-Shtrikman bounds. Extensions of the analytic continuation method and bounds to multicomponent media involve techniques of several complex variables.

Now we compare three extensive sets of data on the complex permittivity of sea ice with the bounds. Measurements were taken in land-fast ice in the Chukchi Sea near Barrow, Alaska (14,004 data points), and in McMurdo Sound, Antarctica (2,382 data points), and in artificial, young sea ice in an outdoor tank in Fairbanks, Alaska (6,403 data points). The permittivity was measured by deploying an array of Stevens Water Monitoring Systems Hydraprobes. Each Hydraprobe is a coaxial probe with a central tine surrounded by three equally spaced outer tines, aligned horizontally, as shown in the photo inset in Figure 20(a). The three outer tines are held at ground potential, and a voltage is applied to the central tine at 50 MHz frequency, resulting in a circularly polarized wave with electric field predominantly in the plane perpendicular to the tines. The complex permittivity measured by the probe represents an average of the components of $\boldsymbol{\epsilon}^*$ in this plane.

In Figure 20(a) the full data set for the Fairbanks ice tank experiment is displayed along with a series of regions $R_1(\phi)$ for the indicated values of the brine porosity $\phi = p_1$. The data have been grouped into intervals of porosity—$[0.02, 0.03]$, $[0.03, 0.04]$, ..., $[0.08, 0.09]$—with a different color for each interval. The corresponding elementary bound has the same color. All the data in each interval lie inside the corresponding bound. The data marked with the gray color lie in the porosity interval $[0.049, 0.051]$, which corresponds to the percolation threshold of about 5%, conjectured in [5].

It is useful to note that tighter bounds [7] on ϵ^* can be obtained if the material is known to have a matrix-particle structure with separated inclusions, like sea ice at temperatures colder than its critical temperature. In this case, the support

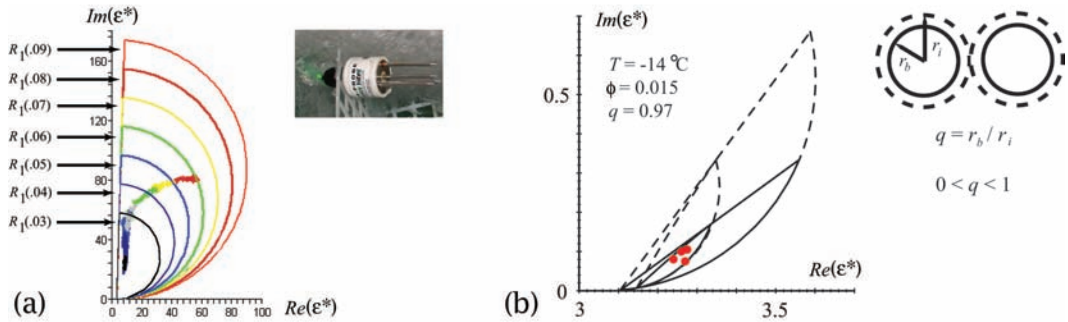


Figure 20. (a) Comparison of data on ϵ^* at 50 MHz for sea ice grown in an ice tank with the elementary bounds, showing excellent correspondence. Data in a brine porosity interval signified by a given color lie inside the region of the same color. The data colored gray have porosities in the interval [0.049,0.051], close to the percolation threshold. The capacitance probe is shown on the right. (b) Matrix-particle bounds on ϵ^* (solid curves), along with the complex elementary and Hashin-Shtrikman bounds (dotted curves), compared with 4.75 GHz data in red.

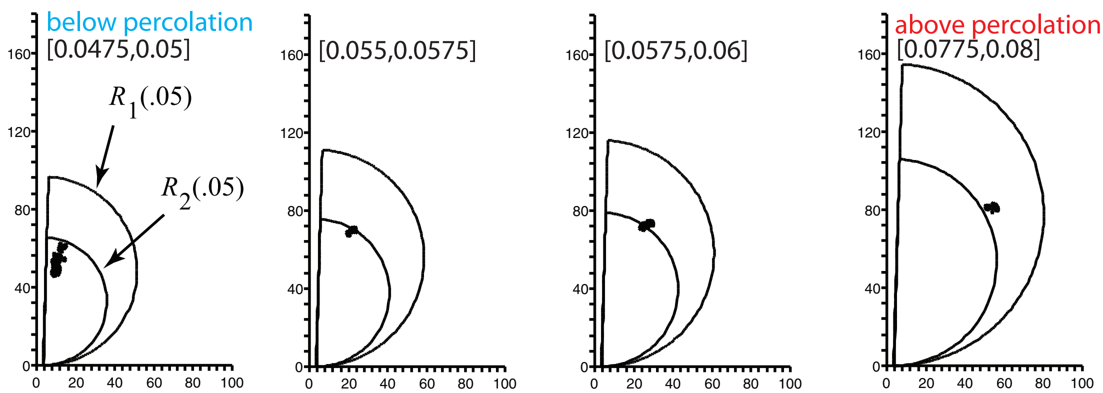


Figure 21. Comparison of ice tank data on the complex permittivity of sea ice at 50 MHz over ranges of ϕ with the elementary (outer) and isotropic (inner) bounds. The data violate, or lie outside, the isotropic bounds as brine volume increases through the percolation threshold and an anisotropic, connected brine structure is formed.

of μ lies in an interval $[s_m, s_M]$, $0 < s_m < s_M < 1$. The more separated the inclusions, the smaller the support interval and the tighter the bounds. Comparison of higher-frequency 4.75 GHz data (red dots) on the complex permittivity ϵ^* of sea ice for $\epsilon_1 = 51 + i45$ and $\epsilon_2 = 3.1$, with a series of bounds [7], is shown in Figure 20(b). The outer dotted curves are complex arithmetic and harmonic mean bounds, and the inner dotted curves are Hashin-Shtrikman bounds. The solid bounds are matrix-particle versions. The parameter q measures inclusion separation and can be bounded using permittivity data.

In Figure 21 we display a series of comparisons of the 50 MHz data in the porosity intervals shown, with corresponding elementary bounds $R_1(\phi_{max})$ and the isotropic bounds $R_2(\phi_{max})$, where ϕ_{max} is the largest porosity in each interval. The development of anisotropy in the dielectric measurements, or violation of the isotropic bounds as brine volume increases through 5%, is consistent with an increasingly connected and vertically elongated brine microstructure. This behavior in the data would likely be accentuated if the electric field were oriented predominantly in the vertical direction.

Inverse bounds for structural parameters

The objective of inverse bounds is to use data about the electromagnetic response of sea ice to recover information about its structural parameters, in particular its brine volume fraction ϕ, or porosity. As mentioned above, the porosity ϕ can be written as a function of the bulk ice salinity S and temperature T, as derived by semi-empirical approaches from the sea ice phase relations. Given salinity information, for example, inverting for porosity yields temperature information. The inverse method [6] we use here yields intervals of uncertainty for the brine volume fraction ϕ. Given an observed value of the complex permittivity

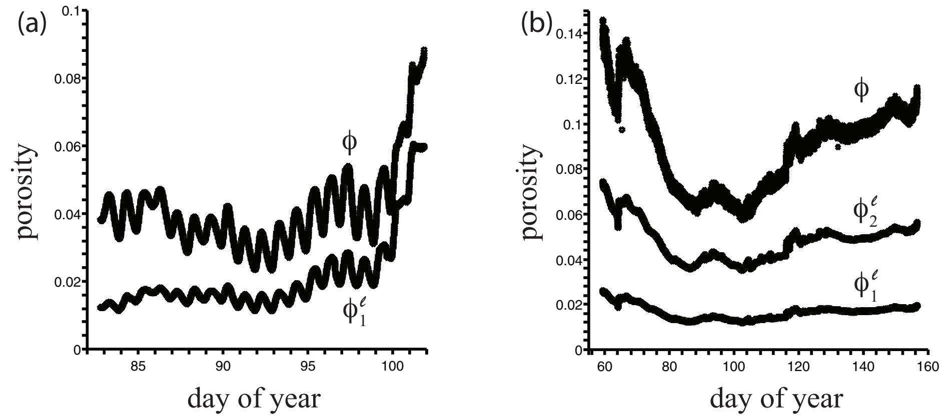


Figure 22. Comparison of the actual brine porosity ϕ with time to the inverse lower bounds for the ice tank data in (a) and for the Barrow data in (b).

ϵ^*, ϕ is increased until the value of ϵ^* touches one boundary of the region R_1 described in the previous section and is then decreased until the value touches the other boundary. This procedure gives a range of values $\phi_1^\ell \le \phi \le \phi_1^u$, with

$$\text{(20)} \quad \phi_1^\ell = |f|^2 \frac{Im(\bar{s})}{Im(f)}, \qquad \phi_1^u = 1 - \frac{|g|^2 Im(\bar{t})}{Im(g)},$$

where f is the known value of $F(s)$ and g is the known value of $G(t) = 1 - \epsilon^*/\epsilon_1$ with $t = 1 - s$. If the material is further assumed to be statistically isotropic, second-order inverse bounds $\phi_2^\ell \le \phi \le \phi_2^u$ can be obtained. We mention recent progress by E. Cherkaev and colleagues on developing methods for reconstructing μ from bulk property measurements. Such methods will likely be useful in obtaining more detailed information about sea ice microstructure, as they have been for bone microstructure and in monitoring osteoporosis [1].

In Figure 22(a) the actual brine porosity $\phi(t)$ as a function of time t for the ice tank experiment is compared with the lower inverse bound ϕ_1^ℓ. Due to the high contrast in the materials, the corresponding upper bound is quite large and currently provides little information. We see that the actual data lie above the lower bound and that the variations in the reconstructed lower bound follow the variations in the actual porosity. In Figure 22(b) the actual brine porosity for the Barrow data is compared with the lower inverse bounds ϕ_1^ℓ and ϕ_2^ℓ. The inverse bounds are obeyed, and the variations in $\phi(t)$ are reflected in the reconstructions.

Measurements of Fluid and Electrical Transport in Antarctic Sea Ice

During September and October of 2007, Adam Gully (University of Utah math student) and I participated in a six-week voyage into the Antarctic sea ice pack. The Sea Ice Physics and Ecosystem eXperiment (SIPEX), conducted off the coast of East Antarctica, was a major component of Australia's project contributions to the worldwide International Polar Year activities of 2007–2008. An extensive science program conducted by forty-five scientists from ten nations was led jointly by the Antarctic Climate & Ecosystems Cooperative Research Centre (ACE CRC) and the Australian Antarctic Division. Valuable information was obtained that will help develop more accurate global climate models and will also help us understand how polar ecosystems might respond to future climate change.

Researchers employed a suite of cutting-edge technologies, including airborne laser altimetry and radar, and an under-ice remotely operated vehicle (ROV). For the first time ever in Antarctica, helicopter flights covered thousands of kilometers tracking over the sea ice, gathering large-scale information about the height of the snow and ice surfaces above sea level. The helicopter data were complemented by information gathered from ice coring and other work directly on the ice. The combined information will help validate satellite altimetry data, which will eventually be used to monitor changes in sea ice thickness around Antarctica.

Biologists combined classical ice coring techniques with the ROV to study the underside of the sea ice to map the distribution of sea ice algae. The ROV observations showed high concentrations of Antarctic krill living on the underside of the sea ice as well as in cracks between ice floes. Other scientists worked on ice algae physiology and sea ice biogeochemistry to better understand processes in the Antarctic sea ice zone during the transition from winter to summer. Oceanographers studied the water mass properties and currents beneath the sea ice and found that, contrary to what was expected, the patterns of sea ice drift appear to be affected more by ocean currents than by wind.

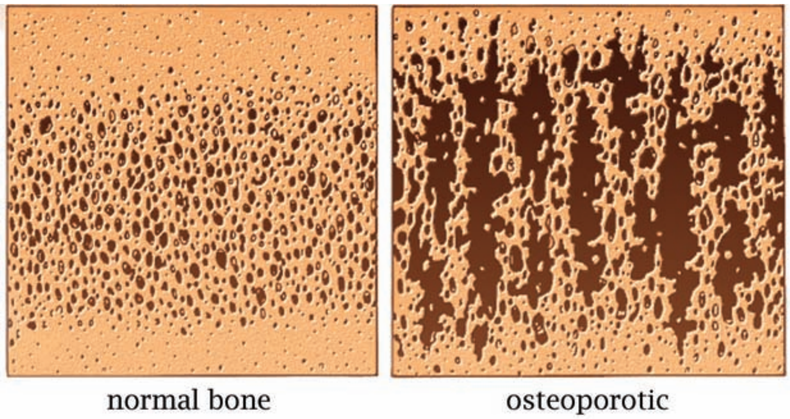


Figure 23. Bone porosity can be estimated from torsional modulus data using reconstruction of the spectral measure, similar to estimating brine porosity in sea ice from permittivity data. Such work will help in monitoring osteoporosis [1]**. The math doesn't care if it's sea ice or bone!**

Gully and I conducted field studies of the fluid and electrical transport properties of first-year sea ice in conjunction with our mathematical modeling work. We made the first fluid permeability measurements of Antarctic pack ice and conducted fluid tracer experiments on extracted blocks of sea ice. In close connection with these fluid measurements, we used surface impedance tomography via a Wenner electrode array to reconstruct the conductivity profile of the ice. We also modified our Wenner configuration to directly measure the vertical conductivity component of extracted cores—a first for sea ice. These measurements will allow the fluid permeability data to be directly related to sea ice electrical properties. Our electrical work, joint with voyage leader Tony Worby, was done in the same exact locations and configurations as EM induction thickness measurements conducted by Japanese sea ice scientist Kazu Tateyama for later correlation. We are still in the process of analyzing the data and preparing it for publication, so our results will be presented elsewhere.

Conclusions and Future Challenges in Sea Ice Modeling

Numerous challenges remain in the modeling of sea ice for climate applications. Many important processes are only crudely represented, such as snow-ice formation, albedo evolution, aspects of the snow pack overlying the ice cover, and the parameterization of ice strength and dynamic-driven ice ridging and rafting. Figure 25 shows dire model predictions for the decline of the summer Arctic sea ice pack, and much work remains to refine these predictions based on our increasing knowledge of important processes in sea ice and how the pack interacts with the ocean and atmosphere.

Our work on fluid transport is central to a broad range of problems in the geophysics and biology of sea ice, yet fundamental studies have been lacking. We have developed a unified theory of the fluid permeability of sea ice and its dependence on microstructural evolution with temperature. The theory ties together different approaches to estimating effective composite behavior and closely captures laboratory and Arctic field data. We have also made progress in CT imaging of sea-ice microstructure, quantifying the thermal evolution of pore connectivity and morphology at the (sub)granular scale, which in turn has informed the theoretical advances. Our work on sea ice electrical properties helps provide a theoretical basis for new techniques in remote sensing, in particular those which help to monitor the impact of climate change.

Our investigations demonstrate how to link information about the microscale structure to macroscopic transport properties, which in turn constrain global-scale processes of climatological and biological importance. The results yield simple parameterizations of fluid transport through sea ice in terms of porosity ϕ, and therefore the state variables T and S, and can help to improve the representation of sea ice in climate, biogeochemical, and geophysical models. Future studies will have to examine how these findings impact the simulation of heat and fluid flow through sea ice in large-scale models, which currently oversimplify such processes.

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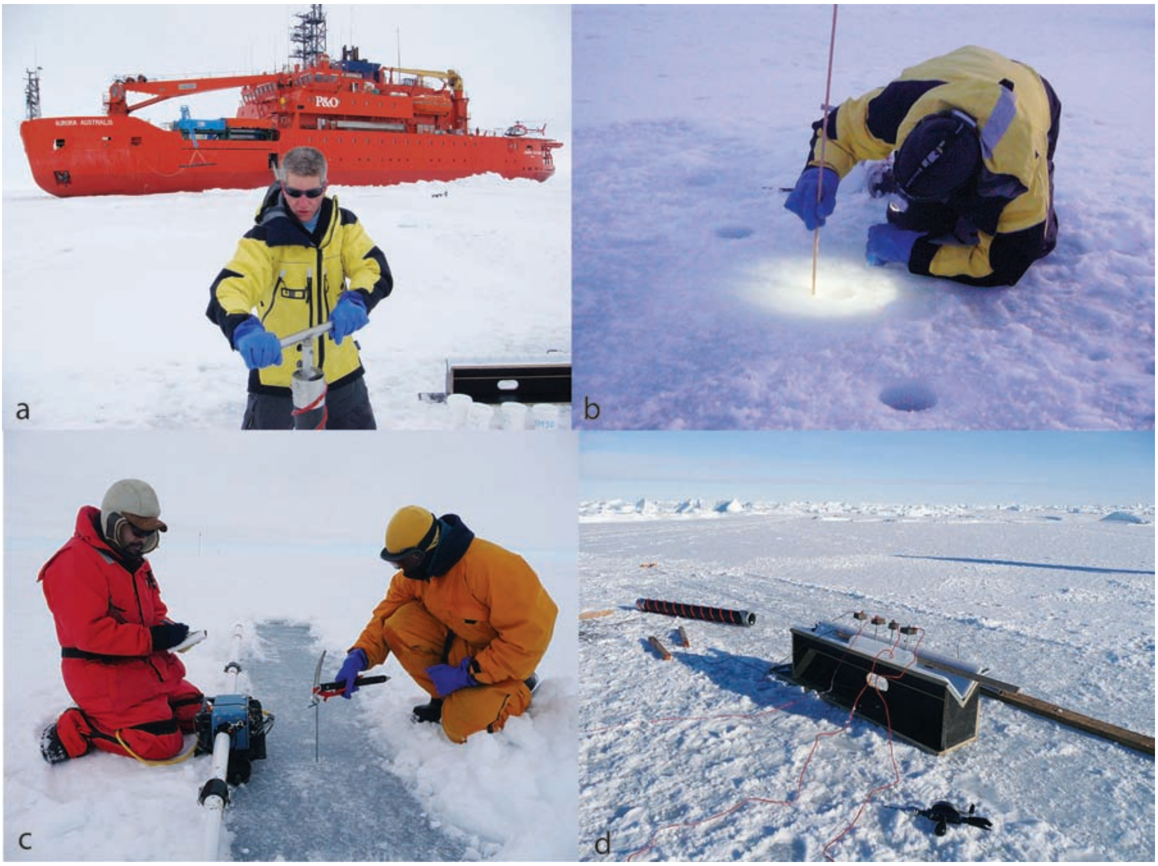

Figure 24. The author taking a sea ice core in (a) and making a permeability measurement in (b) by measuring the rate at which sea water fills in the partial hole through the ice. (c) Adam Gully (right) sets up a Wenner array, while Kazu Tateyama takes thickness measurements with an EM induction instrument. (d) The Wenner array was adapted to take direct measurements of the vertical conductivity of extracted sea ice cores.

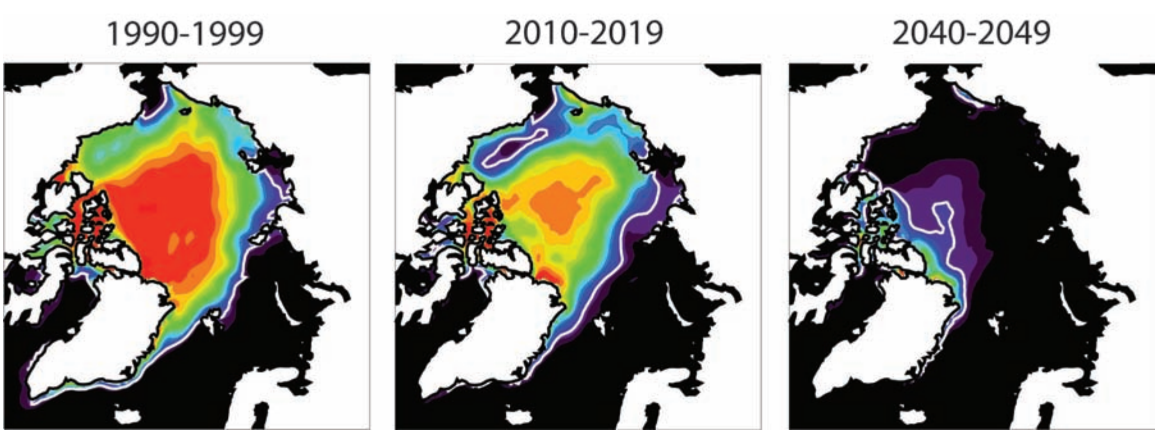

Figure 25. Coupled climate model simulations from the Community Climate System Model, version 3, showing September sea ice concentration averaged for three different time periods: (left to right) 1990–1999, 2010–2019, and 2040–2049. The white line indicates the simulated ice extent, defined as the 15% ice concentration contour. This and other similar climate models suggest significant sea ice loss will occur over the next fifty years, and near ice-free September conditions may be reached in the Arctic by the mid-twenty-first century. Some scientists predict such a decline over an even shorter period of time.

Figure 26. Antarctica is not only of great scientific interest, but is a place of unique beauty and exciting adventures.

Elizabeth Hunke, and Cecilia Bitz for helpful discussions and input and for providing some of the figures shown here. We are grateful for the support provided by the Division of Atmospheric Sciences, the Division of Mathematical Sciences, and the VIGRE and REU Programs at the U.S. National Science Foundation (NSF), as well as the Australian Antarctic Division and the Antarctic Climate & Ecosystems Cooperative Research Centre.

To find out more: If you are interested in finding out more about math and sea ice, you can visit the author's webpage, `http://www.math.utah.edu/~golden`. There you will also be able to link to a video on the SIPEX expedition to Antarctica produced by the author, located under `http://www.math.utah.edu/~golden/media.html`. A list of additional references for this article is located under `http://www.math.utah.edu/~golden/publications.html`. If you are interested in learning about how math is being used to address the big questions in climate science, including more about sea ice, you are encouraged to visit the Math Awareness Month April 2009 website at `http://www.mathaware.org` on "Mathematics and Climate".

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Mathematics and the Internet: A Source of Enormous Confusion and Great Potential

Walter Willinger, David Alderson, and John C. Doyle

For many mathematicians and physicists, the Internet has become a popular real-world domain for the application and/or development of new theories related to the organization and behavior of large-scale, complex, and dynamic systems. In some cases, the Internet has served both as inspiration and justification for the popularization of new models and mathematics within the scientific enterprise. For example, scale-free network models of the preferential attachment type [8] have been claimed to describe the Internet's connectivity structure, resulting in surprisingly general and strong claims about the network's resilience to random failures of its components and its vulnerability to targeted attacks against its infrastructure [2]. These models have, as their trademark, power-law type node degree distributions that drastically distinguish them from the classical Erdős-Rényi type random graph models [13]. These "scale-free" network models have attracted significant attention within the scientific community and have been partly responsible for launching and fueling the new field of *network science* [42, 4].

To date, the main role that mathematics has played in network science has been to put the physicists' largely empirical findings on solid grounds by providing rigorous proofs of some of their more highly publicized claims [14, 15, 16, 23, 11, 25]. The alleged scale-free nature of the Internet's topology has also led to mathematically rigorous results about the spread of viruses over scale-free graphs of the preferential attachment type, again with strong and unsettling implications such as a zero epidemic threshold [11, 25]. The relevance of the latter is that in stark contrast to more homogeneous graphs, on scale-free networks of the preferential attachment type, even viruses with small propagation rates have a chance to cause an epidemic, which is about as bad as it can get from an Internet security perspective. More recently, the realization that large-scale, real-world networks such as the Internet evolve over time has motivated the mathematically challenging problem of developing a theory of graph sequences and graph limits [17, 19, 20]. The underlying idea is that properly defined graph limits can be expected to represent viable models for some of the enormous dynamic graph structures that arise in real-world applications and seem too unwieldy to be described via more direct or explicit approaches.

The generality of these new network models and their impressive predictive ability notwithstanding, surprisingly little attention has been paid in the mathematics and physics communities to parallel developments in the Internet research arena, where the various non-rigorous and rigorous results derived from applying the scale-free modeling paradigm to the Internet have been scrutinized using available measurements or readily available domain knowledge. A driving force behind these Internet-centric validation efforts has been the realization that—because of its engineered architecture, a thorough understanding

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of its component technologies, and the availability of extensive (but not necessarily very accurate) measurement capabilities—the Internet provides a unique setting in which most claims about its properties, structure, and functionality can be unambiguously resolved, though perhaps not without substantial efforts. In turn, models or theories that may appeal to a more mathematically inclined researcher because of their simplicity or generality, but result in incorrect, misleading, or wrong claims about the Internet, can and will be identified and labeled accordingly, but it may take considerable time (and efforts) to expose their specious nature.

In this article, we take a closer look at what measurement-based Internet research in general, and Internet-specific validation efforts in particular, have to say about the popular scale-free modeling paradigm and the flurry of mathematical developments it has inspired. In particular, we illustrate why and how in the case of the Internet, scale-free network models of the preferential attachment type have become a classic lesson in how errors of various forms occur and can add up to produce results and claims that create excitement among non-networking researchers, but quickly collapse under scrutiny with real data or when examined by domain experts. These opposite reactions have naturally been a source of great confusion, but the main conclusion is neither controversial nor should it come as a big surprise: the scale-free modeling paradigm is largely inconsistent with the engineered nature of the Internet and the design constraints imposed by existing technology, prevailing economic conditions, and practical considerations concerning network operations, control, and management.

To this end, we document the main sources of errors regarding the application of the scale-free modeling approach to the Internet and then present an alternative approach that represents a drastic departure from traditional network modeling. In effect, we motivate here the development of a novel modeling approach for Internet-like systems that (1) respects the highly designed nature of the network; (2) reflects the engineering intuition that exists about a great many of its parts; (3) is fully consistent with a wide range of measurements; and (4) outlines a mathematical agenda that is more challenging, more relevant, and ultimately more rewarding than the type of mathematics motivated by an alluring but largely misguided approach to Internet modeling based on scale-free graphs of the preferential attachment type. In this sense, this article demonstrates the great potential that the Internet has for the development of new, creative, and relevant mathematical theories, but it is also a reminder of a telling comment attributed to S. Ulam [12] (slightly paraphrased, though), who said *"Ask not what mathematics can do for [the Internet]; ask what [the Internet] can do for mathematics."*

The Scale-free Internet Myth

The story recounted below of the scale-free nature of the Internet seems convincing, sound, and almost too good to be true. Unfortunately, it turned out to be a complete myth, but has remained a constant source of enormous confusion within the scientific community.

Somewhat ironically, the story starts with a highly-cited paper in the Internet research arena by Faloutsos et al. [27]. Relying on available measurements and taking them at face value, the paper was the first to claim that the (inferred) node degree distributions of the Internet's router-level topology as well as AS-level topology are power-law distributions with estimated α-parameters between 1 and 2. To clarify, by *router-level topology*, we mean the Internet's physical connectivity structure, where nodes are physical devices such as routers or switches, and links are the connections between them. These devices are further organized into networks known as *Autonomous Systems (ASes)*, where each AS is under the administrative control of a single organization such as an Internet Service Provider (ISP), a company, or an educational institution. The relationships among ASes, when organized as a graph, produce what is known as the Internet's *AS-level topology*. Note that a link between two nodes in the AS-level topology represents a type of business relationship (either peering or customer-provider). Also, in contrast to the router-level topology that is inherently physical, the AS topology is a logical construct that reflects the Internet's administrative boundaries and existing economic relationships.

These reported power-law findings for the Internet were quickly picked up by Barabási et al., who were already studying the World Wide Web (WWW) and then added the Internet to their growing list of real-world network structures with an apparently striking common characteristic; that is, their vertex connectivities (described mathematically in terms of node *degrees*) *"follow a scale-free power-law distribution"* [8, 3]. This property is in stark contrast to the Poissonian nature of the node degrees resulting from the traditional Erdős-Rényi random graphs [13] that have been the primary focus of mathematical graph theory for the last 50 years. Naturally, it has fueled the development of new graph models that seek to capture and reproduce this ubiquitously reported power-law relationship, thereby arguing in favor of these models as more relevant for representing real-world network structures than the classical random graph models. In fact, much of the initial excitement in the nascent field of network science can be attributed to an early and appealingly simple class of network models that was proposed by Barabási and Albert [8] and turned out to have surprisingly strong predictive capabilities.

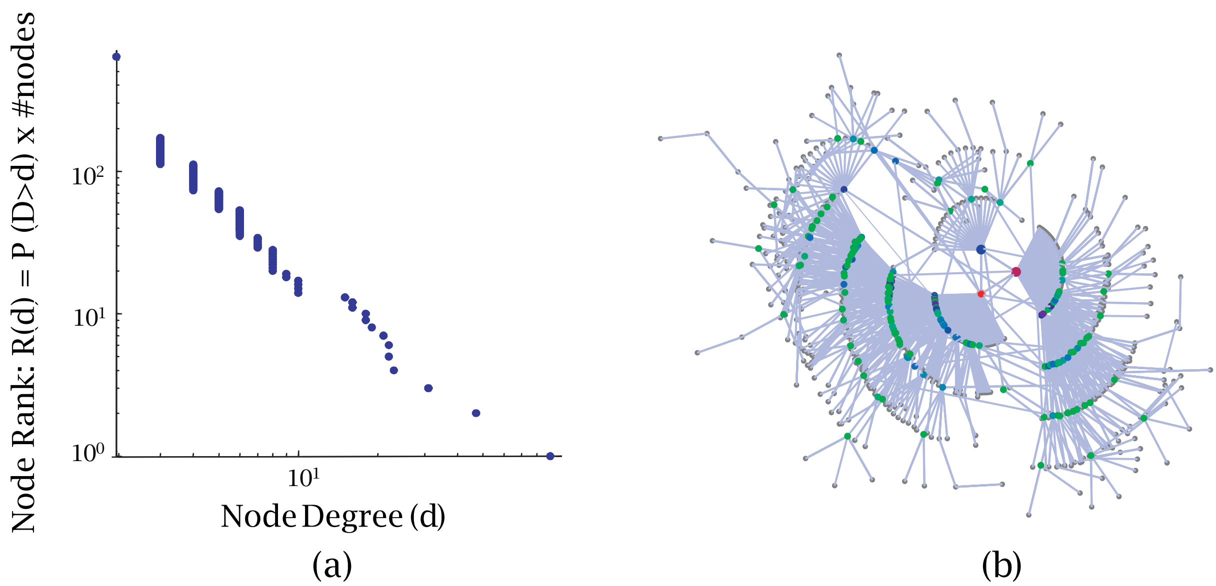


Figure 1. Scale-free networks of the preferential attachment type. (b) A toy example of a scale-free network of the preferential attachment type generated to match a power-law type degree distribution (a).

In short, Barabási and Albert [8] described a network growth model in which newly added vertices connect preferentially to nodes in the existing graph that are already well connected. This *preferential attachment* mechanism had been studied over the previous 75 years by Yule [54], Luria and Delbrück [38], and Simon [49], but it was its rediscovery and application to networks by Barabási and Albert that recently popularized it. Although many variants of the the basic Barabási-Albert construction have been proposed and studied, we will focus in the following on the original version described in [8], mainly because of its simplicity and because it already captures the most important properties of this new class of networks, commonly referred to as *scale-free networks.* The term scale-free derives from the simple observation that power-law node degree distributions are free of scale—most nodes have small degree, a few nodes have very high degree, with the result that the average node degree is essentially non-informative. A detailed discussion of the deeper meanings often associated with scale-free networks is available in [34]. To avoid confusion and to emphasize the fact that preferential attachment is just one of many other mechanisms that is capable of generating scale-free graphs (i.e., graphs with power-law node degree distributions), we will refer here to the network models proposed in [8] as *scale-free networks of the preferential attachment (PA) type* and show an illustrative toy example with associated node degree distribution in Figure 1.

The excitement generated by this new class of models is mainly due to the fact that, despite being generic and largely oblivious to system-specific details, they share some key properties that give them remarkable predictive power. These properties were originally reported in [2], put on mathematically solid footing by Bollobás and Riordan in [14, 15, 16], and explain the key aspects of the structure and behavior of these networks. For one, a hallmark of their structure is the presence of "hubs"; that is, centrally located nodes with high connectivity. Moreover, the presence of these hubs makes these networks highly vulnerable to attacks that target the hub nodes. At the same time, these networks are extremely resilient to attacks that knock out nodes at random, since a randomly chosen node is likely to be one of the low-degree nodes that constitute the bulk of the nodes in the power-law node degree distribution, and the removal of such a node has typically minimal impact on the network's overall connectivity or performance.

This property—simultaneous resilience to random attacks but high vulnerability to targeted worst-case attacks (i.e., attacks against the hub nodes)—featured prominently in the original application of scale-free networks of the PA type to the Internet [2]. The underlying argument follows a very traditional and widely-used modeling approach. First, as reported in [27], the Internet has node degrees that follow a power-law distribution or are scale-free. Second, scale-free networks of the PA type are claimed to be valid models of the Internet because they are capable of reproducing the observed scale-free node degree distributions. Lastly, when abstracted to a scale-free model of the

PA type, the Internet automatically inherits all the emergent features of the latter, most notably the presence of hub nodes that are critical to overall network connectivity and performance and are largely responsible for the network's failure tolerance and attack vulnerability. In this context, the latter property has become known as the "Achilles' heel of the Internet" and has been highly publicized as a success story of network science—the discovery of a fundamental weakness of the Internet that went apparently unnoticed by the engineers and researchers who have designed, deployed, and studied this large-scale, critical complex system.

The general appeal of such surprisingly strong statements is understandable, especially given the simplicity of scale-free networks of the PA type and the fact that, as predictive models, they do not depend on the particulars of the system at hand, i.e., underlying technology, economics, or engineering. As such, they have become the embodiment of a highly popular statistical physics-based approach to complex networks that aims primarily at discovering properties that are universal across a range of very diverse networks. The potential danger of this approach is that the considered abstractions represent simplistic toy models that are too generic to reflect features that are most important to the experts dealing with these individual systems (e.g., critical functionality).

Deconstructing the Scale-free Myth

Given that the scale-free story of the Internet is grounded in real measurement data and based on a widely-accepted modeling approach, why is it so far from the truth? To explain and trace the various sources of errors, we ask the basic question; i.e., *"Do the available measurements, their analysis, and their modeling efforts support the claims that are made in* [2]*?"* To arrive at a clear and simple answer to this question, we address below the issues of data hygiene and data usage, data analysis, and mathematical modeling (including model selection and validation).

Know your data

A very general but largely ignored fact about Internet-related measurements is that what we can measure in an Internet-like environment is typically not the same as what we really want to measure (or what we think we actually measure). This is mainly because as a decentralized and distributed system, the Internet lacks a central authority and does not support third-party measurements. As a result, measurement efforts across multiple ASes become nontrivial and often rely on engineering hacks that typically do not yield the originally desired data but some substitute data. Moreover, using the latter at face value (i.e., as if they were the data we originally wanted) and deriving from them results

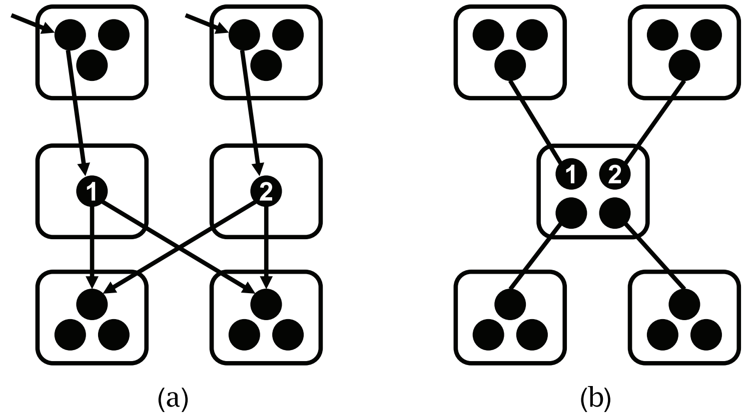


Figure 2. The IP alias resolution problem. Paraphrasing Fig. 4 of [50]**,** `traceroute` **does not list routers (boxes) along paths but IP addresses of input interfaces (circles), and alias resolution refers to the correct mapping of interfaces to routers to reveal the actual topology. In the case where interfaces 1 and 2 are aliases, (b) depicts the actual topology while (a) yields an "inflated" topology with more routers and links than the real one.**

that we can trust generally involves a leap of faith, especially in the absence of convincing arguments or evidence that would support an "as-is" use of the data.

Internet-specific connectivity measurements provide a telling example. To illustrate, consider the data set that was used in [27] to derive the reported power-law claim for the (inferred) node degrees of the Internet's router-level topology.[1] That dataset was originally collected by Pansiot and Grad [44] for the explicitly stated purpose *"to get some experimental data on the shape of multicast trees one can actually obtain in [the real] Internet …"* [44]. The tool of choice was `traceroute`, and the idea was to run `traceroute` between a number of different host computers dispersed across the Internet and glue together the resulting Internet routes to glean the shape of actual multicast trees. In this case, the engineering hack consisted of relying on `traceroute`, a tool that was never intended to be used for the stated purpose, and a substantial leap of faith was required to use Pansiot and Grad's data set beyond its original purpose and rely on it to infer the Internet's router-level topology [27].

For one, contrary to popular belief, running `traceroute` between two host computers does *not* generate the list of compliant (i.e., Internet Protocol

[1]*While the arguments and reasons differ for the data sets used in* [27] *to derive the power-law claim for the Internet's AS-level topology, the bottom line is the same—the available measurements are not of sufficient quality for the purpose for which they are used in* [27] *(see for example* [31]*).*

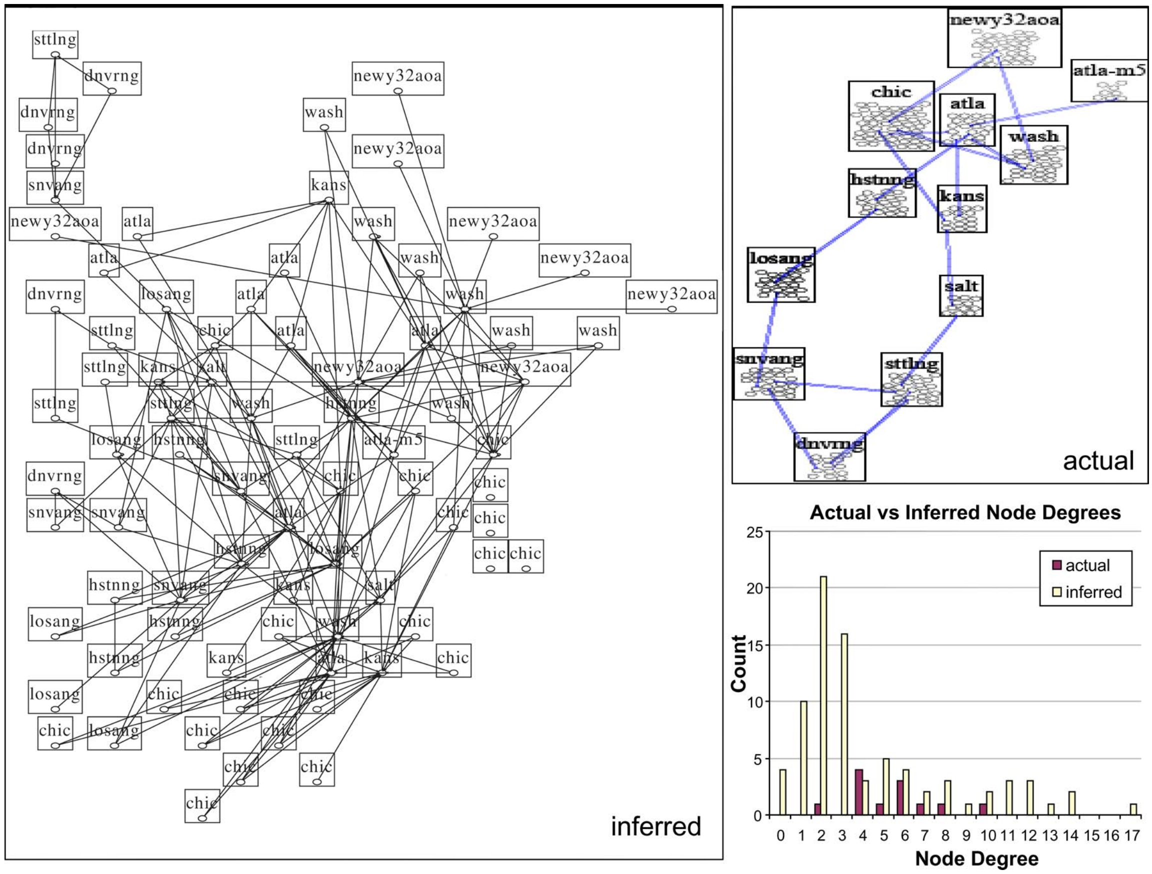
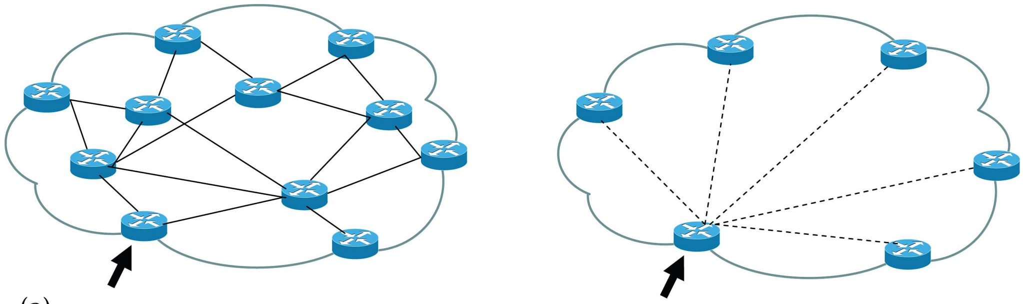


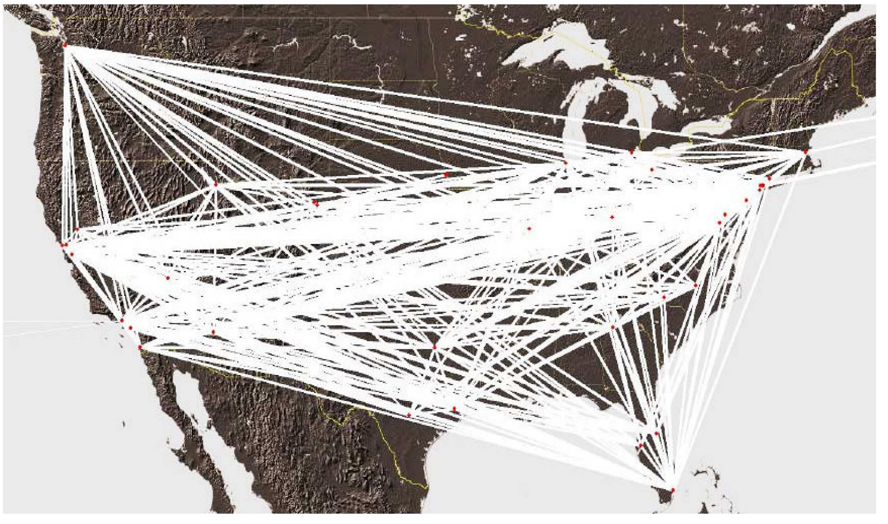
Figure 3. The IP alias resolution problem in practice. This is re-produced from [48] and shows a comparison between the Abilene/Internet2 topology inferred by Rocketfuel (left) and the actual topology (top right). Rectangles represent routers with interior ovals denoting interfaces. The histograms of the corresponding node degrees are shown in the bottom right plot. © 2008 ACM, Inc. Included here by permission.

(IP)-speaking) routers encountered en route from the source to the destination. Instead, since IP routers have multiple interfaces, each with its own IP address, what `traceroute` really generates is the list of (input interface) IP addresses, and a very common property of `traceroute`-derived routes is that one and the same router can appear on different routes with different IP addresses. Unfortunately, faithfully mapping interface IP addresses to routers is a difficult open problem known as the *IP alias resolution problem* [51, 28], and despite continued research efforts (e.g., [48, 9]), it has remained a source of significant errors. While the generic problem is illustrated in Figure 2, its impact on inferring the (known) router-level topology of an actual network (i.e., Abilene/Internet2) is highlighted in Figure 3—the inability to solve the alias resolution problem renders in this case the inferred topology irrelevant and produces statistics (e.g., node degree distribution) that have little in common with their actual counterparts.

Another commonly ignored problem is that `traceroute`, being strictly limited to IP or layer-3, is incapable of tracing through opaque layer-2 clouds that feature circuit technologies such as *Asynchronous Transfer Mode (ATM)* or *Multiprotocol Label Switching (MPLS)*. These technologies have the explicit and intended purpose of hiding the network's physical infrastructure from IP, so from the perspective of `traceroute`, a network that runs these technologies will appear to provide direct connectivity between routers that are separated by local, regional, national, or even global physical network infrastructures. The result is that when `traceroute` encounters one of these opaque layer-2 clouds, it falsely "discovers" a high-degree node that is really a logical entity—a network potentially spanning many hosts or great distances—rather than a physical node of the Internet's router-level topology. Thus, reports of high-degree hubs in the core of the router-level Internet, which defy common engineering sense, can often be easily identified as simple artifacts of



(a)



(b)

Figure 4. How traceroute detects fictitious high-degree nodes in the network core. (a) The actual connectivity of an opaque layer-2 cloud, i.e., a router-level network running a technology such as ATM or MPLS (left) and the connectivity inferred by traceroute probes entering the network at the marked router (right). (b) The Rocketfuel-inferred backbone topology of AS3356 (Level3), a Tier-1 Internet service provider and leader in the deployment of MPLS (reproduced from [50]**) © 2002 ACM, Inc. Included here by permission.**

an imperfect measurement tool. While Figure 4(a) illustrates the generic nature of this problem, Figure 4(b) illuminates its impact in the case of an actual network (i.e., AS3356 in 2002), where the inferred topology with its highly connected nodes says nothing about the actual physical infrastructure of this network but is a direct consequence of `traceroute`'s inability to infer the topology of an MPLS-enabled network.

We also note that from a network engineering perspective, there are technological and economic reasons for why high-degree nodes in the core of the router-level Internet are nonsensical. Since a router is fundamentally limited in terms of the number of packets it can process in any time interval, there is an inherent tradeoff in router configuration: it can support either a few high-throughput connections or many low-throughput connections. Thus, for any given router technology, a high-connectivity router in the core will either have poor performance due to its slow connections or be prohibitively expensive relative to other options. Conversely, if one deploys high-degree devices at the router-level, they are necessarily located at the edge of the network where the technology exists to multiplex a large number of relatively low-bandwidth links. Unfortunately, neither the original `traceroute`-based study of Pansiot and Grad nor any of the larger-scale versions that were subsequently performed by various network research groups have the ability to detect those actual high-degree nodes. The simple reason is that these traditional `traceroute` studies lack access to a sufficient number of participating host computers in any local end-system to reveal their high connectivity. Thus, the irony of `traceroute` is that the high-degree nodes it detects in the network core are necessarily fictitious and represent entire opaque layer-2 clouds, and if there are actual high-degree nodes in the network, existing technology relegates them to the edge of the network where no generic `traceroute`-based measurement experiment will ever detect them.

Lastly, the nature of large-scale traceroute experiments also makes them susceptible to a type of measurement bias in which some points of the network are oversampled, while others are undersampled. Ironically, although this failure of traceroute experiments has received the most attention in the theoretical computer science and applied mathematics communities [32, 1] (most likely, because this failure is the most amenable to mathematical treatment), it is the least significant from a topology modeling perspective.

In view of these key limitations of traceroute, it should be obvious that starting with the Pansiot and Grad data set, traceroute-based measurements cannot be taken at face value and are of no or little use for inferring the Internet's router-level topology. In addition, the arguments provided above show why domain knowledge in the form of such traceroute-specific "details" like IP aliases or layer-2 technology matters when dealing with issues related to data hygiene and why ignoring those details prevents us from deriving results from such data that we can trust. Ironically, Pansiot and Grad [44] detailed many of the above-mentioned limitations and shortcomings of their measurements. Unfortunately, [27] failed to revive these issues or recognize their relevance. Even worse, the majority of subsequent papers in this area typically cite only [27] and no longer [44].

Know your statistic

The inherent inability of traceroute to reveal unambiguously the actual node degree of any router (i.e., the number of different interfaces) due to the IP alias resolution problem, combined with the fundamental difficulties of the tool to correctly infer even the mere absence or presence of high-degree nodes (let alone their actual degrees) makes it impossible to describe accurately statistical entities such as node degree distributions. Thus, it should come as no surprise that taking traceroute-derived data sets "as is" and then making them the basis for any fitting of a particular parameterized distribution (e.g., power-law distribution with index α as in [27]) is statistical "overkill", irrespective of how sophisticated a fitting or corresponding parameter estimation technique has been used. Given the data's limitations, even rough rules-of-thumb such as a Pareto-type 80/20 rule (i.e., 80% of the effects come from 20% of the causes) cannot be justified with any reasonable degree of statistical confidence.

It is in this sense that the claims made in [27] and subsequent papers that have relied on this data set are the results of a data analysis that is not commensurate with the quality of the available data. It is also a reminder that there are important differences between analyzing high-quality and low-quality data sets, and that approaching the latter the same way as the former is not only bad statistics but also bad science, and doing so bolsters the popular notion that "there are lies, damned lies, and statistics." Unfortunately, the work required to arrive at this conclusion is hardly glamorous or newsworthy, especially when compared to the overall excitement generated by an apparent straight-line behavior in the easily obtainable log-log plots of degree vs. frequency. Even if the available measurements were amenable to such an analysis, these commonly-used and widely-accepted log-log plots are not only highly non-informative, but have a tendency to obscure power-law relationships when they are genuine and fabricate them when they are absent (see for example [34]). In the case of the data set at hand, the latter observation is compounded by the unreliable nature of the traceroute-derived node degree values and shows why the power-law claims for the vertex connectivities of the Internet's router-level topology reported in [27] cannot be supported by the available measurements.

When modeling is more than data-fitting

We have shown that the data set used in [27] turns out to be thoroughly inadequate for deriving and modeling power-law properties for the distribution of node degrees encountered in the Internet's router-level topology. As a result, the sole argument put forward in [2] for the validity of the scale-free model of the PA type for the Internet is no longer applicable, and this in turn reveals the specious nature of both the proposed model and the sensational features the Internet supposedly inherits from the model.

Even if the node degree distribution were a solid and reliable statistic, who is to say that matching it (or any other commonly considered statistics of the data) argues for the validity of a proposed model? In the case of scale-free models of the PA type, most "validation" follows from the ability of a model to replicate an observed degree distribution or sequence. However, it is well known in the mathematics literature that there can be many graph realizations for any particular degree sequence [47, 29, 35, 10] and there are often significant structural differences between graphs having the same degree sequence [6]. Thus, two models that match the data equally well with respect to some statistics can still be radically different in terms of other properties, their structures, or their functionality. A clear sign of the rather precarious current state of network-related modeling is that the same underlying data set can give rise to very different, but apparently equally "good" models, which in turn can give rise to completely opposite scientific claims and theories concerning one and the same observed phenomenon. Clearly, modeling and especially model validation has to mean more than being able to match the data if we want to be confident that the results that we drive from our models are valid.

At this point, it is appropriate to recall a quote attributed to G. E. P. Box, who observed that *"All models are wrong, but some models are useful."* Without being more specific about which models are deemed useful and why, this comment is of little practical value. A more constructive piece of advice that is more directly aligned with what we envision modeling should mean in the presence of imprecise data is from B. B. Mandelbrot [39], who observed *"If exactitude is elusive, it is better to be approximately right than certifiably wrong."*

For complex network systems whose measured features suffer from the types of fundamental ambiguities, omissions, and/or systematic errors outlined above, we argue that network modeling must move beyond efforts that merely match particular statistics of the data. Such efforts are little more than exercises in data-fitting and are particularly ill-advised whenever the features of interest cannot be inferred with any reasonable statistical confidence from the currently available measurements. For systems such as the router-level Internet, we believe this to be a more scientifically grounded and constructive modeling approach. For one, given the known deficiencies in the available data sets, matching a particular statistic of the data may be precisely the wrong approach, unless that statistic has been found to be largely robust with respect to these deficiencies. Moreover, it eliminates the arbitrariness associated with determining which statistics of the data to focus on. Indeed, it treats all statistics equally. A model that is "approximately right" can be expected to implicitly match most statistics of the data (at least approximately).

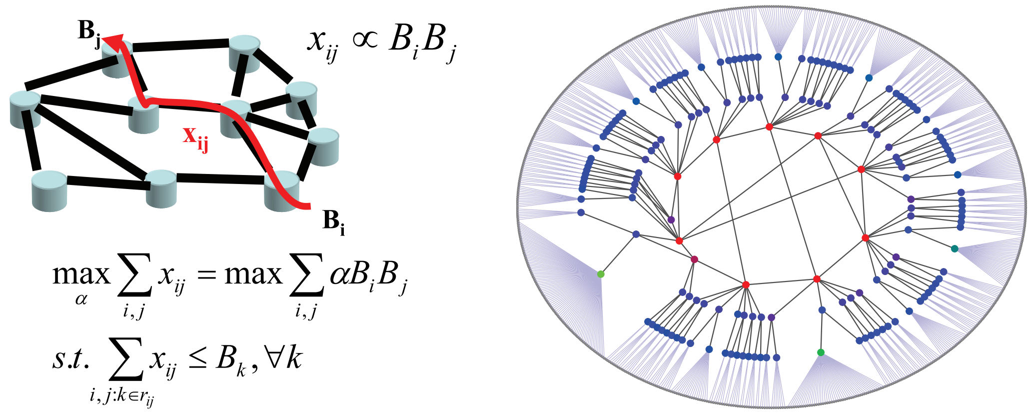
If we wish to increase our confidence in a proposed model, we ought also to ask what new types of measurements are either already available (but have not been used in the present context) or could be collected and used for validation. Here, by "new" we do not mean "same type of measurements as before, just more." What we mean are completely new types of data, with very different semantic content, that have played no role whatsoever in the entire modeling process up to this point. A key benefit of such an approach is that the resulting measurements are used primarily to "close-the-loop", as advocated in [53], and provide a statistically clean separation between the data used for model selection and the data used for model validation—a feature that is alien to most of today's network-related models. However, a key question remains: *What replaces data-fitting as the key ingredient and driver of the model selection and validation process so that the resulting models are approximately right and not certifiably wrong?* The simple answer is: *rely on domain knowledge and exploit the details that matter when dealing with a highly engineered system such as the Internet.* Note that this answer is in stark contrast to the statistical physics-based approach that suggests the development of a system such as the Internet is governed by robust self-organizing phenomena that go beyond the particulars of the individual systems (of interest) [8].

A first-principles approach to internet modeling

If domain knowledge is the key ingredient to build "approximately right" models of the Internet, what exactly is the process that helps us achieve our goal? To illustrate, we consider again the router-level topology of the Internet, or more specifically, the physical infrastructure of a regional, national, or international Internet Service Provider (ISP).

The first key observation is that the way an ISP designs its physical infrastructure is certainly not by a series of (biased) coin tosses that determine whether or not two nodes (i.e., routers) are connected by a physical link, as is the case for the scale-free network models of the PA type. Instead, ISPs design their networks for a purpose; that is, their decisions are driven by objectives and reflect trade-offs between what is feasible and what is desirable. The mathematical modeling language that naturally reflects such a decision-making process is *constrained optimization.* Second, while in general it may be difficult if not impossible to define or capture the precise meaning of an ISP's purpose for designing its network, an objective that expresses a desire to provide connectivity and an ability to carry an expected traffic demand efficiently and effectively, subject to prevailing economic and technological constraints, is unlikely to be far from the "true" purpose. In view of this, we are typically not concerned with a network design that is "optimal" in a strictly mathematical sense and is also likely to be NP-hard, but in a solution that is "*heuristically optimal*" in the sense that it results in "good" performance. That is, we seek a solution that captures by and large what the ISP can afford to build, operate, and manage (i.e., economic considerations), given the hard constraints that technology imposes on the network's physical entities (i.e., routers and links). Such models have been discussed in the context of highly organized/optimized tolerances/tradeoffs (HOT) [18, 26]. Lastly, note that in this approach, randomness enters in a very specific manner, namely in terms of the uncertainty that exists about the "environment" (i.e., the traffic demand that the network is expected to carry), and the heuristically optimal network designs are expected to exhibit *strong robustness properties* with respect to changes in this environment.

Figure 5 shows a toy example of an ISP router-level topology that results from adopting the



(a) (b)

Figure 5. Generating networks using constrained optimization. (a) Engineers view network structure as the solution to a design problem that measures performance in terms of the ability to satisfy traffic demand while adhering to node and arc capacity constraints. (b) A network resulting from heuristically optimized tradeoffs (HOT). This network has very different structural and behavioral properties, even when it has the same number of nodes, links, and degree distribution as the scale free network depicted in Fig. 1.

mathematical modeling language of constrained optimization and choosing a candidate network as a solution of an heuristically optimal network design problem. Despite being a toy example, it is rich enough to illustrate the key features of our engineering-driven approach to network modeling and to contrast it with the popular scale-free network models of the PA type. It's toy nature is mainly due to a number of simplifying assumptions we make that facilitate the problem formulation. For one, by simply equating throughput with revenues, we select as our objective function the maximum throughput that the network can achieve for a given traffic demand and use it as a metric for quantifying the performance of our solutions. Second, considering an arbitrary distribution of end-user traffic demand x_i, we assume a *gravity model* for the unknown traffic demand; that is, assuming shortest-path routing, the demands are given by the traffic matrix X, where for the traffic X_{ij} between routers i and j we have $X_{ij} = \rho x_i x_j$, for some constant ρ. Lastly, we consider only one type of router and its associated technologically feasible region; that is, (router degree, router capacity)-pairs that are achievable with the considered router type (e.g., CISCO 12416 GSR), and implicitly avoid long-haul connections due to their high cost.

The resulting constrained optimization problem can be written in the form

$$\max_{\rho} \quad \sum_{i,j} X_{i,j}$$
$$s.t. \quad RX \leq B$$

where X is the vector obtained by stacking all the demands $X_{ij} = \rho x_i x_j$; R is the routing matrix obtained by using standard shortest path routing and defined by $R_{kl} = 1$ or 0, depending on whether or not demand l passes through router k; and B is the vector consisting of the router degree-bandwidths constraints imposed by the technologically feasible region of the router at hand. While all the simplifying assumptions can easily be relaxed to allow for more realistic objective functions, more heterogeneity in the constraints, or more accurate descriptions of the uncertainty in the environment, Figure 5 illustrates the key characteristics inherent in a heuristically optimal solution of such a problem. First, the cost-effective handling of end user demands avoids long-haul connections (due to their high cost) and is achieved through traffic aggregation starting at the edge of the network via the use of high-degree routers that support the multiplexing of many low-bandwidth connections. Second, this aggregated traffic is then sent toward the "backbone" that consists of the fastest or highest-capacity routers (i.e., having small number of very high-bandwidth connections) and that forms the network's mesh-like core. The result is a network that has a more or less pronounced backbone, which is fed by tree-like access networks, with additional connections at various places to provide a degree of redundancy and robustness to failures.

What about power-law node degree distributions? They are clearly a non-issue in this engineering-based first-principles approach, just as they should be, based on our understanding illustrated earlier that present measurement techniques are incapable of supporting them. Recognizing their irrelevance is clearly the beginning of the end of the scale-free network models of the PA type as far as the Internet is concerned. What about replacing power-laws by the somewhat more plausible assumption of high variability in node degrees? While the answer of the scale-free modeling approach consists of tweaks to the PA mechanism to enforce an exponential cut-off of the power-law node degree distribution at the upper tail, the engineering-based approach demystifies high-variability in node degrees altogether by identifying its root cause in the form of high variability in end-user bandwidth demands (see [33] for details). In view of such a simple physical explanation of the origins of node degree variability in the Internet's router-level topology, Strogatz' question, paraphrasing Shakespeare's Macbeth, "... power-law scaling, full of sound and fury, signifying nothing?" [52] has a resounding affirmative answer.

Great Potential for Mathematics

Given the specious nature of scale-free networks of the PA type for modeling Internet-related connectivity structures, their rigorous mathematical treatment and resulting highly-publicized properties have lost much of their luster, at least as far as Internet matters are concerned. Considering again our example of the router-level Internet, neither the claim of a hub-like core, nor the asserted error tolerance (i.e., robustness to random component failure) and attack vulnerability (i.e., Achilles' heel property), nor the often-cited zero epidemic threshold property hold. In fact, as illustrated with our HOT-based network models, intrinsic and unavoidable tradeoffs between network performance, available technology, and economic constraints necessarily result in network structures that are in all important ways exactly the opposite of what the scale-free models of the PA type assert. In this sense, the HOT toy examples represent a class of network models for the Internet that are not only consistent with various types of measurements and in agreement with engineering intuition, but whose rigorous mathematical treatment promises to be more interesting and certainly more relevant and hence more rewarding than that of the scale-free models of the PA type.[2]

The Internet's robust yet fragile nature

Because high-degree nodes in the router-level Internet can exist only at the edge of the network, their removal impacts only local connectivity and has little or no global effect. So much for the widely-cited discovery of the Internet's Achilles' heel! More importantly, the Internet is known to be extremely robust to component failures, but this is *by design*[3] and involves as a critical ingredient the Internet Protocol (IP) that "sees failures and routes traffic around them." Note that neither the presence of protocols nor their purpose play any role in the scale-free approach to assessing the robustness properties of the Internet. At the same time, the Internet is also known to be very fragile, but again in a sense that is completely different from and has nothing in common with either the sensational Achilles' heel claim or the zero epidemic threshold property, both of which are irrelevant as far as the actual Internet is concerned. The network's true fragility is due to an original trust model[4] that has been proven wrong almost from the get-go and has remained broken ever since. While worms, viruses, or spam are all too obvious and constant reminders of this broken trust model, its more serious and potentially lethal legacy is that it facilitates the malicious exploitation or hijacking of the very mechanisms (e.g., protocols) that ensure the network's impressive robustness properties. This "robust yet fragile" tradeoff is a fundamental aspect of an Internet architecture whose basic design dates back some 40 years and has enabled an astonishing evolution from a small research network to a global communication infrastructure supporting mission-critical applications.

One of the outstanding mathematical challenges in Internet research is the development of a theoretical foundation for studying and analyzing this robustness-fragility tradeoff that is one of the single most important characteristics of complexity in highly engineered systems. To date, this tradeoff has been largely managed with keen engineering insights and little or no theoretical backing, but as the Internet scales even further and becomes ever more heterogeneous, the need for a relevant mathematical theory replacing engineering intuition becomes more urgent. The difficulties in developing such a theory are formidable as the "typical" behavior of a system such as the Internet is often quite simple, inviting naive views and models like the scale-free network models of the PA type that ignore any particulars of the underlying system, inevitably cause confusion, result in misleading claims, and provide simple explanations that may look reasonable at first sight but turn out to be simply wrong. Only extreme circumstances or rare accidents not easily replicable in laboratory experiments or simulations reveal the enormous internal complexity in systems such as the Internet, and any relevant mathematical theory has to respect

[2] *The Fall 2008 Annual Program of the Institute for Pure and Applied Mathematics (IPAM) on "Internet Multi-Resolution Analysis: Foundations, Applications, and Practice" focused on many of the challenges mentioned in this section; for more details, check out* `http://www.ipam.ucla.edu/programs/mra2008/`.

[3] *Being robust to component failures was the number one requirement in the original design of the Internet* [24].

[4] *The original Internet architects assumed that all hosts can be trusted* [24].

the underlying architectural design and account for the various protocols whose explicit purpose is in part to hide all the complexity from the user of the system [5].

Network dynamics and system function

Real networks evolve over time in response to changes in their environment (e.g., traffic, technology, economics, government regulation), and currently proposed network models such as the scale-free models of the PA type cannot account for such interactions. They either ignore the notion of network *function* (i.e., the delivery of traffic) altogether or treat networks as strictly open-loop systems in which modeling exists largely as an exercise in data-fitting. In stark contrast to the scale-free models of the PA type, the proposed HOT-based network models make the dependence of network structure on network traffic explicit. This is done by requiring as input a traffic demand model in the form of a traffic matrix (e.g., gravity model). A particular network topology is "good" only if it can deliver traffic in a manner that satisfies demand. When viewed over time, changes in the environment (e.g., traffic demands), constraints (e.g., available technologies), or objectives (e.g., economic conditions) are bound to impact the structure of the network, resulting in an intricate feedback loop between network traffic and network structure.

The task at hand is to develop a mathematical framework that enables and supports modeling network evolution in ways that account for this feedback loop between the structure of the networks and the traffic that they carry or that gets routed over them. This new modeling paradigm for networks is akin to recent efforts to model the network-wide behavior of TCP or the TCP/IP protocol stack as a whole: the modeling language is (constrained) optimization; a critical ingredient is the notion of separation of time scales; heuristic solution methods (with known robustness properties to changes in the environment) are preferred over mathematically optimal solution techniques (which are likely to be NP-hard); and the overall goal is to transform network modeling from an exercise in data-fitting into an exercise in reverse-engineering. In this sense, relevant recent theoretical works includes *network utility maximization* (e.g., see [30, 36, 37]), *layering as optimization decomposition* (e.g., see [21, 22]), and *the price of anarchy* (e.g., see [45]).

In view of this objective, developing a mathematical framework for studying sequences and limits of graphs that arise in a strictly open-loop manner (e.g., see [17, 19, 20]), while of independent mathematical interest, is of little relevance for studying and understanding real-world networks such as the Internet, unless it is supported by strong and convincing validation efforts. This difference in opinions is fully expected: while mathematicians and physicists tend to view the enormous dynamic graph structures that arise in real-world applications as too complex to be described by direct approaches and therefore invoke randomness to model and analyze them, Internet researchers generally believe they have enough domain knowledge to understand the observed structures in great detail and tend to rely on randomness for the sole purpose of describing genuine uncertainty about the environment. While both approaches have proven to be useful, it is the responsibility of the mathematician/physicist to convince the Internet researcher of the relevance or usefulness of their modeling effort. The scale-free models of the PA type are an example where this responsibility has been badly lacking.

Multiscale network representations

Multiscale representations of networks is an area where Ulam's paraphrased quote *"Ask not what mathematics can do for [the Internet]; ask what [the Internet] can do for mathematics"* is highly appropriate. On the one hand, there exists a vast literature on mathematical multi-resolution analysis (MRA) techniques and methodologies for studying complex objects such as high-dimensional/semantic-rich data and large-scale structures. However, much less is known when it comes to dealing with highly irregular domains such as real-world graph structures or with functions or distributions defined on those domains. In fact, from an Internet perspective, what is needed is an MRA specifically designed to accommodate the vertical (i.e., layers) and horizontal (i.e., administrative or geographic domains) decompositions of Internet-like systems and capture in a systematic manner the "multi-scale" nature of the temporal, spatial, and functional aspects of network traffic over corresponding network structures. In short, the mathematical challenge consists of developing an MRA technology appropriate for dealing with meaningful multi-scale representations of very large, dynamic, and diverse Internet-specific graph structures; for exploring traffic processes associated with those structures; and for studying aggregated spatio-temporal network data representations and visual representations of them.

The appeal of an Internet-specific MRA is that the Internet's architecture supports a number of meaningful and relevant multi-scale network representations with associated traffic processes. For example, starting with our example of the router-level Internet (and associated hypothetical traffic matrix), aggregating routers and the traffic they handle into Points-of-Presences, or PoPs, yields the PoP-level Internet and PoP-level traffic matrix. Aggregating PoPs and the traffic they handle into Autonomous Systems (ASes) or domains produces the AS-level Internet and corresponding AS-level traffic matrix. Aggregating even further, we can group ASes that belong to the same Internet Service Provider (ISP) or company/institution and obtain the ISP-level Internet. While the router- and PoP-level Internet are inherently physical representations of the Internet, the AS- and ISP-level structures are examples of logical or virtual constructs where nodes and links say little or nothing about physical connectivity. At the same time, the latter are explicit examples that support a meaningful view of the Internet as a "network of networks" (see below). With finer-resolution structures and traffic matrices also of interest and of possible use (e.g., BGP prefix-level, IP address-level), the expectations for an Internet-specific MRA technique are that it is capable of recovering these multiple representations by respecting the architectural, administrative, and technological aspects that give rise to this natural hierarchical decomposition and representation of the Internet. While traditional wavelet-based MRA techniques have proven to be too rigid and inflexible to meet these expectations, more recent developments concerning the use of *diffusion wavelets* (e.g., see [40, 41]) show great promise and are presently explored in the context of Internet-specific structures.

Networks of networks

Changing perspectives, we can either view the Internet as a "network of networks" (e.g., AS-level Internet) or consider it as one of many networks that typically partake in the activities of enterprises: transportation of energy, materials, and components; power grid; supply chains, and control of transportation assets; communication and data networks. The networks' activities are correlated because they are invoked to support a common task, and the networks are interdependent because the characteristics of one determine the inputs or constraints for another. They are becoming even more correlated and interdependent as they shift more and more of their controls to be information-intensive and data-network-based. While this "networks of networks" concept ensures enormous efficiency and flexibility, both technical and economical, it also has a dark side—by requiring increasingly complex design processes, it creates vastly increased opportunities for potentially catastrophic failures, to the point where national and international critical infrastructure systems are at risk of large-scale disruptions due to intentional attacks, unintentional (but potentially devastating) side effects, the possibility of (not necessarily deliberate) large cascading events, or their growing dependence on the Internet as a "central nervous system".

This trend in network evolution poses serious questions about the reliability and performability of these critical infrastructure systems in the absence of an adequate theory [46]. Thus the long-term goal of any mathematical treatment of networked systems should be to develop the foundation of a nascent theory in support of such a "networks of networks" concept. To this end, the Internet shows great promise to serve as a case study to illustrate how early verbal observations and arguments with deep engineering insight have led via an interplay with mathematics and measurements to increasingly formal statements and powerful theoretical developments that can be viewed as a precursor of a full-fledged "network of networks" theory.

Conclusion

Over the last decade, there has been a compelling story articulated by the proponents of network science. Advances in information technology have facilitated the collection of petabyte scale data sets on everything from the Internet to biology to economic and social systems. These data sets are so large that attempts even to visualize them are nontrivial and often yield nonsensical results. Thus the "Petabyte Age" requires new modeling approaches and mathematical techniques to identify hidden structures, with the implication that these structures are fundamental to understanding the systems from which the vast amounts of measurements are derived. In extreme cases, this perspective suggests that the ubiquity of petabyte scale data on *everything* will fundamentally change the role of experimentation in science and of science as a whole [7].

In this article we have presented a retrospective view of key issues that have clouded the popular understanding and mathematical treatment of the Internet as a complex system for which vast amounts of data are readily available. Foremost among these issues are the dangers of taking available data "at face value" without a deeper understanding of the idiosyncracies and ambiguities resulting from domain-specific collection and measurement techniques. When coupled with the naive but commonly-accepted view of validation that simply argues for replicating certain statistical features of the observed data, such an "as is" use of the available data reduces complex network modeling to mere "data fitting", with the expected and non-informative outcome that given sufficient parameterization, it is always possible to match a model to any data set without necessarily capturing any underlying hidden structure or key functionality of the system at hand.

For systems whose measured features are subject to fundamental ambiguities, omissions, and/or systematic errors, we have proposed an alternative approach to network modeling that emphasizes data hygiene (i.e., practices associated with determining the quality of the available data and assessing their proper use) and uses constrained optimization as modeling language to account for the inherent objectives, constraints, and domain-specific environmental conditions underlying the growth and evolution of real-world complex networks. We have shown that in the context of the router-level Internet, this approach yields models that not only respect the forces shaping the real Internet but also are robust to the deficiencies inherent in available data.

In this article, the Internet has served as a clear case study, but the issues discussed apply more generally and are even more pertinent in contexts of biology and social systems, where measurement is inherently more difficult and more error prone. In this sense, the Internet example serves as an important reminder that despite the increasing ubiquity of vast amounts of available data, the "Garbage In, Gospel Out" extension of the phrase "Garbage In, Garbage Out" remains as relevant as ever; no amount of number crunching or mathematical sophistication can extract knowledge we can trust from low-quality data sets, whether they are of petabyte scale or not. Although the Internet story may seem all too obvious in retrospect, managing to avoid the same mistakes in the context of next-generation network science remains an open challenge. The consequences of repeating such errors in the context of, say, biology are potentially much more grave and would reflect poorly on mathematics as a discipline.

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WHAT IS...

Measure Rigidity?

Manfred Einsiedler

Measure rigidity is a commonly used shorthand term for rigidity of invariant measures. Here the term rigidity does not have a formal mathematical definition. Rather, it is an informal description of the frequently appearing phenomenon that, for certain mathematical objects, the only examples have much more algebraic structure than was originally demanded. A simple example would be the statement that we have rigidity of continuous homomorphisms of the real line. This is a shorthand way of saying that all continuous homomorphisms of the additive group $\mathbb{R}$ to itself are in fact linear. (A more subtle result is that one even has rigidity of measurable homomorphisms of the real line.) As the requirement to be a homomorphism is much weaker than the requirement to be linear, this is indeed an example where additional algebraic structure is forced. One may say that a linear map cannot be perturbed to a nonlinear continuous homomorphism or that linear maps are rigid.

To explain what an invariant measure is, we need to introduce a transformation on a space—a dynamical system. Let X be a metric space. For example, we could set $X = \mathbb{T} = \mathbb{R}/\mathbb{Z}$, which is often called the torus or the circle group and consists of the cosets $x = r + \mathbb{Z}$ for $r \in \mathbb{R}$. Let us agree that a measure μ on a metric space X is simply a way of assigning to every continuous function $f \in C(X)$ a number—its integral $\int f(x)d\mu(x)$ or $\int f d\mu$ with respect to μ—so that the usual properties of an integral hold: we require that $f \to \int f d\mu$ is linear and that $f \geq 0$ implies $\int f d\mu \geq 0$. In fact, we will be studying probability measures on X, and so the integral of the constant function 1_X is one. The Riemann integral $\int_0^1 f(r+\mathbb{Z})dr$ for $f \in C(\mathbb{R}/\mathbb{Z})$ would in this sense define a probability measure, which is called the Lebesgue measure $m_{\mathbb{T}}$.

Now let $T : X \to X$ be a continuous map. A probability measure μ on X is invariant if $\int f d\mu = \int f \circ T d\mu$. One should think of T as the time evolution of the dynamical system, of f as the outcome of a physical experiment, and of the integral as the expected value for the outcome of f. Then the invariance of μ is simply the requirement that the expected value of the outcome is the same now and one time unit later. The set $\mathcal{M}(T)$ of invariant probability measures depends crucially on the transformation T. For many maps T this set $\mathcal{M}(T)$ is rather large, and it is impossible to give a reasonable description. However, sometimes we also have rigidity of invariant measures: the set of invariant measures shows a surprising amount of structure. We will give examples of both scenarios.

Let us start by studying the case of the circle rotation defined by $R_\alpha(x) = x+\alpha$, where $x \in \mathbb{T}$, $\alpha \in \mathbb{R}$ is a given number, and addition is understood as modulo $\mathbb{Z}$. Let us assume $\alpha \in \mathbb{R} \setminus \mathbb{Q}$ is irrational, as this makes the dynamical system more interesting. The standard substitution rule of the Riemann integral shows in fact that the Lebesgue measure $m_{\mathbb{T}}$ is invariant under R_α. We claim that $m_{\mathbb{T}}$ is the only R_α-invariant measure. One way to see this uses the characters $e_n(x) = e^{2\pi i n x} \in C(\mathbb{T})$ with $n \in \mathbb{Z}$. Suppose μ is an unknown R_α-invariant probability measure. Then by definition $\int e_n d\mu = \int e_n \circ R_\alpha d\mu$. For $n = 0$ this contains no new information, as $e_0 = 1_{\mathbb{T}}$ is constant. So assume $n \neq 0$; then we have $e_n(R_\alpha(x)) = e^{2\pi i n\alpha} e_n(x)$, which shows that $\int e_n \circ R_\alpha d\mu = e^{2\pi i n\alpha} \int e_n d\mu$. As $\alpha \in \mathbb{R} \setminus \mathbb{Q}$ we have $e^{2\pi i \alpha n} \neq 1$, and so $\int e_n d\mu = 0$. This agrees with the value $\int e_n dm_{\mathbb{T}}$ that the Lebesgue measure $m_{\mathbb{T}}$ assigns to e_n. Hence we have $\int f d\mu = \int f dm_{\mathbb{T}}$ for any finite linear combination of characters e_n. As the latter can be used to approximate any other continuous function uniformly (by the Stone-Weierstrass theorem), one sees that $\mu = m_{\mathbb{T}}$. This is the most basic example of rigidity of invariant measures and also the strongest form of it: If $\mathcal{M}(T)$ contains only one measure, then T is called uniquely ergodic.

One may ask why one should care about rigidity of invariant probability measures. The answer lies in a simple construction. Let $T : X \to X$ be an arbitrary continuous map from a compact metric space X to itself. Then notice that for any $f \in C(X)$, $x \in X$, and any large N, the ergodic (time) average $\frac{1}{N}\sum_{n=0}^{N-1} f(T^n x)$ is bounded by $\|f\|_\infty = \max\{|f(x)| : x \in X\}$ and is close to being

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invariant under T in the sense that the difference between the average for f and for $f \circ T$ is bounded by $\frac{2\|f\|_\infty}{N}$. Fixing a function f, we can choose a subsequence N_k along which the ergodic average for f converges—we may think of the limit as $\int f d\mu$ for some $\mu \in \mathcal{M}(T)$. To completely define μ on all functions $f \in C(X)$, one has to continue picking subsequences until one finally arrives at a subsequence for which the ergodic average converges for all functions. In a sense, the measure μ describes the statistical behavior of the orbit $x, T(x), T^2(x), \ldots$, at least for certain very long stretches of time. Combined with measure rigidity this can have very interesting consequences. If T is uniquely ergodic and $\mu \in \mathcal{M}(T)$, then no matter which subsequence (of a subsequence, etc.) one may have picked, the ergodic average for that subsequence must converge to $\int f d\mu$ because μ is the only T-invariant measure. However, this independence of the subsequence means that $\lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} f(T^n x) = \int f d\mu$, both for every $f \in C(X)$ and for every $x \in X$. One may summarize this by saying that unique ergodicity implies equidistribution (with respect to μ) of the orbit $x, T(x), T^2(x), \ldots$ for any $x \in X$. Not only does this help to describe the closure of the orbit (optimally as being equal to the support of μ), but it also asymptotically describes (in terms of μ) the amount of time the orbit will spend in various parts of the space.

Another example of a uniquely ergodic transformation is $S : (x_1, x_2) \to (x_1 + 2\alpha, x_2 + x_1)$ on $\mathbb{T}^2$ where $\alpha \in \mathbb{R} \setminus \mathbb{Q}$. Furstenberg proved the unique ergodicity of S and used the orbit of $(\alpha, 0)$ to derive the equidistribution of the sequence $\alpha, 4\alpha, 9\alpha, \ldots, n^2\alpha, \ldots$ in $\mathbb{T}$, which re-proved a theorem of Weyl. The examples above are still quite simple, but a similar understanding of invariant probability measures for more general classes of transformations can be a very powerful tool. In fact, Marina Ratner proved a very general measure classification theorem (concerning unipotent group actions of quotients of Lie groups) and derived from it equidistribution and orbit closure theorems; the orbit closure theorem is known as Raghunathan's conjecture. The measure classification is indeed a good example of rigidity: by assumption the measure is known to be invariant under a small subgroup, and in the end it is known to be a highly structured measure (called algebraic or Haar) for which the support is the orbit of a bigger group. However, unlike the above cases, in general there will be many different invariant probability measures. These theorems and their extensions by Dani, Eskin, Margulis, Mozes, Ratner, Shah, Tomanov, and others have found many applications, in particular in number theory.

Another transformation on $\mathbb{T}$ is the times-two map $T_2(x) = 2x$ for $x \in \mathbb{T}$. Here the Lebesgue measure $m_{\mathbb{T}}$ is again invariant under T_2, and so is the measure defined by $\int f d\delta_0 = f(0)$. Moreover, we can in fact apply our above construction to produce many invariant probability measures. Represent $x \in [0, 1)$ by its binary expansion, and notice that application of T_2 corresponds to shifting the binary expansion of x. Choosing this infinite sequence in the digits 0 and 1, we may ensure, for example, that we never see the finite sequence 000 or 111. Then the above construction will lead to T_2-invariant probability measures that have support disjoint from $(-\frac{1}{8}, \frac{1}{8}) + \mathbb{Z} \subset \mathbb{T}$. For this transformation the abundance of T_2-orbits of different types (any sequence in 0s and 1s defines an orbit) is reflected in the abundance of T_2-invariant measures.

A highly interesting question was raised by Furstenberg around 1967. He showed that the orbit set $\{2^k 3^\ell (x) : k, \ell \geq 1\}$ is dense in $\mathbb{T}$ whenever the starting point $x \in \mathbb{T} \setminus \mathbb{Q}$ is irrational; here the orbit is taken with respect to the semigroup generated by T_2 and the times-three map T_3. As we have discussed, there is often a correspondence between orbits and invariant measures. Hence it is natural to ask the following: What are the probability measures on $\mathbb{T}$ that are at the same time invariant under T_2 and under T_3? Certain rational numbers $r \in \mathbb{Q}$ are periodic for both T_2 and T_3, and with these one can easily define invariant probability measures. Also, we know that the Lebesgue measure is invariant. Are these (and their convex combinations) the only ones? The best-known result towards this conjecture is due to Dan Rudolph, and several generalizations have been obtained by Kalinin, A. Katok, Lindenstrauss, Spatzier, and me. Similar to Raghunathan's conjecture, these generalized conjectures are phrased for dynamical systems defined on quotients of Lie groups. However, in this case the dynamical system is defined by (several commuting) diagonal matrices instead of unipotent matrices. Even though Furstenberg's question and its generalizations are still open, the partial results have already found several applications. The most striking of these may be Lindenstrauss's proof of the equidistribution of the arithmetic Laplace-eigenfunctions (Hecke-Maaß cusp forms) on certain quotients of the hyperbolic plane.

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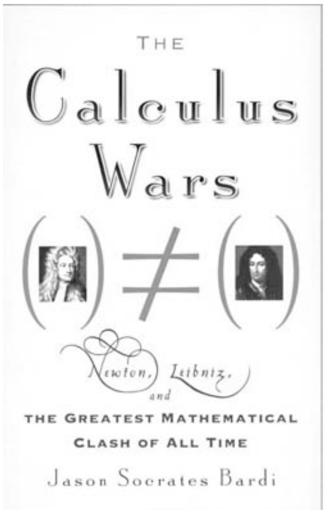
Book Review



The Calculus Wars

Reviewed by Brian E. Blank

The Calculus Wars: Newton, Leibniz, and the Greatest Mathematical Clash of All Time
Jason Socrates Bardi
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According to a consensus that has not been seriously challenged in nearly a century, Gottfried Wilhelm Leibniz and Isaac Newton independently coinvented calculus. Neither would have countenanced history's verdict. Maintaining that he alone invented calculus, Leibniz argued that his priority should be recognized for the good of mathematics. As he reasoned [12, p. 22], "It is most useful that the true origins of memorable inventions be known, especially of those that were conceived not by accident but by an effort of meditation. The use of this is not merely that history may give everyone his due and others be spurred by the expectation of similar praise, but also that the art of discovery may be promoted and its method become known through brilliant examples." Newton believed that Leibniz, for all his fustian rhetoric, was a plagiarist. More importantly to Newton, Leibniz was a second inventor. As Newton framed the issue [15: VI, p. 455], "Second inventors have no right. Whether Mr Leibniz found the Method by himself or not is not the Question… We take the proper question to be,… who was the first inventor of the method." Probity and principle, he argued, demanded a correct answer: "To take away the Right of the first inventor, and divide it between him and that other [the second inventor], would be an Act of Injustice."

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There is no doubt that Newton's discoveries preceded those of Leibniz by nearly a decade. Stimulated by the Lucasian Lectures Isaac Barrow delivered in the fall of 1664, Newton developed his calculus between the winter of 1664 and October 1666. Two preliminary manuscripts were followed by the so-called *October 1666 tract*, a private summation that was not printed until 1962. Because of Newton's dilatory path to publication, word of his calculus did not spread beyond Cambridge until 1669. In that year, Newton, reacting to the rediscovery of his infinite series for $\log(1+x)$, composed a short synopsis of his findings, the *De analysi per aequationes numero terminorum infinitas*. The *De analysi* was written near the end of an era in which scientific discoveries were often first disseminated by networking rather than by publication. Henry Oldenburg, Secretary of the Royal Society, and John Collins, government clerk and de facto mathematical advisor to Oldenburg, served as the principal hubs of correspondence in England. Dispatched by Barrow on behalf of a "friend of mine here, that hath a very excellent genius," the *De analysi* reached Collins in the summer of 1669. "Mr. Collins was very free in communicating to able Mathematicians what he had receiv'd," Newton later remarked.

The second thread of the calculus controversy can be traced to 1673, the year that Leibniz took up infinitesimal analysis. During a two-month visit to London early in that year, Leibniz made contact with several English mathematicians and purchased Barrow's *Lectiones opticæ* and *Lectiones geometricæ*. However, Leibniz neither met Collins nor gained access to Newton's *De analysi* before returning to Paris. The first intelligence of Newton that Leibniz is certain to have received was contained in a report prepared by Collins that Oldenburg transmitted in April 1673. In this summary of English mathematics, Collins referred to

Newton's work on series and asserted that Newton had a general method for calculating a variety of geometric objects such as planar area, arc length, volume, surface area, and center of gravity. Undeterred, and without the benefit of any details of Newton's methods, Leibniz proceeded with his own investigations. Influenced by Pascal's calculation of a moment of a circular arc, Leibniz quickly discovered the "transmutation" formula $\int_0^x y\,dx = xy - \int_0^x (x \cdot dy/dx)\ dx$. By the fall of 1673 he had used this identity to obtain his celebrated series, $\pi/4 = 1 - 1/3 + 1/5 - 1/7 + 1/9 - \cdots$. Although his progress continued somewhat fitfully, Leibniz was in possession of the basic skeleton of calculus by the end of November 1675 [10, pp. 175, 187–200].

It is when we backtrack half a year that the tale becomes tangled. In April 1675 Oldenburg sent Leibniz a report from Collins that contained Newton's series for $\sin(x)$ and $\arcsin(x)$ as well as James Gregory's series for $\tan(x)$ and $\arctan(x)$. Leibniz's reply to Oldenburg was not candid: he professed to have found no time to compare these expansions with formulas he claimed to have obtained several years earlier. He neither divulged his avowed results nor followed through with his promise of a further response. After Collins and Oldenburg pressed the matter by conveying the same series a second time, Leibniz offered to share his infinite sum for $\pi/4$ in exchange for derivations of Newton's formulas. Yielding to the entreaties of Oldenburg and Collins, Newton, who likely had not previously heard of Leibniz, consented to participate in the correspondence using Oldenburg as an intermediary. His letter of 13 June 1676, now known as the *Epistola prior*, was amicable and informative.

Leibniz reciprocated with a few of his own discoveries, as he had promised. He also asked for a further explanation of the methods Newton employed in the calculation of series. This request occasioned Newton's second letter for Leibniz, the *Epistola posterior* of 24 October 1676. Historians who read between the lines of this nineteen page manuscript are divided in their assessments of Newton's state of mind. Derek Whiteside speaks of Newton's "friendly helpfulness". A. Rupert Hall finds no word that would "upset the most tender recipient." However, Richard Westfall states, "An unpleasant paranoia pervaded the *Epistola posterior*." Certainly there is evidence that Newton's guard was up. One sentence in the letter sent to Oldenburg, for example, is heavily crossed out. The less carefully obliterated passage in the copy Newton retained reveals his admission that he had not previously known of Leibniz's series for $\pi/4$. Additionally, by rendering two critical passages as insoluble anagrams, Newton concealed the scope of his fluxional calculus. In the cover letter for Oldenburg that accompanied the *Epistola posterior*, Newton declared his intention to terminate the correspondence. Two days later, still brooding on the matter, Newton directed Oldenburg, "Pray let none of my mathematical papers be printed w$^{\text{th}}$out my special licence."

What Newton learned decades later was that while he was crafting the *Epistola posterior*, methodically deciding what to disclose and what to secrete in code, Leibniz was back in London rummaging through the hoard of documents that Collins maintained. Leibniz emerged from the archive with thirteen pages of notes concerning the series expansions he found in Newton's *De analysi*, but he took away nothing pertaining to Newton's fluxional calculus, of which he had no need, having already found an equivalent. A few weeks later Newton, in one of the last letters he would ever send to Collins, declared his intention to keep his mathematical discoveries private, letting them "ly by till I am out of y$^{\text{e}}$ [the] way." Prudently steering clear of Newton's ire, Collins did not mention the access he had already granted Leibniz. For his part, Leibniz saw no need to breathe a word of it. A long, quiet period was broken in October 1684 when Leibniz staked his claim to calculus by publishing his *Nova methodus pro maximis et minimis* [18, pp. 121–131]. With this paper, which did not allude to Newton, the seeds of a poisonous priority dispute were sown. In the words of Moritz Cantor, it "redounded to the discredit of all concerned."

Historians and sociologists of science have long been fascinated with *multiple discoveries*—clusters of similar or identical scientific advances that occur in close proximity if not simultaneously. Such discoveries are even more noteworthy when they exemplify the phenomenon of *convergence*—the intersection of research trajectories that have different initial directions. Throw in a priority dispute, charges of plagiarism, and two men of genius, one vain, boastful, and unyielding, the other prickly, neurotic, and unyielding, one a master of intrigue, the other a human pit bull, each clamoring for bragging rights to so vital an advance as calculus, and the result is a perfect storm. The entire affair—*the* most notorious scientific dispute in history—has been exhaustively scrutinized by scholars. Three of the most prominent, Hall, Westfall, and Joseph Hofmann, have given us thorough analyses ([8], [20], [10], respectively). Now there is a new account, *The Calculus Wars*, which, according to its author, Jason Socrates Bardi, "is the first book to tell the story of the calculus wars in a popular form." Passages such as "He [Leibniz] began to read more Latin than a busload of pre-law students at a debate camp" and "Newton became a sort of Greta Garbo of the science world" attest to the popular form of Bardi's narrative. There is also truth to Bardi's priority claim: Westfall's account is embedded in a thick biography, Hofmann's requires a mastery of calculus, and Hall's is too comprehensive to be considered popular. Each of these three earlier books exhibits the considerable skills of its author, and yet each becomes mired in the tiresome, repetitive nature of the feud. Thus, in his review of Hofmann's volume, André Weil regretted that its readers had not been "spared a great deal of dull material" [19]. And is there a reader of Hall's *Philosophers At War* who does not applaud when the author, near the end of his story, disregards a petty accusation against Newton and exclaims, "Who can care?"

It is possible, then, that a skimpier treatment of the quarrel might form the center of an attractive, useful book. We would expect such a book, despite its abridgment, to cover the essential elements of the dispute. We would expect it to be informed by the historical research undertaken in the quarter century since the publication of the previous accounts. We would expect a diminution of detail, not of accuracy. And we would expect the squabble, given its barren nature, to serve primarily as a vehicle for illuminating either the mathematics that sparked the war or the remarkable men who prosecuted it. As we will see, *The Calculus Wars* does not meet any of these expectations. Moreover, with its frequent misspellings, its many sentences that would not pass muster in a high school writing class, and its abundance of typographical errors, *The Calculus Wars* falls short of a reader's most basic requirements.

[R]esearch reveals that even in the ancient, academic wrangle between Leibniz and Newton, truth was the first casualty of war.

In this review we present a more or less chronological outline of the developing tensions between Leibniz and Newton, noting along the way several of Bardi's more egregious missteps. We then turn our attention to Bardi's treatment of the mathematics in this story, a treatment that is as unsatisfactory for his intended readers as it is for readers of the *Notices*. Our last major discussion concerns the second front in the war between Leibniz and Newton, their confrontation over physics and metaphysics. It is this battleground that has been the subject of the most recent historical study. That research reveals that even in the ancient, academic wrangle between Leibniz and Newton, truth was the first casualty of war.

Trouble in *The Calculus Wars* begins immediately: the first sentence of the preface gives Newton's year of death as 1726. Bardi changes this to 1727 on page 237, but only a few lines later he states that Newton was interred on 28 March 1726, a date he later repeats. The actual chronology is this: in the English calendar of Newton's era, a calendar that marked 25 March as the legal first day of the new year, Newton died on 20 March 1726 and was buried eight days later on 28 March 1727. To avoid confusing readers with such a timeline, historians often state dates as if 1 January initiated the new year, a practice that this review follows. Bardi seems to have been confounded by the differing conventions of his sources. The year he twice gives for Newton's funeral is wrong by any standard.

In the second paragraph of the preface, Bardi tells us that Leibniz and Newton fought a brutal public battle "to the ends of their lives." In the case of Newton this statement is not true. Historians differ on when the calculus wars began, but they are unanimous about when the squabbling ended. After Leibniz died, his supporters continued to spar with Newton and his allies. However, in February 1723 when an old and infirm Newton, weary at last of the incessant bickering, chose not to respond to a mendacious letter of Johann Bernoulli, the priority dispute finally came to an end [5, p. 557], [7, pp. 66, 597], [8, p. 241], [20, p. 792].

In the fourth paragraph of the preface Bardi tells us that "He [Newton] preferred to circulate private copies of his projects among his friends, and did not publish any of his calculus work until decades after its inception." The first clause of this assertion is false. The second clause is true, but Bardi contradicts it when he states that "Barrow helped Newton publish." Only a few lines later Bardi reverses course again when he writes, "The problem was, he [Newton] didn't publish." In fact, Newton did *not* circulate his mathematical work. He lived for sixty years after writing the *October 1666 tract* and during that time he permitted fewer than ten mathematicians to *view* his manuscripts [16: I, pp. xvii, 11]. Barrow *encouraged* Newton to publish but did not succeed in overcoming Newton's disinclination. When the sixty-two year old Newton published his first mathematical work, Barrow had been dead for twenty-seven years.

If we put this inconsistency behind us and advance one page, we find Bardi derailing anew when he declares that the Great Fire, which ravaged London in 1666, was a "seminal event in the calculus wars." The idea here, advanced by Hofmann [10, p. 43] and Whiteside [16: I, p. xv; II, p. 168; III pp. 5–10], is that publishers, devastated by their losses of stock, could not afford to issue slow-selling mathematical titles. It is true that the conflagration brought the publishing industry close to ruin. As Collins wrote to Newton in 1672, "Latin Booksellers here are averse to y^{e} Printing of Mathematicall Bookes … and so when such a

Coppy is offered, in stead of rewarding the Author they rather expect a Dowry with y[e] Treatise." Nevertheless, there are compelling reasons for rejecting the conclusion that Newton's publishing prospects were seriously impacted [6, pp. 71–73], [20, p. 232]. For one thing, the indefatigable Collins was not daunted. Thinking that Newton might be encountering resistance to publication in Cambridge, he offered his services in London: "I shall most willingly affoard my endeavour to have it well done here." Deploying a procrastinator's armamentarium of excuses to avoid publication—a need for revision, a wish to further develop the material, a shortage of time due to the demands of other activities—Newton never gave Collins the chance.

Even if we were to grant the impossibility of bringing a book-length mathematical work to press, we would still dismiss Bardi's argument that, "If he [Newton] were writing a popular pamphlet or clever little handbill, it could have been a different story." The implication that Newton had to write a weighty tome to secure his priority is untenable: when Leibniz advanced his priority claim, a six page article did the job. Newton could have taken a similarly decisive step. Moreover, his Lucasian salary was generous and he did not even depend on it; the cost of self-publication would have been "trifling" to Newton, as Hall has noted [8, p. 22]. Additionally, Newton had a *certain* opportunity for publication and refused it. Both Barrow and Collins urged him to append his *De analysi* to Barrow's *Lectiones opticæ*, which *was* going to press, the Great Fire notwithstanding. On 12 February 1670 Collins wrote optimistically to James Gregory, "I believe Mr. Newton… will give way to have it printed with Mr. Barrows Lectures." Alas, the young Newton was as obstinate as the old Newton, who boasted, "They could not get me to yield." The absence of Newton's calculus from the Barrovian lectures Leibniz purchased during his first visit to London must be attributed to a Newtonian quirk of character, not an incendiary twist of fate.

The next phase of the prehistory of the calculus dispute was the 1676 correspondence between the two future adversaries. Referring to the *Epistola prior*, Bardi writes, "There was nothing in the letter that was not already known to Leibniz in some form or another. Nothing." One page later Bardi flatly contradicts this unequivocal, emphatic declaration when he admits, "Leibniz was blown away by the *Epistola prior.*" Indeed, as we have observed, there was a great deal Leibniz could have learned about infinite series from Newton in 1676. The gap between them was made even more apparent by the *Epistola posterior.* In an astonishing display of one-upmanship, Newton pointed out that, for suitable choices of its parameters, the rational function $1/(e + fz + gz^2)$ provides not only the series communicated by Leibniz, but also "this series

$$1 + \frac{1}{3} - \frac{1}{5} - \frac{1}{7} + \frac{1}{9} + \frac{1}{11} - \frac{1}{13} - \frac{1}{15} \qquad \text{etc.}$$

for the length $[\pi/(2\sqrt{2})]$ of the quadrantal arc of which the chord is unity." Bardi reports that Newton was "superlative with his praise" of Leibniz in the *Epistola posterior.* Although the quotations Bardi provides do contain superlatives, they merely demonstrate that when the normally plainspoken Newton was on his best behavior, he was capable of embroidering his formal correspondence with the customary encomiums of the era. By not delving deeper, Bardi leaves his readers with the wrong impression. Immediately following the quoted superlatives, Newton continued, "[Leibniz's] letter… leads us also to hope for very great things from him." Having pinged our faint praise radar, Newton follows through with an unsurpassable masterpiece of the art of damning: "Because three methods of arriving at series of that kind had already become known to me, I could scarcely expect a new one to be communicated." To ensure that Collins was not misled by the correspondence, Newton confided privately, "As for y[e] apprehension y[t] [that] M. Leibniz's method may be more general or more easy then [sic] mine, you will not find any such thing.… As for y[e] method of Transmutations in general, I presume he has made further improvements then [sic] others have done, but I dare say all that can be done by it may be done better without it." To Oldenburg Newton confessed a fear that he had been "too severe in taking notice of some oversights in M. Leibniz letter." Nevertheless, Newton could not refrain from adding, "But yet they being I think real oversights I suppose he cannot be offended at it."

Six serene years followed this exchange. Employment brought Leibniz to provincial Germany where he was occupied with the mundane duties of his position, Librarian to the Duke of Hanover. In Newton's case, near isolation resulted from the deaths of Barrow and Oldenburg in quick succession in 1677. Preferring total isolation, Newton lost no time severing his correspondence with Collins. As he later explained, "I began for the sake of a quiet life to decline correspondencies by Letters about Mathematical & Philosophical matters finding them [sic] tend to disputes and controversies." His tranquility received a jolt in mid-June 1684 when he received a presentation copy of *Exercitatio Geometrica*, a fifty page tract authored by David Gregory, nephew of the deceased James Gregory (whose unpublished papers were the source of much of the material). The *Exercitatio* contained several of Newton's results as well as an announcement that more would follow.

Newton reacted with alacrity to this new threat to his priority. To secure his right of first discovery, he began a manuscript, the *Matheseos Universalis Specimina*, intended to explicate both his method of fluxions and its history. In the opening lines of the *Specimina*, Newton relegated the Gregorys to the status of second inventors: "A certain method of resolving problems by convergent series devised by me about eighteen years ago had, by my very honest friend Mr. John Collins, around that time been announced to Mr. James Gregory … as being in my possession.… From his papers… David Gregory also learnt this method of calculation and developed it in a neat and stimulating tract: in this he revealed… what he himself had taken from his predecessor and what his predecessor had received from Collins." Having parried one challenge, Newton attempted to forestall an expected second challenge by making public his correspondence with Leibniz. He explained that those letters would serve readers better than a fresh treatment "since in them is contained Leibniz' extremely elegant method, far different from mine, of attaining the same series—one about which it would be dishonest to remain silent while publishing my own."

Newton started work on the *Specimina* in late June 1684 but his impulse to publish was soon quelled. In July he put down his pen in mid-equation to embark on a new exposition of infinite series. In August he abruptly abandoned that manuscript too. According to tradition, it was at precisely that time that Edmond Halley visited Cambridge to pose the question that diverted Newton and precipitated the *Principia*. Here, perhaps, we may perceive a mischievous intervention of fate. The second thread of the calculus dispute, after its own extended hiatus, was becoming intricately interlaced with the first. With a window of less than two months for establishing an unencumbered claim to calculus, Newton became preoccupied with the planets. Leibniz, whose complacency had also been jolted by a publication from an unexpected source, had just submitted the discoveries that he had withheld for nearly nine years.

At the height of the calculus quarrel, in a rationalization that contained only part of the truth, Leibniz explained that his hand had been forced by a sequence of papers pertaining to tangents, extrema, and quadratures that appeared in the *Acta Eruditorum* beginning in December 1682 [1, p. 117]. The author of the articles Leibniz cited was Ehrenfried Walther von Tschirnhaus, a mathematician whose travels had taken him to London in May 1675 and then to Paris in August 1675. While in England, Tschirnhaus purchased Barrow's lectures and met Collins, who acquainted him with Newton's work. On arriving in Paris, Tschirnhaus entered into a close, working relationship with Leibniz. It was a collaboration that would have some significance in the priority dispute, for Newton deduced—mistakenly, it should be added—that Tschirnhaus passed on to Leibniz what he had obtained from Collins. To the contrary, Tschirnhaus, acknowledging an indebtedness only to Barrow, appropriated and published techniques of calculus that he had learned from Leibniz [11]. In reply to the protestations of Leibniz, Otto Mencke, the founding editor of the *Acta Eruditorum*, urged Leibniz to submit his own exposition [15: II, p. 397]. Referring to the tract of David Gregory, Mencke added, "I now have a reliable report that someone in England has undertaken to attribute publicly to Professor Newton of Cambridge a quadrature of the circle." Leibniz did not dally. Mencke's letter, which was dated 6 July 1684, was answered before the month was over. Leibniz reassured Mencke, "As far as Mr. Newton is concerned… I have succeeded by another way… One man makes one contribution and another man another." Along with his reply he enclosed his contribution, the hastily composed *Nova methodus*. The irony is worth noting: Leibniz's completion of the *Nova methodus* and Newton's abandonment of the *Specimina* were simultaneous.

In the autumn of 1691 David Gregory would play another crucial role in the calculus dispute when he sought Newton's approval of a paper on integration. That second jolt from David Gregory prompted Newton to draft his *De quadratura curvarum*, the revised version of which would become Newton's first mathematical publication a dozen years later. It is clear that both Tschirnhaus and David Gregory influenced the development of the calculus dispute in important ways. Bardi confines his notice of Tschirnhaus to one paragraph, stating that "Newton knew vaguely of Leibniz before their exchange, since he was familiar with one of Leibniz's fellow Germans, Ehrenfried Walther von Tschirnhaus." Where does the idea that Newton learned of Leibniz from Tschirnhaus come from? As for David Gregory, Bardi mentions him only twice: once to quote his hearty praise for Newton's *Principia* and once to say that he was the teacher of John Keill, a later participant in the dispute. Bardi *attempts* to mention David's part in Newton's first publication of fluxions, but botches it by stating that the *De quadratura* came about "only after the Scottish mathematician James Gregory had sent Newton his own method." After the correction of *James* to *David*, Bardi's Escher-like sentence becomes true if "his own" is understood to refer to Newton!

In March 1693 Leibniz initiated a direct exchange of letters with Newton in which he raised the subject of colors. Newton was then between drafts of the *De quadratura* and planning his *Opticks*, a work he did not intend for immediate publication, as he told Leibniz, "for fear that disputes and controversies may be raised against me by ignoramuses." (Robert Hooke was the particular ignoramus Newton had in mind.) Given the remark in the *Specimina* that Newton made concerning acknowledgment, he must have already judged Leibniz to be dishonest. Nevertheless, his letter to Leibniz was cordial. This surface amity was not seriously disturbed by a reckless insinuation of Nicolas Fatio de Duillier, who in 1699 declared that "Newton was the first and by many years the most senior inventor of the calculus… As to whether Leibniz, its second inventor, borrowed anything from him, I prefer to let those judge who have seen Newton's letters." By being the first to publicly suggest plagiarism, Fatio is historically noteworthy, but he was not "a key player in the calculus wars," as asserted by Bardi, who nonetheless consistently misspells *Duillier*.

The death of Hooke in 1703 cleared Newton's path to publication: his *Opticks* went to press in 1704 with the *De quadratura* appended. In the January 1705 issue of the *Acta Eruditorum*, an anonymous review authored by Leibniz proclaimed, "Instead of the Leibnizian differences, Mr. Newton employs, and has always employed fluxions, which are almost the same… He has made elegant use of these… just as Honoré Fabri in his *Synopsis geometrica* substituted the progress of motions for the method of Cavalieri." Here, for the first time, one of the disputants had publicly disparaged the other. Although Leibniz would deny both his authorship of the review and any imputation of plagiarism, Newton recognized both the style and the insult. Writing anonymously many years later, Newton complained, "The sense of the words is that… Leibniz was the first author of this method and Newton had it from Leibniz, substituting fluxions for differences." To Newton, "This Accusation gave a Beginning to this present Controversy."

Even after Leibniz's indiscretion, open warfare was not inevitable. Had it not been for John Keill, the future Savilian Professor of Astronomy at Oxford, Newton may never have seen the offending review. In a paper that appeared in 1710, Keill asserted that "beyond any shadow of doubt" Newton first discovered the "celebrated arithmetic of fluxions." Keill then charged that "the same arithmetic… was afterwards published by Mr Leibniz in the *Acta Eruditorum* having changed the name and the symbolism." Because Keill and Leibniz were both fellows of the Royal Society, the body that published Keill's paper, it was to the Society that Leibniz turned for redress. By demanding a retraction [15: V, p.96], Leibniz crossed the point of no return, for Newton was the Society's president. Keill defended himself by directing Newton to the insinuations Leibniz had inserted into his 1705 review of *Opticks*. Newton advised the secretary of the Royal Society, "I had not seen those passages before but upon reading them I found that I have more reason to complain… than Mr Leibniz has to complain of Mr Keill."

And complain Newton did. Having convened a committee of the Society in March 1712 to investigate the priority issue, he furnished the committee with those documents necessary for establishing his priority. He also drafted the report of the committee and had the Society print the report in the form of a book, the *Commercium Epistolicum*, for international circulation. His conclusion was that "Mr Newton was the first inventor [of the calculus] and… Mr Keill in asserting the same has been noways injurious to Mr Leibniz." A few years later, Newton composed and published the lengthy *Account of the Book entituled Commercium Epistolicum*, which he also translated into Latin for the benefit of continental mathematicians. Unstinting in his efforts, Newton had De Moivre prepare a French version, saw to its publication, and arranged for its positive review. Not yet assuaged some six years after the death of Leibniz, Newton modified the *Commercium* so that his priority was even more evident in its second edition. "Finally, Newton was the prolific author his contemporaries had wanted him to be for so many years," Bardi quips in his best line.

There is no need to describe the remaining battles, but one further error in *The Calculus Wars* should be corrected. In 1713 a condemnation of Newton known as the *Charta Volans* circulated widely. To buttress his charge of plagiarism, the anonymous author of the *Charta Volans* quoted from a letter written by an unnamed "leading mathematician" (*primarii Mathematici*), who referred to an article written by a "certain eminent mathematician" (*eminente quodam Mathematico*). It is a measure of the tedium of the priority dispute that we must distinguish between two translated adjectives that are so similar. Despite all the subterfuge, the authorship of the *Charta Volans* was never in doubt: everybody knew that it had been penned by Leibniz. Because the referenced article was written by Johann Bernoulli, the identity of the "eminent" mathematician was not a mystery either. That left only the "leading" mathematician in question and before long there was a prime suspect. To quote Hall [8, p. 200], "When [Johann] Bernoulli's identity as the 'leading mathematician'… began to be guessed… the joke went around that he had praised himself as the 'very eminent mathematician.' " Bardi loses his way in this comedy of concealed identities, stating, "Leibniz would later be mocked for calling himself an eminent mathematician."

Even after Leibniz's indiscretion, open warfare was not inevitable.

There is little mention of mathematics in *The Calculus Wars,* but when such discussions are unavoidable, the results are invariably frustrating. Leibniz is said to have "solved with ease a problem Descartes was unable to solve in his lifetime." Might Bardi's reader not want to know what that problem was? A mere sixteen pages after mentioning this 1684 application of calculus, Bardi tells us that an article Jakob Bernoulli published in 1690 "was an important document because… it was the *first* in a long series that applied calculus to solving problems in mathematics." The reviewer's italics point to an inconsistency, but the main grievance is that Bardi has tantalized us once again. It surely would not have transcended the bounds of a popular work to cite and even briefly discuss the isochrone problem [18, pp. 260–264]. Against this background, complaints about inaccurate sentences such as "Calculus makes solving quadrature problems trivial" and meaningless phrases such as "to draw a line perpendicular to any point on the curve" will seem futile.

Bardi's characterizations of the mathematics of Newton and Leibniz are particularly misleading. "Newton's big breakthrough," Bardi informs us, "was to view geometry in motion. He saw quantities as flowing and generated by motion." This point of view was not a breakthrough at all: the kinematic construction of curves was a standard method of analysis well before Newton [2, pp. 75–81, 174–177]. Barrow, who mentioned its prior use by Marin Mersenne and Evangelista Torricelli, recognized its value in his geometric lectures because it allowed him to analyze conics without resorting to cases. During the calculus controversy, Newton recollected, "Its probable that D^{r} Barrows Lectures might put me upon considering the generation of figures by motion" [16: I, pp. 11 (n. 26), 150, 344]. He also encountered the method in the appendices of Frans van Schooten's second Latin edition of the *Géométrie* of Descartes, which he started to study in the summer of 1664 [16: I, pp. 146, 371 (n. 11)]. According to another of Bardi's claims, Leibniz "developed calculus more than had Newton." Bardi should have offered some evidence for this assertion since it is contrary to the opinion of the leading expert on Newton's mathematics [16: VII, p. 20], an opinion that is endorsed by other prominent scholars [8, p. 136], [20, p. 515]. To quote Hall, "Well before 1690… [Newton] had reached roughly the point in the development of the calculus that Leibniz, the two Bernoullis, L'Hospital, Hermann and others had by joint efforts reached in print by the early 1700s."

Near the end of his epilogue, Bardi suggests the possibility that neither Newton nor Leibniz deserves all the credit he was seeking. "In some ways, the development of calculus owes just as much all those who came before [sic]." This is a pertinent consideration and it is too bad Bardi drops the issue as soon as he has raised it. Both Lagrange and Laplace, for example, deemed Pierre de Fermat to be the first inventor of the differential calculus [4]. John Mark Child, who edited and interpreted the *Lectiones Geometricæ* of Isaac Barrow, proposed the following set of minimal requirements for a first inventor: "A complete set of standard forms for both the differential and integral sections of the subject, together with rules for their combination, such as for a product, a quotient, or a power of a function, and also a recognition and demonstration of the fact that differentiation and integration are inverse operations." Using these criteria, Child proclaimed that Isaac Barrow was the first inventor of calculus [3]. Historians nowadays augment Child's list—and thereby exclude Barrow—by requiring an awareness that the diverse problems that made up the "research front" of seventeenth century infinitesimal analysis could all be tackled in a comprehensive, algorithmic manner using symbolic language.

It is unfortunate that we know so little of Barrow's influence on either Newton or Leibniz. In a letter of 1716 [15: VI, p. 310], Leibniz argued, "It is possible that Mr. Barrow knew more than he ever published and gave insights to Mr. Newton which we do not know of. And if I were as bold as some, I could assert on the basis of these suspicions, without further evidence, that Newton's method of fluxions, whatever it may be, was taught to him by Mr. Barrow." Leibniz was artful enough to talk only of what Barrow *might* have known, for by then it had often been alleged that the Leibnizian calculus was merely a symbolic recasting of Barrow's published work. For example, Collins, who knew exactly what had passed from Newton to Leibniz, never suggested that Leibniz plagiarized Newton, but he did wonder whether results of Leibniz were "learnt or… derived from Dr Barrows Geometrick Lectures" [15: II, p.241]. Tschirnhaus *did* believe that Leibniz took from Barrow [10, pp. 76, 173]. So did Jakob Bernoulli, who in 1691 asserted, "To speak frankly, whoever has understood Barrow's calculus… will hardly fail to know the other discoveries of Mr. Leibniz, considering that they were based on that earlier discovery, and do not differ from them, except perhaps in the notation of the differentials and in some abridgment of the operation of it."

In more modern times, Margaret Baron has remarked that Leibniz's notes "suggest that he had been dipping into Barrow's *Lectiones,*" whereas Jacqueline Stedall has stated that Leibniz studied Barrow "intensively" [2, p. 288], [18, p. 119]. Since the topic of this review is priority, the key question for us is, *When?* Hofmann [10, pp. 76-78] and Dietrich Mahnke [13] have argued that Leibniz did not read Barrow until the winter of 1675, which is to say, after he had completed his outline of calculus. (Interested readers must judge for themselves how persuasive those arguments are—the reviewer is not convinced by them, but his position is not that of an expert and he offers it only so that he does not appear as weaselly as Fatio.) Leibniz himself strongly and repeatedly denied any debt to Barrow. To Jakob Bernoulli he wrote that he had filled some hundreds of sheets with calculations based on characteristic triangles before the publication of Barrow's *Lectiones* [8, p. 76]. That chronology is impossible. In a letter to another correspondent [15: VI, p. 310], Leibniz wrote, "As far as I can recall, I did not see the books of M. Barrow until my second voyage to England." That recollection is wrong.

Hofmann explained the frequent infelicities of Leibniz's accounts by saying "He must have meant… " or "He wrote in haste… ." Child either cited "memory lapses" or proposed other scenarios to avoid the "brutal" conclusion that Leibniz lied. For André Weil, however, Leibniz's say-so obviated the need for any explanation: "In the early stages he [Leibniz] could have learned a good deal from Barrow's *Lectiones geometricæ*; but, by the time he read them, he found little there that he could not do better. At any rate he says so… In the absence of any serious evidence to the contrary, who but the surliest of British die-hards would choose to disbelieve him?" In the same review [19], Weil repeats, "As he [Leibniz] says, he could well have derived some of his inspiration from Barrow, had he read him at the right moment; there is no point in disputing this fact. He could have; but he says he did not; so he did not, and that is all."

In the time that has passed since Weil's pronouncement, serious evidence undermining Leibniz's good faith *has* been uncovered. In February 1689, two years after the *Principia* became available, Leibniz published a fifteen-page paper, the *Tentamen de motuum coelestium causis*, concerning the planetary orbits. Leibniz's background story for this article was that he had not yet seen the *Principia* because of his travels. He asserted in the *Tentamen* (and elsewhere) that his knowledge of the *Principia* was limited to an epitome that had appeared in the June 1688 *Acta*. Leibniz implied that his work was done some time before that review, but that he had suppressed it until such time as he would have the chance to test his ideas against the most recent astronomical observations. The publication of Newton's theories, he continued, stimulated him to allow his notes to appear "so that sparks of truth should be struck out by the clash and sifting of arguments." To his mentor, Christiaan Huygens, Leibniz drafted a letter in which he affirmed that he did not see the *Principia* before April 1689 (by which time the *Tentamen* had already appeared).

It is likely that Newton, who had had a copy of the *Principia* sent to Leibniz immediately after its publication, was unaware of the *Tentamen* until after 1710. Newton sensed plagiarism when he was apprised of it, but he charged Leibniz with bad manners instead: "Through the wide exchange of letters which he had with learned men, he could have learned the principal propositions contained in that book [the *Principia*] and indeed have obtained the book itself. But even if he had not seen the book itself [before writing the *Tentamen*], he ought nevertheless to have seen it before he published his own thoughts concerning these same matters, and this so that he might not err through… stealing unjustly from Newton what he had discovered, or by annoyingly repeating what Newton had already said before." It now appears that Newton's suspicions were justified. In the 1990s Domenico Bertoloni Meli discovered and presented compelling evidence to reject the cover story Leibniz prepared for the *Tentamen*. Meli concluded that "Leibniz formulated his theory in autumn 1688 [i.e., *after* the *Acta* review], and the *Tentamen* was based on direct knowledge of Newton's *Principia*, not only of Pfautz's review" [14, p. 9]. Given that Meli's book appears in the bibliography of *The Calculus Wars*, it is difficult to understand why Bardi repeats Leibniz's fabrications about the *Tentamen* as if they have never come into question.

There have been several recent reminders that priority disputes remain topical.

We may contrast Bardi's neglect of a serious matter with his excited denunciation of a trivial yet iconic anecdote: "The legend of Isaac Newton and the apple… is probably completely fabricated. Perhaps the only thing that is true about it is that Newton loved apples. The story is no more true than the one about the alligators in the sewers of New York." In fact, Isaac Newton himself told the apple anecdote to at least four close friends and relatives in 1726 and 1727 [7, p.29], [20, p.154]. If Bardi had any reason to think Newton fibbed or confabulated, then he should have shared it. Having repudiated one of the best-known stories of science, Bardi proceeds to spoil one of the best-known quotations of science by paraphrasing it as "Joseph-Louis Lagrange… called Newton the greatest and the luckiest of all mortals for what he accomplished." Bardi does not explain why Lagrange considered Newton so lucky. The missing explanation can be found in the actual quotation, "… et le plus heureux; on *ne* trouve *qu'une fois* un système du monde à établir." This insider's appreciation of priority (with Lagrange's implicit regret that he, unlike Newton, had the misfortune to follow a Newton) could have been a perfect insight for Bardi's readers, but instead it is just one more mystery.

There have been several recent reminders that priority disputes remain topical. Shortly before this review was suggested, the United States Senate tabled a patent reform act that would have aligned the U.S. with the rest of the world in recognizing claims based on first-to-file status rather than first-to-invent. During the writing of this review, Hollywood released *Flash of Genius*, a film that depicted the zeal with which an inventor will battle for recognition. At the same time, a Nobel Prize committee arrived at its own resolution of the most acrimonious scientific dispute of recent years, the fight over the discovery of the human immunodeficiency virus. The battle between Newton and Leibniz, one imagines, should continue to interest many readers. *The Calculus Wars*, however, is an appalling book that cannot be recommended to them. For those seeking a popular treatment of the priority dispute, either one of the reliable, comprehensive biographies of Newton [5], [20] would make a good choice. Those who require a more detailed treatment will continue to consult Hall [8]. If a concise outline is preferred, then Hall has written that too [9]. Newton's correspondence [15], which was edited with an eye to the controversy, is the ultimate resource for a thorough understanding of the affair.

In a bygone era a reviewer might have passed over *The Calculus Wars* in preference to subjecting it to an excoriating review. Incompetent books could safely be allowed to sink quietly into oblivion. Search engines and the Internet have changed that. Because Bardi has had a book published, he is now an expert on its subject as far as the World Wide Web is concerned. At the time of this writing, the Wikipedia page devoted to the priority dispute cites both *Philosophers at War* and *The Calculus Wars*. No indication is given there that one book is authoritative and the other is not. Google finds Bardi's unseemly assessment of the career of John Pell, an appraisal that was out-of-date before he started writing his book, just as easily as it finds the reasoned reappraisal of Pell given in [17]. At the time of this writing, WorldCat locates *The Calculus Wars* in more than twice as many libraries as Meli's book and nearly as many as Hall's. If readers of the future are not to be swamped with misinformation, then the searches that turn up specious books must also find them critically reviewed.

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The Formation of Scholars: Rethinking Doctoral Education for the Twenty-First Century

Reviewed by David Manderscheid

The Formation of Scholars: Rethinking Doctoral Education for the Twenty-First Century
George E. Walker, Chris M. Golde, Laura Jones, Andrea Conklin Bueschel, and Pat Hutchings
Jossey-Bass, 2008
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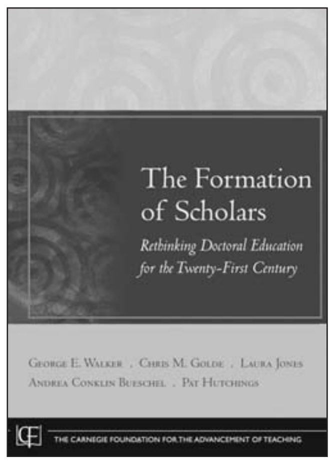
Doctoral education in the mathematical sciences in the United States is the envy of the world. When reforms are suggested, a common reply is, "If it ain't broke, why fix it?" Witness the controversy that the NSF-DMS VIGRE program stirred up. (VIGRE stands for Vertical Integration of Research and Education, a program of the Division of Mathematical Sciences of the National Science Foundation.)

Is all well, however? Outside the top programs there is often a scramble to get enough good graduate students. Moreover, even the top programs, not to mention the others, often have trouble attracting well-qualified U.S. citizens, especially those from groups underrepresented in the discipline. Shouldn't we be concerned? Aren't we losing our best and brightest to other fields?

This book begins with an epigraph that is often attributed to Will Rogers: "Even if you are on the right track you will get run over if you just sit there." The authors argue that the academy is just sitting there when it comes to doctoral education in the United States, and they provide suggestions on how to avoid getting run over.

The book is an outgrowth of the Carnegie Initiative on the Doctorate (CID), a program that ran from



2001 to 2005. The CID concentrated on the disciplines of chemistry, education, English, history, mathematics, and neuroscience, with eleven mathematics departments represented among the eighty-four programs overall. The purpose of the project was primarily to encourage the participant departments to improve the effectiveness of their programs, but it also contained a research agenda, the determination of where problems might be nationally, and how these might be addressed. Further background on the initiative can be found at `http://www.carnegiefoundation.org/cid` and in a *Notices* article by Allyn Jackson [Jackson, 2003]. The website includes information about outcomes of the initiative at many of the participating institutions. It also includes thought pieces written by Hyman Bass, University of Michigan, and Tony Chan, then University of California, Los Angeles, and now NSF, for the CID. These pieces also appeared in the *Notices* [Bass, 2003], [Chan, 2003].

The title *The Formation of Scholars: Rethinking Doctoral Education for the Twenty-First Century* was carefully chosen. The authors chose the word "formation" to emphasize the scholar's professional identity in all its dimensions, as opposed to just the acquisition of specialized, perhaps even esoteric, knowledge.

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The authors identify three key themes in the formation of scholars: scholarly integration, intellectual community, and stewardship. By scholarly integration they mean close ties between teaching and research. Put more baldly, they employ the catch phrase "scholarship segregated is scholarship impoverished." By intellectual community the authors mean more than social activities; they mean "the hidden curriculum" that sends powerful messages that determine, among other things, roles and the extent to which true creativity is welcomed. With stewardship the authors emphasize a moral and ethical duty of scholars both to their own work *and* to their discipline. These three themes are weaved throughout the discussion.

The authors argue that a key first step in reform is reflection. A department should discuss the purpose of its graduate program and how well they are meeting that purpose. For example, do you define your mission to be preparing researchers of the highest caliber? If so, do all of your students go on to positions at AMS Group I departments? If not, then should your statement of purpose include, for example, more scholarly integration of teaching and research? Maybe you would go so far as to include scholarly analysis of learning outcomes in a Ph.D. dissertation, as the Department of Chemistry at the University of Michigan has done.

The authors emphasize that there is no one purpose of graduate education, but whatever a department's purpose is, it should be well defined and decisions should flow from that purpose. Once you know what your goal is, the next question to ask is how your current program helps you achieve that goal.

For example, you might ask how your qualifying or comprehensive exams serve your purpose. If, for example, part of your purpose is to train broadly educated mathematicians, then what do the qualifying exams do toward that end that tests in courses do not? Perhaps they only increase time to degree. If the answer is to fail-out students who should not go on to get the degree, then one might ask whether that purpose could be better accomplished in course work. You might also ask whether the time qualifying exams consume could be better spent. For example, perhaps students could start work on a research project and present on that in either an exam format or a portfolio. Would that better suit your purpose?

The above examples of alternatives to current practices are taken from the book, and there are many more based on practice at departments studied, both mathematics and otherwise. They are often examples of what the authors call *authentic practices*, that is, training that better emulates what will be expected of the graduate as a member of the profession.

An example of a practice that is suggested but that to my knowledge has yet to be fully implemented in a mathematics department comes in considering the role of the dissertation advisor. The relationship of the dissertation advisor to the advisee is often described as one of apprenticeship. The authors argue that this tends to be a reproductive model that leads to both intellectual and social conformity. They argue that apprenticeship to a faculty mentor should become apprenticeship to several faculty mentors. They reason that scholarship and research of the future will involve many people, seldom the isolated scholar. Of the disciplines in the CID, mathematics students identified having the fewest number of mentors, with the exception of chemistry students (page 95). Is this a good thing? Does it serve the purpose of your program?

This model of apprenticeship to several is part of a sense of a community of scholars. This is more than a community of mathematicians founded around annual picnics, holiday parties, or softball and volleyball teams—things that, I hasten to add, I am all for. Rather, it is an intellectual community with a shared responsibility among faculty members, the authors argue. Faculty should hold one another accountable for the education of students.

I would take this one step further than the authors. I believe that students must be viewed as a more important part of the intellectual community. This is something the authors do not stress. As future stewards of the discipline, graduate students should be trained to help their fellow students succeed. Peer-to-peer mentoring can often be successful where a faculty member was unsuccessful. Moreover, it spreads the intellectual burden further, thus requiring less of faculty. Contrary to this, we often hear arguments for more and more fellowship money for students so that they can spend more time on research. This can often work counter to integration into the intellectual community.

The authors point out that academic administrators have a key role to play in implementing the changes departments might come up with as a part of their reflection. They go on to say that, for CID participants, "The changes that took place in departments might be characterized as incremental changes rather than radical reforms. Moreover they were implemented at a measured pace, with respect for departmental decision-making traditions, rather than being quick fixes slapped into place. In part because the Carnegie Foundation gave no money to departments (although some programs received modest institutional support for travel, graduate students, research assistants, or surveys), programmatic changes were a response to locally defined needs. Historically, there have been many reform initiatives in which the (often laudable) activities have been entirely predicated on external funding, but many do not persist or become institutionalized once the seed money disappears [Bacchetti and Ehrlich, 2006]. We believe that changes prompted by the CID will stick because these were choices made by the department itself, rather than as a response to an external agent" (page 165). As a dean I view this as the "if they really want to do it, they can do it on their own and thus they don't need the college's scarce resources" approach.

On the other hand, there is, for example, the approach of the NSF Division of Mathematical Sciences through its workforce program. So where does the truth lie?

Of course the truth is somewhere in between. I agree that reform needs to be a grassroots movement from faculty, staff, and students as a fundamental cultural shift is necessary. But oftentimes these shifts require more than just will. So how do you convince your administration or the NSF? I think the answer is similar for both. Further, most deans think alike on this point and it is no secret—but it is amazing to me at least—how few department chairs provide us the information we want/need to invest scarce resources in a department. We are looking for evidence that you have done your homework, that you have reflected on the issues, that you have well-thought-out plans, and that you have ways to evaluate the success of these plans if you are able to implement them. We are also looking for evidence that you have done all that you can do on your own. Everyone could use more resources, but what have you done to conserve the resources that you have? How high a priority is this to your department, as evidenced by a serious investment of your own time and resources? These issues can require serious and often delicate discussions, but progress is made not just by adding on but sometimes also by cutting back.

I'll step off my soapbox now and return to the book at hand. I highly recommend it to all faculty members interested in improving their graduate programs. It is a relatively quick read. Indeed, portions most relevant to other disciplines can often be skipped, although the cross-cultural enlightenment these sections can provide can be both amusing and worthwhile. The authors and the Carnegie Foundation for the Advancement of Teaching are to be commended.

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About the Cover

Real analysis in polar coordinates

The cover image shows Ken Golden (author of the article on the mathematics of sea ice in this issue) measuring the fluid permeability of first year Antarctic sea ice. He writes about it:

"The fluid permeability of porous sea ice controls brine drainage, melt pond evolution, surface flooding, and snow-ice formation. Melt pond evolution, for example, in turn controls sea ice reflectance, a key parameter in understanding the dramatic decline of the summer Arctic ice pack. Snow-ice formation, a dominant process in the Antarctic, may well become more prevalent in the Arctic as the ice pack thins. The permeability also controls microbial colonization and nutrient fluxes, which are important in gauging the response of polar ecosystems to climate change. The permeability measurements I made on this expedition were the first such measurements done in the Antarctic ice pack. The photograph was taken by Jan Lieser."

—Bill Casselman, Graphics Editor
(`notices-covers@ams.org`)

Homage to Henri Cartan (1904–2008)

Michèle Audin

On August 13, 2008, Henri Cartan died at the age of 104. He was, in more ways than one, an icon of French mathematics in the twentieth century.

In 1986 the Palais de la Découverte, France's premier science museum, presented an exposition on the "Whitehead link", an intertwining of two "knots" (the black loop and the gray figure-eight that one sees in the photograph at right), neither of which is knotted but each of which cannot be separated from the other.

This property interests mathematicians and, in the spirit of Jacques Lacan (1901–81), psychoanalysts. The exposition was thus conceived by a psychoanalyst, Jean-François Chabaud, and a mathematician, Henri Cartan. One sees Cartan in the accompanying picture at the Palais performing a demonstration of the fact that one can exchange the gray and black components, unraveling the gray to make it into a circle, a procedure that has the secondary effect of transforming the black component into a figure-eight. Cartan is accompanied by his wife (who died in December 2008); Jean Brette of the department of mathematics of the Palais is in the back. At the time, Henri Cartan was 82 years old and retired, but he continued to be interested in mathematics and, as one sees, its popularization.

Born in 1904, Henri Cartan was the eldest son of Élie Cartan (1869–1951) and Marie-Louise Bianconi (1880–1950). Élie Cartan was a mathematician, one of the founders of differential geometry, and the one who introduced Lie groups into this branch of mathematics. Among the four Cartan children, two became mathematicians, Henri and Hélène (1917–52), and entered the École Normale Supérieure (ENS) in rue d'Ulm; Louis became a physicist (1909–43, he was deported to Germany as a participant in the resistance and beheaded); and Jean became a composer (1906–32, he died of tuberculosis). There was a great deal of music in the Cartan home, and Henri was a very good pianist.

An Exceptional Mathematician

Henri Cartan entered the ENS in 1923; he got to know Jean Delsarte (1903–68), André Weil (1906–98), and Claude Chevalley (1909–84), as well as other members of a group that, in December 1934, began to lay the foundations for a treatise that would become *Les éléments de mathématiques* by Nicolas Bourbaki, the collective pseudonym under which the group signed its books.

Henri Cartan specialized in complex analysis. After the crucial works of Charles-Émile Picard (1856–1941), Paul Montel (1876–1975), and others, one knew "everything" about the functions of a single variable. Cartan was one of the founders of the theory of functions of several complex variables and analytic spaces, and he was the one responsible for the use of sheaves in that theory. He was also the author of important contributions to algebraic topology, homological algebra (with his American friend Samuel Eilenberg (1913–98)), and potential theory.

From 1929 to 1975 Cartan held various university posts, from Lille to Orsay by way of Strasbourg (just before the war), as well as ENS, where he taught generations of *normaliens* from 1940 to 1965. Through his contributions to the work of Bourbaki (he was, for example, the inventor of filters), through his mathematical work, through his teaching and the seminar that carried his name

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This article is a translation of "Hommage Henri Cartan (1904–2008)" by Michèle Audin, which was originally published in Découverte *(the magazine of the Palais de la Découverte), number 359, November–December 2008, pages 14–15.*

Hélène Cartan

In the Cartan family, the little sister, Hélène, was, like her older brother, Henri, a very good pianist and a mathematician. Born in 1917, she entered the École Normale Supérieure in rue d'Ulm in 1937 and passed the *agrégation* in mathematics in 1940, getting the best rank of all the young women (there was no *agrégation* for young men in wartime). She became a teacher in a secondary school. In 1942, she published a *Comptes Rendus* note, which was presented to the French Academy of Sciences by her father Élie Cartan. The paper proved necessary and sufficient conditions for a connected topological space E to be homeomorphic to a circle: E minus any point is connected, E minus two points is not, and E is either compact or locally connected and contains a non-enumerable dense subset. Hélène contracted a very bad and contagious form of tuberculosis, which prohibited her from teaching, and she spent several years in a sanatorium. In 1952, after going out for a walk one day, she disappeared for a few days and was eventually found dead in the river Isère.

—M. A.

Photograph © Palais de la Découverte, C. Rousselin.

Henri Cartan in 1986, at age 82, with his wife Nicole, at the "Whitehead link" exposition.

Nicole Cartan

Nicole Cartan, born in 1916, was the daughter of physicist Pierre Weiss. She married Henri Cartan in 1935 and they had five children. She died in December 2008, a few months after the death of her husband.

—M. A.

from 1948 to 1964, and through his books, he had an enormous and lasting influence on the French mathematical school: one can count the majority of living French mathematicians as his "grandchildren" or "great-grandchildren".

A Man of His Century

Henri Cartan also had an influence on international affairs, notably in Germany, where at the end of the war he re-established scientific contact with some of his colleagues, for example Heinrich Behnke (1898–1979). Cartan first visited Germany in 1931; he returned in 1946, giving a lecture (and playing the piano) at the mathematics institute in Oberwolfach.

All of this was consistent with his European convictions. He was also a great defender of human rights. His defense of Leonid Plioutch in 1974 and of José-Luis Massera (1915–2002) during the years 1970–80 come to mind. But Cartan did not wait until his retirement to take an interest in the fate of his colleagues. Recall the battle he took on as president of the Société Mathématique de France in 1950, with McCarthyism in full swing, to get the American authorities to give visas to Jacques Hadamard (1865–1963) and Laurent Schwartz (1915–2002) to attend the International Congress of Mathematicians in Cambridge, Massachusetts (where Schwartz received the Fields Medal).

One can also recall the assistance he brought, in occupied France, to the mathematician Jacques Feldbau (1914–45), who was persecuted by anti-Semitic laws (persecuted even to his death).

The twentieth century has passed, Henri Cartan is dead. He leaves us his heritage, visible and invisible, in the living mathematics of the twenty-first century.

Editor's Note: The *Notices* plans to publish a more extensive obituary for Henri Cartan in the future. For further information about Cartan, see the interview conducted by Allyn Jackson in the August 1999 issue of the *Notices* (`http://www.ams.org/notices`), as well as the articles about Cartan collected by the Société Mathématique de France on the occasion of his 100th birthday (`http://smf.emath.fr/VieSociete/Rencontres/JourneeCartan/`).

Encounters with Mischa Cotlar

John Horváth

Before I begin my tale, I want to call the reader's attention to the volume *Analysis and Partial Differential Equations* [20], edited by Cora Sadosky in honor of the seventieth birthday of Mischa Cotlar. There can be found the biography of Mischa, three essays about his personality, an analysis of his mathematical works, and the list of his publications until 1989.

The First Encounter

From 1948 to 1951 I lived in the *Colegio de España* at the University City in Paris. Since the Spanish civil war the college was under the administration of the French government and housed several refugees from Spain. Among them there were some mathematicians who had settled in Argentina, for instance Manuel Balanzat, professor at the University of Cuyo in San Luis and a disciple of Luis A. Santaló. I must add that I had already heard about Santaló in Budapest in 1946 when László Fejes-Tóth presented Santaló's proof of the isoperimetric inequality in his course on geometry.

There were also some Portuguese mathematicians, opposed to the Salazar regime and established in Brazil. It is from all these that I first heard the names of Antonio Monteiro, Leopoldo Nachbin, and Mischa Cotlar.

In May 1951, on my way from Paris to Bogotá, I visited the United States for a few weeks. With my fellow student Steve Gaal we traveled to New Haven to see Shizuo Kakutani at Yale University. Kakutani told us that Cotlar was in New Haven, supported by a Guggenheim fellowship and with the intention of obtaining a Ph.D. in mathematics from Yale. Kakutani gave us some information about Mischa's life: That he had arrived in Uruguay from Russia as a child, that later on he had earned a living playing the piano in the dives of the port of Montevideo, that he never went to school. Kakutani added that Mischa was a self-taught mathematician who had been publishing articles since 1936, when he was twenty-three years old. Cotlar's interest in ergodic theory had brought him to New Haven, to study with Kakutani. Indeed, Kakutani was considered at that time the foremost expert in the subject, so much so that he undertook to give a panoramic lecture on it at the International Congress of Mathematicians held in 1950 in Cambridge, Massachusetts [17]. I recall an amusing detail from those times before Xerox machines, before computers, and before email: Kakutani had a single copy of the text of his lecture, which had not yet been published. I asked him to lend it to me, which he did after I swore to return it.

Mischa Cotlar in Chicago, 1952.

Going back to May 1951, Cotlar came to dinner with us and afterwards, I witnessed the first example of his unbelievable humility. I had to return to New York, and the colleagues accompanied me to the railroad station. Cotlar, who was eleven years older than I, insisted on carrying my suitcase! After

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Mischa Cotlar died January 16, 2007, in Buenos Aires. He was an exceptional mathematician and human being. Generations of mathematicians in Venezuela, Argentina, and other Latin American countries grew under his guidance. He was one of the world experts in harmonic analysis and operator theory.

—Josefina Alvarez
New Mexico State University

I arrived in Bogotá, I learned that Yale suddenly had realized that it could not award a doctoral degree to a person without a high school diploma. When Marshall Stone, who had met Cotlar in Buenos Aires, found out about the problem, he suggested that Cotlar should go to the University of Chicago, where such official chicaneries were treated more lightly.

Thus, Cotlar arrived in the realm of Antoni Zygmund and Alberto P. Calderón, at the time when the object of study of the analysts was the theory of singular integrals. In a stroke of genius, Cotlar combined the topic of singular integrals with his own interest in ergodic theory, writing his doctoral dissertation, about which I will speak later, under the title "A unified theory of Hilbert transformations and of ergodic theorems".

Encounters in South America

My next personal meeting with Mischa was in July 1954 on the occasion of the Second Symposium on Some Mathematical Problems Which Are Being Studied in Latin America.

In the city of Mendoza, at the foothills of the Andes, the National University of Cuyo had established a mathematical institute, and Cotlar was appointed its director upon his return from the United States. The institute organized the symposium, and the inaugural lecture was given at the San Juan College of Mendoza by Julio Rey Pastor, a leading figure of Latin American mathematics. The following day we were transported by bus to an elegant hotel in Villavicencio, higher up in the mountains, where we were lodged and where the other talks of the symposium took place. Cotlar gave an excellent expository lecture titled "The Moment Problem and the Theory of Hermitian Operators" [6]. Let me mention that the first simposium had taken place in Punta del Este, Uruguay, in December 1951 and that Cotlar's lecture had been "On the Fundaments of the Ergodic Theory" [5].

The institute published its own journal, *Revista Matemática Cuyana.* The often cited second number of the first volume includes four contributions by Cotlar. The first three articles present auxiliary results, which are then applied in the fourth one. A footnote states that the essential parts of the last three articles are taken from the doctoral dissertation of the author, University of Chicago, 1953. The first article [8] contains one of the best known results of Cotlar and is the origin of the concept of *quasi-orthogonality*. Elias M. Stein consecrates to it a large part of the seventh chapter of his book [21]. The main theorem can be stated as follows:

> Let $\mathcal{A}$ be a commutative normed ring, let T_k, $1 \le k \le n$, be elements of $\mathcal{A}$, and set $T = \sum_{k=1}^{n} T_k$. If for $1 \le i, j \le n$ we have the quasi-orthogonality condition
>
> $$\|T_i T_j\| \le 2^{-|i-j|} \tag{1}$$
>
> and furthermore, $\|T_i\| \le 1$, then $\|T^k\| \le 2^{3k} k k^{k^{3/4}} n$. An immediate consequence is that if $\{T_i\}_{i \in \mathbb{N}}$ is a sequence of hermitian operators on a prehilbert space that satisfy (1), and if $\|T_i\| \le C$, then $\sum_{i=1}^{n} T_i$ converges in norm as $n \to \infty$ to an operator T such that $\|T\| \le 5C$.

The original proof of this result, based on a combinatorial argument, is complicated. A little later, Béla Szőkefalvi-Nagy found a simpler proof [23]. Both Cotlar and Stein generalized the result to the case where the operators do not commute, and Stein, in collaboration with A. W. Knapp, used it in the theory of semi-simple Lie groups [22]. The inequality plays an important role in works by Calderón-Vaillancourt [3] and by Coifman-Meyer [4].

A classical theorem of Marcel Riesz has as a particular case the following:

> Let $\mathcal{D}$ be a function space that is dense in every space L^p, for instance the step functions, for $1 \le p \le \infty$. If $T : \mathcal{D} \to \mathcal{D}$ is a linear operator that satisfies the condition (C_r),
>
> $$\|Tf\|_r \le M_r \|f\|_r$$
>
> for $r = 1$ and $r = p$, then (C_r) holds for all r between 1 and p. Furthermore, the optimal bound M_r is a logarithmically convex function of r, i.e.,
>
> $$M_r \le M_1^{\frac{r-p}{1-p}} M_p^{\frac{r-1}{p-1}}.$$

It is because of this inequality that the theorem of Marcel Riesz used to be called "convexity theorem".

There are situations where the operator T does not satisfy (C_1) but nevertheless it satisfies (C_r) for $1 < r \le p$; the most important example is the Hilbert transform,

$$Hf(x) = \frac{1}{\pi} vp \int_{\mathbb{R}} \frac{f(t)}{x-t} dt.$$

One can ask whether replacing $\|Tf\|_1$ in (C_1) by a smaller quantity, the condition so obtained does not still imply that (C_r) holds for $1 < r \le p$. An answer can be obtained from the inequality of Chebišov,

$$\|f\|_1 \ge \int_{\{|f|\ge\lambda\}} |f| \ge \lambda |\{|f| \ge \lambda\}|,$$

where $\{|f| \ge \lambda\} = \{x \in \mathbb{R} : |f(x)| \ge \lambda\}$ for $\lambda > 0$ and $|E|$ is the Lebesgue measure of the set E. This inequality suggests replacing (C_1) by (C_1^*),

$$|\{|Tf| \ge \lambda\}| \le \frac{M_1}{\lambda} \|f\|_1 .$$

Indeed, jointly with many other similar conditions, Cotlar found that (C_1^*) together with (C_p) imply that (C_r) is valid for $1 < r < p$. When Zygmund read Cotlar's result, he told Cotlar that condition (C_1^*) had already been discovered by his Polish student Jozef Marcinkiewicz, who announced it without a proof in the *Comptes Rendus* of the Paris Academy [19] in 1939, a short time before the Red Army murdered him in the Katin forest. On this topic there is a letter of Cotlar to Jaak Peetre in [15], pp. 46-47. The proof of Marcinkiewicz's theorem was published by Zygmund only in 1956 [25]. I think that the note of Marcinkiewicz is the first place where the expression "interpolation of linear operators" occurs.

Similarly to the preceding, if the linear operator $T : \mathcal{D} \to \mathcal{D}$ satisfies the conditions

$$\|Tf\|_1 \le K_1 \|f\|_1 \tag{2}$$

$$\left|\{|Tf|^2 > \lambda\}\right| \le \frac{K_2}{\lambda} \|f\|_2^2 \tag{3}$$

for every $\lambda > 0$, then $\|Tf\|_p \le K_p \|f\|_p$ for $1 < p < 2$. Cotlar proceeds to weaken condition (2), replacing $\|Tf\|_1$ by a "modified norm", which is not a norm, and it is considerably smaller than $\|Tf\|_1$. To define it, he chooses $\mathcal{D}$ in a special manner and introduces the concept of generalized support $S_L(f)$ defined with the help of an operator $L : \mathcal{D} \to \mathcal{D}$, which possesses some of the properties of the identity operator. All this leads to the main theorem of [9].

The third article in the *Revista Matemática Cuyana* [10] deals with generalizations of inequalities concerning the maximal operator of Hardy and Littlewood,

$$\Lambda f(x) = \sup_{Q(x)} \frac{1}{|Q(x)|} \int_{Q(x)} |f(t)|\, dt,$$

where $Q(x)$ denotes cubes in $\mathbb{R}^n$ with center x and with edges parallel to the coordinate axes.

Let $M : \mathcal{D} \to \mathcal{D}$ be an operator that satisfies

$$|M(f+g)(x)| \le |M(f)(x)| + |Mg(x)|,$$

and let $T : \mathcal{D} \to \mathcal{D}$ be another operator that satisfies an analogous condition. Cotlar defines two local subordination conditions between M and T:

We write $|M| \preceq O_1 |T|$ if for $f \in \mathcal{D}$ and every $x \in \mathbb{R}^n$ there exists a cube $Q(x)$ such that

$$|Mf(x)| \le O_1 \frac{1}{|Q(x)|} \int_{Q(x)} |Tf(t)| dt.$$

On the other hand, $|M| \ll O_1 |T|$ means that

$$|Mf(x)| \le O_1 \frac{1}{|Q(x)|} \int_{Q(x)} |T(\varphi_{Q(x)} f)(t)|\, dt,$$

where φ_E is the characteristic function of the set E.

Using the notation $|M_\alpha f| = |Mf|^\alpha$ and $|T_\alpha f| = |Tf|^\alpha$, Cotlar proves that if $|M_\alpha| \ll |T_\alpha|$ and if $\|Tf\|_p \le K_p \|f\|_p$ for $p > \alpha$, then

$$\|Mf\|_q \le K_q \|f\|_q$$

for every $q > p$.

Furthermore, if

$$|\{|Tf| > \lambda\}| \le \frac{O_p}{\lambda^p} \int |f|^p\, dx$$

for $p > \alpha$, then this "weak condition" is also satisfied by M. Similar results are true also for the other condition of subordination.

Cora Sadosky ([20], p. 772) writes: "The paper deals mainly with maximal operators that are to a given operator T what the Hardy-Littlewood maximal operator is to the identity operator I, and with other forms of 'localization' of operators. It also gives maximal theorems in product spaces, pioneering much later work in the subject." She mentions in particular the inequality

$$T_* f(x) \le C(Tf)^*(x) + \|T\| f^*(x),$$

which Coifman and Meyer ([4], p. 95) call "Cotlar's inequality" and say that it is the "heart of the proof" of their Theorem 21 on the convergence almost everywhere of an operator with Calderón-Zygmund kernel K. In this inequality, g^* is the Hardy-Littlewood function Λg and

$$T_* f(x) = \sup_{\varepsilon > 0} \left| \int_{|x-y|\ge\varepsilon} K(x,y) f(y) dy \right|.$$

Both Coifman-Meyer ([4], p. 102) and Sadosky emphasize the particular property of the inequality, namely that T figures on both sides.

The fourth and last part of this group of articles [11] uses the tools forged in the first three to deduce the most important contributions of this issue of the *Revista Matemática Cuyana*. Cotlar considers an integrable function K on $\mathbb{R}^n$ and defines

$$K_j(x) = \frac{1}{2^{nj}} K\left(\frac{x}{2^j}\right).$$

for $j \in \mathbb{Z}$. Cotlar considers as well $\Omega = \{P\}$, a "space" equipped with a measure μ and $\{\sigma_x : x \in \mathbb{R}^n\}$, a group of measure-preserving transformations, $\sigma_x : \Omega \to \Omega$, that is, transformations satisfying $\mu(\sigma_x(E)) = \mu(E)$ for every $x \in \mathbb{R}^n$ and $E \subset \Omega$ measurable. Moreover, he assumes that $\sigma_x \circ \sigma_y = \sigma_{x+y}$. If f is a μ-measurable function defined on Ω, Cotlar defines

$$H_m f(P) = \sum_{j=-m}^{m} \int_{\mathbb{R}^n} f(\sigma_x P) K_j(x) dx,$$

Mischa Cotlar and his wife, Yanny Frenkel, in Caracas, 2001.

and asks whether $H_m f$ converges to a function Hf when $m \to \infty$, either at almost every point $P \in \Omega$ or in the mean.

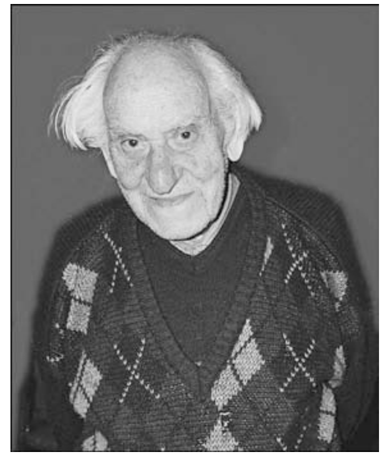
When $\Omega = \mathbb{R}^n$, $\sigma_x(t) = x + t$, $K(x) = \omega(x)|x|^{-n}$ and $\int_{|x|=1} \omega(x)dx = 0$, the operator H_m is an n-dimensional generalization of the Hilbert transformation. When Ω is a general measure space, $K(x) = -1$ for $|x| < 1$ and $K(x) = 1$ for $1 \le |x| \le 2$, then H_m is the ergodic operator.

The function $H_m f$ converges to Hf in the mean as $m \to \infty$ when $f \in L^p(\Omega, \mu)$ and $p > 1$: this is the theorem of John von Neumann. The function $H_m f$ also converges to Hf at almost every point: this is the ergodic theorem of George Birkhoff. A section of the article studies the case when instead of $\mathbb{R}^n$ one considers a locally compact abelian group.

The mathematical institute of which Cotlar was the director was disbanded after less than two years; of the *Revista Matemática Cuyana* only three issues were published. In 1957 Mischa was appointed professor of mathematics in the School of Sciences of the University of Buenos Aires. With the collaboration of Cora Ratto de Sadosky, he published the series *Cursos y Seminarios de Matemática*, for which he obtained the contributions of a group of outstanding mathematicians, for instance Laurent Schwartz, Jean-Pierre Kahane, Alberto P. Calderón, Guido Weiss, and Stephen Vági.

Cotlar himself was the author of three volumes in the series. The first and thickest, at 353 pages, has the title *Continuity Conditions for Potential and Hilbert Operators* [12]. I had the pleasure of reviewing this work for *Zentralblatt für Mathematik* (Zbl. **99**, 377) and started my report with the words: "This is a vivid and highly readable account of the recent theory of potential operators and singular integrals, due mainly to Sobolev, Thorin, Calderón-Zygmund, and the author." I lent the volume to Jacques-Louis Lions, who was visiting the University of Maryland and who at that exact time was working in the theory of interpolation of linear operators. He returned it to me saying: "He who wrote this, knows very much." Cora Sadosky mentions that Béla Szőkefalvi-Nagy, who was an editor of the collection *Ergebnisse der Mathematik und Ihrer Grenzgebiete*, suggested publishing an English translation of the volume. It is regrettable that this project has never materialized.

The other two volumes of the *Cursos y Seminarios de Matemática* written by Cotlar are Number 11, *Introduction to the Theory of Representation of Groups* ([13]) and Number 15, *Equipping with Hilbert Spaces* ([14]).



Mischa Cotlar in Buenos Aires, 2006.

Encounters in the United States

In 1966 the military entered the University of Buenos Aires and brutally beat teachers as well as students. About four hundred faculty members resigned and the golden period of mathematics in Buenos Aires ended. Mischa first went to Montevideo, and in 1967 he was appointed professor at Rutgers University. His friends believed that this appointment would suit him because, among other reasons, his disciple and coauthor Ricardo Ricabarra was at that time professor at the nearby University of Delaware. Let me add that Dover is also near College Park, so I and my colleagues at the University of Maryland were happy to see Ricabarra frequently at our functional analysis seminar on Tuesday evenings.

During the time that Cotlar was a professor at Rutgers University, he also spent some time in Nice at the invitation of Jean Dieudonné. However, Cotlar was homesick for the Hispano-American atmosphere. In 1971 he went to Caracas, then spent two years in Argentina between the University of Buenos Aires and the University of La Plata, and he finally settled in Caracas in 1974.

In 1972 the Center for Research in Applied Mathematics, Systems and Services of the Autonomous National University of Mexico invited me to teach a summer course on locally convex spaces. Lucien Waelbroeck invited me to teach a course on the same subject at the Summer School on Topological Vector Spaces in Brussels in September. The courses given at this summer school have been published ([24] and [18]). The first problem I wanted to discuss in my course was the extension of the so-called Hahn-Banach theorem to ordered semi-groups. I consulted the article of Georg Aumann [1] which begins with the following sentence: "The theorems of M. Cotlar [7] on the extension of additive monotone functions on partially ordered semi-groups, can be generalized without essential change of the method of proof and so obtain a notable round out form." In my lecture I mentioned this reference to the work of Cotlar, and after the class Carlos Berenstein, who had just received his Ph.D. with Leon Ehrenpreis, came to talk with me. He told me that he was Argentinean and that there were other Argentineans in the audience who all were happy to hear Cotlar's name quoted. I must add that after this summer school, Benno Fuchssteiner furthered the theory of operators on ordered semi-groups. His results can be found in [16], as well as in the Lecture Notes ([24], pp. 45–46) and in the updated edition of the book on topological vector spaces by Bourbaki ([2], II, p. 78, Exerc. 7).

In 1975 the collaboration between Mischa Cotlar and Cora Sadosky began. Together, and occasionally with other coauthors, they published more than fifty papers. Due to their collaboration, Cotlar visited Washington almost every year, since Cora was and still is a professor at Howard University.

At the University of Maryland we profited from Cotlar's visits by inviting him to lecture in our departmental colloquium and in the seminar of Israel Gohberg, who for several years spent a considerable amount of time in College Park. Gohberg and Cotlar had many common interests, in particular Toeplitz operators. When we invited Cotlar to lecture to us, the usual answer of this very humble man was: "Why do you people invite such an ignorant person to lecture?"

Acknowledgment

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Nebraska Department Wins Exemplary Program Award

Allyn Jackson

The University of Nebraska–Lincoln has been chosen for the 2009 AMS Award for Exemplary Program or Achievement in a Mathematics Department. In one sense, it's easy to see why. Each year the department brings in two hundred female math majors from around the country for the Nebraska Conference for Undergraduate Women in Mathematics. An even bigger program in the department is the American Mathematics Competitions, which coordinates a nationwide series of contests that draw 400,000 participants, culminating in selection of the USA team for the International Mathematical Olympiad. Yet another program is All Girls/All Math, a week-long summer mathematics camp for talented high school girls from all over the nation. The department runs a major professional development effort for middle school teachers, a Research Experiences for Undergraduates program, a summer bridge program for students about to enter graduate school—the list goes on.

In this 2008 photo from the UNL department are (left to right) graduate student Deanna Dreher, graduate student Raegan Higgins, faculty member Mark Walker, and graduate student Suanne Au. Higgins is now a visiting assistant professor at Texas Tech University.

Photographs courtesy of the Department of Mathematics at the University of Nebraska–Lincoln.

And yet if you ask members of the department about the key to its success, they don't automatically bring up these high-profile programs. "Of course the big programs make a difference," said David Manderscheid, dean of the College of Arts and Sciences and a member of the department. "There are going to be those critical moments, those impact moments, where a student will say, 'I could be a mathematician' or 'I'm good at mathematics'—these might come from big events like the Nebraska Conference for Undergraduate Women or All Girls/All Math. But it's also the day-to-day things—when a student is sitting in the lounge and a faculty member stops by and helps the student for no reason other than the fact that they both love mathematics. That's part of the fabric of the department."

An unusual synergy has taken root in the Nebraska department: the smaller, quieter day-to-day activities that nurture students and faculty provide a foundation out of which larger, more ambitious activities grow naturally, and these larger programs, in turn, provide a way to leverage the department's enthusiasm and shared sense of purpose to increase the impact it has locally and nationally. The department has achieved this synergy by a careful integration of its three main missions: research, teaching, and educational outreach. It is for this extraordinary success, operating on so many levels, that Nebraska has received the award from the American Mathematical Society.

Putting Students at the Center

"I was incredibly impressed when I came for an interview here and met so many very happy and excited graduate students," said Judy Walker, who joined the faculty in 1996. "It was overwhelming for me and made me want to come here." Walker was referring to an unusual component of the interviewing process in the Nebraska department: each job candidate has a closed-door meeting with graduate students only, no faculty. The students' opinions are passed on to the department chair and become one factor weighed in hiring decisions. "This gives the candidate a much better sense of what the graduate program is like, and it gives students insight into the candidates," she said.

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Brimming with energy and enthusiasm, Walker epitomizes a new breed of faculty member, the "complete mathematician", as Manderscheid put it, one who is deeply involved in research as well as in teaching and outreach. A mathematician like Walker might not have been so impressed had she visited the department in the 1980s. At that time it was a fairly typical Group III state university mathematics department (it moved up to Group II in the 1995 ranking by the National Research Council). Like many others, the graduate program was more oriented toward discovering who the top students were than at aiming to help all students succeed. A lot of the students dreaded the qualifying examination and put it off as long as possible. Many took eight or nine years to get their degrees. There was extra teaching capacity in the faculty, but student numbers were such that it was not worthwhile to add more graduate courses. During the 1980s the department produced twenty-three Ph.D.'s, which was a decline from the numbers in the 1960s and 1970s but still respectable for a department of its type.

But when W. James Lewis became department chair in 1988, something about that 1980s cohort of new doctorates jumped out at him: it contained no women, even though females made up about 20 percent of the graduate student population. "Jim decided that this was unacceptable, and he challenged the department to do something about it," said John Meakin, the current chair, who succeeded Lewis in 2003. Lewis realized that encouraging women in the graduate program was not only the right thing to do but could also engender a sense of purpose that had the potential to transform the department. Thus began a slow, continuous process of making the department a place where talented people—women and men, faculty and students—are given the encouragement and support they need to excel. As faculty member Sylvia Wiegand explained it, "The focus on encouraging more women graduate students to study and get Ph.D.'s in mathematics actually made the department more helpful and more nurturing to *all* graduate students and in the process made it a better place for everybody."

Some steps in that process were tangible, organizational changes. Improving financial support for graduate students was made a priority, and today nearly all students are supported by the department. Another change was in the handling of the qualifying examination. At the beginning of Lewis's tenure, the exam functioned essentially as a barrier to progress toward the degree: students so feared the exam that they put off taking it until the end of their second or third year. Because it had no desire to make the exam easier, the department instead set up Qualifying Exam Workshops, led by senior graduate students, to prepare new students and reduce the fear level. Today students attempt the exam much earlier, many in their first year. Another change was the establishment of a Graduate Student Advisory Board, which elects its own representatives and serves as an intermediary between students and the department when problems arise.

Participants in the UNL summer program All Girls/All Math.

In addition to these organizational changes, the department has taken smaller, informal steps to create a supportive environment for students. Whenever a woman mathematician comes to visit in the department, a brown bag lunch is set up so that she can meet with and talk to students. Some of the faculty, in particular the husband-and-wife team of Roger and Sylvia Wiegand, are highly sociable and hold parties for faculty and students alike. Little things like providing food for students in the Qualifying Exam Workshops and cookies for the Graduate Student Seminar heighten the social enjoyment and help students feel included.

Perhaps the most important factor is the student-centered atmosphere created by the entire faculty. "The faculty in the department were very friendly and extremely supportive," said Raegan Higgins, who finished her Ph.D. in 2008 and is now a visiting professor at Texas Tech University. "I've received help from professors with whom I've never taken a course. Their doors were always open. Yes, professors had their advisees, but all the students were *math* graduate students, and the faculty members worked to make sure we were successful."

Students: Like Junior Faculty

Walker finished her Ph.D. at the University of Illinois in 1996, and she remembers the anxiety among the students there, all of whom were nervous about the difficulties of the 1990s job market. "Here at Nebraska, the students did not have that fear," she recalled. "They felt they had the preparation

Nebraska Conference for Undergraduate Women in Mathematics 2009.

that would get them good jobs. And we had 100 percent employment." The department creates opportunities for students to build a portfolio of professional accomplishments, such as participating in a teaching seminar or being in charge of their own courses or figuring out how to implement calculator use in a lower-level course. In addition, students can get involved helping out with the department's many programs, such as All Girls/All Math, the Qualifying Exam Workshops, or Math Day, which brings in about 1,400 students from high schools around the state for a day of math competitions, puzzles, and games. "We think quite a bit about how we are going to market students when they are looking for jobs," remarked Roger Wiegand. "The students can put these things on their résumés." By participating in so many aspects of departmental life, graduate students begin to function like junior faculty members.

Graduate students have played an especially significant role in Math in the Middle, the department's professional development program for teachers. The program is run by Jim Lewis and has a multiyear, US$5.6 million grant from the Education and Human Resources directorate of the National Science Foundation. Through this program, the mathematics department has partnered with the university's teacher education department and a network of schools across the state to produce an innovative middle school mathematics curriculum and to enhance the mathematical background of teachers. The program also brings outstanding teachers to the university to earn master's degrees in mathematics, to set them on the path to becoming leaders in their schools. In addition to improving middle school education in the state, the program employs Ph.D. students in the mathematics department, providing them with experiences that hone their own teaching skills and giving them substantial interaction with schoolteachers. "This has helped a whole lot of them get jobs," Lewis remarked. "Employers see them as standing out from the crowd because they have some experience with teacher education."

A Math in the Middle workshop.

All of this means that Nebraska has become an attractive place to get a Ph.D. in mathematics, and the department has in recent years supported about twenty new students each year (though this number is expected to decline a bit in coming years due to economic conditions and other factors). About 80 percent of the students are U.S. citizens. "People are sort of beating down the doors to get into our graduate program right now," Meakin said. "They are attracted by the environment, and they see a supportive culture. Word's got around. They want to be part of it." During the period 1994–2008, the department graduated ninety-three Ph.D.'s, including thirty-nine women and sixty-six U.S. citizens or residents. All of them got offers for appropriate jobs within a few months of finishing.

In past years recruiting for the Ph.D. program was fairly local, but now Nebraska gets applications from students who are also considering places like Illinois, Michigan, Minnesota, Purdue, and Rutgers. The result is a crop of applicants with stronger backgrounds—and sometimes different expectations. "Some are coming from very competitive, world-class schools," Walker explained. "But we are trying to maintain a noncompetitive, supportive environment. It's a challenge for us." Another challenge the department faces is recruiting more students from underrepresented minority groups, an issue Walker hopes to focus on now that she is graduate recruiting chair. The graduate student body has not been devoid of minority students, however, and two African-American women received Ph.D.'s in the department in 2008.

Although it has devoted much attention to its Ph.D. program, the department has not lost sight of undergraduate education. "We have fabulous undergraduate students," said Walker. Because Lincoln is the flagship campus of the state university system, it draws the best students from across the state. "My favorite course to teach is a senior-level course," Walker remarked. "The students are just so good." The department has run a very successful NSF-supported Research Experiences for Undergraduates site since 2002, as well as another NSF-supported program called Research for Undergraduates in Theoretical Ecology (RUTE). In addition, it runs an innovative summer bridge program called IMMERSE for students who have graduated from non-Ph.D.-granting colleges and have been accepted into a graduate program in mathematics. In IMMERSE, Nebraska faculty and graduate students work with early-career faculty from small colleges to organize intensive courses in algebra and analysis. The courses focus on working through research papers rather than through textbooks. Primarily to support IMMERSE, the department has a US$2.5 million, five-year grant through the NSF's "Mentoring through Critical Transitions Points" program.

Participants in the IMMERSE summer bridge program.

Five Couples, Ten Productive Faculty

"Seeing female role models in the department, preferably more than one, is important for inspiring women graduate students to succeed in graduate school," said Sylvia Wiegand, who for many years was the only tenured woman on the faculty. When there is only one woman, Wiegand explained, she has the burden of representing all women mathematicians, and therefore her limitations are magnified in the eyes of some students. "Many women mathematicians I know have brought this up to me," she remarked. "So it is much better to have more than one!" Early on in its drive to get more women on the faculty, the department ran up against an important reality in hiring women mathematicians, namely, the two-body problem. "If we were going to be successful in hiring women in the Midwest, in an isolated location, we had to recognize how frequently women come as a part of a couple," Lewis said. "It's just a reality about university life. So a department that is resistant to dual-career issues is also a department that de facto is resistant to women."

To solve two-body problems, the department has taken advantage of a university program that provides partial salary for a spouse for a few years until a permanent position can be arranged. The department now has a total of seven women in tenured or tenure-track positions (out of a total of about forty) and five dual-career couples. Because of its growing reputation as a successful place with a thriving graduate program, the department has been able to attract excellent couple-candidates. One example is Judy Walker and her husband, Mark, both of whom have become leaders in the department. In every case the couples hired have worked out very well. With the five couples, Judy Walker said, the department has "ten very productive members of the faculty. It was not a sacrifice by any means."

The department is also conscious of the need to nurture and inspire the faculty. Partly this is a question of leadership, and in this regard the department has been fortunate. "We have had many years of strong, competent chairs who are really interested in building up the department rather than padding their own résumés," Roger Wiegand remarked. The chair can set the course and the tone for the department and provide recognition and tangible rewards for a variety of activities that support the department's goals in integrating research, teaching, and educational outreach. This does not mean that every faculty member must be deeply involved in all three aspects. But it does mean that the department as a whole is deeply involved in all three—and that all three are recognized and rewarded. Rewards might come in the form of salary raises or release time from teaching. At Nebraska the usual teaching load is four courses per year, but in fact because so many faculty are involved in activities that require release time, the average teaching load is actually closer to three courses per year.

A Model Department

"Everybody thinks their own childhood is normal until they get out into the world," said Graham Leuschke, who was a Ph.D. student in the Nebraska department from 1995 to 2000 and is now on the faculty of Syracuse University. "I was phenomenally lucky to get my training in how a math department works from Nebraska. I haven't seen it duplicated (or even really approached) anywhere else, but I use it as a model in how comfortable a department can be." In 1998 Nebraska received the Presidential Award for Excellence in Science, Mathematics and Engineering Mentoring, specifically for its success in mentoring women Ph.D.'s. But as Leuschke's remarks indicate, the commitment to improving the atmosphere for women helped the men students as well and in fact had a salutary effect on everything the department did. The department has accumulated many other accolades, including about twenty campus teaching awards to department faculty and a US$25,000 award to the department overall for its outstanding teaching.

Back in the late 1980s, when Nebraska made the decision to devote energy to supporting and encouraging more women students in its Ph.D. program, it could not have known exactly where that path would lead. The department has learned a great deal along the way. "There were some graduate students who we couldn't harm even if we tried," said Lewis. "And there were some who we couldn't help—they were not going to get past the master's. And then there were a lot in the middle, and how we mentored them, how we challenged them, how we supported them would make a difference in whether they discovered their capacity to be mathematicians." This was an investment in the department's future: in its students, their enthusiasm, talent, and energy. The investment has been a wise one.

A UNL Ph.D. Looks Back

In my second-to-last semester as an undergraduate at Nebraska, I checked in with my undergraduate advisor, Gordon Woodward in the math department, for the first time. He suggested graduate school, which I had never considered. Gordon sent me to talk with the department chair (Jim Lewis), who was extremely encouraging, and Jim sent me to talk with the graduate chair (David Logan), who thought he could arrange TA support for me. That I could continue to be a student (which I enjoyed) and someone would pay my tuition *and* pay me to do a little teaching seemed like a deal worth considering.

The atmosphere in the graduate program was (and is, from what I hear) incredibly supportive and not particularly competitive, though it was challenging. It's a friendly place. My classmates were my friends and constituted my social circle while I was in graduate school. Many remain close friends.

That there was a "critical mass" of women in the program made a huge difference, both because women students didn't feel isolated and because the faculty then didn't see it as unusual to have women in class or for advisees and thus didn't treat us differently. They might have provided an occasional extra bit of encouragement for some of us, but they didn't, for instance, avoid talking about math with us because they didn't know how to talk to women professionally, which seems to be a problem women students elsewhere have encountered.

There was constant support from everyone in the department. Sylvia Wiegand was one of the few women on the faculty at the time and an algebraist, so she was an inspiration. I took many courses from her husband, Roger Wiegand, who also provided travel support (through one of his grants) for me to attend a week-long workshop on commutative algebra in Barcelona during my last summer in the graduate program. My advisor, Brian Harbourne, helped me connect the commutative algebra I had learned to the algebraic geometry I was fascinated by. While Jim, Roger, and Brian are the people who probably went out of their way the most to support me, I think everyone in the department was supportive just in the sense of treating the graduate students with respect and providing opportunities for, rather than barriers to, our growth as mathematicians.

I see Nebraska's math program as one that grows stronger mathematically all the time and as a great place to send my own students who are interested in continuing their study of mathematics (one earned an M.S. there, another is currently in the Ph.D. program).

Nebraska has a great math department because it has great people *in* the math department—great mathematicians, but great *people* too. One thing I consistently saw at UNL was a recognition that different people could best contribute in different ways, and all those ways were valued. It came as a huge and disappointing shock to me when I later learned that some places simply didn't think that organizing lower-division classes, training TAs, and doing a great job teaching a huge number of freshmen were particularly important activities for the department.

I think my story is perhaps typical of many of the department's graduate students, at least in the timeframe I was there: students who were strong academically but not necessarily savvy about the academic world. Going to graduate school was one of the best decisions I ever made, and staying put at UNL was definitely the way to go for me.

—*Stephanie Fitchett, University of Northern Colorado*

What Is New in LaTeX? II. TeX Implementations, Evolution or Revolution

G. Grätzer

This is the second of a series of columns updating the mathematical community about some current developments in TeX and TeXing.

—Andy Magid

The Advice of the Red Queen

Donald Knuth coded TeX on a mainframe computer. It migrated quickly to Unix, and most mathematicians used TeX on a terminal attached to a mainframe or a Unix workstation.

TeX was conceived as one of four applications necessary to produce a mathematical article or book. At the command line you invoked an *editor* to type (edit) the source file and then you quit the editor. Again, at the command line, you instructed TeX to typeset the source file; this produced the `dvi` file. Another application invoked at the command line presented the `dvi` file on the monitor and/or printed it (on a remote printer). You noted the changes you wanted to make, opened the source file with the editor, made the changes, and the cycle continued.

Personal computers became powerful enough for a TeX implementation in the mid 1980s. Michael Doob [2] and [3] gives a snapshot in 1990. Reporting on a Mac implementation, Textures, Doob (who has a Unix background) writes, "The fact that you can edit your source file, run TeX, and preview the result from within the Textures program is both unique and very helpful." Such implementations we then called "integrated". For a detailed review of two products as of 1994 (Textures and PCTeX for Windows), see George Grätzer [4].

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There was a recurring complaint about my LaTeX books before the fourth edition [5] because I assumed that the reader had a TeX implementation and knew how to use it. Readers wanted me to guide them through installation and use of TeX. But this was not possible. Norman Walsh's book [8] in 1994 lists fourteen TeX implementations for Mac and Windows, in addition to the older Unix and Linux implementations (which were made even more diverse by the dozens of editors, some quite TeX capable). Each one of these required special instructions to obtain (many utilizing `ftp` downloads), install, and start to use.

LaTeX's strength lies in the hundreds of packages that are available to perform specific tasks. In the previous editions of [5], I had to describe how to obtain a package and how to install it.

Fast forward to today. [5] now has an appendix giving detailed instructions on how to install WinEdt and MiKTeX for Windows and TeXShop with TeX Live for the Mac. In my opinion, this covers 90% plus of all users and *almost all* PC and Mac users. And Unix and Linux users do not need my help, they are all technical experts…

It is equally dramatic that these two installations give you not only a TeX implementation, but also all conceivable LaTeX packages—in excess of 1,000! No more where to get it from, how to unpack it, and where to put it. In addition, these implementations give you everything from TeX to LaTeX to pdfTeX, pdfLaTeX, even XeTeX and XeLaTeX (see [7]).

Evolution or revolution. You decide. Things are progressing really quickly to make the installation and use of LaTeX easy. As the Red Queen famously said to Alice (in Lewis Carroll [1]), "Now *here* you see, it takes all the running *you* can do, to keep in the same place. If you want to get somewhere else, you must run at least twice as fast as that!"

How We Got Here

The foundation: CTAN

In the 1980s there were a number of `ftp` sites that provided TEX-related material, such as LATEX packages. In 1991 at a EuroTEX conference, the idea of a unified site was proposed: CTAN, the Comprehensive TEX Archive Network. It was built in 1992 by Rainer Schöpf and Joachim Schrod in Germany, Sebastian Rahtz in the UK, and George Greenwade in the U.S. Today, it is maintained by Rainer Schöpf, Joachim Schrod, Robin Fairbairns, and Jim Hefferon. The main CTAN nodes serve downloads of more than 6 TB per month, not counting the seventy-five mirror sites worldwide.

teTEX

In 1994 Thomas Esser decided to build a complete TEX distribution for Unix users; he called it teTEX. He aimed at creating a distribution that

- provides a TEX system that consists only of free software;
- is simple to install, use, and maintain;
- includes as much useful documentation as possible.

Esser maintained by himself this forerunner of the TEX installations. Since each release took longer to prepare, in 2006 he stopped supporting teTEX, and suggested that the teTEX users should switch to TEX Live.

MiKTEX

Maybe the best current TEX implementation for Windows is MiKTEX, developed by Christian Schenk in 2000. MiKTEX updates itself by downloading new versions of previously installed components and packages. It introduced "download on demand": it will download and install any package that has not yet been installed but is requested by the current document.

MiKTEX 2.8—with some nice new features—should be available by the time this article appears. But Schenk writes, "Now that TEX Live has a nice package manager of its own, I do not expect much interest in the MiKTEX package manager for Linux."

TEX Live

TEX Live is a collection of TEX installations that has been developed by an extraordinary collaboration of TUG (TEX User Group) and the German, Dutch, and Polish TEX Users Groups (DANTE e.V., NTG, and GUST) over the last twelve years. It is developed to work with Windows, Mac OS X, and most flavors of Unix and Linux. At the start, for a number of years, it was under the direction of Sebastian Rahtz; presently this role is assumed by Karl Berry.

The origin of the project was the 4AllTEXCD for MS-DOS users developed in 1993 by the Dutch TEX Users Group. TEX Live started out with the goal of extending this project to include all major operating systems. This proved to be too ambitious at the time. The discussions, however, led to the formation of the TUG Technical Council working group on a TEX Directory Structure (TDS), necessary for the creation of consistent and manageable collections of TEX support files. A complete draft of the TDS was published in the December 1995 issue of *TUGboat*, and the recommendations therein formed the foundation of TEX Live.

The major signposts of TEX Live are

- The first edition appeared in May 1996.
- Karl Berry launched a major new release of Web2c, which included nearly all the features that Thomas Esser had added in teTEX. The second edition in 1997 was based on Web2c, with the addition of the configuration script of teTEX.
- The fourth edition included a complete Windows setup.
- The major change for TEX Live 5 in 2000 was the removal of all nonfree software.
- Mac OS X support was added in the seventh edition of 2002.
- TEX Live 2006–07 added XƎLATEX.

TEX Live 2008 is the result of a major redesign and re-implementation of the TEX Live infrastructure. Among the many benefits is a file manager that allows easy upgrades (which MiKTeX has provided for many years). LuaTEX has been added, which provides an excellent scripting language. The new script `tlmgr` manages TEX Live after the initial installation. It handles package updates, language files, and local additions. For a complete listing of new, changed, and deleted features, and the acknowledgments, see the documentation

```
http://www.tug.org/texlive/doc/
 texlive-en/texlive-en.html#news
```

```
http://www.tug.org/texlive/doc/
 texlive-en/texlive-en.html#ack
```

I hope that the hundreds of volunteers who contributed to TEX Live do not resent the very sketchy presentation of this huge project or the omission of their names and contributions due to space limitations.

Helping the Users

Curiously, as the TEX installations get so much more technically complicated, the main direction of the development is clear: from the user's perspective, everything is getting so much simpler.

On my Mac, I download MacTeX (which includes TEXShop and TEX Live). I get a standard Mac installation dialog box: I confirm the location of the installation on my hard disk, click on install, and I am done. Compare this with the thirty-page installation manuals of just a few years ago.

Consider the extreme technical difficulties Jonathan Kew had to overcome to code XƎTEX (see [7]). The benefit to a user is that any font installed on the computer can be used by simply typing the name of the font. What could be simpler? Compare this with the difficulty of installing and using a new font in a typical Unix setup.

But real progress is being made on a different front as well. LaTeX has a well-deserved reputation of having a steep learning curve. A lot more attention is being paid now to helping the beginner.

Richard Koch came up with some interesting ideas for TeXShop. In the `Help` menu, you can choose *George Grätzer's Short Course*, and you get Part I of the book [5]. You can also choose *TeXShop Demos* to get video presentations for novices on how to use TeXShop.

In my books, I often point out that the gentlest learning curve for LaTeX is provided by a knowledgeable friend, who sits down with you and gets you started, and who is available most any time to help you out. Of course, few of us have friends with so much free time to share.

Probably, the next best thing is a video presentation. I got so encouraged by Koch's idea that I turned the *Short Course* into a series of video presentations. You can find it at

```
http://www.ctan.org/tex-archive/info/
 Math_into_LaTeX-4/
```

I think this idea of Koch's is the future of user assistance: to provide a series of video presentations on

- how to get started with the application,
- how to get started with LaTeX,
- the main features of TeX implementation.

There is an example topic of the third type: for book writers, Koch introduced a very helpful new feature in TeXShop; at the click of the mouse, index entries pale to a very light yellow, making the marked-up book easy to edit. A two-minute video presentation demonstrates how to set this feature up by modifying the tool bar and how to activate/deactivate this feature.

TeX Live now comes with extensive documentation (in many languages). It also provides the command line utilities `texdoc` and `texdoctk` (the latter with a GUI interface) to help you find what you need.

WinEdt 6 provides access to this documentation and its `Help` menu provides an interface to the *Short Course* and its video presentations.

Intermediate users of WinEdt will find a ready-to-compile LaTeX thesis, an AMS article, and a document that demonstrates how to include graphics, how to use the `hyperref` package, and how to deal with international characters.

On the Horizon

This is the first year that TeX Live appears with file management features. This lays the foundation, but we need more.

We need to download and install packages on demand (as required by the document). Once we have that, we could get away from the one gigabyte size download for the initial installation. At the first installation, we should get maybe a hundred or so of the most often used packages, and the rest should come as necessary. Of course, the full download could always be an option if you go on vacation with a notebook computer to a remote place with no Internet connection.

Even better, at the first installation, you should be able to choose

- minimal installation,
- full installation,
- selective installation.

The last would provide you with a list of packages, and you select the ones you need installed. Any re-installation should only update the packages you chose to have in the first place.

Although XƎTEX takes care of your need for text fonts, math font support is presently lacking. This is okay, since very few math fonts are available today. However, there are some math fonts on the horizon. Once they are ready, it would be nice to be able to invoke them with the same ease as we can now invoke text fonts.

Coming soon to a computer near you: the STIX project, with thousands of math symbols. We shall discuss this in Part III of this series.

Acknowledgments

In Parts I and II, I was helped by many TeX experts: Barbara Beeton, Karl Berry, Michael Doob, Jim Hefferon, Victor Ivrii, Jonathan Kew, Richard Koch, Herb Schultz, Aleksander Simonic. For their patient teaching of a nontechnical LaTeX user, thanks.

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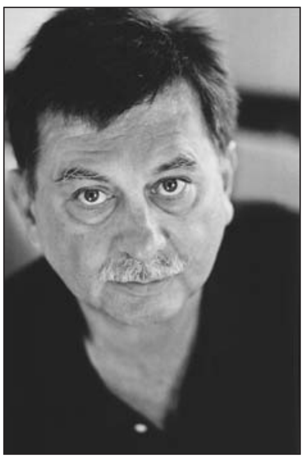
2009 JPBM Communications Award



George Csicsery

The 2009 Communications Award of the Joint Policy Board for Mathematics (JPBM) was presented at the Joint Mathematics Meetings in Washington, DC, in January 2009.

The JPBM Communications Award is presented annually to reward and encourage journalists and other communicators who, on a sustained basis, bring accurate mathematical information to nonmathematical audiences. JPBM represents the AMS, the American Statistical Association, the Mathematical Association of America, and the Society for Industrial and Applied Mathematics. The award carries a cash prize of US$1,000.

Previous recipients of the JPBM Communications Award are: James Gleick (1988), Hugh Whitemore (1990), Ivars Peterson (1991), Joel Schneider (1993), Martin Gardner (1994), Gina Kolata (1996), Philip J. Davis (1997), Constance Reid (1998), Ian Stewart (1999), John Lynch and Simon Singh (special award, 1999), Sylvia Nasar (2000), Keith J. Devlin (2001), Claire and Helaman Ferguson (2002), Robert Osserman (2003), Barry Cipra (2005), Roger Penrose (2006), Steven H. Strogatz (2007), and Carl Bialik (2008).

The 2009 JPBM Communications Award was presented to George Csicsery. The text that follows presents the selection committee's citation, a brief biographical sketch, and the recipient's response upon receiving the award.

Citation

The 2009 JPBM Communications Award is awarded to George Csicsery for his extraordinary body of work showing the process of mathematical thinking through the medium of film.

George Csicsery is an artist who has employed his talents to communicate the beauty and fascination of mathematics and the passion of those who pursue it. This began with the film *N is a Number: A Portrait of Paul Erdős* (1993), which has been broadcast in Hungary, Australia, The Netherlands, Japan, and the United States. In 2008 he completed the biographical documentary *Julia Robinson and Hilbert's Tenth Problem*, and *Hard Problems: The Road to the World's Toughest Math Contest*, a documentary on the preparations and competition of the U.S. International Mathematical Olympiad team in 2006. Other recent works include *Invitation to Discover* (2002), made for the Mathematical Sciences Research Institute, and *porridge pulleys and Pi* (2003), a 30-minute piece on mathematicians Hendrik Lenstra and Vaughan Jones which premiered at Téléscience in Montreal, Canada, in November 2003. Through his films, George Csicsery expresses the excitement experienced by mathematically gifted individuals, and he has delighted mathematicians, students, and the public with his intriguing stories told through the media of film.

Biographical Sketch

George Paul Csicsery, a writer and independent filmmaker since 1968, was born in Germany in 1948 and immigrated to the United States in 1951. As an undergraduate at the University of California, Berkeley, in 1969, he made three short experimental films, and these led to a masters degree in cinema from San Francisco State College. He has directed 26 films, including dramatic shorts, performance films, and documentaries.

His screenplay, *Alderman's Story*, a period epic set during King Philip's War in New England in 1675, was awarded first prize at the Rhode Island International Film Festival Screenplay

Competition in 2005. His articles, reviews, and interviews have appeared in film journals, newspapers, and many other publications. He has taught cinema at the Film Arts Foundation, San Francisco State University, and at the University of California, Davis. Csicsery's films on historical, ethnographic, and cultural subjects include works on pirates, prostitutes, romance novel writers, policemen, scouts, and Transylvanian folk musicians.

In 1988 Csicsery's career took a dramatic turn when he began work on a biographical film about Paul Erdős. *N is a Number: A Portrait of Paul Erdős*, is still his best known and most popular work, with broadcasts, screenings, and DVD copies in constant demand worldwide. That film led to more projects on mathematical subjects. *Invitation to Discover* about MSRI, and *porridge pulleys and Pi*, about Hendrik Lenstra and Vaughan Jones, were both completed in 2003. *The Right Spin* (2005), the story of astronaut Michael Foale's role in saving the Mir space station in 1997, was made for Mathematics Awareness Month. *Julia Robinson and Hilbert's Tenth Problem*, a one-hour biographical documentary, premiered in January 2008 at the Joint Mathematics Meetings. *Hard Problems: The Road to the World's Toughest Math Contest* also premiered there in January 2008. The feature documentary about American students at the 2006 International Mathematical Olympiad (IMO) was produced with the Mathematical Association of America. Csicsery is currently completing a new project for MSRI and a film of interviews with mathematician Paul Halmos for the MAA. Both are scheduled for 2009 release on DVD. Future projects include films about Ronald Graham and coincidence.

Response

It is a great honor to receive this award, especially when I look at the list of previous recipients—Martin Gardner, Constance Reid. Wow! I am hardly in their league.

It is tremendously satisfying to be recognized in this fashion, especially because making films about mathematicians has often been such an uphill and lonely battle. The most frequent question I get when I try to explain what I do is, "Who will be interested in a film about mathematicians?" Believe it or not, the first person to ask me that question was Paul Erdős. And I had no acceptable answer for him until after *N is a Number* was broadcast in five countries and had sold 4,000 copies in VHS.

Why did I start making films about mathematicians? My standard explanation was that I was a refugee from the social sciences looking for terra firma, and mathematicians seemed interested in actually finding out if something is true or not. More recently, I've developed another theory. There are more people in mathematics than in any other field who claim that they don't know anything. And to someone who really doesn't know anything, that is almost like having a community.

The task of explaining mathematics and mathematicians on film would be impossible without the patience and passion for the subject that I've encountered. Mathematicians are the most enthusiastic expositors of their subject, helping me look for ways to translate complex ideas under the severe time limitations imposed by the medium of film.

The list of people behind the successful completion of the films being recognized is a long one, starting with Charles L. Silver, Ronald Graham, Paul Erdős, Hyman Field, and includes Don Albers of MAA publications, and Klaus Peters, who both provided early opportunities for distribution of my films. David Eisenbud and Bob Osserman at MSRI became key advocates. Jim Carlson and the Clay Mathematics Institute, along with Will Hearst, provided financial support for the film about Julia Robinson and Hilbert's Tenth Problem. I wish to thank Constance Reid, Martin Davis, and Yuri Matiyasevich for their heroic efforts as protagonists in that film. *Hard Problems* was a dream project brought to me by Joe Gallian, who then raised the funding. It was supported by everyone at MAA and AMC, especially Steve Dunbar and Tina Straley. There are many others, including all of the people on the film production teams—cinematographers, editors, sound engineers, composers, and assistants. They all deserve this recognition and my thanks.

2009 Award for an Exemplary Program or Achievement in a Mathematics Department

The Award for an Exemplary Program or Achievement in a Mathematics Department was established by the AMS Council in 2004 and was given for the first time in 2006. The purpose is to recognize a department that has distinguished itself by undertaking an unusual or particularly effective program of value to the mathematics community, internally or in relation to the rest of society. Departments of mathematical sciences in North America that offer at least a bachelor's degree in mathematical sciences are eligible. For the first three awards (2006–2008), the prize amount was US$1,200. After an endowment by an anonymous donor in 2008, the amount became US$5,000 starting with the 2009 prize.

The award is presented by the AMS Council acting on the recommendation of a selection committee. For the 2009 award, the members of the selection committee were: Steven A. Bleiler (chair), Amy Cohen, William B. Jacob, and Michael E. Moody.

The previous recipients of the award are Harvey Mudd College (2006), the University of California, Los Angeles (2007), and the University of Iowa (2008).

The recipient of the 2009 Award for an Exemplary Program or Achievement in a Mathematics Department is the THE MATHEMATICS DEPARTMENT AT THE UNIVERSITY OF NEBRASKA-LINCOLN. What follows is the selection committee's citation.

Citation

The Department of Mathematics at the University of Nebraska-Lincoln is granted the 2009 Award for Exemplary Program or Achievement in a Mathematics Department.

For well over a decade, the Department of Mathematics at the University of Nebraska-Lincoln has been at the forefront of efforts to increase the representation of women in the profession, has improved the mentoring and early career training of students, and has achieved a national leadership position in the mathematical education of teachers, all while concurrently growing its doctoral program in both size and national profile.

The effect on the profession of the department's efforts is evidenced by its continuing organization of outreach programs and conferences such as All Girls/All Math; the Nebraska Conference for Undergraduate Women in Mathematics; and Nebraska IMMERSE, a summer bridge program for students from non-Ph.D.-granting colleges accepted into a mathematics graduate program at a U.S. university. The department has forged a strong partnership with its counterparts in the University of Nebraska-Lincoln Department of Teaching Learning and Teacher Education, and through its participation in the Carnegie Initiative on the Doctorate. The department has made substantial contributions to the national dialog on graduate education.

The Society is pleased to recognize the Department of Mathematics' highly successful integrated approach to its multiple missions of research, teaching, and education outreach through this award.

MAA Prizes Presented in Washington, DC

At the Joint Mathematics Meetings in Washington, DC, in January 2009, the Mathematical Association of America (MAA) presented several prizes.

Gung and Hu Award for Distinguished Service

The Yueh-Gin Gung and Dr. Charles Y. Hu Award for Distinguished Service to Mathematics is the most prestigious award made by the MAA. It honors distinguished contributions to mathematics and mathematical education, in one particular aspect or many, and in a short period or over a career.

ROBERT MEGGINSON received the 2009 Gung and Hu Award. Megginson has devoted a great deal of time and energy to addressing the problem of underrepresentation of minorities in mathematics. He has worked on the problem through committees of professional societies and through programs on college and university campuses, as well as through direct interaction with minority students. For example, beginning in 1992, he helped design and teach programs for precollege students at Turtle Mountain Community College, a tribally controlled college of the Turtle Mountain Chippewa Nation in North Dakota. He has also mentored many undergraduate and graduate students from varied backgrounds who have gone on to receive degrees in mathematically based disciplines.

Megginson was co-chair of the MAA Committee on Minority Participation in Mathematics, as well as chair of the MAA's Coordinating Council on Human Resources, of the Human Resources Advisory Committee of the Mathematical Sciences Research Institute (MSRI) in Berkeley, and of the subcommittee of the AMS Committee on the Profession charged with identifying successful diversity programs. Currently Megginson chairs the Committee on Opportunities in Science of the American Association for the Advancement of Science. His service in helping underrepresented students succeed in mathematics and science is complemented and enriched by many of his other professional activities, such as serving as co-principal investigator on several grants that have helped fund MAA's National Research Experiences for Undergraduates Program. At the University of Michigan, he designed, directed, and implemented a reformed precalculus program that emphasized cooperative learning, and he was director of the Michigan mathematics laboratory, a walk-in tutoring service.

Megginson has been on the faculty of the University of Michigan since 1992. His mathematical area is functional analysis, specifically the geometry of Banach spaces. He served as deputy director of MSRI from 2002 through 2004, after which he returned to the University of Michigan, where he is currently Arthur F. Thurnau Professor of Mathematics and Associate Dean for Undergraduate and Graduate Education in the College of Literature, Science, and the Arts.

For his record of mentoring students and other works on underrepresentation, Megginson was one of ten individuals who were honored at the White House with the 1997 U.S. Presidential Award for Excellence in Science, Mathematics, and Engineering Mentoring.

Haimo Awards for Teaching

The Deborah and Franklin Tepper Haimo Awards for Distinguished College or University Teaching were established in 1991. These awards honor college or university teachers who have been widely recognized as extraordinarily successful and whose teaching effectiveness has been shown to have had influence beyond their own institutions.

The 2009 Haimo Awards were presented to MICHAEL BARDZELL, DAVID PENGELLEY, and VALI SIADAT.

Michael Bardzell has had a pronounced impact, both locally at Salisbury University in Maryland and well beyond. At Salisbury he has involved in research many students with a variety of backgrounds. Over the last twelve years, his students have presented their work at various national and regional venues, and his students have won awards for their presentations. A variety of publications have resulted from their work. Together with faculty from five institutions, Bardzell received two grants from the National Science Foundation, one of which focused on visualizing abstract mathematics and included his organization of two summer undergraduate research retreats at New College of Florida. The grants have also led to a set of laboratory exercises that help students study and explore abstract algebra, dynamical systems, and number theory. In his twelve years at Salisbury, Bardzell has taught twenty-four different courses, many of which he designed. These include a capstone course, two cross-listed courses with computer science, and four graduate courses for in-service teachers. In partnership with school districts in Maryland and Delaware, he and his colleagues have developed a series of workshops for high school teachers in geometry, in real world mathematics, and in algebra. Bardzell won the Distinguished Faculty Award of Salisbury in 2001 and the MAA MD-DC-VA Sectional Teaching Award in 2007.

For the past twenty years, David Pengelley has been continually reinventing his teaching, and the mathematical community has benefited greatly from those innovations. At the beginning of the calculus reform movement, he and his colleagues developed a program for using major multi-step problems to engage students in imaginative thinking, to challenge them to integrate ideas, and to express them in a written report. These teaching innovations have been disseminated in the MAA volume, *Student Research Projects in Calculus*, a best-seller. Pengelley is passionate about using primary historical sources in teaching. At New Mexico State University, he developed honors courses based on primary sources, leading to two coauthored textbooks, and has disseminated this pedagogy through many national and international presentations and minicourses. Pengelly has also done his own original research in the history of mathematics. NSF has supported Pengelley's innovations through seven multi-year grants spanning twenty years. In 2007 he won a Faculty Outstanding Achievement Award from the College of Arts and Sciences at New Mexico State University, and in both 1993 and 2008 he won the MAA Southwestern Section Teaching Award. On campus, Pengelley is an extremely popular and successful teacher. His teaching methods and their connections to the history of mathematics have been disseminated through a wide variety of publications and talks, nationally and internationally, and even through a broadcast interview with the BBC.

Vali Siadat of City College of Chicago, Richard J. Daley Campus, cares deeply about the success of his students and does whatever it takes to help them achieve their educational goals. He is best known for his Keystone Project, a synergistic teaching program that focuses on frequent assessment, constant feedback, and student support. In a controlled experiment with 800 entering college students, 63% of Keystone students passed an "Elements of Algebra" class, while only 18% passed in the control group. An interesting concomitant result was that the Keystone students also achieved significant positive gains on a standardized reading test whereas those in the control group did not. Siadat has also been a leader in Project Access, a NASA-funded mathematics-based summer program for low-income and minority students from middle schools and high schools. As a scholar with two doctorates, one in pure mathematics and another in mathematics education, he has done research to develop innovative approaches in pedagogy and improvement of teaching of undergraduate mathematics. Among his awards are the Distinguished Professor Award of Richard J. Daley College in 1999-2000, the 1999 Exemplary Initiatives in the Classroom Award from the National Council of Instructional Administrators, and the MAA Illinois Section's Distinguished Teaching Award in 2002. In 2005 he received the Carnegie Foundation for the Advancement of Teaching Illinois Professor of the Year Award.

Chauvenet Prize

The Chauvenet Prize recognizes a member of the MAA who has written an outstanding expository article. First awarded in 1925, the prize is named for William Chauvenet, who was a professor of mathematics at the United States Naval Academy.

Harold Boas of Texas A&M University won the 2009 Chauvenet Prize for his article "Reflections on the arbelos", *American Mathematical Monthly*, **113** (2006), no. 3, 236–249. As with most good mathematical stories, this fascinating piece begins with a problem, a geometry problem that was sent to the author by a young mathematician and that deals with a mathematical figure known as an "arbelos". (An arbelos is the region bounded by three semicircles, tangent in pairs, with diameters on the same line.) The subject's long history, on which much of this erudite and beautifully written paper dwells, is smoothly interwoven with interesting results and elegant proofs. Reflections and inversions in lines and circles are key tools. The classical remarkable theorem, attributed to Pappus by default, concerning an infinite chain in an arbelos, is described. The proof by Pappus was a tour de force of Euclidean geometry, while the modern proof using inversion is elegantly simple. Connections are made with Pythagorean triangles (i.e., right triangles that are similar to triangles having sides with integral lengths), and also with the Gothic arch. The author made a surprising discovery. Textbooks in solid mechanics deal with "Mohr's circle", which comes up in analyzing shear stress. The relevant linear-algebra theorem is that the range of a certain mapping is an arbelos. In addition to Pappus, Jacob Steiner, William Thomson (Lord Kelvin), and Leon Bankoff play roles in the story. (Bankoff was a dentist with a strong interest in mathematics, including the arbelos.) This paper stands out as a model of expository excellence.

Euler Book Prize

The Euler Book Prize is given to the author(s) of an outstanding book about mathematics. The prize was given for the first time in 2007, to commemorate the 300th anniversary of the birth of Leonhard Euler.

SIOBHAN ROBERTS was awarded the 2009 Euler Prize for her book *King of Infinite Space: Donald Coxeter, the Man Who Saved Geometry* (Walker and Company, 2006). This book by Siobhan Roberts gives an intimate and engaging portrait of one of the most influential mathematicians of the last century. It also provides a mathematical history of those years, including the currents set in motion by Hilbert's 23 problems, the influence of Bourbaki, and the unexpected applications of mathematics to computer science, communications, information, crystallography, medical research, environmental studies, as well as in art—Coxeter's work directly inspired*Circle Limit III* by M.C. Escher. Above all, it gives a superbly readable account, in personal terms, of the search for beauty that sets mathematics in motion and of the synergy of individual and group efforts that make it happen. It's an engaging page-turner, even for nonmathematically trained readers, and it will offer them an insider's look at the world of mathematics and the people who create it. The scope of Roberts' research and scholarship is impressive and fully documented in fine print with 74 pages of endnotes, a 14-page bibliography, and eight appendices. *King of Infinite Space* will fascinate the general reader with its detailed and frank account of Coxeter's personal life. It will also strike a special chord with mathematicians, because it honors the spirit of wonder and openness that Coxeter embodied in his approach to mathematics.

Certificates of Meritorious Service

Each year the MAA presents Certificates of Meritorious Service for service at the national level or for service to a section of the MAA. Those honored in 2009 are: Carl C. Cowen (Purdue University), Indiana Section; Richard Anderson (Louisiana State University), Louisiana-Mississippi Section (posthumous award); John Fuelberth (Wayne State College), Nebraska-Southeast South Dakota Section; John R. Michel (Marietta College), Ohio Section; and David R. Stone (Georgia Southern University), Southeastern Section.

—From MAA announcements

AWM Awards Given in Washington, DC

The Association for Women in Mathematics (AWM) presented two awards at the Joint Mathematics Meetings in Washington, DC, in January 2009.

Louise Hay Award

Established in 1991, the annual Louise Hay Award recognizes outstanding achievements in any area of mathematics education. Louise Hay was widely recognized for her contributions to mathematical logic and her devotion to students.

The 2009 Hay Award was presented to Deborah Loewenberg Ball of the University of Michigan. Ball was honored for her important research into the mathematics needed by elementary school teachers, her ability to communicate mathematics to children and related understandings to diverse communities of adults, and her effective national and international leadership.

Deborah Ball presents a unique combination of highly integrated talents and accomplishments—long experience and continued engagement as an accomplished elementary mathematics teacher; original, rigorous, and prolific contributions on the frontiers of research in mathematics education; a high standing and respect among research mathematicians; and visionary intellectual and administrative leadership to reform the institutions of mathematics teacher education in this country. She is currently dean of the School of Education at the University of Michigan.

One of Ball's primary research interests is the mathematical knowledge needed for teaching (MKT). Her investigations of what MKT is, how it may be measured, and how teachers' knowledge of it impacts the learning of children are providing a foundation for reforms of the mathematics education and development of teachers.

Ball was a major contributor to several NRC projects, notably the one that produced the widely-cited report *Adding It Up*. She was one of the few educators on the Glenn Commission, otherwise populated mainly by members of Congress and business leaders. She headed the subgroup on teaching of the National Mathematics Advisory Panel, whose report was recently released. She chaired the International Commission on Mathematics Instruction Study 15 on the Professional Education and Development of Teachers of Mathematics. Deborah Hughes Hallett of the University of Arizona (and an earlier recipient of the Hay Award) wrote, "Over the last decade, Deborah has been extraordinarily effective in promoting real collaboration and communication. In countless presentations, videotapes, and live demonstrations, she has displayed the insight a mathematics educator brings to an elementary school classroom."

Schafer Prize

The Alice T. Schafer Prize for Excellence in Mathematics by an Undergraduate Woman was established in 1990. The prize is named in honor of Alice T. Schafer, one of the founders of AWM and one of its past presidents.

The 2009 Schafer Prize was presented to Maria Monks of the Massachusetts Institute of Technology. Monks, a junior mathematics major, has already written six research papers; one has been accepted for publication by the *Journal of Combinatorial Theory Series A*, three have been submitted to leading research journals, and the other two are in nearly final form. On five of these six papers she is the sole author. Her outstanding work is already so widely known that she gets invitations to speak at mathematics meetings and in research departments. At the same time, Monks does exceptional work in her classes at MIT and has achieved a perfect grade point average. She has furthermore contributed phenomenal service to the mathematics community, for example by coaching the USA China Girls' Math Olympiad team.

Monks' outstanding research abilities, her exceptional coursework, and her great leadership in the mathematics community make her this year's winner of the Schafer prize. In her response upon receiving the prize, she particularly thanked her father, who she says "opened my eyes to the beauty of mathematics and served as a coach, teacher and mentor throughout my childhood, inspiring me to pursue my love of mathematics to the best of my ability."

Additionally, the accomplishments of three outstanding young women, all senior mathematics majors, were recognized by honorable mentions: Doris Dobi, Massachusetts Institute of Technology; Nicole Larsen, Georgia Institute of Technology; and Ila Varma, California Institute of Technology.

—From AWM announcements

Mathematics People

Wood and Wickelgren Awarded AIM Five-Year Fellowships

Melanie Matchett Wood of Princeton University and Kirsten Graham Wickelgren of Stanford University have been named recipients of the 2009 American Institute of Mathematics (AIM) Five-Year Fellowship.

Wood will complete her Ph.D. at Princeton in 2009 under the direction of Manjul Bhargava; her thesis is titled *Moduli Spaces for Rings and Ideals*. She conducts research at the interface of number theory and algebraic geometry, constructing geometric moduli spaces whose points correspond to number-theoretic objects of interest. She represented the United States on the International Mathematics Olympiad team in 1998 and 1999. As an undergraduate at Duke University, she was a Putnam Fellow in 2002 and also won the Alice Schafer Prize that year. In 2003 she won the Morgan Prize for best original mathematical research by an undergraduate.

Wickelgren will receive her Ph.D. in 2009 from Stanford University under the direction of Gunnar Carlsson. She conducts research at the interface of topology and arithmetic and algebraic geometry. In her thesis she examines cohomological obstructions to the existence of sections of the map on étale fundamental groups induced by the structure map of a curve over a number field.

The Five-Year Fellowship covers sixty months of full-time research and provides funds for travel and equipment. The compensation is US$4,000 per month.

—From an AIM announcement

Sarkar Selected as Clay Research Fellow

Sucharit Sarkar of Princeton University has been awarded a 2009 Clay Research Fellowship by the Clay Mathematics Institute (CMI). He will receive his Ph.D. from Princeton University in 2009 under the guidance of Zoltán Szabó. His research area is low-dimensional topology, and he is currently working on Heegaard Floer homology for three-manifolds and knots inside three-manifolds.

Clay Research Fellows are appointed for terms ranging from two to five years; graduating doctoral students and mathematicians within three years of receiving the doctoral degree are eligible for the fellowships. The primary selection criteria for the fellowship are the exceptional quality of the candidate's research and the candidate's promise to become a mathematical leader.

—From a CMI announcement

Gordina Awarded Michler Prize

Maria Gordina of the University of Connecticut has been awarded the third annual Ruth I. Michler Memorial Prize by the Association for Women in Mathematics (AWM) and Cornell University. She will spend the fall semester of the 2009–2010 academic year at Cornell. She received her Ph.D. from Cornell University, investigating holomorphic functions and the heat kernel measure. Her primary research interests involve heat kernel measures and their properties in the context of infinite-dimensional nonlinear spaces.

The Michler Prize is awarded annually to a woman recently promoted to associate professor or an equivalent position in the mathematical sciences. It consists of a residential fellowship in the Cornell University mathematics department without teaching obligations. The amount of the award is US$45,000, with an additional travel allowance provided by the Cornell University mathematics department.

—From an AWM announcement

Bozeman Receives AAAS Mentor Award

Sylvia T. Bozeman of Spelman College has received the 2008 Mentor Award from the American Association for the Advancement of Science for her commitment to increasing the number of African-American women with doctorates in mathematics.

Bozeman earned her Ph.D. from Emory University. Since she became chair of the mathematics department at Spelman, twenty Spelman mathematics graduates have received doctoral degrees in mathematics or mathematics education. Half of those students attribute their decisions to attend graduate school directly to Bozeman's encouragement and mentoring. She also is cofounder and codirector of the Enhancing Diversity in Graduate Education (EDGE) program, a joint effort between Spelman College and Bryn Mawr College to improve retention rates of female students in mathematics graduate programs. Between 1998 and 2006, 105 women completed the program; 14 of these, including 5 who are African-American, received their doctoral degrees in mathematics.

The AAAS Mentor Award honors members of the AAAS who have mentored and guided significant numbers of underrepresented students toward Ph.D. degrees in the sciences and who have demonstrated scholarship, activism, and community building on behalf of underrepresented groups, including women of all racial or ethnic groups; African-American, Native American, and Hispanic men; and people with disabilities. This award recognizes individuals in the early or midcareer stage who have mentored students for less than twenty-five years. The recipient receives US$5,000 and a commemorative plaque.

—From an AAAS announcement

ASL Karp and Sacks Prizes Awarded

Zlil Sela of Hebrew University has been awarded the 2008 Karp Prize of the Association for Symbolic Logic (ASL) "for his fundamental work connecting logic with geometric group theory. Among the consequences of his work are a proof that the class of finitely generated, torsion-free hyperbolic groups is closed under elementary equivalence and a proof that any two nonabelian free groups are elementarily equivalent." The prize consists of a cash award and is given every five years for an outstanding paper or book in the field of symbolic logic.

Inessa Epstein of the California Institute of Technology and Dilip Raghavan of the University of Toronto have been awarded the 2008 Sacks Prize for the most outstanding doctoral dissertations in mathematical logic. Epstein received her Ph.D. in 2008 from the University of California, Los Angeles, under the supervision of Greg Hjorth. According to the prize citation, in her thesis, *Some Results on Orbit Inequivalent Actions of Non-amenable Groups*, she "solves one of the most important problems in measurable group theory, the resolution of which involves a combination of deep results from different branches of mathematics." Raghavan received his Ph.D. in 2008 from the University of Wisconsin at Madison under the supervision of Bart Kastermans and Ken Kunen. According to the citation, his thesis, *Madness and Set Theory*, "uses modern methods associated with independence proofs to obtain, just using *ZFC*, results on almost disjoint (MAD) families, that in particular solve a twenty-year-old problem of Van Douwen."

The Sacks Prize was established to honor Gerald Sacks of the Massachusetts Institute of Technology and Harvard University for his unique contribution to mathematical logic. It consists of a cash award plus five years' free membership in the ASL.

—From an ASL announcement

Heineman Prize Awarded

The 2009 Dannie Heineman Prize in Mathematical Physics has been awarded jointly to Alain Rouet of the Science & Tec company, Carlo Becchi of the University of Genoa, Igor Tyutin of Lebedev Physical Institute, and Raymond Stora of the Centre National de la Recherche Scientifique for their "discovery and exploitation of the BRST symmetry for the quantization of gauge theories providing a fundamental and essential tool for subsequent developments."

The prize carries a cash award of US$10,000 and is presented in recognition of outstanding publications in the field of mathematical physics. The prize was established in 1959 by the Heineman Foundation for Research, Educational, Charitable, and Scientific Purposes, Inc., and is administered jointly by the American Institute of Physics (AIP) and the American Physical Society (APS). The prize is presented annually.

—From an APS announcement

National Academy of Engineering Elections

Howard A. Stone, Vicky Joseph Professor of Engineering and Applied Mathematics at Harvard University, has been elected to the National Academy of Engineering (NAE) for his work on the "development of fundamental concepts and novel applications in microfluidics and for improving the understanding of small-scale, viscous-flow phenomena."

—From an NAE announcement

Mathematics Opportunities

NSF Postdoctoral Research Fellowships

The National Science Foundation (NSF) awards Mathematical Sciences Postdoctoral Research Fellowships (MSPRF) for appropriate research in areas of the mathematical sciences, including applications to other disciplines. Awardees are permitted to choose research environments that will have maximal impact on their future scientific development. Awards are made in the form of either Research Fellowships or Research Instructorships. The Research Fellowship option provides full-time support for any eighteen academic-year months in a three-year period, in intervals not shorter than three consecutive months. The Research Instructorship option provides a combination of full-time and half-time support over a period of three academic years, usually one academic year full time and two academic years half time. Under both options, the award includes six summer months; however, no more than two summer months of support may be received in any calendar year. Under both options, the stipend support for twenty-four months (eighteen academic-year months plus six summer months) will be provided within a forty-eight-month period.

The deadline for proposals is **October 21, 2009.** See `http://www.nsf.gov/pubs/2008/nsf08582/nsf08582.htm`.

—From an NSF announcement

International Mathematics Competition for University Students

The sixteenth International Mathematics Competition (IMC) for University Students will be held July 25–30, 2009, at Eötvös Loránd University in Budapest, Hungary. Participating universities are invited to send several students and one teacher; individual students are welcome. Students completing their first, second, third, or fourth years of university education are eligible. The competition will consist of two sessions of five hours each. Problems will come from the fields of algebra, analysis (real and complex), and combinatorics. The working language will be English.

The deadline for registration is **May 31, 2009.** For details, see the website `http://www.imc-math.org.uk/` or contact John Jayne, University College London, Gower Street, London WC1E 6BT, United Kingdom; telephone: +44-20-7679-7322; email: `j.jayne@ucl.ac.uk`.

—John Jayne, University College London

AMS Graduate Student Blog

The AMS Graduate Student Blog is a new blog by and for math graduate students, managed by Frank Morgan, AMS vice president, and professor of mathematics at Williams College. "Graduate students are the future of the AMS, and they have a lot to talk about," says Morgan. The blog explores issues such as organizing a reading seminar, how to give a good mathematics talk, advice for beginning teaching assistants, navigating seminars, and finding an advisor. Graduate students are invited to post comments, questions, and advice, and to join its Graduate Student Editorial Board. Hosted by Williams College, the blog is at `http://mathgradblog.williams.edu/`.

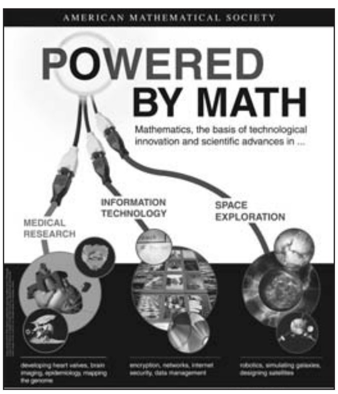
—Allyn Jackson

Inside the AMS

From the AMS Public Awareness Office

• *Powered by Math* poster. Download the pdf file of a small version of this poster (printable on 8.5 by 11 inches or A4-size paper), or request a copy on the AMS Printable Posters webpage at `http://www.ams.org/posters/`.

• **The AMS at the annual AAAS Meeting.** The Public Awareness Office hosted an exhibit at the American Association for the Advancement of Science (AAAS) annual meeting held in Chicago, IL, February 12–16, 2009. The Society was the only mathematics exhibitor at this meeting, which draws scientists from many disciplines, science writers and media, students, and families. On display were a sampling of AMS books, the Calendar of Mathematical Imagery, the Mathematics Awareness Month 2009 "Mathematics and Climate" theme poster, and a selection from the 70-plus Mathematical Moments that show the role mathematics plays in science, nature, technology, and human culture. Read about the mathematics sessions on the program at `http://www.ams.org/ams/aaas2009.html`.



• **Feature Column.** Recent columns include "No Static at All: Frequency Modulation and Music Synthesis", by David Austin; "The Mathematics of Rainbows", by Bill Casselman; and "People Making a Difference", by Joe Malkevitch. Visit the page `http://www.ams.org/featurecolumn/`.

• **Mathematical Moments translated into French.** There are now eleven Mathematical Moments translated into French: Préserver le passé, Enterrer le dioxyde de carbone, Déceler les photos truquées, Prédire l'onde de tempête, S'aventurer dans l'Inconnu…, La Réalisation des Concepts, Reconnaissance de la Parole, Ciblage des Tumeurs, Révolutionner l'ordinateur, Les moteurs de recherche, and Expérimenter avec le coeur. Also recently added to the collection are Mathematical Moments in Greek, Arabic, and Hebrew. All can be viewed and downloaded at `http://www.ams.org/mathmoments/`.

—*Annette Emerson and Mike Breen*
AMS Public Awareness Officers
`paoffice@ams.org`

Epsilon Awards for 2009

The AMS Epsilon Fund for Young Scholars was established in 1999 to provide financial assistance to summer programs in the United States and Canada for mathematically talented high school students. These programs have provided many talented youngsters with their first serious mathematical experiences. The name for the fund was chosen in remembrance of the late Paul Erdős, who was fond of calling children "epsilons".

The AMS has chosen ten summer mathematics programs to receive Epsilon grants for the summer of 2009. The grants will support program expenses and student scholarships and, in some cases, scholarships only. The programs were chosen on the basis of mathematical excellence and enthusiasm. Award amounts were governed by the varying financial needs of each program and totaled US$100,000.

The 2009 grants are awarded to: Achievement in Mathematics Program (AMP), Lamar University; All Girls/All Math, University of Nebraska, Lincoln; Hampshire College Summer Studies in Mathematics (HCSSiM), Hampshire College; MathPath, Colorado College, Colorado Springs; Michigan Math and Science Scholars Summer Program, University of Michigan, Ann Arbor; PROMYS (Program in Mathematics for Young Scientists), Boston University; PROTaSM (Puerto Rico Opportunities for Talented Students in Mathematics), University of Puerto Rico, Mayagüez Campus; Research Science Institute, Massachusetts Institute of Technology; Ross Mathematics Program, Ohio State University, Columbus; Texas State University Honors Summer Math Camp, Texas State University, San Marcos.

The grants for summer 2009 are paid for by the AMS Epsilon Fund for Young Scholars. The AMS Epsilon Fund for Young Scholars has been funded by contributions of AMS members and friends; the goal of the endowment is to provide at least US$100,000 in support each summer.

For further information about the Epsilon Fund for Young Scholars, visit the website `http://www.ams.org/giving-to-ams/` or contact `development@ams.org`. Information about how to apply for Epsilon grants is available at `http://www.ams.org/employment/epsilon.html`. A fairly comprehensive listing of summer programs for mathematically talented high school students (including those with and without Epsilon grants) is available at `http://www.ams.org/employment/mathcamps.html`.

—*AMS Development Office*

Reference and Book List

*The **Reference** section of the* Notices *is intended to provide the reader with frequently sought information in an easily accessible manner. New information is printed as it becomes available and is referenced after the first printing. As soon as information is updated or otherwise changed, it will be noted in this section.*

Contacting the *Notices*

The preferred method for contacting the *Notices* is electronic mail. The editor is the person to whom to send articles and letters for consideration. Articles include feature articles, memorial articles, communications, opinion pieces, and book reviews. The editor is also the person to whom to send news of unusual interest about other people's mathematics research.

The managing editor is the person to whom to send items for "Mathematics People", "Mathematics Opportunities", "For Your Information", "Reference and Book List", and "Mathematics Calendar". Requests for permissions, as well as all other inquiries, go to the managing editor.

The electronic-mail addresses are `notices@math.ou.edu` in the case of the editor and `notices@ams.org` in the case of the managing editor. The fax numbers are 405-325-7484 for the editor and 401-331-3842 for the managing editor. Postal addresses may be found in the masthead.

Upcoming Deadlines

April 24, 2009: Proposals for 2010 NSF-CBMS Regional Conferences. See `http://www.cbmsweb.org/NSF/2010_call.htm`, or contact: Conference Board of the Mathematical Sciences, 1529 Eighteenth Street, NW, Washington, DC 20036; telephone: 202-293-1170; fax: 202-293-3412; email: `lkolbe@maa.org` or `rosier@georgetown.edu`.

May 8, 2009: Applications for AWM Travel Grants. See `http://www.awm-math.org/travelgrants.html`; telephone: 703-934-0163; email: `awm@awm-math.edu`. The postal address is: Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

Where to Find It

A brief index to information that appears in this and previous issues of the *Notices*.

AMS Bylaws—*November 2007, p. 1366*

AMS Email Addresses—*February 2009, p. 278*

AMS Ethical Guidelines—*June/July 2006, p. 701*

AMS Officers 2008 and 2009 Updates—*May 2009, p. 651*

AMS Officers and Committee Members—*October 2008, p. 1122*

Conference Board of the Mathematical Sciences—*September 2008, p. 980*

IMU Executive Committee—*December 2008, p. 1441*

Information for *Notices* Authors—*June/July 2008, p. 723*

Mathematics Research Institutes Contact Information—*August 2008, p. 844*

National Science Board—*January 2009, p. 67*

New Journals for 2006, 2007—*June/July 2008, p. 725*

NRC Board on Mathematical Sciences and Their Applications—*March 2009, p. 404*

NRC Mathematical Sciences Education Board—*April 2009, p. 511*

NSF Mathematical and Physical Sciences Advisory Committee—*February 2009, p. 278*

Program Officers for Federal Funding Agencies—*October 2008, p. 1116 (DoD, DoE); December 2007, p. 1359 (NSF); December 2008, p. 1440 (NSF Mathematics Education)*

Program Officers for NSF Division of Mathematical Sciences—*November 2008, p. 1297*

Stipends for Study and Travel—*September 2008, p. 983*

May 15-June 15, 2009: Proposals for DMS Workforce Program in the Mathematical Sciences. See http://www.nsf.gov/funding/pgm_summ.jsp?pims_id=503233.

May 15, 2009: Applications for National Academies Research Associateship Programs. See http://www7.nationalacademies.org/rap/ or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

May 31, 2009: Registration for the International Mathematics Competition for University Students (IMC). See "Mathematics Opportunities" in this issue.

June 1, 2009: Applications for the September program of the Christine Mirzayan Science and Technology Policy Graduate Fellowship Program of the National Academies. See http://www7.nationalacademies.org/policyfellows; or contact The National Academies Christine Mirzayan Science and Technology Policy Graduate Fellowship Program, 500 Fifth Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202-334-1667; email: policyfellows@nas.edu.

June 2, 2009: Proposals for NSF's Enhancing the Mathematical Sciences Workforce in the Twenty-First Century program. See http://www.nsf.gov/publications/pub_summ.jsp?ods_key=nsf05595.

June 30, 2009: Applications for Fermat Prize for Mathematics Research. Contact Prix Fermat de Recherche en Mathématiques, Service Relations Publiques, Université Paul Sabatier, 31062 Toulouse Cedex 9, France, or see the website http://www.math.ups-tlse.fr/Fermat/.

August 4, 2009: Letters of intent for NSF Project ADVANCE Institutional Transformation (IT) and Institutional Transformation Catalyst (IT-Catalyst) awards. See http://www.nsf.gov/pubs/2009/nsf09504/nsf09504.htm?govDel=USNSF_25.

August 4, 2009: Full proposals (by invitation only) for NSF Partnerships for International Research and Education (PIRE). See http://www.nsf.gov/pubs/2009/nsf09505/nsf09505.htm?govDel=USNSF_25#awd_info.

August 15, 2009: Applications for National Academies Research Associateship Programs. See http://www7.nationalacademies.org/rap/ or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

September 14, 2009: Full proposals for NSF Integrative Graduate Education and Research Training (IGERT). See http://www.nsf.gov/pubs/2009/nsf09519/nsf09519.htm?govDel=USNSF_25.

October 1, 2009: Applications for AWM Travel Grants. See http://www.awm-math.org/travelgrants.html; telephone: 703-934-0163; email: awm@awm-math.edu. The postal address is: Association for Women in Mathematics, 11240 Waples Mill Road, Suite 200, Fairfax, VA 22030.

October 21, 2009: Proposals for NSF Postdoctoral Research Fellowships. See "Mathematics Opportunities" in this issue.

November 1, 2009: Applications for the January program of the Christine Mirzayan Science and Technology Policy Graduate Fellowship Program of the National Academies. See http://www7.nationalacademies.org/policyfellows; or contact The National Academies Christine Mirzayan Science and Technology Policy Graduate Fellowship Program, 500 Fifth Street, NW, Room 508, Washington, DC 20001; telephone: 202-334-2455; fax: 202-334-1667; email: policyfellows@nas.edu.

November 12, 2009: Full proposals for NSF Project ADVANCE Institutional Transformation (IT) and Institutional Transformation Catalyst (IT-Catalyst) awards. See http://www.nsf.gov/pubs/2009/nsf09504/nsf09504.htm?govDel=USNSF_25.

November 15, 2009: Applications for National Academies Research Associateship Programs. See http://www7.nationalacademies.org/rap/ or contact Research Associateship Programs, National Research Council, Keck 568, 500 Fifth Street, NW, Washington, DC 20001; telephone 202-334-2760; fax 202-334-2759; email: rap@nas.edu.

Book List

The Book List highlights books that have mathematical themes and are aimed at a broad audience potentially including mathematicians, students, and the general public. When a book has been reviewed in the Notices, *a reference is given to the review. Generally the list will contain only books published within the last two years, though exceptions may be made in cases where current events (e.g., the death of a prominent mathematician, coverage of a certain piece of mathematics in the news) warrant drawing readers' attention to older books. Suggestions for books to include on the list may be sent to* notices-booklist@ams.org.

*Added to "Book List" since the list's last appearance.

An Abundance of Katherines, by John Green. Dutton Juvenile Books, September 2006. ISBN-13:978-0-5254-7688-7. (Reviewed October 2008.)

The Annotated Turing: A Guided Tour Through Alan Turing's Historic Paper on Computability and the Turing Machine, by Charles Petzold. Wiley, June 2008. ISBN-13: 978-04702-290-57.

The Archimedes Codex, by Reviel Netz and William Noel. Weidenfeld and Nicolson, May 2007. ISBN-13: 978-0-29764-547-4. (Reviewed September 2008.)

The Best of All Possible Worlds: Mathematics and Destiny, by Ivar Ekeland. University Of Chicago Press, October 2006. ISBN-13: 978-0-226-19994-8. (Reviewed March 2009.)

The Book of Numbers: The Secret of Numbers and How They Changed the World, by Peter J. Bentley. Firefly Books, February 2008. ISBN-13: 978-15540-736-10.

**The Calculus Wars: Newton, Leibniz, and the Greatest Mathematical Clash of All Time*, by Jason Socrates Bardi. Thunder's Mouth Press, April 2007. ISBN-13: 978-15602-5992-3. (Reviewed in this issue.)

The Cat in Numberland, by Ivar Ekeland. Cricket Books, April 2006. ISBN-13: 978-0-812-62744-2. (Reviewed January 2009.)

Crossing the Equal Sign, by Marion D. Cohen. Plain View Press, January 2007. ISBN-13: 978-18913-866-95.

Digital Dice, by Paul J. Nahin. Princeton University Press, March 2008. ISBN-13: 978-06911-269-82.

Dimensions, by Jos Leys, Etienne Ghys, and Aurélien Alvarez. DVD, 117 minutes. Available at `http://www.dimensions-math.org`.

Discovering Patterns in Mathematics and Poetry, by Marcia Birken and Anne C. Coon. Rodopi, February 2008. ISBN-13: 978-9-0420-2370-3.

The Drunkard's Walk: How Randomness Rules Our Lives, by Leonard Mlodinow. Pantheon, May 2008. ISBN-13: 978-03754-240-45.

Einstein's Mistakes: The Human Failings of Genius, by Hans C. Ohanian. W. W. Norton, September 2008. ISBN-13: 978-0393062939.

Embracing the Wide Sky: A Tour Across the Horizons of the Human Mind, by Daniel Tammet. Free Press, January 2009. ISBN-13: 978-14165-696-95.

Emmy Noether: The Mother of Modern Algebra, by M. B. W. Tent. A K Peters, October 2008. ISBN-13: 978-15688-143-08.

Euclidean and Non-Euclidean Geometries: Development and History, fourth revised and expanded edition, by Marvin Jay Greenberg. W. H. Freeman, September 2007. ISBN-13: 978-0-7167-9948-1.

Euler's Gem: The Polyhedron Formula and the Birth of Topology, by David S. Richeson. Princeton University Press, September 2008. ISBN-13: 97-80691-1267-77.

Fifty Mathematical Ideas You Really Need to Know, by Tony Crilly. Quercus, 2007. ISBN-13: 978-18472-400-88.

Fighting Terror Online: The Convergence of Security, Technology and the Law, by Martin Charles Golumbic. Springer, 2008. ISBN: 978-0-387-73577-1.

Five-Minute Mathematics, by Ehrhard Behrends (translated by David Kramer). AMS, May 2008. ISBN-13: 978-08218-434-82.

Geekspeak: How Life + Mathematics = Happiness, by Graham Tattersall. Collins, September 2008. ISBN-13: 978-00616-292-42.

Geometric Folding Algorithms: Linkages, Origami, Polyhedra, by Erik D. Demaine and Joseph O'Rourke. Cambridge University Press, July 2007. ISBN-13: 978-05218-57574.

Geometric Origami, by Robert Geretschläger. Arbelos, October 2008. ISBN-13: 978-09555-477-13.

The Golden Section: Nature's Greatest Secret (Wooden Books), by Scott Olsen. Walker and Company, October 2006. ISBN-13: 978-08027-153-95.

Group Theory in the Bedroom, and Other Mathematical Diversions, by Brian Hayes. Hill and Wang, April 2008. ISBN-13: 978-08090-521-96. (Reviewed February 2009.)

Guesstimation: Solving the World's Problems on the Back of a Cocktail Napkin, by Lawrence Weinstein and John A. Adam. Princeton University Press, April 2008. ISBN-13: 978-0-6911-2949-5.

Hexaflexagons, Probability Paradoxes, and the Tower of Hanoi: Martin Gardner's First Book of Mathematical Puzzles and Games, by Martin Gardner. Cambridge University Press, September 2008. ISBN-13: 978-0-521-73525-4.

**The Housekeeper and the Professor*, by Yoko Ogawa. Picador, February 2009. ISBN-13: 978-03124-278-01.

How Math Explains the World: A Guide to the Power of Numbers, from Car Repair to Modern Physics, by James D. Stein. Collins, April 2008. ISBN-13: 978-00612-417-65.

How to Think Like a Mathematician: A Companion to Undergraduate Mathematics, by Kevin Houston. Cambridge University Press, March 2009. ISBN-13: 978-05217-197-80.

Impossible?: Surprising Solutions to Counterintuitive Conundrums, by Julian Havil. Princeton University Press, April 2008. ISBN-13: 978-0-6911-3131-3.

The Indian Clerk, by David Leavitt. Bloomsbury USA, September 2007. ISBN-13: 978-15969-1040-9. (Reviewed September 2008.)

Irreligion: A Mathematician Explains Why the Arguments for God Just Don't Add Up, by John Allen Paulos. Hill and Wang, December 2007. ISBN-13: 978-0-8090-591-95. (Reviewed August 2008.)

Is God a Mathematician? by Mario Livio. Simon & Schuster, January 2009. ISBN-13: 978-07432-940-58.

Kiss My Math: Showing Pre-Algebra Who's Boss, by Danica McKellar. Hudson Street Press, August 2008. ISBN-13: 978-1594630491.

The Last Theorem, by Arthur C. Clarke and Frederik Pohl. Del Rey, August 2008. ISBN-13: 978-0345470218.

**Leonhard Euler and His Friends: Switzerland's Great Scientific Expatriate*, by Luis-Gustave du Pasquier (translated by John S. D. Glaus). CreateSpace, July 2008. ISBN: 978-14348-332-73.

**Lewis Carroll in Numberland: His Fantastical Mathematical Logical Life: An Agony in Eight Fits*, by Robin Wilson. W. W. Norton & Company, ISBN-13: 978-03930-602-70.

Logic's Lost Genius: The Life of Gerhard Gentzen, by Eckart Menzler-Trott, Craig Smorynski (translator), Edward R. Griffor (translator). AMS-LMS, November 2007. ISBN-13: 978-0-8218-3550-0.

The Map of My Life, by Goro Shimura. Springer, September 2008. ISBN-13: 978-03877-971-44.

Mathematical Omnibus: Thirty Lectures on Classic Mathematics, by Dmitry Fuchs and Serge Tabachnikov. AMS, October 2007. ISBN-13: 978-08218-431-61. (Reviewed December 2008).

The Mathematician's Brain, by David Ruelle. Princeton University Press, July 2007. ISBN-13 978-0-691-12982-2. (Reviewed November 2008.)

Mathematics and the Aesthetic: New Approaches to an Ancient Affinity, edited by Nathalie Sinclair, David Pimm, and William Higginson. Springer, November 2006. ISBN-13: 978-03873-052-64. (Reviewed February 2009.)

Mathematics and Democracy: Designing Better Voting and Fair-Division Procedures, by Steven J. Brams. Princeton University Press, December 2007. ISBN-13: 978-0691-1332-01.

Mathematics at Berkeley: A History, by Calvin C. Moore. A K Peters, February 2007. ISBN-13: 978-1-5688-1302-8. (Reviewed November 2008.)

**Mathematics Emerging: A Sourcebook 1540–1900*, by Jacqueline Stedall. Oxford University Press, November 2008. ISBN-13: 978-01992-269-00.

Mathematics in Ancient Iraq: A Social History, by Eleanor Robson.

Princeton University Press, August 2008. ISBN13: 978-06910-918-22.

**Mathematics in India*, by Kim Plofker. Princeton University Press, January 2009. ISBN-13: 978-06911-206-76.

The Mathematics of Egypt, Mesopotamia, China, India, and Islam: A Sourcebook, by Victor J. Katz et al. Princeton University Press, July 2007. ISBN-13: 978-0-6911-2745-3.

Measuring the World, by Daniel Kehlmann. Pantheon, November 2006. ISBN 0-375-42446-6. (Reviewed June/July 2008.)

More Mathematical Astronomy Morsels, by Jean Meeus. Willmann-Bell, 2002. ISBN 0-943396743.

More Sex Is Safer Sex: The Unconventional Wisdom of Economics, by Steven E. Landsburg. Free Press, April 2007. ISBN-13: 978-1-416-53221-7. (Reviewed June/July 2008.)

Number and Numbers, by Alain Badiou. Polity, June 2008. ISBN-13: 978-07456-387-82.

Number Story: From Counting to Cryptography, by Peter M. Higgins. Springer, February 2008. ISBN-13: 978-1-8480-0000-1.

The Numbers Behind NUMB3RS: Solving Crime with Mathematics, by Keith Devlin and Gary Lorden. Plume, August 2007. ISBN-13: 978-04522-8857-7. (Reviewed March 2009.)

**The Numbers Game: The Commonsense Guide to Understanding Numbers in the News, in Politics, and in Life*, by Michael Blastland and Andrew Dilnot. Gotham, December 2008. ISBN-13: 978-15924-042-30.

The Numerati, by Stephen Baker. Houghton Mifflin, August 2008. ISBN-13: 978-06187-846-08.

One to Nine: The Inner Life of Numbers, by Andrew Hodges. W. W. Norton, May 2008. ISBN-13: 978-03930-664-18.

Origami, Eleusis, and the Soma Cube: Martin Gardner's Mathematical Diversions, by Martin Gardner. Cambridge University Press, September 2008. ISBN-13: 978-0-521-73524-7.

Our Days Are Numbered: How Mathematics Orders Our Lives, by Jason Brown. McClelland and Stewart, to appear April 2009. ISBN-13: 978-07710-169-67.

A Passion for Discovery, by Peter Freund. World Scientific, August 2007. ISBN-13: 978-9-8127-7214-5.

Plato's Ghost: The Modernist Transformation of Mathematics, by Jeremy Gray. Princeton University Press, September 2008. ISBN-13: 978-06911-361-03.

The Princeton Companion of Mathematics, edited by Timothy Gowers (June Barrow-Green and Imre Leader, associate editors). Princeton University Press, November 2008. ISBN-13: 978-06911-188-02.

Professor Stewart's Cabinet of Mathematical Curiosities, by Ian Stewart. Basic Books, December 2008. ISBN-13: 978-0-465-01302-9.

Pursuit of Genius: Flexner, Einstein, and the Early Faculty at the Institute for Advanced Study, by Steve Batterson. A K Peters, June 2006. ISBN 1-56881-259-0. (Reviewed August 2008.)

Pythagorean Crimes, by Tefcros Michalides. Parmenides Publishing, September 2008. ISBN-13: 978-19309-722-78. (Reviewed January 2009.)

Random Curves: Journeys of a Mathematician, by Neal Koblitz. Springer, December 2007. ISBN-13: 978-3-5407-4077-3.

**Recountings: Conversations with MIT Mathematicians*, edited by Joel Segel. A K Peters, January 2009. ISBN-13: 978-15688-144-90.

Reminiscences of a Statistician: The Company I Kept, by Erich Lehmann. Springer, November 2007. ISBN-13: 978-0-387-71596-4.

Rock, Paper, Scissors: Game Theory in Everyday Life, by Len Fisher. Basic Books, November 2008. ISBN-13: 978-04650-093-81.

Roots to Research: A Vertical Development of Mathematical Problems, by Judith D. Sally and Paul J. Sally Jr. AMS, November 2007. ISBN-13: 978-08218-440-38. (Reviewed December 2008.)

Sacred Mathematics: Japanese Temple Geometry, by Fukagawa Hidetoshi and Tony Rothman. Princeton University Press, July 2008. ISBN-13: 978-0-6911-2745-3.

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Mathematics Calendar

May 2009

* 4–6 **Workshop on Pairings in Arithmetic Geometry and Cryptography**, Institute for Experimental Mathematics of the University of Duisburg-Essen, Essen, Germany.

Description: The subject of the workshop is bilinear structures on ideal class groups of curves over finite fields. Besides the theoretical background, computational aspects as well as applications to public key cryptography, will be in the center of the talks. We plan to have four introductory minicourses (two hours each) as well as lectures about the state of the art and new perspectives. Everyone interested in these topics is heartily invited to come!

Information: Visit `http://www.exp-math.uni-essen.de/zahlentheorie/pairings09/index.html`.

* 4–9 **The Power of Analysis: A conference in honor of Charles Fefferman on the occasion of his 60th birthday**, Princeton University (Mathematics Department), Princeton, New Jersey.

Description: Conference (sponsored by AIM, the NSF, CMI, and the Princeton University Mathematics Department) honoring Charles Fefferman on the occasion of his 60th birthday. To be held in McDonnell Hall (part of the Fine Hall–Jadwin Hall complex at Princeton University). Registration requested (via conference website).

Information: `http://www.math.princeton.edu/conference/fefferman09/`.

* 4–13 **The Seventh Annual Spring Institute on Noncommutative Geometry and Operator Algebras**, Department of Mathematics, Vanderbilt University, Nashville, Tennessee.

Description: The Seventh Annual Spring Institute on Noncommutative Geometry and Operator Algebras is a combination of spring school and international conference. During the school part of the meeting, several mini-courses on a variety of topics from noncommutative geometry, operator algebras and related topics will be given by leading experts. This year's topic is "Geometry over F1, noncommutative geometry and Zeta". Mini-courses will be given by David Goss (Ohio State University), Lisa Carbone (Rutgers University), Christophe Soule (IHES), Alain Connes (IHES & Vanderbilt University), Pierre Cartier (IHES), Caterina Consani (John Hopkins University) and Jack Morava (John Hopkins University). The conference is organized by Alain Connes (Director), Dietmar Bisch, Bruce Hughes, Gennadi Kasparov and Guoliang Yu. Students and postdocs are especially encouraged to attend.

Information: `http://www.math.vanderbilt.edu/~ncgoa09/`.

* 6–10 **East Asian Symplectic Conference 2009**, Institute of Mathematics, Academia Sinica, Taipei, Taiwan.

Description: The purpose of the conference is to explore and cross-fertilize the developments of various subjects in Symplectic Geometry and Topology for the researchers in East Asia.

Confirmed speakers: Meng-Kiat Chuah (NTHU, Taiwan), Urs Frauenfelder (SNU, Korea), Kenji Fukaya (Kyoto-Univ., Japan), Young-Hoon Kiem (SNU, Korea), Bumsig Kim (KIAS, Korea), Otto van Koert

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This section contains announcements of meetings and conferences of interest to some segment of the mathematical public, including ad hoc, local, or regional meetings, and meetings and symposia devoted to specialized topics, as well as announcements of regularly scheduled meetings of national or international mathematical organizations. A complete list of meetings of the Society can be found on the last page of each issue.

An announcement will be published in the *Notices* if it contains a call for papers and specifies the place, date, subject (when applicable), and the speakers; a second announcement will be published only if there are changes or necessary additional information. Once an announcement has appeared, the event will be briefly noted in every third issue until it has been held and a reference will be given in parentheses to the month, year, and page of the issue in which the complete information appeared. Asterisks (*) mark those announcements containing new or revised information.

In general, announcements of meetings and conferences carry only the date, title of meeting, place of meeting, names of speakers (or sometimes a general statement on the program), deadlines for abstracts or contributed papers, and source of further information. If there is any application deadline with respect to participation in the meeting, this fact should be noted. All communications on meetings and conferences in the mathematical sciences should be sent to the Editor of the *Notices* in care of the American Mathematical Society in Providence or electronically to `notices@ams.org` or `mathcal@ams.org`.

In order to allow participants to arrange their travel plans, organizers of meetings are urged to submit information for these listings early enough to allow them to appear in more than one issue of the *Notices* prior to the meeting in question. To achieve this, listings should be received in Providence **eight months** prior to the scheduled date of the meeting.

The complete listing of the Mathematics Calendar will be published only in the September issue of the *Notices*. The March, June/July, and December issues will include, along with new announcements, references to any previously announced meetings and conferences occurring within the twelve-month period following the month of those issues. New information about meetings and conferences that will occur later than the twelve-month period will be announced once in full and will not be repeated until the date of the conference or meeting falls within the twelve-month period.

The Mathematics Calendar, as well as Meetings and Conferences of the AMS, is now available electronically through the AMS website on the World Wide Web. To access the AMS website, use the URL: `http://www.ams.org/`.

(Hokkaido Univ., Japan), Hiroshi Konno (U-Tokyo, Japan), Yng-Ing Lee (NTU, Taiwan), Naichung Conan Leung (CUHK, Hong Kong), Jiang-Hua Lu (HKU, Hong Kong), Yoshiaki Maeda (Keio Univ., Japan), Giovanni Marelli (KIAS, Korea), Stefan Muller (KIAS, Korea), Takeo Nishinou (Tohoku Univ., Japan), Yuichi Nohara (Nagoya-Univ., Japan), Hiroshi Ohta (Nagoya-Univ., Japan), Hajime Ono (TUS, Japan), Kaoru Ono (Hokkaido Univ., Japan), Jongil Park (SNU, Korea), Wei-dong Ruan (KAIST, Korea), Kazushi Ueda (Osaka-Univ., Japan), Xiaowei Wang (CUHK, Hong Kong), Siye Wu (HKU, Hong Kong & CU-Boulder, USA), Daisuke Yamakawa (Kyoto-Univ., Japan), Mei-Lin Yau (NCU, Taiwan).
Information: http://www.math.ncku.edu.tw/~easc2009/.

* 8–9 **The 2009 Annual Meeting of the Michigan Section of the Mathematical Association of America & MichMATYC**, Mathematics Department at Central Michigan University, Mount Pleasant, Michigan.
Information: Visit http://www.cst.cmich.edu/units/mth/MAAMeeting/MAASectMeet09.htm.

* 11–13 **DIMACS Workshop on Algorithmic Mathematical Art: Special Cases and Their Applications**, DIMACS Center, CoRE Building, Rutgers University, Piscataway, New Jersey.
Short Description: There is a significant and growing interest in mathematical or algorithmic art. This is in part due to advancement in computer technology, higher mathematics, sophisticated algorithms, and the community of mathematically- or computationally-minded contributors whose works demonstrate the beauty and potential of mathematics in manifestation of algorithmic art. This DIMACS workshop will be a three-day exploration focusing on some special cases of what may be characterized as algorithmic mathematical art and its applications. Topics include mathematically-rooted, algorithmically-conducted approaches to computer graphics, visualization, painting, sculpturing, drawing, and other visual arts. The workshop will include invited and contributed talks as well as hands-on activities. In particular, the participants will be introduced to Polynomiography and its software at a computer lab. There will be an informal exhibit area for display of artwork of the participants. We encourage participation of mathematicians, scientists, artist, and educators. To contribute a talk, please submit a 1–2 page abstract (pdf format is preferred) to Bahman Kalantari (email: kalantari@cs.rutgers.edu) for refereed evaluation by March 15, 2009. Contributors will be notified of acceptance decisions by mid April. The possibility of a refereed proceedings exists and a decision will be announced at a later time.
Organizers: Bahman Kalantari, Rutgers University, kalantari@cs.rutgers.edu; Helaman Ferguson, http://Helasculpt.com; helamanf@helasculpt.com. Dirk Huylebrouck, Sint Lucas (Brussels), dirk.huylebrouck@architectuur.sintlucas.wenk.be; Radmila Sazdanovic, The George Washington University, radmila@gwmail.gwu.edu.
Local Arrangements: Nicole Clark, DIMACS Center, nicolec@dimacs.rutgers.edu, 732-445-5928.
Information: http://dimacs.rutgers.edu/Workshops/MathArt/.

* 14–17 **Foundational Adventures: Conference in Honor of the 60th Birthday of Harvey M. Friedman**, The Blackwell Inn, The Ohio State University, Columbus, Ohio.
Description: This interdisciplinary conference, with a stellar list of speakers, is made possible by the support of The Ohio State University and the John Templeton Foundation. We have at our disposal some limited travel funds from the National Science Foundation, with the aim of providing opportunities for participation by women, minorities, and persons with disabilities. Details about the application process are available on the conference webpage. The program and the list of titles and abstracts are also accessible from that webpage.
Plenary speakers include: Martin Davis (UC Berkeley and NYU), Solomon Feferman (Stanford), Harvey Friedman (Ohio State), Saul Kripke (CUNY), Anil Nerode (Cornell), Hilary Putnam (Harvard), Gerald Sacks (Harvard and MIT), and Patrick Suppes (Stanford).
Information: http://people.cohums.ohio-state.edu/tennant9/friedman_conference.html.

* 15–17 **Conference on Differential Geometry: Tribute to Professor Kostake Teleman (1933-2007)**, Bucharest University, Faculty of Mathematics and Computer Science, Academiei 14, Bucharest, Romania.
Description: This international conference is devoted to the memory of Kostake Teleman, an outstanding Romanian geometer, professor at the Bucharest University till 2007. The topics of the conference will cover the domains of interest of Professor Teleman: Differential Geometry, Topology, Algebraic Topology, Mathematical Physics and Relativity, focusing on latter developments of his works.
Organizers: Knud Lonsted (Denmark), Andrei Moroianu (CNRS- Ecole Polytechnique), Gabriel-Teodor Pripoae (Univ. Bucharest), Olivian Simionescu-Panait (Univ. Bucharest), Constantin Udriste (Univ. Politehnica, Bucharest), Vladimir Balan (Univ. Politehnica, Bucharest), Paul Popescu (Univ. Craiova), Wladimir Boskoff (Univ. `Ovidius', Constanta).
Registration: The registration is free.
Deadline: The deadline for registration is April 15, 2009.
Information: For further details, please contact Gabriel-Teodor Pripoae: gpripoae@fmi.unibuc.ro or gpripoae@yahoo.com; Visit http://fmi.unibuc.ro/ro/pdf/2009/manifestari/anunt_KTeleman%202009.pdf.

* 15–17 **Twenty Years of Wavelets**, DePaul University, Chicago, Illinois.
Organizers: Jonathan Cohen (DePaul), jcohen@condor.depaul.edu, and Ahmed Zayed (DePaul), azayed@condor.depaul.edu.
Conference Themes: DePaul University is sponsoring a conference on wavelets and their applications. The conference commemorates the twentieth anniversary of a 1989 conference at DePaul University on computational and numerical aspects of harmonic analysis that featured the emerging topic of wavelets. This conference will focus on what has been accomplished in twenty years both in terms of application and theory, and on current areas.
Speakers: Raphy Coifman, Peter Jones, Ingrid Daubechies, John Benedetto, Martin Vetterli, Michael Frazier, Naoki Saito, Greg Beylkin, Victor Wickerhauser, Guido Weiss, Bjorn Jawerth and Mauro Maggioni.
Financial Support: The conference is supported by a grant from the National Science Foundation, DMS-0852170. Partial travel support is available with priority given to junior faculty and graduate students. There will be contributed paper sessions related to the themes of the conference.
Information: Registration, travel, and support information can be obtained directly from the website: http://depaul.edu/~wavelets Contact: wavelets@depaul.edu.

* 18–29 **2009 Georgia International Topology Conference**, University of Georgia, Athens, Georgia.
Description: This will be the seventh in a series of octennial conferences that started in 1961.
Information: http://www.math.uga.edu/~topology/.

* 19–29 **2009 PIMS-MITACS GIMMC/IPSW Events**, University of Calgary, Calgary, Alberta, Canada.
Description: Both events are being organized under the auspices of the Pacific Institute for the Mathematical Sciences and the MITACS Network of Centres of Excellence. Registration began on February 11, 2009, and will end on April 9, 2009.
Information: http://www.pims.math.ca/industrial/workshops/2009-pims-industrial-problem-solving-workshop-and-graduate-industrial-mathemati.

* 24–29 **Workshop on Finsler Geometry and its Applications**, University of Debrecen, Debrecen, Hungary.
Description: The scientific program is devoted to lectures given by the participants on various fields of Finsler geometry, their generalizations, and applications in physics, biology, control theory, finance, psychometry, etc. The workshop focuses on open problems such as

the existence of nontrivial Landsberg metrics. This event is intended to continue the series of workshops on Finsler geometry (Seattle 1995, Edmonton 1998, Iasi 2001, Berkeley 2002, Debrecen 2003, Tianjin 2004, Balatonföldvár 2007) and the Finsler Symposiums held yearly in Japan. At the openings, the 100th anniversary of the late academician Ottó Varga's birthday will be commemorated.
Information: http://www.math.klte.hu/finsler2009.

* 28–30 **Nonlinear Partial Differential Equations. Fuensanta Andreu: In Memoriam**, Facultat de Matematiques, Universitat de Valencia, Burjassot, Spain.
Description: This international conference is a tribute to the memory of Fuensanta Andreu Vaillo, an outstanding person and mathematician who sadly passed away last December 26th at the age of 53.
List of speakers: David Arcoya, Coloma Ballester, Lucio Boccardo, Xavier Cabré, Vicent Caselles, Andrea Dall'Aglio, Jesús Ildefonso Díaz, Noureddine Igbida, Antonio Marquina, Rainer Nagel, Luigi Orsina, Ireneo Peral, Julio Rossi, Mircea Sofonea, Juan Soler.
Information: http://www.uv.es/fuensanta.

June 2009

* 2–5 **Holomorphically symplectic varieties and moduli spaces**, Laboratoire Paul Painlevé at the University of Lille 1, Lille, France.
Description: The conference will focus on irreducible symplectic varieties (ISV) and on moduli problems related to ISVs. One of the aims of the conference is to bring together specialists in both arithmetic and geometric aspects of the theory. The geometric aspect includes the description of families of ISVs' which possess particular geometric properties. The arithmetic aspect is connected to indefinite lattices and associated automorphic forms, which come up via the polarized Hodge structures of the ISVs' or of other varieties with a similar Hodge structure.
Organizers: Valery Gritsenko and Dimitri Markushevich.
Preliminary List of Non-local Participants: F. Bogomolov, U. Bruzzo, F. Catanese, S. Druel, L. Gruson, K. Hulek, M. Lehn, E. Looijenga, C. Madonna, L. Manivel, E. Markman, V. Nikulin, G. Sankaran, C. Sorger, A. Tikhomirov, M. Verbitsky, K.-I. Yoshikawa.
Information: See http://math.univ-lille1.fr/~markushe/MOD2009/.

* 2–6 **The 29th Great Planes Operator Theory Symposium (GPOTS 2009)**, Department of Mathematics, University of Colorado at Boulder, Colorado.
Description: The GPOTS conferences have traditionally brought together a wide range of researchers in operator theory and operator algebras. Moreover, the GPOTS 2009 special meeting will serve as a high-altitude training for the next generation of operator theorists and operator algebraists. The particular topics covered in the conference are (A) C^*-algebras and their classification, (B) noncommutative geometry and index theory, (C) wavelets, frames, and operator theory and (c) subfactors.
Information: http://www.colorado.edu/math/gpots2009.

* 4–6 **Lehigh University Geometry and Topology Conference**, Lehigh University, Bethlehem, Pennsylvania.
Description: Invited talks by Gunnar Carlsson (Stanford), Fred Cohen (Rochester), Panagiota Daskalopoulos (Columbia), Ngaiming Mok (Hong Kong), Wilfried Schmid (Harvard), and Rick Schoen (Stanford) (all confirmed). Participants may give 40-minute contributed talks in geometry or topology. Limited travel support is available for recent Ph.D.'s, current graduate students and members of underrepresented groups.
Information: http://www.lehigh.edu/~dlj0/geotop.html.

* 8–11 **Analysis, Inequalities and Homogenization Theory (AIHT 2009)**, Lulea University of Technology, Lulea, Sweden.
Description: Midnight sun conference in honour of Lars-Erik Persson on the occasion of his 65th birthday.
Information: http://www.math.ltu.se/aiht/.

* 7–12 **35th International Conference: Applications of Mathematics in Engineering and Economics (AMEE'09)**, The Black Sea resort of Sozopol, Bulgaria.
Description: This is the 35th in a series of conferences on the application of mathematics to foundational issues in general science and in engineering sciences. The aim of the conference is to provide a widely selected forum among experts and young scientists to exchange ideas, methods and techniques in the fields of Applied Mathematics and Mathematical Modeling. In addition to the invited lectures there will be sessions for twenty-minute contributed talks. Young researchers are particularly encouraged to present their recent works. The conference post-proceedings will be published only after peer reviewing by the International Conference Proceedings Series of AIP.
Local Organizers: Ketty Peeva, Vesela Pasheva, Adriana Georgieva, Nikola Kaloyanov, Krasimira Prodanova, George Venkov, Yana Stoyanova.
International Programme Committee: Ralitza Kovacheva, Mayer Aladjem, Stefanka Chukova, Maya Flikop, Vladimir Georgiev, Michail Konstantinov, Raytcho Lazarov, Bernadette Miara, Petko Petkov.
Information: http://www.tu-sofia.bg/fpmi/amee/.

* 10–12 (NEW DATE) **Sixth Advanced Course in Operator Theory and Complex Analysis**, Seville, Spain.
Invited Speakers: Gilles Cassier (Université Lille 1, France), Carl C. Cowen (Purdue University, USA), Nikolai G. Makarov (California Institute of Technology, USA), Armen G. Sergeev (Steklov Mathematical Institute, Russia).
Information: http://congreso.us.es/ceacyto/2009.

* 15–19 **Conference on Harmonic Analysis, Geometric Measure Theory and Quasiconformal Mappings**, CRM: Centre de Recerca Matematica, Bellaterra, Spain.
Purpose: Of the conference is to provide researchers working in harmonic analysis, quasiconformal mappings or geometric measure theory with a scientific event designed to promote a deep interaction between the three subjects. The themes dealt with in the conference will be: 1. New developments on distortion of sets under quasiconformal mappings (Lacey, Sawyer, Uriarte) 2. Quasiconformal mappings and PDE, in particular the Calderón inverse problem (Astala, Faraco, Iwaniec, Päivärinta, Zhong) 3. Removability and quasiconformal mappings (Astala, Clop, Mateu, Orobitg, Tolsa) 4. Functions of finite distortion and hyperelastic deformations (David, Iwaniec, Koskela, Saksman) 5. Metric measure spaces, Poincaré's inequality and harmonic functions (Hajlasz, Koskela, Zhong) 6. Human vision and the Heisenberg group (Citti, Sarti). Grants are available for young researchers.
Chair: Pertti Mattila, Univ. of Helsinki, Finland.
Organisers: ESF-EMS-ERCOM.
Information: http://www.esf.org/conferences/09308.

* 15–19 **International Summer School on Operator Algebras and Applications**, Instituto Superior Tecnico, UTL, Lisbon, Portugal.
Description: The main goal of this summer school is to present developments in operator algebras, function algebras and applications and to promote research exchanges in these topics. The school is being organized by the Research Center for Functional Analysis and Applications, at Instituto Superior TÈcnico, Lisbon, Portugal. It consists of four short courses covering the following topics: Crossed product $C*$-algebras, Christopher Phillips, University of Oregon; Regularity conditions for Banach function algebras, Joel Feinstein, University of Nottingham; Spatial discretization of $C*$-algebras, Steffen Roch, TU Darmstadt; $C*$-algebras: Hilbert Modules, Index Theory, K-Homolology, Vladimir Manuilov, Moscow State University. To stimulate the participation of students and young researchers, a small number of scholarships will be available.

* 16–25 **International Workshop on Resonance Oscillations and Stability of Nonsmooth Systems**, Imperial College, London, United Kingdom.

Description: Mathematical research on resonance oscillations in nonsmooth systems has seen a steady growth in attention lately, and due to recent progress a number of technically relevant open problems concerning resonances of nonsmooth mechanical and physical systems have become in reach of being resolved. The object of the meeting is to explore the current power and open problems of the theory of resonance oscillations and stability of nonsmooth systems by bringing together different research groups working in the field as well as by organizing discussions with relevant industrial experts. The first part of the workshop (four days: 16-19 June) will be formal, with a full schedule of invited and contributed talks. In contrast, the 2nd part (four days: 22-25 June) will be more informal, with discussion sessions on specific problems aimed at developing new research collaborations.
Information: http://www.ma.ic.ac.uk/~omakaren/rosns2009/.

* 18–20 **Models, Logics and Higher-Dimensional Categories: A tribute to the work of Mihaly Makkai**, Centre de recherches mathématiques, Université de Montréal, Montreal, QC Canada.
Description: This meeting celebrates the mathematical career of Mihaly Makkai, focussing on his work in model theory, logic, and higher dimensional category theory. Recently there has been a widening consensus that traditional model theory might usefully embrace the categorical methods he pioneered. Traditional model theory is set-based; in contrast, there has been an increasing use of category-theoretic (specifically including sheaf-theoretic) contexts and techniques in recent model-theoretic work in such areas as algebraic geometry, differential algebra, Mordell-Lang theory, etc. We will have contributions from Michael's former and current colleagues and students, including the following: Mike Barr, Victor Harnik, Bradd Hart, Andre Joyal, Hal Kierstead, Julia Knight, Francois Lamarche, Robert Pare, Anand Pillay, Gonzalo Reyes, Charles Steinhorn, Marek Zawadowski.
Information: http://www.crm.umontreal.ca/Makkaifest09/index_e.php.

* 22–25 **Geometric Flows in Mathematics and Theoretical Physics**, Center "Ennio De Giorgi"–Scuola Normale superiore, Pisa, (IT).
Description: The workshop aims to report on the recent interdisciplinary developments about geometric flows with an emphasis on the discussion of problems and new research directions coming from the mathematical and theoretical physics communities.
Speakers will include: J. Bakas, G. Bellettini, H.-D. Cao (to be confirmed), F. Costantino, D. Friedan, J. Gegenberg, D. Glickenstein, J. Isenberg, A. Magni, S. Maillot, L. Ni (to be confirmed), M. Petropoulos, E. Tsatis, S. Vardarajan, E. Wolgar.
Information: For more information, contact Mauro Carfora at: Mauro.Carfora@pv.infn.it; or visit: http://cvgmt.sns.it/MPRicci-Workshop.

* 23–25 **Geometric Topology in 3 and 4 Dimensions**, University of California, Davis, California.
Description: This conference, in honor of Martin Scharlemann, will consist of four, one hour long talks on each of the three days.
Speakers will include: David Bachman (Pitzer College), Dave Gabai (Princeton), Hiroshi Goda, (Tokyo University), Robert Gompf (UT Austin), Cameron Gordon (UT Austin), Ko Honda (University of Southern California), Tao Li (Boston College), Daryll McCullough (University of Oklahoma), Ruifeng Qiu (Dalian University), and Hyam Rubinstein (University of Melbourne).
Information: http://www.math.yale.edu/~jj327/conference/.

* 24–26 **International Workshop on Reliable Methods of Mathematical Modeling (RMMM 2009)**, Humboldt-Universität zu Berlin, Germany.
Aims and Scope: The workshop is organized to bring together specialists developing mathematical and computational methods intended to increase the reliability of the numerical results obtained in various mathematical modeling methods.
Main Topics: Mesh-adaptive numerical methods in various applied problems, a posteriori error control and verification of numerical solutions, validation of mathematical models used in computer simulation, nonlinear computational partial differential equations.
Program Committee: D. Braess, C. Carstensen, W. Dörfler, R. Hoppe, P. Neittaanmaki, R. Rannacher, J. Rappaz, S. Repin, S. Sauter, R. Verfürth.
Organizing Committee: C. Carstensen, P. Neittaanmaki, S. Repin, S. Sauter.
Information: http://www.math.hu-berlin.de/~rmmm2009.

* 24–26 **Sixth Advanced Course in Operator Theory and Complex Analysis**, Seville, Spain.
Invited Speakers: Gilles Cassier (Université Lille 1, France), Carl C. Cowen (Purdue University, USA), Nikolai G. Makarov (California Institute of Technology, USA), Armen G. Sergeev (Steklov Mathematical Institute, Russia).
Information: http://congreso.us.es/ceacyto/2009.

July 2009

* 3–7 **International Conference on Mathematical Control Theory and Mechanics**, Suzdal, Russia.
Description: Steklov Mathematical Institute, Russian Academy of Sciences, and Vladimir State University are organizing an International Conference on Mathematical Control Theory and Mechanics. The Conference will include the following sections: 1. Mathematical problems of Control Theory; 2. Mathematical problems of Mechanics.
Information: http://mctm2009.vlsu.ru.

* 5–10 **The second European Set Theory Meeting: In honor of Ronald Jensen — an ESF-EMS-ERCOM research conference**, Mathematical Research and Conference Center, Bedlewo, Poland.
Scope: Over the last century set theory has developed into a vibrant and important subject. On the one hand, it deals with questions of mathematical logic of deep foundational importance, such as the choice of axioms for mathematics and the questions of relative consistency of mathematical theories. On the other hand, techniques of set theory are applied in many areas of mathematics such as classical analysis, general topology, measure theory, Banach space theory, abstract algebra, ergodic theory and dynamical systems.
Topics: Inner model theory and large cardinals; Descriptive set theory; Combinatorial set theory; Applications of set theory to Banach spaces, measure theory, general topology, and other neighboring areas.
Speakers: Menachem Magidor will give a Special Lecture on the work of Ronald Jensen.
Grants: Available for young researchers. Closing date for applications and abstracts: April 8, 2009/
Chair: Jouko Väänänen, Univ. of Helsinki, FI/Univ. of Amsterdam, NL.
Information: http://www.esf.org/conferences/09306.

* 5–18 **39th Probability Summer School**, Saint-Flour, France.
Description: Three courses will be given: Robert Adler: Topological complexity of smooth random functions. Mireille Bousquet-Melou: Enumerative combinatorics for probability. Alison Etheridge: Some mathematical models from population genetics.
Deadline for registration: April 20, 2009.
Information: http://math.univ-bpclermont.fr/stflour/.

* 15–20 **Dobrushin International Conference**, Institute for the Information Transmission Problems, RAS, Moscow, Russia.
Description: Institute for Information Transmission Problems of the Russian Academy of Sciences and Independent University of Moscow organize a conference to honor the 80-th anniversary of Prof. Roland L. Dobrushin (1929-1995). The areas of emphasis of the conference correspond to the main fields of interest of Roland Dobrushin and

include Probability Theory, Mathematical Physics, Statistical Mechanics, Information Theory and Mathematical Linguistics.

* 20–22 **Future Models for Energy and Water Management— AMSI/MASCOS/UNESCO Industry workshop and short course: Future Models for Energy and Water Management under a Regulated Environment**, Queensland University of Technology, Brisbane, Queensland 4000 Australia.

Description: This three-day event will consist of three 3-hour short courses and three 3-hour workshops on Future Models for Energy and Water Supply under a Regulated Environment with contributed talks in the afternoon. Participate in workshops — offering industry professionals a rare opportunity to share their experience, expertise and discuss problems.

Topics: Likely to be addressed will be: Energy and water economy in a regulated environment, Renewable energy sources, Network modeling and management, Risk management, Risk-return and efficient frontiers, Power system stability, Network optimisation, Demand forecasting, Carbon cost of food, Demand side management.

Information: `http://www.amsi.org.au/energy.php`.

* 20–24 **Vibration and Structural Acoustics Analysis: in conjunction with 3rd International Conference on Integrity, Reliability & Failure**, FEUP/INEGI, University of Porto, Porto, Portugal.

Description: Papers are solicited in the following and related areas: Vibration analysis; analytical and computational structural acoustics and vibration; material systems and technologies for noise and vibration control; passive/semi-active/active control techniques; piezoelectric and viscoelastic damping technologies; shape/position and fluid flow control; nonlinear dynamics and vibrations; vibration-based structural health monitoring/evaluation and damage prognosis; machinery noise/vibration and diagnostics; reciprocating machinery and rotor dynamics; experimental testing in vibration and structural acoustics; experimental modal analysis; modeling of the dynamics and damping of complex systems; noise and vibration on high-speed trains and railways; structural and musical acoustics; applications and case studies in structural acoustics and vibration.

Promoters: J. Dias Rodrigues & C. M. A. Vasques.

Deadlines: Extended Abstracts are due by 28 February 2009 (please see address below), and those accepted will appear in a book which will be made available to delegates of IRFA 2009 during the event. Full papers will also be compiled in a CD-ROM and selected papers will be considered for publication in a number of scientific journals of international reputation.

Information: `http://paginas.fe.up.pt/clme/IRF2009`; email: `jdr@fe.up.pt`; `cvasques@fe.up.pt`.

August 2009

* 1–15 **Groups St Andrews 2009 in Bath**, University of Bath, Bath, United Kingdom.

Topics: The conference aims to cover all aspects of group theory. The short lecture courses are intended to be accessible to postgraduate students, postdoctoral fellows, and researchers in all areas of group theory.

Programme: The speakers below have kindly agreed to give short courses of lectures in week 1. In addition there will be a programme of one-hour invited lectures and short research presentations. There will be three theme days in week 2: a “B.H. Neumann day”, a day to celebrate birthdays of John Cannon and Derek Holt, and an “Engel groups day”.

Principal Speakers: Gerhard Hiss (RWTH, Aachen, Germany), Volodymyr Nekrashevych (Texas A&M, Texas, USA), Dan Segal (All Souls College, Oxford), Eamonn O’Brien (Auckland, New Zealand), Mark Sapir (Vanderbilt, Nashville, USA).

Information: `http://www.groupsstandrews.org/2009/index.html`.

* 9–15 **Reconnect Conference 2009: Reconnecting Teaching Faculty to the Mathematical Sciences Enterprise and Exposing Researchers in Government and Industry to Relevant Current Research, Visual Analytics and its Applications.**, DyDAn Center, Rutgers University, Piscataway, New Jersey.

About the Reconnect Conference: The conference will expose faculty teaching undergraduates to the role of the mathematical sciences in homeland security by introducing them to a current research topic that will be relevant for classroom presentation. The conferences also offer researchers in government or industry the opportunity to learn about recent techniques in emerging application areas relevant to homeland security.

Principal Speaker: Georges Grinstein, University of Massachusetts, Lowell; `grinstein@cs.uml.edu`.

Topics: Will be presented in a week-long series of lectures and activities led by an expert in the field. Participants will be involved in both research activities and in writing materials useful in the classroom or to share with their colleagues, with the possibility of ultimately preparing these materials for publication in either our Technical Report or our Educational Modules Series. These workshops offer the opportunity for junior faculty as well as mid-level and senior faculty and government and industry professionals to explore research questions in a new area of the mathematical sciences. Participants will also acquire materials and gain ideas for seminar presentations and for undergraduate research projects and have the opportunity to network with people from a variety of backgrounds.

Conference Organizer: Fred S. Roberts, Rutgers University (email: `froberts@dimacs.rutgers.edu`).

Registration: Registration Fees, Lodging, Meals and Travel: See website. Anyone may apply. Preference will be given to faculty whose primary job is undergraduate teaching and those working at government labs. Two-year college faculty are welcome to apply. Faculty from groups under-represented in mathematics are also encouraged to apply.

Deadline: Application deadline is April 15, 2009.

Information: To receive more information, visit our website at: `http://dydan.rutgers.edu/Education/reconnect/2009/`. Or, contact the Reconnect Program Coordinator, at: `reconnect@dimacs.rutgers.edu`; tel: (732) 445-4304.

* 10–14 **Nonlinear problems for p-Laplace and Laplace**, Linköping, Sweden.

Description: The aim of the conference is to cover p-harmonic functions, infinity-harmonic functions, p-parabolic functions, water waves and related subjects such as various types of generalizations including quasiminimizers, the porous media equation and the corresponding theories on metric spaces.

Lecture Series: Michael Crandall (Santa Barbara), Juha Kinnunen (Helsinki Tech), Olli Martio (Helsinki), and John Toland (Bath) will give series of three lecture each.

Invited Speakers: Walter Craig (McMaster, Hamilton), Ugo Gianazza (Pavia), Mark Groves (Saarbrücken), Robert Jensen (Loyola, Chicago), Nikolay Kuznetsov (St Petersburg), Peter Lindqvist (Trondheim), Kaj Nystrˆm (Umeå) Xiao Zhong (Jyväskyla).

Information: `http://www.mai.liu.se/TM/conf09/`; email: `conf-p-laplace@mai.liu.se`.

* 17–19 **Measurement, Design, and Analysis Methods for Health Outcomes: Research Offered by the Harvard School of Public Health Center for Continuing Professional Education**, Boston, Massachusetts.

Description: Taught in an interactive classroom setting, this program is geared towards introductory to intermediate learning levels to help participants design, implement, and analyze outcomes studies, and critically review and use outcomes data for clinical decision making, health care planning, and technology development. The course provides participants with an overview of several topics in the exciting new field of health outcomes, equipping newcomers with knowledge of the language and concepts. In an increasingly competitive health care market, outcomes evaluation and research is important for: ∗ Improving patient outcomes, ∗ Controlling costs and allocating resources, ∗ Implementing disease management programs, ∗ Making effective clinical and business decisions, ∗ Developing and marketing health care products and services. Improving your analytical and quantitative understanding and skills can greatly enhance the quality of health outcomes research in the health care and pharmaceutical industries.

Information: `http://www.hsph.harvard.edu/ccpe/programs/MDA.html`. Please be sure to mention your reference code: MDA09-CAL 14.

* 23–25 **Salford Data Mining Conference 2009 (SALFORD 2009)**, San Diego, California.

Description: Post-Conference Training: August 26-28, 2009. Keynotes and panel discussions will address topics of current interest in data mining including: The Credit Crunch: How is Data Mining Helping? Is Data Mining to Blame? Finding Needles in Haystacks: Combating Terrorism, Fraud, and Criminal Activity. Advertising, Politics, Drug Discovery and Telecommunications: How Do These Applications Benefit from Data Mining? Presentation lists from previous conferences: `http://www.salforddatamining.com/prevprog.php`.

Key Links: Conference website: `http://www.salforddatamining.com`.

Conference update list: `http://www.salforddatamining.com/conferenceupdatelist.php`. `http://www.salfordatamining.com/docs/2009conferenceForm.pdf`.

Abstract Submissions: `http://www.salforddatamining.com/2009Abstract.php`.

Post-conference training: `http://www.salforddatamining.com/postConferenceTraining.php`.

Information: `http://www.salforddatamining.com/`.

* 24–26 **2009 Workshop for Young Researchers in Mathematical Biology (WYRMB)**, Mathematical Biosciences Institute, The Ohio State University, Columbus, Ohio.

Description: This workshop is intended to broaden the scientific perspective of young researchers in mathematical biology and to encourage interactions with other scientists. Workshop activities include plenary talks, poster sessions, and discussion forums on "Applying for Jobs" and "Math Bio Jobs in Industry." We cordially invite young researchers to apply for participation in the workshop! All invitees will be expected to present a poster, and this year, a select number will be chosen to give short talks as well. The MBI will plan to cover local expenses for all invitees, but travel expenses may only be available on a competitive basis.

Information: `http://www.mbi.osu.edu/wyrmb/wyrmb2009.html`.

* 24–28 **Mal'tsev Meeting**, Sobolev Institute of Mathematics SB RAS, Novosibirsk, Russia.

Description: This international conference on algebra, mathematical logic, and applications is dedicated to the 100th anniversary of Anatolii Ivanovich Mal'tsev (1909-1967). The programme of the conference will consist of 26 invited talks and contributions in sections. The main topics include group theory, ring theory, universal algebra, mathematical logic, computability theory, theoretical computer science, and related areas of mathematics.

Information: `http://www.math.nsc.ru/conference/malmeet/09/index.html`.

* 30–31 **Oded Schramm Memorial Conference in Probability and Geometry**, Microsoft Research, Redmond, Washington.

Description: A two-day conference to be held at Microsoft Research in honor of Oded Schramm and his mathematics. Speakers at the conference will include Omer Angel (Univ. British Columbia), Itai Benjamini (Weizmann), Mario Bonk (Univ. Michigan), Michael Freedman (Microsoft), Christophe Garban (ENS Paris), Olle Häggström (Chalmers), Zheng-Xu He (Beijing CAS), Gregory F. Lawler (Univ. Chicago), Russell Lyons (Indiana Univ.), Assaf Naor (Courant), Yuval Peres (Microsoft), Gábor Pete (Univ. Toronto), Steffen Rohde (Univ. Washington), Scott Sheffield (MIT), Stanislav Smirnov (Univ. Genève), Wendelin Werner (Orsay), and David B. Wilson (Microsoft).

Information: `http://research.microsoft.com/~schramm/workshop/`.

September 2009

* 3–5 **International Conference on Modern Mathematical Methods in Science and Technology (M3ST '09)**, Poros Image Hotel, Poros Island, Greece.

Topics: Differential equations and mathematical models, numerical analysis, mathematics of computation, applications of mathematics in economy, stochastic analysis, modelling optimization, control theory, image and signal processing.

Invited Speakers: H. Ammari (CNRS, France), G. Bellettini (Roma, Tor Vergata, Italy), N. Bouleau (ENPC, France), G. Dassios (Patras, Greece), P. Imkeller (Humbolt University, Berlin, Germany), O. A. Karakashian (Tennessee, Knoxville, U.S.A.), L. Kirousis Patras, Greece), D. J. N. Limebeer (Imperial College, U.K.), F. Murat (Paris VI, France), E. M. Ouhabaz (Bordeaux, France), G. Papanicolaou (Stanford, U.S.A.), J.-C. Saut (Paris Sud 11, France), A. Tertikas (Heraklion, Crete, Greece), A. E. Tzavaras (Heraklion, Crete, Greece).

Information: `http://www.math.uoa.gr/M3ST09/index.html`.

* 3–6 **International Conference on Theory and Applications in Mathematics and Informatics**, "1 Decembrie 1918" University of Alba Iulia, Alba Iulia, Romania.

Description: The aim of the conference is to bring together mathematicians and informaticians from all over the world and to attract original papers on the following topics: algebra, analysis and complex analysis, topology and geometry, differential equations, probability and statistics, applied mathematics, computer science, intelligence computation, product and process modelling, embedded systems, knowledge engineering, e-education.

Information: `http://www.uab.ro/ictami`.

* 7–12 **Advanced School on Homotopy Theory and Algebraic Geometry**, Mathematical Research Institute, University of Sevilla (IMUS), Sevilla, Spain.

Description: The school is addressed to Ph.D. students and young post-doc researchers working on Algebraic Geometry and related areas. There will be three main minicourses: (1) Derived Algebraic Geometry. (2) Model Categories and Derivators. (3) Cartan-Eilenberg Categories and Descent Categories.

Speakers: Francisco Guillén Santos, University of Barcelona; Bernhard Keller and Georges Maltsiniotis, University of Paris 7; Vicente Navarro Aznar, University of Barcelona; Beatriz Rodríguez González, CSIC-Madrid; Bertrand Toën and Michel Vaquié, University of Toulouse; Gabriele Vezzossi, University of Bologna.

Information: `http://congreso.us.es/htag09`.

* 8–12 **IV International Conference on Mathematical Analysis in Andalucia**, University of Cadiz, Jerez de la Frontera, Spain.

Description: This edition will be dedicated to the memory of Professor Antonio Aizpuru Tomas, full professor in Mathematical Analysis of Cadiz University, who suddenly passed away on May 1, 2008. He was mainly responsible for the development of the studies of mathematics in Cadiz and in the research activities on functional analysis in this university. He was also a beloved person and friend. We kindly invite you to participate in this scientific event which we hope will be of interest to you.

Information: `http://cidama.uca.es`.

* 11–13 **Algebra and Topology in Interaction**, University of California, Davis, California.

Description: In honor of Professor Dmitry Fuchs' 70th Anniversary.

Theme: The main theme of the conference is the interplay of algebra and topology over the past 40 years, since the birth of Gelfand Fuchs cohomology.

Topics: Include current exciting developments in symplectic field theory, representations of infinite dimensional Lie algebras, topological quantum field theory, topological applications of cohomology of infinite dimensional Lie algebras, characteristic classes of foliations, contact homology, Chekanov Eliashberg differential graded algebra, and Legendrian knot theory.

List of speakers: B. Feigin, E. Frenkel, S. Gindikin, A. Givental, M. Khovanov, A. Kirillov, S. Novikov, V. Retakh, C. Roger, G. Segal, S. Tabachnikov, and O. Viro. Mathematicians at all levels are invited to attend. An important goal of the conference is to provide an opportunity for a diverse group of mathematicians including postdoctoral researchers, those with traditionally underrepresented background, graduate students, and faculty from primary undergraduate institutions, to meet and discuss mathematics with the invited leading experts of the field.

Deadline: Those interested in receiving funding for travel should apply by July 31, 2009. This conference is supported by the NSF, MSRI, and UC Davis.

Information: email: `cdani@math.ucdavis.edu`; `http://www.math.ucdavis.edu/research/algetopcon`.

* 27–29 **Symposium on Engineered & Natural Complex Systems**, Toronto, Ontario, Canada.

Topics: Include, but are not limited to, the following: Structure, function and dynamics of complex systems, i.e. data communication networks, cyberspace, transportation networks, organizational networks, power grids, biological, physical, social, ecological, epidemiological and other complex systems & networks; Emergence, multiscale phenomena, self-organization, self-similarity, long range dependence, phase transition, pattern formation, synchronization, robustness, reliability, fragility, interdependence, cooperation, adaptation, evolution; Analysis & control techniques of dynamics & performance, mean field & information theory of complex systems & networks; Cellular automata, agent based & individually based and other models of complex systems & networks and their simulations.

Information: `http://toronto.ieee.ca/tic-sth2009/cfps/IEEE_TIC-STH09_CFP-ENS.pdf`.

October 2009

* 5–9 **International Conference "Kolmogorov readings. General control problems and their applications (GCP-2009)"**, Tambov State University named after G.R. Derzhavin, Institute of Mathematics, Physics, and Computer sciences, Tambov, Russia.

Description: The conference is the 4th one in the series "Kolmogorov readings" gathering international scientists in the city where the outstanding mathematician, A.N. Kolmogorov, was born. Traditionally the conference will mainly focus on general control problems and their applications in natural and human sciences, optimization theory, differential equations and inclusions. There are planned plenary (40 min.) and sectional (20 min.) talks, as well as a school on optimal control aimed to Ph.D students and young researches.

Information: `http://www.tambovopu2009.narod.ru/`.

* 12–14 **The 6th annual International New Exploratory Technologies Conference (NEXT 2009)**, Fudan University, Shanghai, China.

Description: This year's NEXT focuses on four special themes: Productization and Commercialization, Productization of Embedded Software in Products and Services, Renewable Energy Technology, and Exploratory Materials and Technology.

Organizers: Fudan university and University of Turku.

Information: `http://next.utu.fi/2009`.

* 19–22 **International Conference "Discrete Mathematics, Algebra, and their applications" (DIMA09)**, Belarus State University, Minsk, Belarus.

Description: The conference is dedicated to the 80th birthday of Professor Regina Tyshkevich. The research activity of Regina Tyshkevich is connected with two fields: discrete mathematics (graph theory, combinatorics) and algebra (permutation groups, linear groups, matrix algebras). She is the founder of the Belarus school in graph theory, that has gained a worldwide recognition. Professor Tyshkevich was awarded the Belarus State Prize and the title of a Distinguished Worker of Education. The conference topics include (but are not restricted to): Graph theory, combinatorics, discrete optimization, algorithms, data structures and computational complexity, applications of discrete mathematics in computer science, operations research, algebra, topology, probability theory, permutation groups, linear groups and representations, Brauer groups of varieties and algebraic groups.

Information: `http://dima09.bsu.by`.

* 26–28 **SAGA 2009, Fifth Symposium on Stochastic Algorithms, Foundations and Applications**, Hokkaido University, Sapporo, Japan.

Description: The symposium offers the opportunity to present original research on the analysis, implementation, experimental evaluation, and real-world application of stochastic algorithms. The focus of SAGA'09 is on new algorithmic ideas involving stochastic decisions and the design and evaluation of stochastic algorithms within realistic scenarios. Thus, the symposium wants to foster the co-operation between practitioners and theoreticians from this research area.

Topics: Original research papers (including significant work-in-progress and work identifying and exploring directions of future research) or state-of-the-art surveys are invited on all aspects of algorithms employing stochastic components.

Information: `http://www-alg.ist.hokudai.ac.jp/~thomas/SAGA09/saga09.html`.

* 26–30 **Implementing algebraic geometry algorithms**, American Institute of Mathematics, Palo Alto, California.

Description: This workshop, sponsored by AIM and the NSF, will be devoted to developing three packages, algebraic statistics, numerical algebraic geometry, toric algebraic geometry, for the computer algebra system Macaulay 2. Macaulay 2 is a widely used computer algebra system for research and teaching in algebraic geometry and commutative algebra and is one of the leading computer algebra programs for performing such computations.

Information: Visit `http://aimath.org/ARCC/workshops/agalgorithms.html`.

* 26–31 **Autumn School: "Towards a p-adic Langlands Correspondence"**, Mathematical Research Institute, University of Sevilla (IMUS), Sevilla, Spain.

Description: The school is addressed to Ph.D. students and young post-doc researchers working on number theory, arithmetic algebraic geometry and related areas. There will be four main minicourses: (1) Introduction to the theory of representations of p-adic groups. (2) Modular forms, automorphic forms and GL(2). (3) The Langlands program. (4) Towards a modular Langlands correspondence.

Speakers: James Cogdell, Ohio State University; Jean François Dat, Université Paris 6; Guy Henniart, Université Paris Sud; Ariane Mézard Université de Versailles; Vincent Sécherre, Université de Marseille; Shaun Stevens, University of East Anglia; Jose M. Tornero Sánchez, Universidad de Sevilla.

Information: `http://congreso.us.es/planglands09`.

November 2009

* 19–21 **2nd meeting on Optimization Modelization and Approximation Moma 2009**, Hassania School, Public Works Département de Mathématiques et Informatique Km 7, Route d'El Jadida, B.P 8108, Oasis-Casablanca, Morocco.

Description: The scope of this second meeting covers a range of major topics in numerical analysis, optimization, also in approximation and engineering and related disciplines, ranging from theoretical developments to industrial applications and modelling of problems. The themes of the conference include, but are not limited to: Optimization, computational optimization frameworks, optimization modeling, approximation theory, radial basis functions, scattered data approximation, learning machine theory, meshless methods, numerical analysis, modelization. Applications: Image processing, financial computation, medicine and biology.

Information: `http://www-lmpa.univ-littoral.fr/MOMA09/`.

December 2009

* 14–18 **Brownian motion and random matrices**, American Institute of Mathematics, Palo Alto, California.

Description: This workshop, sponsored by AIM and the NSF, will be devoted to beta-generalizations of the classical ensembles in random matrix theory. These are certain tridiagonal and unitary Hessenberg matrices, with an eigenvalue p.d.f. generalizing that of Gaussian Hermitian matrices and Haar distributed unitary matrices.

Information: `http://aimath.org/ARCC/workshops/brownianrmt.html`.

* 19–21 **International Conference on Current Trends in Mathematics**, Allahabad, Uttar Pradesh, India.

Description: The aim of the conference is to introduce undergraduate and Ph.D. students in mathematics as well as post-doctoral researchers to recently emerged trends of mathematics.

Deadline: Submit abstracts with full-length paper to: `complexgeometry18@yahoo.com`: October 20, 2009. Acknowledgement of accepted papers by email: October 25, 2009. For registration: November 15, 2009. All submitted papers will be under peer review and accepted papers will be published in the conference proceedings.

Information: `complexgeometry18@yahoo.com`; Sushil Shukla (email: `ss123a@rediffmail.com`).

January 2010

* 2–4 **International Convention on Mathematical Sciences**, Allahabad, India.

Description: The aim of the conference is to introduce undergraduate and Ph.D. students in mathematics as well as post-doctoral researchers in recently emerged trends of mathematics.

Deadlines: The deadline for submitting abstracts with full-length paper to `complexgeometry18@yahoo.com`: October 20, 2009. Acknowledgement of accepted papers by email: October 25, 2009. For registration: November 15, 2009. All submitted papers will be under peer review and accepted papers will be published in the conference proceedings.

Information: Contact: `complexgeometry18@yahoo.com`.

* 24–26 **International Conference on Analysis and Applications (ICAA10)**, Sultan Qaboos University, Muscat, Oman.

Description: The aim of this conference is to reflect the current state of the art in the study of analysis with the hope to promote scientific exchange among analysts all over the world. The main goal of this conference is to discuss new developments and future directions in analysis. Its particular focus is on the active participation of all who attend to promote a spirit of training, learning and communicating. The conference will consist of plenary talks and contributed talks of 25 minutes (20 + 5 for discussion) in parallel special sessions. All areas of analysis-related mathematics, especially topology, complex analysis, real and functional analysis, numerical analysis, and applications of analysis to other areas of mathematics and the sciences, will be covered. For any inquiries please contact us at: `http://icaa10@squ.edu.om`.

Information: `http://www.squ.edu.om/Portals/87/Conference/ICAA10/conference2010/ICAA10.htm`.

February 2010

* 8–11 **The International Symposium on Stochastic Models in Reliability Engineering, Life Sciences, and Operations Management**, Sami Shamoon College of Engineering, Bialik/Basel Sts., Beer Sheva, 84100, Israel

Description: The SMRLO'Ò10 will serve as a forum for discussing different issues of Stochastic Models and Methods in Reliability Engineering, Life Sciences, and Operations Management and their applications. The idea of the symposium is to assemble researchers and practitioners from universities, institutions, industries, businesses and government, working in these fields. Theoretical issues and applied case-studied, presented on the symposium, will range from academic considerations to operational applications. There will be invited talks, plenary sessions, parallel sessions, posters and exhibitions. The talks will be selected by the program committee and will be included in the symposium proceedings. Selected papers after review and revision will be published in special issues of international journals.

Information: Tel: +972-8-6475-642; fax: +972-8-6475-643; `http://info.sce.ac.il/i/SMRLO10`.

March 2010

* 15–19 **Localization techniques in equivariant cohomology**, American Institute of Mathematics, Palo Alto, California.

Description: This workshop, sponsored by AIM and the NSF, will be devoted to localization techniques in equivariant cohomology. Localization techniques in equivariant cohomology are a powerful tool in computational algebraic topology in the context of a topological space with the action of a Lie group.

Information: Visit `http://aimath.org/ARCC/workshops/localization.html`.

The following new announcements will not be repeated until the criteria in the next to the last paragraph at the bottom of the first page of this section are met.

December 2010

* 25–27 **International Conference on Current trends in Mathematics**, Allahabad, Uttar Pradesh, India.

Description: The aim of the conference is to introduce undergraduate and Ph.D. students in mathematics as well as post-doctoral researchers in recently emerged trends of mathematics.

Deadline: For submitting abstracts with full-length paper to `complexgeometry18@yahoo.com`: October 20, 2010. Acknowledgement of accepted papers by email: October 25, 2010. For registration: November 15, 2010. All submitted papers will be under peer review and accepted papers will be published in the conference proceeding.

Information: Contact: `complexgeometry18@yahoo.com`.

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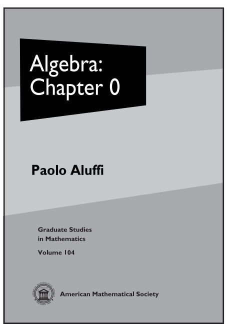
Algebra and Algebraic Geometry

COURSE ADOPTION



Algebra: Chapter 0

Paolo Aluffi, *Florida State University, Tallahassee, FL*

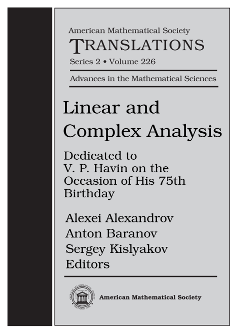
Algebra: Chapter 0 is a self-contained introduction to the main topics of algebra, suitable for a first sequence on the subject at the beginning graduate or upper undergraduate level. The primary distinguishing feature of the book, compared to standard textbooks in algebra, is the early introduction of categories, used as a unifying theme in the presentation of the main topics. A second feature consists of an emphasis on homological algebra: basic notions on complexes are presented as soon as modules have been introduced, and an extensive last chapter on homological algebra can form the basis for a follow-up introductory course on the subject. Approximately 1,000 exercises both provide adequate practice to consolidate the understanding of the main body of the text and offer the opportunity to explore many other topics, including applications to number theory and algebraic geometry. This will allow instructors to adapt the textbook to their specific choice of topics and provide the independent reader with a richer exposure to algebra. Many exercises include substantial hints, and navigation of the topics is facilitated by an extensive index and by hundreds of cross-references.

Contents: Preliminaries: Set theory and categories; Groups, first encounter; Rings and modules; Groups, second encounter; Irreducibility and factorization in integral domains; Linear algebra; Fields; Linear algebra, reprise; Homological algebra; Index.

Graduate Studies in Mathematics, Volume 104

July 2009, approximately 728 pages, Hardcover, ISBN: 978-0-8218-4781-7, LC 2009004043, 2000 *Mathematics Subject Classification:* 00-01; 12-01, 13-01, 15-01, 18-01, 20-01, **AMS members US$71**, List US$89, Order code GSM/104

Analysis



Linear and Complex Analysis

Dedicated to V. P. Havin on the Occasion of His 75th Birthday

Alexei Alexandrov, *Steklov Mathematical Institute at St. Petersburg, Russia*, **Anton Baranov**, *Saint Petersburg State University, St. Petersburg, Russia*, and **Sergey Kislyakov**, *Steklov Mathematical Institute at St. Petersburg, Russia*, Editors

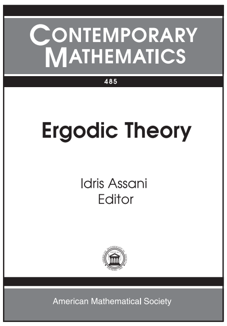
The volume consists of articles by friends and collaborators of a renowned Russian mathematician V. P. Havin, prepared on the occasion of Havin's 75th birthday. The articles in the volume are devoted to areas of analysis where Havin himself worked successfully for many years.

Contents: **A. Aleman** and **C. Sundberg**, Zeros of functions in weighted Bergman spaces; **S. Alesker**, **S. Artstein-Avidan**, and **V. Milman**, A characterizataion of the Fourier transform and related topics; **J. Bourgain**, Geodesic restrictions and L^p-estimates for eigenfunctions of Riemannian surfaces; **S. Favorov** and **L. Golinskii**, A Blaschke-type condition for analytic and subharmonic functions and application to contraction operators; **A. Fryntov** and **L. Nazarov**, New estimates for the length of the Erdős-Herzog-Piranian lemniscate; **P. M. Gauthier** and **M. S. Melnikov**, Compact approximation by bounded functions and functions continuous up to the boundary; **J.-P. Kahane**, Un théorème de Helson pour des séries de Walsh; **X. Massaneda** and **J. Ortega-Cerdà**, Interpolation sequences for the Bernstein algebra; **V. Maz'ya**, Integral and isocapacitary inequalities; **V. V. Peller**, Differentiability of functions of contractions; **A. Poltoratski**, Asymptotic behavior of arguments of Cauchy integrals; **D. Sarason**, Free interpolation in the Nevanlinna class; **K. Seip**, Interpolation by Dirichlet series in H^∞; **M. Solomyak**, Remarks on counting negative eigenvalues of the Schrödinger operator on regular metric trees; **S. Treil**, H^1 and dyadic H^1; **V. Vasyunin** and **A. Volberg**, Monge-Ampère equation and Bellman optimization of Carleson embedding theorems; **A. Volberg** and **P. Yuditskii**, Remarks on

Nehari's problem, matrix A_2 condition, and weighted bounded mean oscillation.

American Mathematical Society Translations—Series 2 (*Advances in the Mathematical Sciences*), Volume 226

June 2009, approximately 269 pages, Hardcover, ISBN: 978-0-8218-4801-2, LC 91-640741, 2000 *Mathematics Subject Classification:* 30-06, **AMS members US$87**, List US$109, Order code TRANS2/226



Ergodic Theory

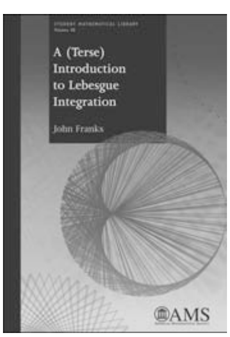
Idris Assani, *University of North Carolina, Chapel Hill, NC*, Editor

This book contains papers written by participants at the two Chapel Hill Ergodic Theory Workshops organized in February 2007 and 2008. The topics covered by these papers help to illustrate the interaction between ergodic theory and related fields such as harmonic analysis, number and probability theories.

Contents: **P. C. Allaart** and **R. D. Mauldin**, Injectivity of the Dubins–Freedman construction of random distributions; **I. Assani** and **Z. Buczolich**, A maximal inequality for the tail of the bilinear Hardy–Littlewood function; **G. Cohen** and **M. Lin**, Almost sure convergence of weighted sums of independent random variables; **J.-P. Conze**, Recurrence, ergodicity and invariant measures for cocycles over a rotation; **N. Chevallier** and **J.-P. Conze**, Examples of recurrent or transient stationary walks in $\mathbb{R}^d$ over a rotation of $\mathbb{T}^2$; **Y. Coudene**, A short proof of the unique ergodicity of horocyclic flows; **D. Lenz**, Aperiodic order via dynamical systems: Diffraction for sets of finite local complexity; **M. Lin** and **M. Weber**, Laws of iterated logarithm for weighted sums of iid random variables; **R. D. Mauldin** and **A. Yingst**, Homeomorphic Bernoulli trial measures and ergodic theory; **J. Rosenblatt**, Distinguishing transformations by averaging methods; **I. Assani**, Some open problems.

Contemporary Mathematics, Volume 485

June 2009, 162 pages, Softcover, ISBN: 978-0-8218-4649-0, LC 2008048524, 2000 *Mathematics Subject Classification:* 28D05, 34C28, 37A05, 37A20, 37A45, 42A16, 47A35, 60F15, 60G50, 62J05, **AMS members US$47**, List US$59, Order code CONM/485



A (Terse) Introduction to Lebesgue Integration

John Franks, *Northwestern University, Evanston, IL*

This book provides a student's first encounter with the concepts of measure theory and functional analysis. Its structure and content reflect the belief that difficult concepts should be introduced in their simplest and most concrete forms.

Despite the use of the word "terse" in the title, this text might also have been called *A (Gentle) Introduction to Lebesgue Integration*. It is terse in the sense that it treats only a subset of those concepts typically found in a substantial graduate-level analysis course. The book emphasizes the motivation of these concepts and attempts to treat them simply and concretely. In particular, little mention is made of general measures other than Lebesgue until the final chapter and attention is limited to R as opposed to R^n.

After establishing the primary ideas and results, the text moves on to some applications. Chapter 6 discusses classical real and complex Fourier series for L^2 functions on the interval and shows that the Fourier series of an L^2 function converges in L^2 to that function. Chapter 7 introduces some concepts from measurable dynamics. The Birkhoff ergodic theorem is stated without proof and results on Fourier series from Chapter 6 are used to prove that an irrational rotation of the circle is ergodic and that the squaring map on the complex numbers of modulus 1 is ergodic.

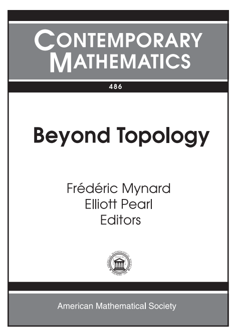
This book is suitable for an advanced undergraduate course or for the start of a graduate course. The text presupposes that the student has had a standard undergraduate course in real analysis.

Contents: The regulated and Riemann integrals; Lebesgue measure; The Lebesgue integral; The integral of unbounded functions; The Hilbert space L^2; Classical Fourier series; Two ergodic transformations; Background and foundations; Lebesgue measure; A non-measurable set; Bibliography; Index.

Student Mathematical Library, Volume 48

July 2009, approximately 205 pages, Softcover, ISBN: 978-0-8218-4862-3, 2000 *Mathematics Subject Classification:* 28A20, 28A25, 42B05, **AMS members US$30**, List US$37, Order code STML/48

Geometry and Topology



Beyond Topology

Frédéric Mynard, *Georgia Southern University, Statesboro, GA*, and **Elliott Pearl**, *Toronto, ON, Canada*, Editors

The purpose of this collection is to guide the non-specialist through the basic theory of various generalizations of topology, starting with clear motivations for their introduction. Structures considered include closure spaces, convergence spaces, proximity spaces, quasi-uniform spaces, merotopic spaces, nearness and filter spaces, semi-uniform convergence spaces, and approach spaces. Each chapter is self-contained and accessible to the graduate student, and focuses on motivations to introduce the generalization of topologies considered, presenting examples where desirable properties are not present in the realm of topologies and the problem is remedied in the more general context. Then, enough material will be covered to prepare the reader for more advanced papers on the topic. While category theory is not the focus of the book, it is a convenient language to study these structures and, while kept as a tool rather than an object of study, will be used throughout the book. For this reason, the book contains an introductory chapter on categorical topology.

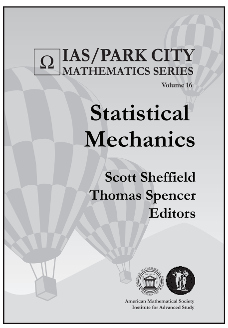
Contents: **R. Lowen**, **M. Sioen**, and **S. Verwulgen**, Categorical topology; **H. L. Bentley**, **E. Colebunders**, and **E. Vandersmissen**, A convenient setting for completions and function spaces; **A. Di Concilio**, Proximity: a powerful tool in extension theory, function spaces, hyperspaces, boolean algebras and point-free geometry; **S.**

Dolecki, An initiation into convergence theory; **M. Erné**, Closure; **H.-P. A. Künzi**, An introduction to quasi-uniform spaces; **R. Lowen** and **C. Van Olmen**, Approach theory; **G. Preuss**, Semiuniform convergence spaces and filter spaces.

Contemporary Mathematics, Volume 486

June 2009, 383 pages, Softcover, ISBN: 978-0-8218-4279-9, LC 2008050812, 2000 *Mathematics Subject Classification:* 54Axx, 54-02, **AMS members US$87**, List US$109, Order code CONM/486

Mathematical Physics



Statistical Mechanics

Scott Sheffield, *Massachusetts Institute of Technology, Cambridge, MA*, and **Thomas Spencer**, *Institute for Advanced Study, Princeton, NJ*, Editors

In recent years, statistical mechanics has been increasingly recognized as a central domain of mathematics. Major developments include the Schramm–Loewner evolution, which describes two-dimensional phase transitions, random matrix theory, renormalization group theory and the fluctuations of random surfaces described by dimers. The lectures contained in this volume present an introduction to recent mathematical progress in these fields. They are designed for graduate students in mathematics with a strong background in analysis and probability.

This book will be of particular interest to graduate students and researchers interested in modern aspects of probability, conformal field theory, percolation, random matrices and stochastic differential equations.

This item will also be of interest to those working in probability.

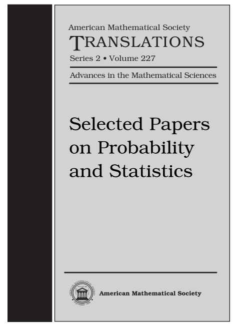
Titles in this series are co-published with the Institute for Advanced Study/Park City Mathematics Institute. Members of the Mathematical Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM) receive a 20% discount from list price.

Contents: D. C. Brydges, Lectures on the renormalisation group; **A. Guionnet**, Statistical mechanics and random matrices; **R. Kenyon**, Lectures on dimers; **G. Lawler**, Schramm-Loewner evolution (*SLE*); **W. Werner**, Lectures on two-dimensional critical percolation.

IAS/Park City Mathematics Series, Volume 16

July 2009, 360 pages, Hardcover, ISBN: 978-0-8218-4671-1, LC 2009003040, 2000 *Mathematics Subject Classification:* 82-01, 82-06, 60-01, 60-06, 30-XX, 05-XX, 15-XX, 81Txx, **AMS members US$60**, List US$75, Order code PCMS/16

Probability



Selected Papers on Probability and Statistics

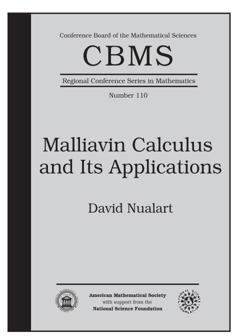
This volume contains translations of papers that originally appeared in the Japanese journal *Sūgaku*. The papers range over a variety of topics in probability theory, statistics, and applications.

This volume is suitable for graduate students and research mathematicians interested in probability and statistics.

Contents: J. Akahori, **M. Izumi**, and **S. Watanabe**, Noises, stochastic flows and E_O-semigroups; **S. Kuriki** and **A. Takemura**, Volume of tubes and distribution of the maxima of Gaussian random fields; **T. Funaki**, Stochastic analysis on large scale interacting systems; **T. Mikami**, Optimal transportation problem as stochastic mechanics; **M. Hayashi**, Quantum estimation and the quantum central limit theorem; **S. Aoki** and **A. Takemura**, Statistics and Gröbner bases–The origin and development of computational algebraic statistics; **S. Tomizawa**, Analysis of square contingency tables in statistics; **M. Akahira**, The structure of higher order asymptotic theory of statistical estimation; **A. Takahashi**, On an asymptotic expansion approach to numerical problems in finance; **K. Kuroda** and **N. Matsuyama**, Actuarial mathematics: Theory and current practice in Japan.

American Mathematical Society Translations—Series 2, Volume 227

June 2009, approximately 272 pages, Hardcover, ISBN: 978-0-8218-4821-0, LC 2009004070, 2000 *Mathematics Subject Classification:* 60-06, 62-06, **AMS members US$91**, List US$114, Order code TRANS2/227



Malliavin Calculus and Its Applications

David Nualart, *The University of Kansas, Lawrence, KS*

The Malliavin calculus was developed to provide a probabilistic proof of Hörmander's hypoellipticity theorem. The theory has expanded to encompass other significant applications.

The main application of the Malliavin calculus is to establish the regularity of the probability distribution of functionals of an underlying Gaussian process. In this way, one can prove the existence and smoothness of the density for solutions of various stochastic differential equations. More recently, applications of the Malliavin calculus in areas such as stochastic calculus for fractional Brownian motion, central limit theorems for multiple stochastic integrals, and mathematical finance have emerged.

The first part of the book covers the basic results of the Malliavin calculus. The middle part establishes the existence and smoothness results that then lead to the proof of Hörmander's hypoellipticity theorem. The last part discusses the recent developments for Brownian motion, central limit theorems, and mathematical finance.

A co-publication of the AMS and CBMS.

Contents: The derivative operator; The divergence operator; The Ornstein-Uhlenbeck semigroup; Sobolev spaces and equivalence of norms; Regularity of probability laws; Support properties. Density of the maximum; Application of Malliavin calculus to diffusion processes; The divergence operator as a stochastic integral; Central limit theorems and Malliavin calculus; Applications of Malliavin calculus in finance; Bibliography; Index.

CBMS Regional Conference Series in Mathematics, Number 110

May 2009, 85 pages, Softcover, ISBN: 978-0-8218-4779-4, LC 2009003082, 2000 *Mathematics Subject Classification:* 60H07; 60H05, 60H10, **All Individuals US$23**, List US$29, Order code CBMS/110

New AMS-Distributed Publications

Algebra and Algebraic Geometry

Generalized Bialgebras and Triples of Operads

Jean-Louis Loday, *Centre National de la Recherche Scientifique, Strasbourg, France*

This book introduces the notion of generalized bialgebra, which includes the classical notion of bialgebra (Hopf algebra) and many others, among them the tensor algebra equipped with the deconcatenation as coproduct. The author proves that, under some mild conditions, a connected generalized bialgebra is completely determined by its primitive part. This structure theorem extends the classical Poincaré–Birkhoff–Witt theorem and Cartier–Milnor–Moore theorem, valid for cocommutative bialgebras, to a large class of generalized bialgebras.

Technically, the author works in the theory of operads which allows him to state his main theorem and permits him to give it a conceptual proof. A generalized bialgebra type is determined by two operads: one for the coalgebra structure C and one for the algebra structure $\mathcal{A}$. There is also a compatibility relation relating the two. Under some conditions, the primitive part of such a generalized bialgebra is an algebra over some sub-operad of $\mathcal{A}$, denoted $\mathcal{P}$. The structure theorem gives conditions under which a connected generalized bialgebra is cofree (as a connected C-coalgebra) and can be reconstructed out of its primitive part by means of an enveloping functor from $\mathcal{P}$-algebras to $\mathcal{A}$-algebras. The classical case is $(C, \mathcal{A}, \mathcal{P}) = (Com, As, Lie)$.

This structure theorem unifies several results, generalizing the PBW and the CMM theorems, scattered in the literature. The author treats many explicit examples and suggests a few conjectures.

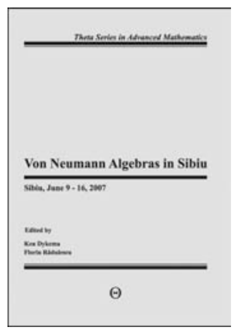
A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Introduction; Algebraic operads; Generalized bialgebra and triple of operads; Applications and variations; Examples; Duplicial bialgebras; Appendix; Bibliography; Index.

Astérisque, Number 320

February 2009, 114 pages, Softcover, ISBN: 978-2-85629-257-0, 2000 *Mathematics Subject Classification:* 16A24, 16W30, 17A30, 18D50, 81R60, **Individual member US$38**, List US$42, Order code AST/320

Analysis



Von Neumann Algebras in Sibiu

Conference Proceedings, Sibiu, June 9–16, 2007

Ken Dykema, *Texas A&M University, College Station, TX*, and **Florin Rădulescu**, *University of Iowa, Iowa City, IA*, Editors

The volume represents the proceedings of the International Workshop on Free Probabilities, Operator Spaces, and von Neumann Algebras, held on June 9–16, 2007, in Sibiu, Romania. It contains five original refereed research papers, as well as an innovative survey by Roberto Longo, presenting a remarkable new perspective on the "one particle structure" of conformal field theory.

A publication of the Theta Foundation. Distributed worldwide, except in Romania, by the AMS.

Contents: **S. T. Belinschi**, C-free convolution for measures with unbounded support; **P. Boivin** and **J. Renault**, A Hausdorff–Young inequality for measured groupoids; **F. Fidaleo**, New results in noncommutative ergodic theory; **R. Longo**, Real Hilbert subspaces, modular theory, $SL(2, \mathbb{R})$ and CFT; **F. Rădulescu**, A non-commutative, analytic version of Hilbert's 17th problem in type II_1 von Neumann algebras; **S. Sakai**, Recent topics on C^*-algebras (consistency and independency) and Kadison–Singer problem.

International Book Series of Mathematical Texts

December 2008, 109 pages, Hardcover, ISBN: 978-973-87899-4-4, 2000 *Mathematics Subject Classification:* 00B25, 46-06, 47-06, **AMS members US$26**, List US$33, Order code THETA/13

General and Interdisciplinary

Current Developments in Mathematics, 2006

Barry Mazur, **Wilfried Schmid**, and **Shing-Tung Yau**, *Harvard University, Cambridge, MA*, and **David Jerison**, **Tomasz Mrowka**, and **Richard P. Stanley**, *Massachusetts Institute of Technology, Cambridge, MA*, Editors

The Current Developments in Mathematics (CDM) conference is an annual seminar, jointly hosted by Harvard University and the Massachusetts Institute of Technology, and devoted to surveying the most recent developments in mathematics. In choosing speakers, the hosts take a broad look at the field of geometry and select geometers who transcend classical perceptions within their field. All speakers are prominent specialists in the fields of algebraic geometry, mathematical physics, and other areas. International Press is pleased to present the full contents of these proceedings in the CDM book series.

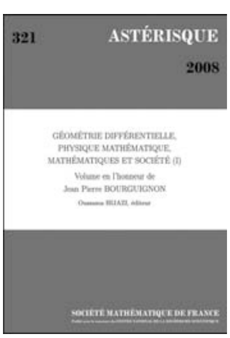
A publication of International Press. Distributed worldwide by the American Mathematical Society.

Contents: L. Clozel, The Sato–Tate conjecture; **S. Gukov** and **E. Witten**, Gauge theory, ramification, and the geometric Langlands program; **D. Li** and **Y. Sinai**, Complex singularities of the Burgers system and renormalization group method; **P. Seidel**, A biased view of symplectic cohomology; **T. Tao**, Global behaviour of nonlinear dispersive and wave equations.

International Press

February 2008, 344 pages, Hardcover, ISBN: 978-1-57146-167-4, 2000 *Mathematics Subject Classification:* 00Bxx, **AMS members US$54**, List US$68, Order code INPR/73

Geometry and Topology



Géométrie Différentielle, Physique Mathématique, Mathématiques et Société (I)

Volume en l'honneur de Jean Pierre Bourguignon

Oussama Hijazi, *Université Henri Poincare, Vandoeuvre-les-Nancy, France*, Editor

This volume, the first in a two-volume set, contains original research articles on various aspects of differential geometry, analysis on manifolds, complex geometry, algebraic geometry, number theory and general relativity.

The articles are based on talks presented at the Conference on Differential Geometry, Mathematical Physics, Mathematics and Society, held in honor of Jean-Pierre Bourguignon on the occasion of his 60th birthday. The conference was held from August 27 to 31, 2007 at the Institut des Hautes Études Scientifiques and at the École Polytechnique.

This item will also be of interest to those working in number theory.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: J. Simons and **D. Sullivan**, Structured bundles define differential K-theory; **N. Hitchin**, Einstein metrics and magnetic monopoles; **K. Liu**, **X. Sun**, and **S.-T. Yau**, Geometry of moduli spaces; **R. L. Bryant**, Gradient Kähler Ricci solitons; **D. Auroux**, Special Lagrangian fibrations, mirror symmetry and Calabi–Yau double covers; **J. Cheeger** and **B. Kleiner**, Characterization of the Radon–Nikodym property in terms of inverse limits; **X. Chen** and **Y. Tang**, Test conguration and geodesic rays; **R. Mazzeo**, Flexibility of singular Einstein metrics; **P. T. Chruściel** and **J. L. Costa**, On uniqueness of stationary vacuum black holes; **H. Omori**, **Y. Maeda**, **N. Miyazaki**, and **A. Yoshioka**, A new nonformal noncommutative calculus: Associativity and finite part regularization.

Astérisque, Number 321

February 2009, 298 pages, Softcover, ISBN: 978-2-85629-258-7, 2000 *Mathematics Subject Classification:* 14J32, 46L65, 53C55, 53D10, 53D12, 53D55, 58G11, 83C57, **Individual member US$56**, List US$62, Order code AST/321

Géométrie Différentielle, Physique Mathématique, Mathématiques et Société (II)

Volume en l'honneur de Jean Pierre Bourguignon

Oussama Hijazi, *Université Henri Poincare, Vandoeuvre-les-Nancy, France*, Editor

This volume, the second in a two-volume set, contains original research articles on various aspects of differential geometry, analysis on manifolds, complex geometry, algebraic geometry, number theory and general relativity.

The articles are based on talks presented at the Conference on Differential Geometry, Mathematical Physics, Mathematics and Society, held in honor of Jean-Pierre Bourguignon on the occasion of his 60th birthday. The conference was held from August 27 to 31, 2007 at the Institut des Hautes Études Scientifiques and at the École Polytechnique.

This item will also be of interest to those working in analysis.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: C. Voisin, Rationally connected 3-folds and symplectic geometry; **S.-Y. Chang** and **P. C. Yang**, The Q-curvature equation in conformal geometry; **J.-M. Bismut**, A survey of the hypoelliptic Laplacian; **G. Tian**, New results and problems on Kahler–Ricci flow; **V. Apostolov**, **D. M. Calderbank**, **P. Gauduchon**, and **C. W. Tønnesen–Friedman**, Extremal Kähler metrics on ruled manifolds and stability; **N. Mok**, Geometric structures on uniruled projective manifolds defined by their varieties of minimal rational tangents; **D. Hoffman** and **B. White**, On the number of minimal surfaces with a given boundary; **P. Sarnak**, Equidistribution and primes; **R. Harvey**, **B. Lawson**, and **J. Werner**, The projective hull of certain curves in $\mathbb{C}^2$.

Astérisque, Number 322

February 2009, 255 pages, Softcover, ISBN: 978-2-85629-259-4, 2000 *Mathematics Subject Classification:* 14J45, 30H05, 32H02, 32M15, 32Q99, 35H10, 53C10, 58A14, 58J20, **Individual member US$78**, List US$87, Order code AST/322

Number Theory

Représentations p-adiques Cristallines et de de Rham dans le Cas Relatif

Olivier Brinon, *Université Paris-Nord, Villetaneuse, France*

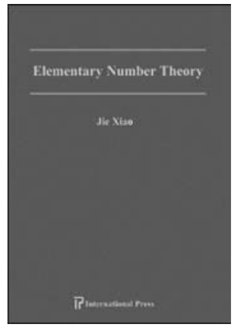
The author defines and studies the notions of de Rham and crystalline smooth p-adic sheaves over "suitable" p-adic bases. To do this, he introduces p-adic period rings (analogous to those of J.-M. Fontaine), which are used to associate differential invariants to them. In the good reduction case, he obtains a fully faithful functor from the category of crystalline smooth p-adic sheaves in that of filtered F-isocrystals on the special fiber.

A publication of the Société Mathématique de France, Marseilles (SMF), distributed by the AMS in the U.S., Canada, and Mexico. Orders from other countries should be sent to the SMF. Members of the SMF receive a 30% discount from list.

Contents: Introduction; Notations, premiéres propriétés; L'anneau C; Rappels sur l'anneau B_{HT} de Hyodo; L'anneau B_{dR}; L'anneau B_{cris}; (φ, ∇)-modules filtrés; Représentations p-adiques; Appendices; Bibliographie; Index.

Mémoires de la Société Mathématique de France, Number 112

December 2008, 159 pages, Softcover, ISBN: 978-2-85629-250-1, 2000 *Mathematics Subject Classification:* 11F80, 11S25, 14F30, **Individual member US$50**, List US$55, Order code SMFMEM/112



Elementary Number Theory

Jie Xiao, *Tsinghua University, Beijing, China*

A self-contained introduction to Number Theory, this volume requires a background knowledge only of some simple properties of the system of integers. The book begins with a few preliminaries on induction principles, followed by a quick review of division algorithm. The second chapter then explores the use of divisors, the greatest (least) common divisor (multiple), the Euclidean algorithm, and linear indeterminate equation. Subsequent chapters deal with prime numbers, congruences, congruent equations, cryptography, Diophantine equations, and Gaussian integers. Each chapter ends with exercises to illustrate the theory and provide practice in the techniques, with answers to even-numbered problems at the end of the book.

A publication of International Press. Distributed worldwide by the American Mathematical Society.

Contents: Basics; Divisibility; Primes; Congruences; Congruent equations; Three additional topics; Solutions to even-numbered exercises.

International Press

December 2006, 84 pages, Hardcover, ISBN: 978-1-57146-163-6, 2000 *Mathematics Subject Classification:* 11-XX, **AMS members US$20**, List US$25, Order code INPR/74

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The AMS strongly supports equal opportunity in employment. Despite increasing participation at many levels, low rates of retention and promotion of women and underrepresented minorities remain a serious concern, particularly at doctoral-granting institutions. Therefore, AMS members, both individual and institutional, are urged to examine frequently their policies and procedures to see in what ways they may facilitate careers in mathematics research for women and underrepresented minorities. Resources can be found at the website: **http://www.ams.org/employment/equalopportunity.html**

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000027

CHILE

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Director
Departamento de Matemáticas
Pontificia Universidad Católica de Chile
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For full consideration, complete application materials must arrive by June 30, 2009.

000018

INDIA

INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH (IISER), MOHALI

Indian Institute of Science Education and Research (IISER) Mohali is in the process of building a school of Mathematics and

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Upcoming deadlines for classified advertising are as follows: June/July 2009 issue–April 28. 2009; August 2009 issue–May 28, 2009; September 2009 issue–June 29, 2009; October 2009 issue–July 29, 2009; November 2009 issue–August 28, 2009; December 2009 issue–September 28, 2009.

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Situations wanted advertisements from involuntarily unemployed mathematicians are accepted under certain conditions for free publication. Call toll-free 800-321-4AMS (321-4267) in the U.S. and Canada or 401-455-4084 worldwide for further information.

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000024

IRELAND

POSTGRADUATE DEGREE PROGRAM

Study for M. Phil or D. Phil by research in Applied Mathematics in Dublin, Ireland. Degrees awarded by U. of Buckingham, UK. Fees €15K pa. See: `http://www.in-caireland.org`.

000025

Meetings & Conferences of the AMS

IMPORTANT INFORMATION REGARDING MEETINGS PROGRAMS: AMS Sectional Meeting programs do not appear in the print version of the *Notices*. However, comprehensive and continually updated meeting and program information with links to the abstract for each talk can be found on the AMS website. See http://www.ams.org/meetings/. Final programs for Sectional Meetings will be archived on the AMS website accessible from the stated URL and in an electronic issue of the *Notices* as noted below for each meeting.

Worcester, Massachusetts

Worcester Polytechnic Institute

April 25–26, 2009

Saturday – Sunday

Meeting #1050

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: February 2009
Program first available on AMS website: March 12, 2009
Program issue of electronic *Notices*: April 2009
Issue of *Abstracts*: Volume 30, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Octav Cornea, Université de Montréal, *Lagrangian submanifolds: From physics to number theory*.

Fengbo Hang, Courant Institute of New York University, *Topology of weakly differentiable maps*.

Umberto Mosco, Worcester Polytechnic Institute, *Fractal spectra between Scylla and Charybdis*.

Kevin Whyte, University of Illinois at Chicago, *A rapid survey of coarse geometry*.

Special Sessions

Algebraic Graph Theory, Association Schemes, and Related Topics, **William J. Martin**, Worcester Polytechnic Institute, and **Sylvia A. Hobart**, University of Wyoming.

Analysis of Weakly Differentiable Maps with Constraints and Applications, **Fengbo Hang**, Courant Institute, New York University, and **Mohammad Reza Pakzad**, University of Pittsburgh.

Discrete Geometry and Combinatorics, **Egon Schulte**, Northeastern University, and **Brigitte Servatius**, Worcester Polytechnic Institute.

Effective Dynamics and Interactions of Localized Structures in Schrödinger Type Equations, **Fridolin Ting**, Lakehead University.

Number Theory, **John T. Cullinan**, Bard College, and **Siman Wong**, University of Massachusetts, Amherst.

Quasi-Static and Dynamic Evolution in Fracture Mechanics, **Christopher J. Larsen**, Worcester Polytechnic Institute.

Real and Complex Dynamics of Rational Difference Equations with Applications, **M. R. S. Kulenovic** and **Orlando Merino**, University of Rhode Island.

Scaling, Irregularities, and Partial Differential Equations, **Umberto Mosco** and **Bogdan M. Vernescu**, Worcester Polytechnic Institute.

Symplectic and Contact Topology, **Peter Albers**, Purdue University/ETH Zurich, and **Basak Gurel**, Vanderbilt University.

The Mathematics of Climate Change, **Catherine A. Roberts** and **Gareth E. Roberts**, College of the Holy Cross, and **Mary Lou Zeeman**, Bowdoin College.

Topological Robotics, **Li Han** and **Lee N. Rudolph**, Clark University.

San Francisco, California

San Francisco State University

April 25–26, 2009

Saturday – Sunday

Meeting #1049

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: February 2009
Program first available on AMS website: March 12, 2009
Program issue of electronic *Notices*: April 2009
Issue of *Abstracts*: Volume 30, Issue 3

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: Expired
For abstracts: Expired

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Yehuda Shalom, University of California Los Angeles, *Bounded generation of arithmetic groups and some recent applications*.

Roman Vershynin, University of Michigan, *Concentration of measure, convex geometry, and random matrices*.

Karen Vogtmann, Cornell University, *Actions of automorphism groups of free groups*.

Efim Zelmanov, University of California San Diego, *Asymptotic properties of finite groups and finite-dimensional algebras*.

Special Sessions

Advances in the Theory of Integer Linear Optimization and its Extensions, **Matthias Koeppe** and **Peter Malkin**, University of California Davis.

Algebra and Number Theory with Polyhedra, **Matthias Beck**, San Francisco State University, and **Christian Haase**, Freie Universität Berlin.

Applications of Knot Theory to the Entanglement of Biopolymers, **Javier Arsuaga**, San Francisco State University, **Kenneth Millett**, University of California Santa Barbara, and **Mariel Vazquez**, San Francisco State University.

Aspects of Differential Geometry, **David Bao**, San Francisco State University, and **Lei Ni**, University of California San Diego.

Banach Algebras, Topological Algebras, and Abstract Harmonic Analysis, **Thomas V. Tonev**, University of Montana-Missoula, and **Fereidoun Ghahramani**, University of Manitoba.

Concentration Inequalities, **Sourav Chatterjee**, University of California Berkeley, and **Roman Vershynin**, University of California Davis.

Geometry and Topology of Orbifolds, **Elizabeth Stanhope**, Lewis & Clark University, and **Joseph E. Borzellino**, California State University San Luis Obispo.

Lie group actions, Teichmüller Flows and Number Theory, **Jayadev Athreya**, Yale University, **Yitwah Cheung**, San Francisco State University, and **Anton Zorich**, Rennes University.

Matroids in Algebra and Geometry, **Federico Ardila**, San Francisco State University, and **Lauren Williams**, Harvard University.

Nonlinear Dispersive Equations, **Sebastian Herr**, University of California Berkeley, and **Jeremy L. Marzuola**, Columbia University.

Nonlinear Partial Differential Equations, **Igor Kukavica**, **Amjad Tuffaha**, and **Mohammed Ziane**, University of Southern California.

Recent Progress in Geometric Group Theory, **Seonhee Lim** and **Anne Thomas**, Cornell University.

Waco, Texas

Baylor University

October 16–18, 2009

Friday – Sunday

Meeting #1051

Central Section
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: August 2009
Program first available on AMS website: September 3, 2009
Program issue of electronic *Notices*: October 2009
Issue of *Abstracts*: Volume 30, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: June 30, 2009
For abstracts: August 25, 2009

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

David Ben-Zvi, University of Texas at Austin, *Title to be announced.*

Alexander A. Kiselev, University of Wisconsin, *Title to be announced.*

Michael C. Reed, Duke University, *Title to be announced.*

Igor Rodnianski, Princeton University, *Title to be announced.*

Special Sessions

Commutative Algebra: Module and Ideal Theory (Code: SS 4A), **Lars W. Christensen**, Texas Tech University, **Louiza Fouli**, University of Texas at Austin, and **David Jorgensen**, University of Texas at Arlington.

Dynamic Equations on Time Scales: Analysis and Applications (Code: SS 1A), **John M. Davis**, **Ian A. Gravagne**, and **Robert J. Marks**, Baylor University.

Fusion Categories and Applications (Code: SS 7A), **Deepak Naidu** and **Eric Rowell**, Texas A&M University.

Harmonic Analysis and Partial Differential Equations (Code: SS 8A), **Susan Fiedlander**, University of Southern California, **Natasa Pavlovic**, University of Texas at Austin, and **Nikolaos Tzirakis**, University of Illinois at Urbana-Champaign.

Interdisciplinary Session on Stochastic Partial Differential Equations (Code: SS 11A), **M. Chekroun**, ENS-Paris and University of California Los Angeles, and **Shouhong Wang** and **Nathan Glatt-Holtz**, Indiana University.

Lie Groups, Lie Algebras, and Representations (Code: SS 6A), **Markus Hunziker**, **Mark Sepanski**, and **Ronald Stanke**, Baylor University.

Mathematical Models of Neuronal and Metabolic Mechanisms (Code: SS 3A), **Janet Best**, Ohio State University, and **Michael Reed**, Duke University.

Mathematical Aspects of Spectral Problems Related to Physics (Code: SS 10A), **Klaus Kirsten**, Baylor University, **Gregory Berkolaiko** and **Stephen Fulling**, Texas A&M University, **Jon Harrison**, Baylor University, and **Peter Kuchment**, Texas A&M University.

Numerical Solutions of Singular or Perturbed Partial Differential Equation Problems with Applications (Code: SS 2A), **Peter Moore**, Southern Methodist University, and **Qin Sheng**, Baylor University.

Recent Developments on Turbulence (Code: SS 9A), **Eleftherios Gkioulekas**, University of Texas-Pan American, and **Michael Jolly**, Indiana University.

Topological Methods for Boundary Value Problems for Ordinary Differential Equations (Code: SS 5A), **Richard Avery**, Dakota State University, **Paul W. Eloe**, University of Dayton, and **Johnny Henderson**, Baylor University.

University Park, Pennsylvania

Pennsylvania State University

October 24–25, 2009

Saturday – Sunday

Meeting #1052

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: August 2009
Program first available on AMS website: September 10, 2009
Program issue of electronic *Notices*: October 2009
Issue of *Abstracts*: Volume 30, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 7, 2009
For abstracts: September 1, 2009

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Invited Addresses

Michael K. H. Kiessling, Rutgers University, *Title to be announced.*

Kevin R. Payne, Universita degli di Milano, *Title to be announced.*

Laurent Saloff-Coste, Cornell University, *Title to be announced.*

Robert C. Vaughan, Penn State University, *Title to be announced.*

Special Sessions

Algebraic Combinatorics (Code: SS 6A), **Peter McNamara**, Bucknell University, and **Mark Skandera**, Lehigh University.

Analytic Number Theory (Code ss 16A), **Angel V. Kumchev**, Towson University, **Michael P. Knapp**, Loyola College, and **Robert C. Vaughan**, Pennsylvania State University.

Automorphisms of Riemann Surfaces and Related Topics (Code ss 15A), **S. Allen Broughton**, Rose-Hulman Institute of Technology, **Anthony Weaver**, Bronx Community College, City University of New York, and **Aaron D. Wootton**, University of Portland.

Combinatorial and Homological Aspects of Commutative Algebra (Code: SS 3A), **Amanda I. Beecher**, United States Military Academy, and **Alexandre B. Tchernev**, University at Albany.

Commutative Algebra and Applications to Algebraic Geometry (Code: SS 11A), **Janet Striuli**, Fairfield University, and **Jooyoun Hong**, Southern Connecticut State University.

Difference Equations and Applications (Code: SS 2A), **Michael A. Radin**, Rochester Institute of Technology.

Geometry of Integrable and Non-Integrable Dynamics (Code: SS 5A), **Boris Khesin**, University of Toronto, and **Mark Levi** and **Sergei Tabachnikov**, Pennsylvania State University.

Heat Kernel Analysis (Code: SS 8A), **Maria Gordina**, University of Connecticut, and **Laurent Saloff-Coste**, Cornell University.

Homotopy Theory (Code: SS 1A), **James Gillespie** and **Mark W. Johnson**, Pennsylvania State University, **Simona Paoli**, University of Haifa, and **Donald Yau**, Ohio State University.

Integrable Systems and Related Areas (Code: SS 4A), **Sam Evans** and **Michael Gekhtman**, University of Notre Dame, and **Luen-Chau Li**, Pennsylvania State University.

Microlocal Analysis and Spectral Theory on Singular Spaces (Code: SS 14A), **Juan B. Gil**, Pennsylvania State University, Altoona, and **Thomas Krainer**, Pennsylvania State University, Altoona.

New Trends in Triangulated Categories and Their Associated Cohomology Theories (Code: SS 12A), **Sunil Kumar Chebolu**, Illinois State University, and **Keir H. Lockridge**, Wake Forest University.

Surface Water Waves (Code: SS 13A), **Bernard Deconinck**, University of Washington, and **Diane Henderson**, Pennsylvania State University.

Symplectic, Contact, and Complex Structures on Manifolds (Code: SS 7A), **Philippe Rukimbira**, **Tedi C. Draghici**, and **Gueo V. Grantcharov**, Florida International University.

Topics in Mathematical Finance (Code: SS 10A), **Nick Costanzino**, **Anna L. Mazzucato**, and **Victor Nistor**, Pennsylvania State University.

q-Series and Related Areas in Enumerative Combinatorics and Number Theory (Code: SS 9A), **David Little**, **James Sellers**, and **Ae Ja Yee**, Pennsylvania State University.

Boca Raton, Florida

Florida Atlantic University

October 30 – November 1, 2009

Friday – Sunday

Meeting #1053

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: August 2009
Program first available on AMS website: September 17, 2009
Program issue of electronic *Notices*: October 2009
Issue of *Abstracts*: Volume 30, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 14, 2009
For abstracts: September 8, 2009

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Spyros Alexakis, Princeton University, *Title to be announced.*

Kai-Uwe Bux, University of Virginia, *Title to be announced.*

Dino J. Lorenzini, University of Georgia, *Title to be announced.*

Eduardo D. Sontag, Rutgers University, *Title to be announced.*

Special Sessions

Applied Partial Differential Equations (Code: SS 10A), **Shar Sajjadi** and **Timothy A. Smith**, Embry Riddle Aeronautical University.

Commutative Ring Theory (Code: SS 3A), **Alan Loper**, Ohio State University, and **Lee C. Klingler**, Florida Atlantic University.

Concentration, Functional Inequalities, and Isoperimetry (Code: SS 2A), **Mario Milman**, Florida Atlantic University, **Christian Houdre**, Georgia Institute of Technology, and **Emanuel Milman**, Institute for Advanced Study.

Constructive Mathematics (Code: SS 1A), **Robert Lubarsky**, **Fred Richman**, and **Martin Solomon**, Florida Atlantic University.

Dynamical Systems (Code: SS 6A), **William D. Kalies** and **Vincent Naudot**, Florida Atlantic University.

Enumerative Combinatorics (Code: SS 4A), **Christian Krattenthaler**, University of Vienna, and **Aaron D. Meyerowitz**, **Heinrich Niederhausen**, and **Wandi Wei**, Florida Atlantic University.

Graded Resolutions (Code: SS 13A), **Christopher Francisco**, Oklahoma State University, and **Irena Peeva**, Cornell University.

Graph Theory (Code: SS 11A), **Zixia Song** and **Yue Zhao**, University of Central Florida.

Harmonic Analysis (Code: SS 5A), **Galia D. Dafni**, Concordia University, and **J. Michael Wilson**, University of Vermont, Burlington.

Homological Aspects of Module Theory (Code: SS 7A), **Andrew R. Kustin**, University of South Carolina, **Sean M. Sather-Wagstaff**, North Dakota State University, and **Janet Vassilev**, University of New Mexico.

Hypercomplex Analysis (Code: SS 12A), **Craig A. Nolder**, Florida State University, and **John Ryan**, University of Arkansas at Fayetteville.

Invariants of Knots and Links (Code: SS 9A), **Heather A. Dye**, McKendree University, **Mohamed Elhamdadi**, University of South Florida, and **Louis H. Kauffman**, University of Illinois at Chicago.

Inverse Problems and Signal Processing (Code: SS 14A), **M. Zuhair Nashed** and **Qiyu Sun**, University of Central Florida.

Partial Differential Equations from Fluid Mechanics (Code: SS 15A), **Chongsheng Cao**, Florida International University, **Jiahong Wu**, Oklahoma State University, and **Baoquan Yuan**, Henan Polytechnic University.

Recent Advances in Probability and Statistics (Code: SS 8A), **Lianfen Qian** and **Hongwei Long**, Florida Atlantic University.

Riverside, California

University of California

November 7–8, 2009

Saturday – Sunday

Meeting #1054

Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: September 2009
Program first available on AMS website: September 24, 2009
Program issue of electronic *Notices*: November 2009
Issue of *Abstracts*: Volume 30, Issue 4

Deadlines

For organizers: Expired
For consideration of contributed papers in Special Sessions: July 21, 2009
For abstracts: September 15, 2009

The scientific information listed below may be dated. For the latest information, see www.ams.org/amsmtgs/sectional.html.

Invited Addresses

Christopher Hacon, University of Utah, *Title to be announced*.

Birge Huisgen-Zimmerman, University of California Santa Barbara, *Title to be announced*.

Jun Li, Stanford University, *Title to be announced*.

Joseph Teran, University of California Los Angeles, *Title to be announced*.

Special Sessions

Algebraic Geometry (Code: SS 1A), **Christopher Hacon**, University of Utah, and **Ziv Ran**, University of California Riverside.

Calabi-Yau Manifolds (Code: SS 15A), **Owen Dearricott**, University of California Riverside, **Jun Li**, Stanford University, and **Bun Wong** and **Yat-Sun Poon**, University of California Riverside.

Fluid Mechanics (Code: SS 5A), **James Kelliher** and **Qi Zhang**, University of California Riverside.

Fractal Geometry, Dynamical Systems, Number Theory and Analysis on Rough Spaces (Code: SS 6A), **Michel L. Lapidus**, University of California Riverside, **Hung Lu**, Hawaii Pacific University, and **Erin P. J. Pearse**, University of Iowa.

Global Riemannian Geometry (Code: SS 14A), **Fred Wilhelm**, University of California Riverside, and **Peter Petersen**, University of California Los Angeles.

History and Philosophy of Mathematics (Code: SS 4A), **Shawnee L. McMurran**, California State University San Bernardino, and **James J. Tattersall**, Providence College.

Homotopy Theory and Higher Algebraic Structures (Code: SS 8A), **John Baez** and **Julie Bergner**, University of California Riverside.

Interactions Between Algebraic Geometry and Noncommutative Algebra (Code: SS 9A), **Kenneth R. Goodearl**, University of California Santa Barbara, **Daniel S. Rogalski**, University of California San Diego, and **James Zhang**, University of Washington.

Knotting Around Dimension Three: A Special Session in Memory of Xiao-Song Lin (Code: SS 11A), **Martin Scharlemann**, University of California Santa Barbara, and **Mohammed Ait Nouh**, University of California Riverside.

Noncommutative Geometry (Code: SS 2A), **Vasiliy Dolgushev** and **Wee Liang Gan**, University of California Riverside.

Operator Algebras (Code: SS 13A), **Marta Asaeda** and **Aviv Censor**, University of California Riverside, and **Adrian Ioana**, Clay Institute and Caltech.

Representation Theory (Code: SS 3A), **Vyjayanthi Chari**, **Wee Liang Gan**, and **Jacob Greenstein**, University of California Riverside.

Representations of Finite Dimensional Algebras (Code: SS 7A), **Frauke Bleher**, University of Iowa, **Birge Huisgen-Zimmermann**, University of California at Santa Barbara, and **Markus Schmidmeier**, Florida Atlantic University.

Research Conducted by Students (Code: SS 10A), **Robert G. Niemeyer** and **Jack R. Bennett**, University of California Riverside.

Stochastic Analysis and Applications (Code: SS 12A), **Michael L. Green**, **Alan C. Krinik**, and **Randall J. Swift**, California State Polytechnic University Pomona.

Seoul, Korea

December 16–20, 2009

Wednesday – Sunday

Meeting #1055

First Joint International Meeting of the AMS and the Korean Mathematical Society.
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: June 2009
Program first available on AMS website: Not applicable
Program issue of electronic *Notices*: Not applicable
Issue of *Abstracts*: Not applicable

Deadlines

For organizers: March 31, 2009
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Francisco, California

Moscone Center West and the San Francisco Marriott

January 13–16, 2010

Wednesday – Saturday

Joint Mathematics Meetings, including the 116th Annual Meeting of the AMS, 93rd Annual Meeting of the Mathematical Association of America (MAA), annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society of Industrial and Applied Mathematics (SIAM).

Associate secretary: Matthew Miller

Announcement issue of *Notices*: October 2009

Program first available on AMS website: November 1, 2009

Program issue of electronic *Notices*: January 2010

Issue of *Abstracts*: Volume 31, Issue 1

Deadlines

For organizers: Expired

For consideration of contributed papers in Special Sessions: July 28, 2009

For abstracts: September 22, 2009

Lexington, Kentucky

University of Kentucky

March 27–28, 2010

Saturday – Sunday

Southeastern Section

Associate secretary: Matthew Miller

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 28, 2009

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

St. Paul, Minnesota

Macalester College

April 10–11, 2010

Saturday – Sunday

Central Section

Associate secretary: Georgia Benkart

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 10, 2009

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

Albuquerque, New Mexico

University of New Mexico

April 17–18, 2010

Saturday – Sunday

Western Section

Associate secretary: Michel L. Lapidus

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 17, 2009

For consideration of contributed papers in Special Sessions: To be announced

For abstracts: To be announced

Newark, New Jersey

New Jersey Institute of Technology

May 22–23, 2010

Saturday – Sunday

Eastern Section

Associate secretary: Steven H. Weintraub

Announcement issue of *Notices*: To be announced

Program first available on AMS website: To be announced

Program issue of electronic *Notices*: To be announced

Issue of *Abstracts*: To be announced

Deadlines

For organizers: November 23, 2009
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Berkeley, California

University of California Berkeley

June 2–5, 2010

Wednesday – Saturday
Eighth Joint International Meeting of the AMS and the Sociedad Matemática Mexicana.
Associate secretary: Susan J. Friedlander
Announcement issue of *Notices*: February 2010
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Syracuse, New York

Syracuse University

October 2–3, 2010

Saturday – Sunday
Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Los Angeles, California

University of California Los Angeles

October 9–10, 2010

Saturday – Sunday
Western Section
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: March 10, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: August 17, 2010

The scientific information listed below may be dated. For the latest information, see `www.ams.org/amsmtgs/sectional.html`.

Special Sessions

Topology and Symplectic Geometry (Code: SS 1A), **Robert Brown** and **Ciprian Manolescu**, University of California Los Angeles, and **Stefano Vidussi**, University of California Riverside.

Notre Dame, Indiana

Notre Dame University

October 29–31, 2010

Friday – Sunday
Central Section
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: February 19, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Richmond, Virginia

University of Virginia

November 6–7, 2010

Saturday – Sunday

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: To be announced
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

New Orleans, Louisiana

New Orleans Marriott and Sheraton New Orleans Hotel

January 5–8, 2011

Wednesday – Saturday

Joint Mathematics Meetings, including the 117th Annual Meeting of the AMS, 94th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).

Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2010
Program first available on AMS website: November 1, 2010
Program issue of electronic *Notices*: January 2011
Issue of *Abstracts*: Volume 32, Issue 1

Deadlines

For organizers: April 1, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Statesboro, Georgia

Georgia Southern University

March 12–13, 2011

Saturday – Sunday

Southeastern Section
Associate secretary: Matthew Miller
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: August 12, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Worcester, Massachusetts

College of the Holy Cross

April 9–10, 2011

Saturday – Sunday

Eastern Section
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: To be announced
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: To be announced
Issue of *Abstracts*: To be announced

Deadlines

For organizers: September 9, 2010
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Boston, Massachusetts

John B. Hynes Veterans Memorial Convention Center, Boston Marriott Hotel, and Boston Sheraton Hotel

January 4–7, 2012

Wednesday – Saturday

Joint Mathematics Meetings, including the 118th Annual Meeting of the AMS, 95th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the

winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Michel L. Lapidus
Announcement issue of *Notices*: October 2011
Program first available on AMS website: November 1, 2011
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 33, Issue 1

Deadlines

For organizers: April 1, 2011
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Diego, California

San Diego Convention Center and San Diego Marriott Hotel and Marina

January 9–12, 2013

Wednesday – Saturday
Joint Mathematics Meetings, including the 119th Annual Meeting of the AMS, 96th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic (ASL), with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Georgia Benkart
Announcement issue of *Notices*: October 2012
Program first available on AMS website: November 1, 2012
Program issue of electronic *Notices*: January 2012
Issue of *Abstracts*: Volume 34, Issue 1

Deadlines

For organizers: April 1, 2012
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Baltimore, Maryland

Baltimore Convention Center, Baltimore Hilton, and Marriott Inner Harbor

January 15–18, 2014

Wednesday – Saturday
Joint Mathematics Meetings, including the 120th Annual Meeting of the AMS, 97th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association for Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Matthew Miller
Announcement issue of *Notices*: October 2013
Program first available on AMS website: November 1, 2013
Program issue of electronic *Notices*: January 2013
Issue of *Abstracts*: Volume 35, Issue 1

Deadlines

For organizers: April 1, 2013
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

San Antonio, Texas

Henry B. Gonzalez Convention Center and Grand Hyatt San Antonio

January 10–13, 2015

Saturday – Tuesday
Joint Mathematics Meetings, including the 121st Annual Meeting of the AMS, 98th Annual Meeting of the Mathematical Association of America, annual meetings of the Association for Women in Mathematics (AWM) and the National Association of Mathematicians (NAM), and the winter meeting of the Association of Symbolic Logic, with sessions contributed by the Society for Industrial and Applied Mathematics (SIAM).
Associate secretary: Steven H. Weintraub
Announcement issue of *Notices*: October 2014
Program first available on AMS website: To be announced
Program issue of electronic *Notices*: January 2015
Issue of *Abstracts*: Volume 36, Issue 1

Deadlines

For organizers: April 1, 2014
For consideration of contributed papers in Special Sessions: To be announced
For abstracts: To be announced

Meetings and Conferences of the AMS

Associate Secretaries of the AMS

Western Section: Michel L. Lapidus, Department of Mathematics, University of California, Surge Bldg., Riverside, CA 92521-0135; e-mail: lapidus@math.ucr.edu; telephone: 951-827-5910.

Central Section: **Susan J. Friedlander**, Department of Mathematics, University of Illinois at Chicago, 851 S. Morgan (M/C 249), Chicago, IL 60607-7045; e-mail: susan@math.nwu.edu; telephone: 312-996-3041. **Georgia Benkart** (after January 31, 2010), University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: benkart@math.wisc.edu; telephone: 608-263-4283.

Eastern Section: **Steven H. Weintraub**, Department of Mathematics, Lehigh University, Bethlehem, PA 18105-3174; e-mail: steve.weintraub@lehigh.edu; telephone: 610-758-3717.

Southeastern Section: **Matthew Miller,** Department of Mathematics, University of South Carolina, Columbia, SC 29208-0001, e-mail: miller@math.sc.edu; telephone: 803-777-3690.

2009 Seoul, Korea Meeting: Georgia Benkart, University of Wisconsin-Madison, Department of Mathematics, 480 Lincoln Drive, Madison, WI 53706-1388; e-mail: benkart@math.wisc.edu; telephone: 608-263-4283.

The Meetings and Conferences section of the *Notices* gives information on all AMS meetings and conferences approved by press time for this issue. Please refer to the page numbers cited in the table of contents on this page for more detailed information on each event. Invited Speakers and Special Sessions are listed as soon as they are approved by the cognizant program committee; the codes listed are needed for electronic abstract submission. For some meetings the list may be incomplete. **Information in this issue may be dated. Up-to-date meeting and conference information can be found at** www.ams.org/meetings/**.**

Meetings:

2009		
April 25–26	Worcester, Massachusetts	p. 671
April 25–26	San Francisco, California	p. 672
October 16–18	Waco, Texas	p. 672
October 24–25	University Park, Pennsylvania	p. 673
October 30–Nov. 1	Boca Raton, Florida	p. 674
November 7–8	Riverside, California	p. 675
December 6–20	Seoul, Korea	p. 675
2010		
January 13–16	San Francisco, California Annual Meeting	p. 676
March 27–28	Lexington, Kentucky	p. 676
April 10–11	St. Paul, Minnesota	p. 676
April 17–18	Albuquerque, New Mexico	p. 676
May 22–23	Newark, New Jersey	p. 676
June 2–5	Berkeley, California	p. 677
October 2–3	Syracuse, New York	p. 677
October 9–10	Los Angeles, California	p. 677
October 29–31	Notre Dame, Indiana	p. 677
November 6–7	Richmond, Virginia	p. 678
2011		
January 5–8	New Orleans, Louisiana Annual Meeting	p. 678
March 12–13	Statesboro, Georgia	p. 678
April 9–10	Worcester, Massachusetts	p. 678
2012		
January 4–7	Boston, Massachusetts Annual Meeting	p. 678
2013		
January 9–12	San Diego, California Annual Meeting	p. 679
2014		
January 15–18	Baltimore, Maryland Annual Meeting	p. 679
2015		
January 10–13	San Antonio, Texas Annual Meeting	p. 679

Important Information Regarding AMS Meetings

Potential organizers, speakers, and hosts should refer to page 89 in the January 2009 issue of the *Notices* for general information regarding participation in AMS meetings and conferences.

Abstracts

Speakers should submit abstracts on the easy-to-use interactive Web form. No knowledge of LaTeX is necessary to submit an electronic form, although those who use LaTeX may submit abstracts with such coding, and all math displays and similarily coded material (such as accent marks in text) must be typeset in LaTeX. Visit http://www.ams.org/cgi-bin/abstracts/abstract.pl. Questions about abstracts may be sent to abs-info@ams.org. Close attention should be paid to specified deadlines in this issue. Unfortunately, late abstracts cannot be accommodated.

Conferences: (see http://www.ams.org/meetings/ for the most up-to-date information on these conferences.)

Co-sponsored conferences:

June 13–July 3, 2009: Mathematics Research Communities, Snowbird, UT (see www.ams.org/amsmtgs/mrc.html for more information).

March 18-21, 2010: First International Conference on Mathematics and Statistics, AUS-ICMS '10, American University of Sharjah, Sharjah, United Arab Emirates (please see http://www.aus.edu/conferences/icms10/ for more information).

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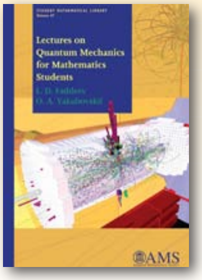

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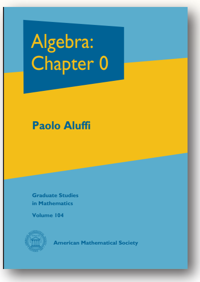

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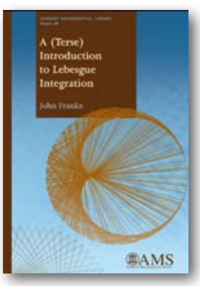

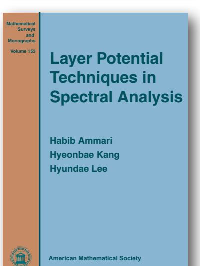

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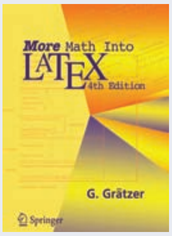

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