

# Engineering Maths

## First Aid Kit

8.7

### Table of Integrals

Engineers usually refer to a table of integrals when performing calculations involving integration. This leaflet provides such a table. Sometimes restrictions need to be placed on the values of some of the variables. These restrictions are shown in the third column.

#### 1. A table of integrals

$f(x)$	$\int f(x) dx$	
$k$ , any constant	$kx + c$	
$x$	$\frac{x^2}{2} + c$	
$x^2$	$\frac{x^3}{3} + c$	
$x^n$	$\frac{x^{n+1}}{n+1} + c$	$n \neq -1$
$x^{-1} = \frac{1}{x}$	$\ln x  + c$	
$e^x$	$e^x + c$	
$e^{kx}$	$\frac{1}{k}e^{kx} + c$	
$\cos x$	$\sin x + c$	
$\cos kx$	$\frac{1}{k} \sin kx + c$	
$\sin x$	$-\cos x + c$	
$\sin kx$	$-\frac{1}{k} \cos kx + c$	
$\tan x$	$\ln(\sec x) + c$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$
$\sec x$	$\ln(\sec x + \tan x) + c$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$
cosec $x$	$\ln(\cosec x - \cot x) + c$	$0 < x < \pi$
$\cot x$	$\ln(\sin x) + c$	$0 < x < \pi$
$\cosh x$	$\sinh x + c$	
$\sinh x$	$\cosh x + c$	
$\tanh x$	$\ln \cosh x + c$	
$\coth x$	$\ln \sinh x + c$	$x > 0$
$\frac{1}{x^2+a^2}$	$\frac{1}{a} \tan^{-1} \frac{x}{a} + c$	$a > 0$
$\frac{1}{x^2-a^2}$	$\frac{1}{2a} \ln \frac{x-a}{x+a} + c$	$ x  > a > 0$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a} \ln \frac{a+x}{a-x} + c$	$ x  < a$
$\frac{1}{\sqrt{x^2+a^2}}$	$\sinh^{-1} \frac{x}{a} + c$	$a > 0$
$\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1} \frac{x}{a} + c$	$x \geq a > 0$
$\frac{1}{\sqrt{x^2+k}}$	$\ln(x + \sqrt{x^2 + k}) + c$	
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1} \frac{x}{a} + c$	$-a \leq x \leq a$

## Exercises

1. In each case, use the Table of Integrals to integrate the given function with respect to  $x$ .

- a)  $x$
- b)  $x^6$
- c)  $x^{-2}$
- d)  $x^{-3}$
- e)  $x^{-1}$ , (be careful!)
- f)  $x^{1/2}$
- g)  $x^{-1/2}$
- h)  $e^{3x}$
- i)  $e^{7x}$
- j)  $e^{-2x}$
- k)  $e^{0.5x}$
- l)  $e^x$
- m)  $e^{-x}$
- n)  $\cos x$
- o)  $\sin x$
- p)  $\sin 3x$
- q)  $\cos 2x$
- r) 5

2. You should be able to use the table when variables other than  $x$  are involved. Use the table to integrate each of the following functions with respect to  $t$ .

- a)  $e^t$ ,      b)  $e^{5t}$ ,      c)  $t^7$ ,      d)  $\sqrt{t}$ ,      e)  $\cos 5t$ ,      f)  $e^{-t}$ .

## Answers

1. a)  $\frac{x^2}{2} + c$ ,    b)  $\frac{x^7}{7} + c$ ,    c)  $\frac{x^{-1}}{-1} + c = -x^{-1} + c$ , or  $-\frac{1}{x} + c$ ,    d)  $\frac{x^{-2}}{-2} + c = -\frac{1}{2}x^{-2} + c$ , or  $-\frac{1}{2x^2} + c$ ,
- e)  $\ln|x| + c$ ,    f)  $\frac{x^{3/2}}{3/2} + c = \frac{2}{3}x^{3/2} + c$ ,    g)  $\frac{x^{1/2}}{1/2} + c = 2x^{1/2} + c$ ,    h)  $\frac{1}{3}e^{3x} + c$ ,    i)  $\frac{1}{7}e^{7x} + c$ ,
- j)  $-\frac{1}{2}e^{-2x} + c$ ,    k)  $2e^{0.5x} + c$ ,    l)  $e^x + c$     m)  $-e^{-x} + c$ ,    n)  $\sin x + c$ ,    o)  $-\cos x + c$ ,
- p)  $-\frac{1}{3}\cos 3x + c$ ,    q)  $\frac{1}{2}\sin 2x + c$ ,    r)  $5x + c$ .
2. a)  $e^t + c$ ,    b)  $\frac{e^{5t}}{5} + c$ ,    c)  $\frac{t^8}{8} + c$ ,    d)  $\frac{2t^{3/2}}{3} + c$ ,    e)  $\frac{\sin 5t}{5} + c$ ,    f)  $-e^{-t} + c$ .