

# **Texas Examinations of Educator Standards™ (TE<sub>x</sub>ES™) Program**

## **Preparation Manual**

### **Mathematics 7–12 (235)**



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## About The Test

Test Name	Mathematics 7–12
Test Code	235
Time	5 hours
Number of Questions	100 multiple-choice questions
Format	Computer-administered test (CAT)

The TExES Mathematics 7–12 (235) test is designed to assess whether a test taker has the requisite knowledge and skills that an entry-level educator in this field in Texas public schools must possess. The 100 multiple-choice questions are based on the Mathematics 7–12 test framework. The test may contain questions that do not count toward the score. Your final scaled score will be based only on scored questions.

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## The Domains

Domain	Domain Title	Approx. Percentage of Test	Standards Assessed
I.	Number Concepts	14%	Mathematics 7–12 I
II.	Patterns and Algebra	33%	Mathematics 7–12 II
III.	Geometry and Measurement	19%	Mathematics 7–12 III
IV.	Probability and Statistics	14%	Mathematics 7–12 IV
V.	Mathematical Processes and Perspectives	10%	Mathematics 7–12 V–VI
VI.	Mathematics Learning, Instruction and Assessment	10%	Mathematics 7–12 VII–VIII

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## The Standards

### **Mathematics 7–12 Standard I**

Number Concepts: The mathematics teacher understands and uses numbers, number systems and their structure, operations and algorithms, quantitative reasoning and technology appropriate to teach the statewide curriculum (Texas Essential Knowledge and Skills [TEKS]) to prepare students to use mathematics.

### **Mathematics 7–12 Standard II**

Patterns and Algebra: The mathematics teacher understands and uses patterns, relations, functions, algebraic reasoning, analysis and technology appropriate to teach the statewide curriculum (TEKS) to prepare students to use mathematics.

### **Mathematics 7–12 Standard III**

Geometry and Measurement: The mathematics teacher understands and uses geometry, spatial reasoning, measurement concepts and principles and technology appropriate to teach the statewide curriculum (TEKS) to prepare students to use mathematics.

### **Mathematics 7–12 Standard IV**

Probability and Statistics: The mathematics teacher understands and uses probability and statistics, their applications and technology appropriate to teach the statewide curriculum (TEKS) to prepare students to use mathematics.

### **Mathematics 7–12 Standard V**

Mathematical Processes: The mathematics teacher understands and uses mathematical processes to reason mathematically, to solve mathematical problems, to make mathematical connections within and outside of mathematics and to communicate mathematically.

### **Mathematics 7–12 Standard VI**

Mathematical Perspectives: The mathematics teacher understands the historical development of mathematical ideas, the relationship between society and mathematics, the structure of mathematics and the evolving nature of mathematics and mathematical knowledge.

### **Mathematics 7–12 Standard VII**

Mathematical Learning and Instruction: The mathematics teacher understands how children learn and develop mathematical skills, procedures and concepts; knows typical errors students make; and uses this knowledge to plan, organize and implement instruction to meet curriculum goals and to teach all students to understand and use mathematics.

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**Mathematics 7–12 Standard VIII**

Mathematical Assessment: The mathematics teacher understands assessment, and uses a variety of formal and informal assessment techniques appropriate to the learner on an ongoing basis to monitor and guide instruction and to evaluate and report student progress.

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## Domains and Competencies

The content covered by this test is organized into broad areas of content called **domains**. Each domain covers one or more of the educator standards for this field. Within each domain, the content is further defined by a set of **competencies**. Each competency is composed of two major parts:

- The **competency statement**, which broadly defines what an entry-level educator in this field in Texas public schools should know and be able to do.
- The **descriptive statements**, which describe in greater detail the knowledge and skills eligible for testing.

### Domain I — Number Concepts

Competency 001: *The teacher understands the real number system and its structure, operations, algorithms and representations.*

The beginning teacher:

- Understands the concepts of place value, number base and decimal representations of real numbers.
- Understands the algebraic structure and properties of the real number system and its subsets (e.g., real numbers as a field, integers as an additive group).
- Describes and analyzes properties of subsets of the real numbers (e.g., closure, identities).
- Selects and uses appropriate representations of real numbers (e.g., fractions, decimals, percents, roots, exponents, scientific notation) for particular situations.
- Uses a variety of models (e.g., geometric, symbolic) to represent operations, algorithms and real numbers.
- Uses real numbers to model and solve a variety of problems.
- Uses deductive reasoning to simplify and justify algebraic processes.
- Demonstrates how some problems that have no solution in the integer or rational number systems have solutions in the real number system.

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Competency 002: *The teacher understands the complex number system and its structure, operations, algorithms and representations.*

The beginning teacher:

- A. Demonstrates how some problems that have no solution in the real number system have solutions in the complex number system.
- B. Understands the properties of complex numbers (e.g., complex conjugate, magnitude/modulus, multiplicative inverse).
- C. Understands the algebraic structure of the complex number system and its subsets (e.g., complex numbers as a field, complex addition as vector addition).
- D. Selects and uses appropriate representations of complex numbers (e.g., vector, ordered pair, polar, exponential) for particular situations.
- E. Describes complex number operations (e.g., addition, multiplication, roots) using symbolic and geometric representations.

Competency 003: *The teacher understands number theory concepts and principles and uses numbers to model and solve problems in a variety of situations.*

The beginning teacher:

- A. Applies ideas from number theory (e.g., prime numbers and factorization, the Euclidean algorithm, divisibility, congruence classes, modular arithmetic, the fundamental theorem of arithmetic) to solve problems.
- B. Applies number theory concepts and principles to justify and prove number relationships.
- C. Compares and contrasts properties of vectors and matrices with properties of number systems (e.g., existence of inverses, non-commutative operations).
- D. Uses properties of numbers (e.g., fractions, decimals, percents, ratios, proportions) to model and solve real-world problems.
- E. Applies counting techniques such as permutations and combinations to quantify situations and solve problems.
- F. Uses estimation techniques to solve problems and judges the reasonableness of solutions.

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## Domain II — Patterns and Algebra

Competency 004: *The teacher uses patterns to model and solve problems and formulate conjectures.*

The beginning teacher:

- A. Recognizes and extends patterns and relationships in data presented in tables, sequences or graphs.
- B. Uses methods of recursion and iteration to model and solve problems.
- C. Uses the principle of mathematical induction.
- D. Analyzes the properties of sequences and series (e.g., Fibonacci, arithmetic, geometric) and uses them to solve problems involving finite and infinite processes.
- E. Understands how sequences and series are applied to solve problems in the mathematics of finance (e.g., simple, compound and continuous interest rates; annuities).

Competency 005: *The teacher understands attributes of functions, relations and their graphs.*

The beginning teacher:

- A. Understands when a relation is a function.
- B. Identifies the mathematical domain and range of functions and relations and determines reasonable domains for given situations.
- C. Understands that a function represents a dependence of one quantity on another and can be represented in a variety of ways (e.g., concrete models, tables, graphs, diagrams, verbal descriptions, symbols).
- D. Identifies and analyzes even and odd functions, one-to-one functions, inverse functions and their graphs.
- E. Applies basic transformations [e.g.,  $k f(x)$ ,  $f(x) + k$ ,  $f(x - k)$ ,  $f(kx)$ ,  $|f(x)|$ ] to a parent function,  $f$ , and describes the effects on the graph of  $y = f(x)$ .
- F. Performs operations (e.g., sum, difference, composition) on functions, finds inverse relations and describes results symbolically and graphically.
- G. Uses graphs of functions to formulate conjectures of identities [e.g.,  $y = x^2 - 1$  and  $y = (x - 1)(x + 1)$ ,  $y = \log x^3$  and  $y = 3 \log x$ ,  $y = \sin(x + \frac{\pi}{2})$  and  $y = \cos x$ ].

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Competency 006: *The teacher understands linear and quadratic functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.*

The beginning teacher:

- A. Understands the concept of slope as a rate of change and interprets the meaning of slope and intercept in a variety of situations.
- B. Writes equations of lines given various characteristics (e.g., two points, a point and slope, slope and y-intercept).
- C. Applies techniques of linear and matrix algebra to represent and solve problems involving linear systems.
- D. Analyzes the zeros (real and complex) of quadratic functions.
- E. Makes connections between the  $y = ax^2 + bx + c$  and the  $y = a(x - h)^2 + k$  representations of a quadratic function and its graph.
- F. Solves problems involving quadratic functions using a variety of methods (e.g., factoring, completing the square, using the quadratic formula, using a graphing calculator).
- G. Models and solves problems involving linear and quadratic equations and inequalities using a variety of methods, including technology.

Competency 007: *The teacher understands polynomial, rational, radical, absolute value and piecewise functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.*

The beginning teacher:

- A. Recognizes and translates among various representations (e.g., written, tabular, graphical, algebraic) of polynomial, rational, radical, absolute value and piecewise functions.
- B. Describes restrictions on the domains and ranges of polynomial, rational, radical, absolute value and piecewise functions.
- C. Makes and uses connections among the significant points (e.g., zeros, local extrema, points where a function is not continuous or not differentiable) of a function, the graph of the function and the function's symbolic representation.
- D. Analyzes functions in terms of vertical, horizontal and slant asymptotes.
- E. Analyzes and applies the relationship between inverse variation and rational functions.

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- F. Solves equations and inequalities involving polynomial, rational, radical, absolute value and piecewise functions using a variety of methods (e.g., tables, algebraic methods, graphs, use of a graphing calculator) and evaluates the reasonableness of solutions.
  - G. Models situations using polynomial, rational, radical, absolute value and piecewise functions and solves problems using a variety of methods, including technology.

Competency 008: *The teacher understands exponential and logarithmic functions, analyses their algebraic and graphical properties and uses them to model and solve problems.*

The beginning teacher:

- A. Recognizes and translates among various representations (e.g., written, numerical, tabular, graphical, algebraic) of exponential and logarithmic functions.
- B. Recognizes and uses connections among significant characteristics (e.g., intercepts, asymptotes) of a function involving exponential or logarithmic expressions, the graph of the function and the function's symbolic representation.
- C. Understands the relationship between exponential and logarithmic functions and uses the laws and properties of exponents and logarithms to simplify expressions and solve problems.
- D. Uses a variety of representations and techniques (e.g., numerical methods, tables, graphs, analytic techniques, graphing calculators) to solve equations, inequalities and systems involving exponential and logarithmic functions.
- E. Models and solves problems involving exponential growth and decay.
- F. Uses logarithmic scales (e.g., Richter, decibel) to describe phenomena and solve problems.
- G. Uses exponential and logarithmic functions to model and solve problems involving the mathematics of finance (e.g., compound interest).
- H. Uses the exponential function to model situations and solve problems in which the rate of change of a quantity is proportional to the current amount of the quantity [i.e.,  $f'(x) = kf(x)$ ].

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Competency 009: *The teacher understands trigonometric and circular functions, analyzes their algebraic and graphical properties and uses them to model and solve problems.*

The beginning teacher:

- A. Analyzes the relationships among the unit circle in the coordinate plane, circular functions and the trigonometric functions.
- B. Recognizes and translates among various representations (e.g., written, numerical, tabular, graphical, algebraic) of trigonometric functions and their inverses.
- C. Recognizes and uses connections among significant properties (e.g., zeros, axes of symmetry, local extrema) and characteristics (e.g., amplitude, frequency, phase shift) of a trigonometric function, the graph of the function and the function's symbolic representation.
- D. Understands the relationships between trigonometric functions and their inverses and uses these relationships to solve problems.
- E. Uses trigonometric identities to simplify expressions and solve equations.
- F. Models and solves a variety of problems (e.g., analyzing periodic phenomena) using trigonometric functions.
- G. Uses graphing calculators to analyze and solve problems involving trigonometric functions.

Competency 010: *The teacher understands and solves problems using differential and integral calculus.*

The beginning teacher:

- A. Understands the concept of limit and the relationship between limits and continuity.
- B. Relates the concept of average rate of change to the slope of the secant line and relates the concept of instantaneous rate of change to the slope of the tangent line.
- C. Uses the first and second derivatives to analyze the graph of a function (e.g., local extrema, concavity, points of inflection).
- D. Understands and applies the fundamental theorem of calculus and the relationship between differentiation and integration.
- E. Models and solves a variety of problems (e.g., velocity, acceleration, optimization, related rates, work, center of mass) using differential and integral calculus.
- F. Analyzes how technology can be used to solve problems and illustrate concepts involving differential and integral calculus.

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## Domain III — Geometry and Measurement

Competency 011: *The teacher understands measurement as a process.*

The beginning teacher:

- A. Applies dimensional analysis to derive units and formulas in a variety of situations (e.g., rates of change of one variable with respect to another) and to find and evaluate solutions to problems.
- B. Applies formulas for perimeter, area, surface area and volume of geometric figures and shapes (e.g., polygons, pyramids, prisms, cylinders, cones, spheres) to solve problems.
- C. Recognizes the effects on length, area or volume when the linear dimensions of plane figures or solids are changed.
- D. Applies the Pythagorean theorem, proportional reasoning and right triangle trigonometry to solve measurement problems.
- E. Relates the concept of area under a curve to the limit of a Riemann sum.
- F. Uses integral calculus to compute various measurements associated with curves and regions (e.g., area, arc length) in the plane, and measurements associated with curves, surfaces and regions in three-space.

Competency 012: *The teacher understands geometries, in particular Euclidian geometry, as axiomatic systems.*

The beginning teacher:

- A. Understands axiomatic systems and their components (e.g., undefined terms, defined terms, theorems, examples, counterexamples).
- B. Uses properties of points, lines, planes, angles, lengths and distances to solve problems.
- C. Applies the properties of parallel and perpendicular lines to solve problems.
- D. Uses properties of congruence and similarity to explore geometric relationships, justify conjectures and prove theorems.
- E. Describes and justifies geometric constructions made using compass and straightedge, reflection devices and other appropriate technologies.
- F. Demonstrates an understanding of the use of appropriate software to explore attributes of geometric figures and to make and evaluate conjectures about geometric relationships.
- G. Compares and contrasts the axioms of Euclidean geometry with those of non-Euclidean geometry (i.e., hyperbolic and elliptic geometry).

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Competency 013: *The teacher understands the results, uses and applications of Euclidian geometry.*

The beginning teacher:

- A. Analyzes the properties of polygons and their components.
- B. Analyzes the properties of circles and the lines that intersect them.
- C. Uses geometric patterns and properties (e.g., similarity, congruence) to make generalizations about two- and three-dimensional figures and shapes (e.g., relationships of sides, angles).
- D. Computes the perimeter, area and volume of figures and shapes created by subdividing and combining other figures and shapes (e.g., arc length, area of sectors).
- E. Analyzes cross-sections and nets of three-dimensional shapes.
- F. Uses top, front, side and corner views of three-dimensional shapes to create complete representations and solve problems.
- G. Applies properties of two- and three-dimensional shapes to solve problems across the curriculum and in everyday life.

Competency 014: *The teacher understands coordinate, transformational and vector geometry and their connections.*

The beginning teacher:

- A. Identifies transformations (i.e., reflections, translations, glide-reflections, rotations, dilations) and explores their properties.
- B. Uses the properties of transformations and their compositions to solve problems.
- C. Uses transformations to explore and describe reflectional, rotational and translational symmetry.
- D. Applies transformations in the coordinate plane.
- E. Applies concepts and properties of slope, midpoint, parallelism, perpendicularity and distance to explore properties of geometric figures and solve problems in the coordinate plane.
- F. Uses coordinate geometry to derive and explore the equations, properties and applications of conic sections (i.e., lines, circles, hyperbolas, ellipses, parabolas).
- G. Relates geometry and algebra by representing transformations as matrices and uses this relationship to solve problems.
- H. Explores the relationship between geometric and algebraic representations of vectors and uses this relationship to solve problems.

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## Domain IV — Probability and Statistics

Competency 015: *The teacher understands how to use appropriate graphical and numerical techniques to explore data, characterize patterns and describe departures from patterns.*

The beginning teacher:

- A. Selects and uses an appropriate measurement scale (i.e., nominal, ordinal, interval, ratio) to answer research questions and analyze data.
- B. Organizes, displays and interprets data in a variety of formats (e.g., tables, frequency distributions, scatter plots, stem-and-leaf plots, box-and-whisker plots, histograms, pie charts).
- C. Applies concepts of center, spread, shape and skewness to describe a data distribution.
- D. Understands measures of central tendency (i.e., mean, median, mode) and dispersion (i.e., range, interquartile range, variance, standard deviation).
- E. Applies linear transformations (i.e., translating, stretching, shrinking) to convert data and describes the effect of linear transformations on measures of central tendency and dispersion.
- F. Analyzes connections among concepts of center and spread, data clusters and gaps, data outliers and measures of central tendency and dispersion.
- G. Supports arguments, makes predictions and draws conclusions using summary statistics and graphs to analyze and interpret one-variable data.

Competency 016: *The teacher understands concepts and applications of probability.*

The beginning teacher:

- A. Understands how to explore concepts of probability through sampling, experiments and simulations and generates and uses probability models to represent situations.
- B. Uses the concepts and principles of probability to describe the outcomes of simple and compound events.
- C. Determines probabilities by constructing sample spaces to model situations.
- D. Solves a variety of probability problems using combinations and permutations.
- E. Solves a variety of probability problems using ratios of areas of geometric regions.
- F. Calculates probabilities using the axioms of probability and related theorems and concepts such as the addition rule, multiplication rule, conditional probability and independence.

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- G. Understands expected value, variance and standard deviation of probability distributions (e.g., binomial, geometric, uniform, normal).
  - H. Applies concepts and properties of discrete and continuous random variables to model and solve a variety of problems involving probability and probability distributions (e.g., binomial, geometric, uniform, normal).

Competency 017: *The teacher understands the relationships among probability theory, sampling and statistical inference and how statistical inference is used in making and evaluating predictions.*

The beginning teacher:

- A. Applies knowledge of designing, conducting, analyzing and interpreting statistical experiments to investigate real-world problems.
- B. Analyzes and interprets statistical information (e.g., the results of polls and surveys) and recognizes misleading as well as valid uses of statistics.
- C. Understands random samples and sample statistics (e.g., the relationship between sample size and confidence intervals, biased or unbiased estimators).
- D. Makes inferences about a population using binomial, normal and geometric distributions.
- E. Describes and analyzes bivariate data using various techniques (e.g., scatterplots, regression lines, outliers, residual analysis, correlation coefficients).
- F. Understands how to transform nonlinear data into linear form to apply linear regression techniques to develop exponential, logarithmic and power regression models.
- G. Uses the law of large numbers and the central limit theorem in the process of statistical inference.
- H. Estimates parameters (e.g., population mean and variance) using point estimators (e.g., sample mean and variance).
- I. Understands principles of hypotheses testing.

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## Domain V — Mathematical Processes and Perspectives

Competency 018: *The teacher understands mathematical reasoning and problem solving.*

The beginning teacher:

- A. Understands the nature of proof, including indirect proof, in mathematics.
- B. Applies correct mathematical reasoning to derive valid conclusions from a set of premises.
- C. Uses inductive reasoning to make conjectures and uses deductive methods to evaluate the validity of conjectures.
- D. Uses formal and informal reasoning to justify mathematical ideas.
- E. Understands the problem-solving process (i.e., recognizing that a mathematical problem can be solved in a variety of ways, selecting an appropriate strategy, evaluating the reasonableness of a solution).
- F. Evaluates how well a mathematical model represents a real-world situation.

Competency 019: *The teacher understands mathematical connections both within and outside of mathematics and how to communicate mathematical ideas and concepts.*

The beginning teacher:

- A. Recognizes and uses multiple representations of a mathematical concept (e.g., a point and its coordinates, the area of a circle as a quadratic function of the radius, probability as the ratio of two areas, area of a plane region as a definite integral).
- B. Understands how mathematics is used to model and solve problems in other disciplines (e.g., art, music, science, social science, business).
- C. Translates mathematical ideas between verbal and symbolic forms.
- D. Communicates mathematical ideas using a variety of representations (e.g., numeric, verbal, graphical, pictorial, symbolic, concrete).
- E. Understands the use of visual media, such as graphs, tables, diagrams and animations, to communicate mathematical information.
- F. Uses appropriate mathematical terminology to express mathematical ideas.

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## Domain VI — Mathematical Learning, Instruction and Assessment

Competency 020: *The teacher understands how children learn mathematics and plans, organizes and implements instruction using knowledge of students, subject matter and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).*

The beginning teacher:

- A. Applies research-based theories of learning mathematics to plan appropriate instructional activities for all students.
- B. Understands how students differ in their approaches to learning mathematics.
- C. Uses students' prior mathematical knowledge to build conceptual links to new knowledge and plans instruction that builds on students' strengths and addresses students' needs.
- D. Understands how learning may be enhanced through the use of manipulatives, technology and other tools (e.g., stop watches, rulers).
- E. Understands how to provide instruction along a continuum from concrete to abstract.
- F. Understands a variety of instructional strategies and tasks that promote students' abilities to do the mathematics described in the TEKS.
- G. Understands how to create a learning environment that provides all students, including English-language learners, with opportunities to develop and improve mathematical skills and procedures.
- H. Understands a variety of questioning strategies to encourage mathematical discourse and to help students analyze and evaluate their mathematical thinking.
- I. Understands how to relate mathematics to students' lives and to a variety of careers and professions.

Competency 021: *The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.*

The beginning teacher:

- A. Understands the purpose, characteristics and uses of various assessments in mathematics, including formative and summative assessments.
- B. Understands how to select and develop assessments that are consistent with what is taught and how it is taught.

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- C. Understands how to develop a variety of assessments and scoring procedures consisting of worthwhile tasks that assess mathematical understanding, common misconceptions and error patterns.
  - D. Understands the relationship between assessment and instruction and knows how to evaluate assessment results to design, monitor and modify instruction to improve mathematical learning for all students, including English-language learners.

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## Approaches to Answering Multiple-Choice Questions

The purpose of this section is to describe multiple-choice question formats that you will typically see on the Mathematics 7–12 test and to suggest possible ways to approach thinking about and answering them. These approaches are intended to supplement and complement familiar test-taking strategies with which you may already be comfortable and that work for you. Fundamentally, the most important component in assuring your success on the test is knowing the content described in the test framework. This content has been carefully selected to align with the knowledge required to begin a career as a Mathematics 7–12 teacher.

The multiple-choice questions on this test are designed to assess your knowledge of the content described in the test framework. In most cases, you are expected to demonstrate more than just your ability to recall factual information. You may be asked to think critically about the information, to analyze it, consider it carefully, compare it with other knowledge you have or make a judgment about it.

When you are ready to respond to a single-selection multiple-choice question, you must choose one of four answer options. Leave no questions unanswered. Questions for which you mark no answer are counted as incorrect. Your score will be determined by the number of questions for which you select the correct answer.

**NOTE:** The Definitions and Formulas are provided on-screen for this exam. A copy of this reference material can be found in this preparation manual. This exam requires you to bring a graphing calculator to the test center. Refer to the examination’s information page on the Texas Educator Certification Examination Program website for a list of approved calculator models.

The Mathematics 7–12 test is designed to include a total of 100 multiple-choice questions. Your final scaled score will be based only on scored questions. The questions that are not scored are being pilot tested to collect information about how these questions will perform under actual testing conditions. These pilot questions are not identified on the test.

### How to Approach Unfamiliar Question Formats

Some questions include introductory information such as a map, table, graph or reading passage (often called a stimulus) that provides the information the question asks for. New formats for presenting information are developed from time to time. Tests may include audio and video stimulus materials such as a movie clip or some kind of animation, instead of a map or reading passage.

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Tests may also include interactive types of questions. These questions take advantage of technology to assess knowledge and skills that go beyond what can be assessed using standard single-selection multiple-choice questions. If you see a format you are not familiar with, **read the directions carefully**. The directions always give clear instructions on how you are expected to respond.

For most questions, you will respond by clicking an oval to choose a single answer choice from a list of options. Other questions may ask you to respond by:

- **Typing in an entry box.** You may be asked to enter a text or numeric answer. Some questions may have more than one place to enter a response.
- **Clicking check boxes.** You may be asked to click check boxes instead of an oval when more than one choice within a set of answers can be selected.
- **Clicking parts of a graphic.** In some questions, you will choose your answer by clicking on location(s) on a graphic such as a map or chart, as opposed to choosing from a list.
- **Clicking on sentences.** In questions with reading passages, you may be asked to choose your answer by clicking on a sentence or sentences within the reading passage.
- **Dragging and dropping answer choices into “targets” on the screen.** You may be asked to choose an answer from a list and drag it into the appropriate location in a table, paragraph of text or graphic.
- **Selecting options from a drop-down menu.** This type of question will ask you to select the appropriate answer or answers by selecting options from a drop-down menu (e.g., to complete a sentence).

Remember that with every question, you will get clear instructions on how to respond.

## Question Formats

You may see the following types of multiple-choice questions on the test:

- Single Questions
- Clustered Questions

On the following pages, you will find descriptions of these commonly used question formats, along with suggested approaches for responding to each type.

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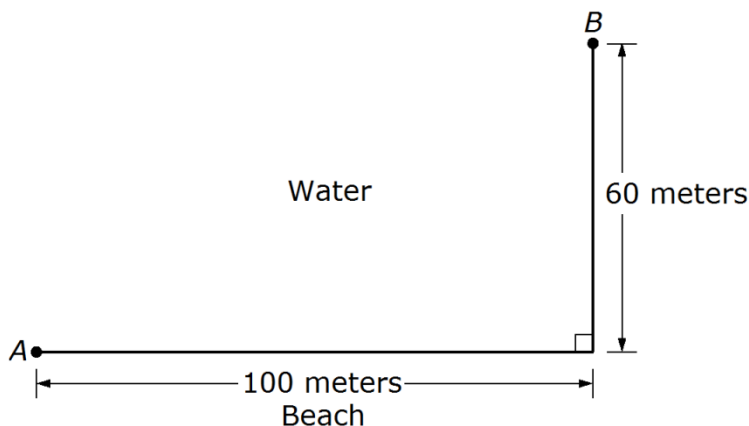
## Single Questions

The single-question format presents a direct question or an incomplete statement. It can also include a graphic, table or a combination of these. Four answer options appear below the question.

The following question is an example of the single-question format. It tests knowledge of Mathematics 7–12 Competency 010: *The teacher understands and solves problems using differential and integral calculus.*

### Example

**Use the diagram below to answer the question that follows.**



A lifeguard sitting on a beach at point  $A$  sees a swimmer in distress at point  $B$ . The lifeguard can run at a rate of 3 meters per second and can swim at a rate of 1.5 meters per second. To minimize the amount of time it takes to reach the swimmer, how far along the beach should the lifeguard run before entering the water?

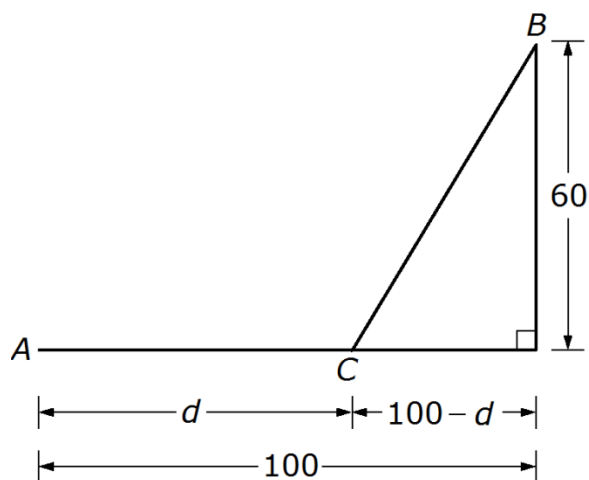
- A. 40 meters
- B. 65 meters
- C. 73 meters
- D. 100 meters

### Suggested Approach

Read the question carefully and critically. Think about what it is asking and the situation it is describing. Eliminate any obviously wrong answers, select the correct answer choice and mark your answer.

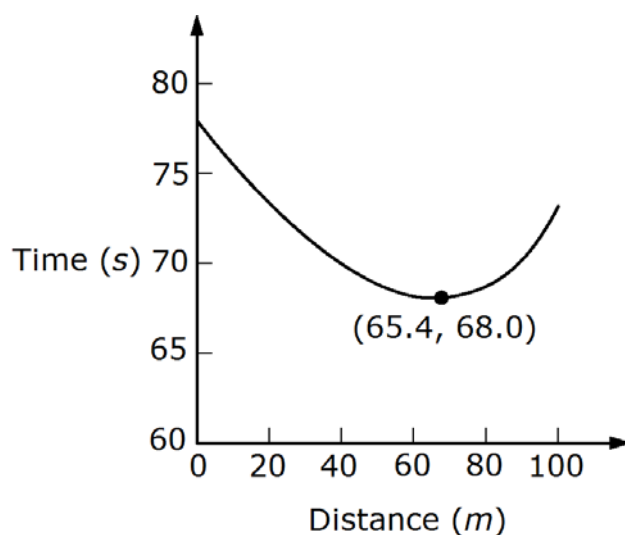
In analyzing this problem, redrawing the diagram to highlight the important information may be helpful.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.



Let  $d$  represent the distance in meters that the lifeguard runs along the beach. Then by an application of the Pythagorean theorem, the distance traveled in water is represented by  $\sqrt{60^2 + (100 - d)^2}$ . Because  $distance = rate \times time$  and the lifeguard can run at 3 meters per second and swim at 1.5 meters per second, the time it takes the lifeguard to run along the beach,  $t_b$ , can be represented by  $\frac{d}{3}$ , and the time it takes the lifeguard to swim in the water,  $t_w$ , can be represented by  $\frac{\sqrt{60^2 + (100 - d)^2}}{1.5}$ . Thus, the total time,  $t$ , it takes the lifeguard to travel to the swimmer can be represented by  $t_b + t_w$ . To solve the problem, we need to find the value of  $d$  that minimizes the function  $t = t_b + t_w = \frac{d}{3} + \frac{\sqrt{60^2 + (100 - d)^2}}{1.5}$ . This can be done using either differential calculus or a graphing approach. We will use a graphing approach. A graphing calculator can be used to produce a graph similar to the one that follows.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.



Using the capabilities of the calculator, you see that the minimum value of the function  $t$  occurs when  $d$  is approximately 65 meters, or option B.

Option A results from dividing 60 by 1.5, which is the time required to swim 60 meters. Option C results from misusing parentheses when entering the equation for

$t$  into the graphing utility; i.e., entering  $\sqrt{\frac{60^2 + (100 - d)^2}{1.5}}$  instead of

$\frac{\sqrt{60^2 + (100 - d)^2}}{1.5}$ . Option D results from minimizing the function

$t_w = \frac{\sqrt{60^2 + (100 - d)^2}}{1.5}$  instead of the expression for  $t$ , the total time required to reach the swimmer.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.



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## Clustered Questions

Clustered questions are made up of a stimulus and two or more questions relating to the stimulus. The stimulus material can be a graph of one or more mathematical functions, geometric designs, charts, data tables, equations or any other information necessary to answer the questions that follow.

You can use several different approaches to respond to clustered questions. Some commonly used strategies are listed below.

- Strategy 1** Skim the stimulus material to understand its purpose, its arrangement and/or its content. Then read the questions and refer again to the stimulus material to obtain the specific information you need to answer the questions.
- Strategy 2** Read the questions *before* considering the stimulus material. The theory behind this strategy is that the content of the questions will help you identify the purpose of the stimulus material and locate the information you need to answer the questions.
- Strategy 3** Use a combination of both strategies. Apply the “read the stimulus first” strategy with shorter, more familiar stimuli and the “read the questions first” strategy with longer, more complex or less familiar stimuli. You can experiment with the sample questions in this manual and then use the strategy with which you are most comfortable when you take the actual test.

Whether you read the stimulus before or after you read the questions, you should read it carefully and critically. You may want to note its important points to help you answer the questions.

As you consider questions set in educational contexts, try to enter into the identified teacher’s frame of mind and use that teacher’s point of view to answer the questions that accompany the stimulus. Be sure to consider the questions only in terms of the information provided in the stimulus — not in terms of your own experiences or individuals you may have known.

NOTE: After clicking on a link, right click and select “Previous View” to go back to original text.

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### Example

First read the stimulus (a learning expectation from the statewide curriculum).

**Use the student expectation below from the Texas Essential Knowledge and Skills (TEKS) to answer the questions that follow.**

The student uses characteristics of the quadratic parent function to sketch the related graphs and makes connections between the  $y = ax^2 + bx + c$  and the  $y = a(x - h)^2 + k$  symbolic representations of quadratic functions.

Now you are prepared to respond to the first of the two questions associated with this stimulus. The first question tests knowledge of Mathematics 7–12 Competency 020: *The teacher understands how children learn mathematics and plans, organizes and implements instruction using knowledge of students, subject matter and statewide curriculum (Texas Essential Knowledge and Skills [TEKS]).*

1. Which of the following algebraic techniques will students need to know to symbolically convert a quadratic function of the form  $y = ax^2 + bx + c$  into the form  $y = a(x - h)^2 + k$  ?
  - A. Solving systems of equations
  - B. Completing the square
  - C. Solving quadratic equations
  - D. Simplifying polynomial expressions

### Suggested Approach

You are asked to identify the algebraic technique that students should use to convert the expression  $y = ax^2 + bx + c$  into the expression  $y = a(x - h)^2 + k$ . The following steps show how this conversion can be achieved.

First rewrite the expression  $y = ax^2 + bx + c$  as  $y = a\left(x^2 + \frac{b}{a}x\right) + c$  by factoring  $a$  from the quantity  $ax^2 + bx$ . Next, take one-half the coefficient of the linear term, square it and add this quantity inside the parentheses while adding the product of the quantity's additive inverse and  $a$  outside of the parentheses. Note that this is equivalent to adding  $\frac{b^2}{4a}$  and  $-\frac{b^2}{4a}$  to the same side of the equation as follows:

$$y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - \frac{b^2}{4a}$$

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

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Notice that the quantity inside the parentheses is a perfect square and can be factored.

$$y = a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{4a}$$

This expression is equivalent to  $y = a(x - h)^2 + k$ , with  $h = -\frac{b}{2a}$  and

$k = \frac{4ac - b^2}{4a}$ , which are the  $x$ - and  $y$ -coordinates of the vertex of the graph of

$y = ax^2 + bx + c$ . This algebraic method of converting the first expression into the second is known as completing the square. Therefore, **option B is correct**.

Option A, solving systems of equations, is not helpful in this situation because the student is being asked to rewrite an equation, not solve it. Option C is incorrect because the student is being asked to rewrite a quadratic equation, not solve it.

Finally, although one can simplify the expression  $y = a(x - h)^2 + k$  and compare it to  $y = ax^2 + bx + c$ , this approach is ineffective when applied in the opposite direction, which makes option D incorrect.

Now you are ready to answer the next question. The second question measures Mathematics 7-12 Competency 021: *The teacher understands assessment and uses a variety of formal and informal assessment techniques to monitor and guide mathematics instruction and to evaluate student progress.*

2. Which of the following exercises best assesses student understanding of the expectation from the statewide curriculum (TEKS)?
  - A. Use a graphing calculator to graph the function  $y = x^2 - 4x + 3$ , and use the graph to find the zeros of the function
  - B. Write a real-world word problem that is modeled by the function  $y = x^2 - 4x + 3$ , and relate the zeros of the function to the graph of  $y = x^2 - 4x + 3$
  - C. Describe how the graph of  $y = (x - 3)(x - 1)$  is related to the graph of  $y = x^2 - 4x + 3$
  - D. Describe how the graph of  $y = x^2$  is related to the graph of  $y = x^2 - 4x + 3$

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

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### ***Suggested Approach***

You are asked to select an activity that would best assess student understanding of converting a function of the form  $y = ax^2 + bx + c$  into the form

$y = a(x - h)^2 + k$  and then analyzing the graph of this function in relation to the quadratic parent function  $y = x^2$ . Carefully read each of the responses to determine how well they assess student understanding of this topic.

Option A asks the student to enter a quadratic function into a graphing calculator and then use the capabilities of the graphing calculator to estimate the zeros of the function. This is a method of using technology to solve a quadratic equation, and hence is incorrect.

Option B asks the student to create a problem that can be modeled by a specific quadratic equation and to relate the graph of the equation to the problem. This assessment would be useful for evaluating student understanding of applications of quadratic functions but not for assessing understanding of the two different symbolic representations of the quadratic function. Option B is therefore incorrect.

Option C assesses understanding of the fact that a factored quadratic function has the same graph as the expanded, or unfactored, quadratic function. Option C would not assess the given learning expectation and is therefore incorrect.

Option D assesses student understanding of how the graph of  $y = x^2$  is related to that of a more complicated quadratic function involving a linear term and a constant term. Expressing the function  $y = x^2 - 4x + 3$  in the form  $y = (x - 2)^2 - 1$  allows a student to determine by inspection that the vertex is at  $(2, -1)$ . This implies that the graph of  $y = x^2 - 4x + 3$  can be obtained by translating the graph of  $y = x^2$  two units in the positive  $x$ -direction and one unit in the negative  $y$ -direction.

This analysis of the four choices should lead you to select **option D as the best response**.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

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## Multiple-Choice Practice Questions

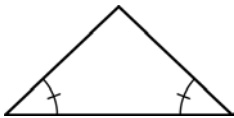

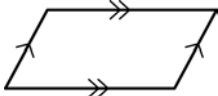
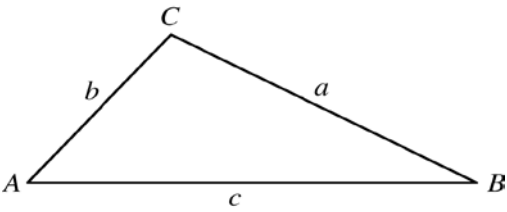
This section presents some sample test questions for you to review as part of your preparation for the test. To demonstrate how each competency may be assessed, each sample question is accompanied by the competency that it measures. While studying, you may wish to read the competency before and after you consider each sample question. Please note that the competency statements do not appear on the actual test.

For each sample test question, there is a correct answer and a rationale for each answer option. Please note that the sample questions are not necessarily presented in competency order.

The sample questions are included to illustrate the formats and types of questions you will see on the test; however, your performance on the sample questions should not be viewed as a predictor of your performance on the actual test.

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

## Definitions and Formulas for Mathematics 7–12

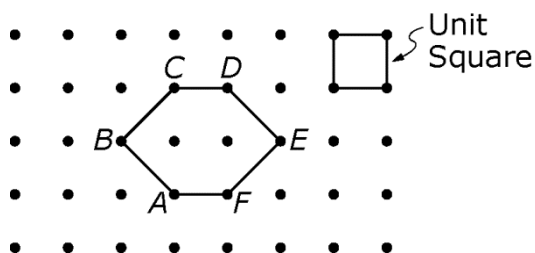
<p style="text-align: center;"><b>CALCULUS</b></p> <p><b>First Derivative:</b> <math>f'(x) = \frac{dy}{dx}</math></p> <p><b>Second Derivative:</b> <math>f''(x) = \frac{d^2y}{dx^2}</math></p> <p style="text-align: center;"><b>PROBABILITY</b></p> <p><math>P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)</math></p> <p><math>P(A \text{ and } B) = P(A)P(B A) = P(B)P(A B)</math></p>	<p style="text-align: center;"><b>ALGEBRA</b></p> <p><math>i</math> <span style="margin-left: 100px;"><math>i^2 = -1</math></span></p> <p><math>A^{-1}</math> <span style="margin-left: 100px;">inverse of matrix <math>A</math></span></p> <p><math>A = P\left(1 + \frac{r}{n}\right)^{nt}</math> <span style="margin-left: 20px;">Compound interest, where <math>A</math> is the final value <math>P</math> is the principal <math>r</math> is the interest rate <math>t</math> is the term <math>n</math> is the number of divisions within the term</span></p> <p><math>[x] = n</math> <span style="margin-left: 100px;">Greatest integer function, where <math>n</math> is the integer such that <math>n \leq x &lt; n + 1</math></span></p>
<p style="text-align: center;"><b>GEOMETRY</b></p> <p style="text-align: center;"><b>Congruent Angles</b></p>  <p style="text-align: center;"><b>Congruent Sides</b></p>  <p style="text-align: center;"><b>Parallel Sides</b></p>  <p style="text-align: center;"><b>Circumference of a Circle</b></p> <p style="text-align: center;"><math>C = 2\pi r</math></p>	<p style="text-align: center;"><b>VOLUME</b></p> <p><b>Cylinder:</b> (area of base) <math>\times</math> height</p> <p><b>Cone:</b> <math>\frac{1}{3}</math> (area of base) <math>\times</math> height</p> <p><b>Sphere:</b> <math>\frac{4}{3}\pi r^3</math></p> <p><b>Prism:</b> (area of base) <math>\times</math> height</p> <p style="text-align: center;"><b>AREA</b></p> <p><b>Triangle:</b> <math>\frac{1}{2}</math> (base <math>\times</math> height)</p> <p><b>Rhombus:</b> <math>\frac{1}{2}</math> (diagonal<sub>1</sub> <math>\times</math> diagonal<sub>2</sub>)</p> <p><b>Trapezoid:</b> <math>\frac{1}{2}</math> height (base<sub>1</sub> + base<sub>2</sub>)</p> <p><b>Sphere:</b> <math>4\pi r^2</math></p> <p><b>Circle:</b> <math>\pi r^2</math></p> <p><b>Lateral surface area of cylinder:</b> <math>2\pi rh</math></p>
<p style="text-align: center;"><b>TRIGONOMETRY</b></p> <p><b>Law of Sines:</b> <math>\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}</math></p> <p><math>c^2 = a^2 + b^2 - 2ab \cos C</math></p> <p><b>Law of Cosines:</b> <math>b^2 = a^2 + c^2 - 2ac \cos B</math></p> <p><math>a^2 = b^2 + c^2 - 2bc \cos A</math></p>	

### End of Definitions and Formulas

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

COMPETENCY 001

Use the figure below to answer the question that follows.



1. The figure above represents a geoboard, and each unit square has area 1. Which of the following quantities associated with hexagon  $ABCDEF$  is an integer?
  - A. The length of  $\overline{BD}$
  - B. The area of triangle  $BCE$
  - C. The area of hexagon  $ABCDEF$
  - D. The distance from  $B$  to the midpoint of  $\overline{BE}$

Answer and Rationale

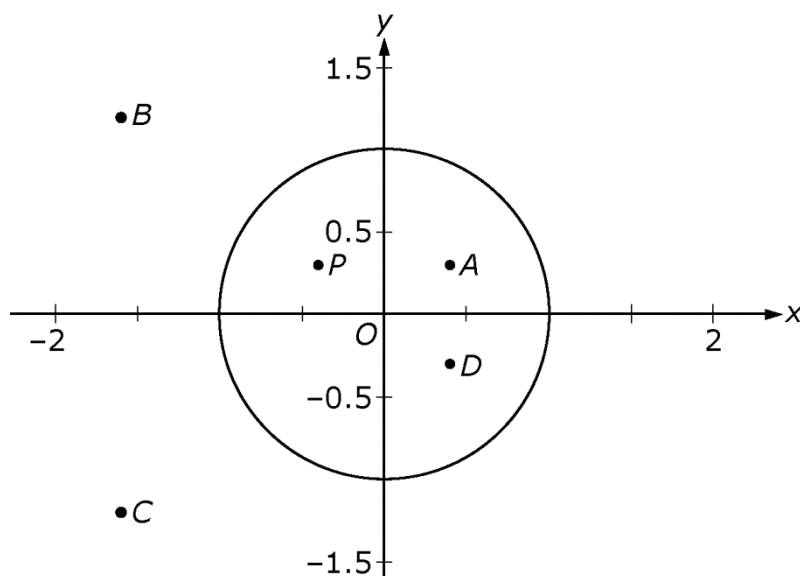
COMPETENCY 001

2.  $S$  is the set of all positive integers that can be written in the form  $2^n \cdot 3^m$ , where  $n$  and  $m$  are positive integers. If  $a$  and  $b$  are two numbers in  $S$ , which of the following must also be in  $S$ ?
  - A.  $a + b$
  - B.  $\sqrt{ab}$
  - C.  $\frac{a}{b}$
  - D.  $ab$

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

Use the figure below to answer the question that follows.



3. The figure above shows a unit circle in the complex plane. Which of the following points could represent the multiplicative inverse of the complex number represented by point  $P$ , which has coordinates  $(-0.4, 0.3)$ ?
- A.  $A$
  - B.  $B$
  - C.  $C$
  - D.  $D$

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.



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COMPETENCY 003

4. Olivia traveled 25 miles in 30 minutes, and then she traveled for an additional 20 minutes. If her average speed for the entire trip was 36 miles per hour (mph), what was her average speed for the final 20 minutes of the trip?
- A. 15 mph
  - B. 20 mph
  - C. 25 mph
  - D. 30 mph

Answer and Rationale

COMPETENCY 003

5. A researcher measured the length of an object to be  $k$  centimeters, where  $k < 0.00001$ . The researcher expressed the value of  $k$  in the form  $a \times 10^b$ , where  $a$  is a real number and  $b$  is an integer. Which of the following could be true about  $a$  and  $b$  in this situation?
- A.  $-1 \leq a < 0$  and  $b < -1$
  - B.  $1 \leq a < 10$  and  $b < -1$
  - C.  $1 \leq a < 10$  and  $b > 1$
  - D.  $\frac{1}{2} \leq a < 1$  and  $b > 1$

Answer and Rationale

COMPETENCY 004

6. If  $\{a_n\}_{n=1}^{\infty}$  is a sequence such that  $a_1 = 1$ ,  $a_2 = 3$ , and  $a_{n+3} = \frac{a_{n+1}}{a_{n+2}}$  for all integers  $n \geq 0$ , what is the value of  $a_4$ ?
- A. 9
  - B. 7
  - C. 1
  - D.  $\frac{1}{3}$

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

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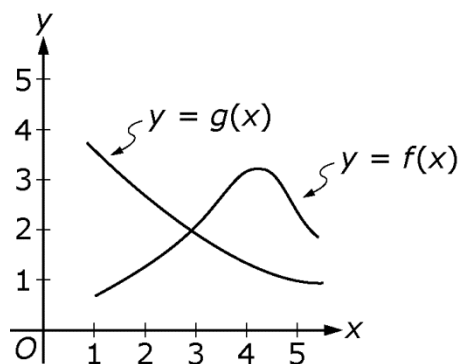
COMPETENCY 004

7. A certain finite sequence of consecutive integers begins with  $-13$ . If the sum of all the terms of the sequence is 45, how many terms are there in the sequence?
- A. 27  
B. 28  
C. 29  
D. 30

Answer and Rationale

COMPETENCY 005

**Use the graphs below to answer the question that follows.**



8. The graphs of the functions  $f$  and  $g$  are shown in the  $xy$ -plane above. For which of the following values of  $x$  is the value of  $g(x)$  closest to the value of  $f(2)$ ?
- A. 1  
B. 2  
C. 3  
D. 4

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

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COMPETENCY 005

9. Let  $f$  be the function defined by  $f(x) = -x + \frac{1}{x}$  for all  $x \neq 0$ . Which of the following must be true?

A.  $f(-x) = -f(x)$

B.  $f(-x) = f(x)$

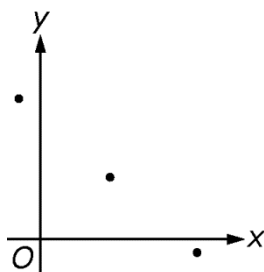
C.  $f\left(\frac{1}{x}\right) = f(x)$

D.  $f\left(\frac{1}{x}\right) = -\frac{1}{f(x)}$

Answer and Rationale

COMPETENCY 006

**Use the graph below to answer the question that follows.**



10. A nonvertical line in the  $xy$ -plane can be represented by an equation of the form  $y = mx + b$ , where  $m$  and  $b$  are constants. If line  $\ell$  contains the three points shown, which of the following statements about  $m$  and  $b$  is true for line  $\ell$ ?

A.  $m > 0$  and  $b > 0$

B.  $m > 0$  and  $b < 0$

C.  $m < 0$  and  $b > 0$

D.  $m < 0$  and  $b < 0$

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

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COMPETENCY 006

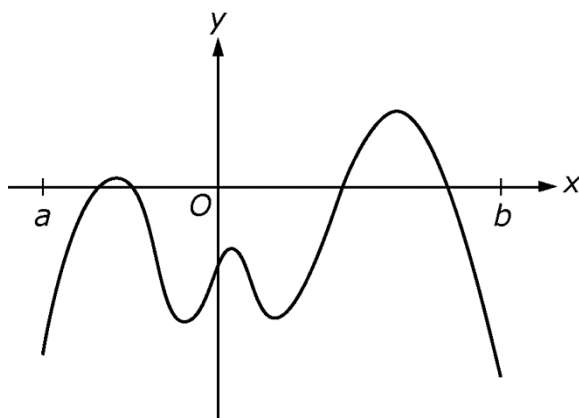
11. Let  $f$  be the function defined for all real numbers  $x$  by  $f(x) = (x - a)^2 + b$ , where  $a$  and  $b$  are constants such that  $0 < a < b$ . The function  $f$  is one-to-one on which of the following intervals?

- A.  $0 < x < b$
- B.  $0 < x < 2a$
- C.  $-b < x < b$
- D.  $-b < x < a$

Answer and Rationale

COMPETENCY 007

**Use the graph below to answer the question that follows.**



12. The  $xy$ -plane above shows the graph of  $y = f(x)$  on the closed interval  $[a, b]$ , where  $f$  is a polynomial with real coefficients. The function  $f$  is strictly increasing for all  $x < a$  and is strictly decreasing for all  $x > b$ . Which of the following statements about  $f$  is true?
- A.  $f$  has 6 real zeros and degree at least 6.
  - B.  $f$  has 4 real zeros and degree at least 6.
  - C.  $f$  has 4 real zeros and degree at most 5.
  - D.  $f$  has 4 real zeros and degree at most 4.

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

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COMPETENCY 007

**Use the equation below to answer the question that follows.**

$$y = x + 3 - \frac{1}{x - 2}$$

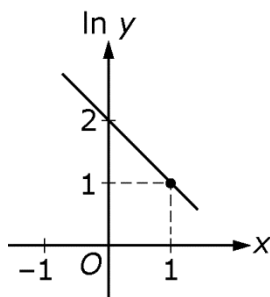
13. Which of the following is an equation of one of the asymptotes of the graph, in the  $xy$ -plane, of the equation above?

- A.  $x = -3$
- B.  $x = 1$
- C.  $y = x - 2$
- D.  $y = x + 3$

Answer and Rationale

COMPETENCY 008

**Use the figure below to answer the question that follows.**



14. If  $x$  and  $\ln y$  are related by the line shown above, which of the following equations gives  $y$  in terms of  $x$ ?

- A.  $y = e^x + 2$
- B.  $y = 2e^x$
- C.  $y = e^{x+2}$
- D.  $y = e^{2-x}$

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

**Use the formula and information below to answer the question that follows.**

In a bank account in which interest is compounded continuously, the amount  $A$  in the account is given by  $A = Pe^{rt}$ , where  $P$  is the initial deposit,  $r$  is the annual interest rate, and  $t$  is the time in years.

15. Felicia opens a bank account that pays interest compounded continuously at the annual rate of 2.5%. Her initial deposit is \$2000, and there will be no other transactions until the amount in her account is \$2500. Based on the formula given above, how many years, to the nearest whole number of years, will it take until she has \$2500 in the account?
- A. 9
  - B. 10
  - C. 11
  - D. 12

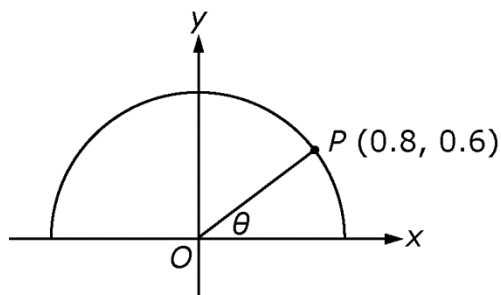
Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

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COMPETENCY 009

Use the figure below to answer the question that follows.



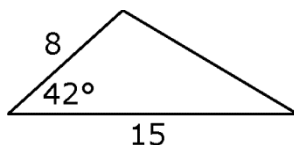
16. In the  $xy$ -plane above, point  $P$  lies on the semicircle with center  $O$ . What is the value of  $\theta$ ?

- A.  $\cos^{-1} 0.6$
- B.  $\sin^{-1} 0.75$
- C.  $\sin^{-1} 0.8$
- D.  $\tan^{-1} 0.75$

Answer and Rationale

COMPETENCY 009

Use the figure below to answer the question that follows.



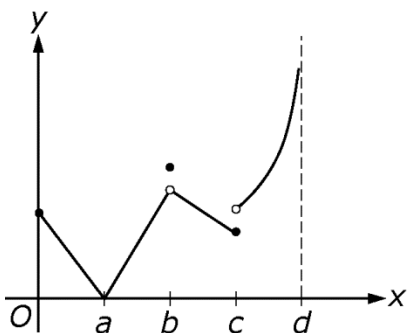
17. What is the area of the triangle above?

- A.  $60 \sin 42^\circ$
- B.  $60 \cos 42^\circ$
- C.  $120 \sin 42^\circ$
- D.  $120 \tan 42^\circ$

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

Use the graph below to answer the question that follows.



18. The graph of the function  $f$  on the interval  $0 \leq x < d$  is shown above, where  $\lim_{x \rightarrow d^-} f(x) = +\infty$ . For which of the following values of  $x$  does  $f$  have a removable discontinuity?

- A.  $a$
- B.  $b$
- C.  $c$
- D.  $d$

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.



Use the information below to answer the question that follows.

The surface area of a roof is measured in squares of shingles. Each square of shingles covers 100 square feet. However, shingles are sold in bundles and priced per bundle. It takes 3 bundles of shingles to make a square of shingles.

19. A certain roof consists of 2 rectangular sides, each having dimensions 15 feet by 60 feet. Based on the information above, if shingles cost \$28.99 per bundle, which of the following represents the total cost of the shingles for the roof?

- A.  $\frac{(3)(2)(15)(60)(\$28.99)}{100}$
- B.  $\frac{(2)(15)(60)(\$28.99)(100)}{3}$
- C.  $\frac{(2)(15)(60)(\$28.99)}{(3)(100)}$
- D.  $\frac{(3)(\$28.99)(100)}{(2)(15)(60)}$

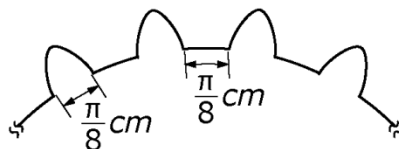
Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

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COMPETENCY 011

Use the figure below to answer the question that follows.



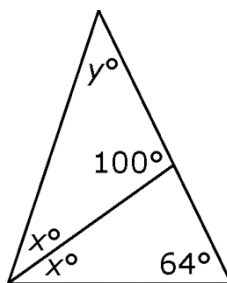
20. The figure shows a portion of a gear that has cogs evenly spaced around the circumference of a wheel. Each cog is  $\frac{\pi}{8}$  centimeters wide, and there is a space of  $\frac{\pi}{8}$  centimeters between consecutive cogs. If the diameter of the wheel is 9 centimeters, how many cogs are on the wheel?

- A. 12
- B. 18
- C. 24
- D. 36

Answer and Rationale

COMPETENCY 012

Use the figure below to answer the question that follows.



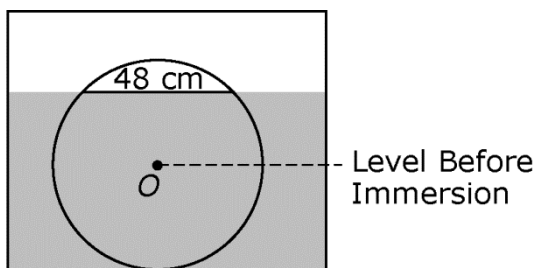
21. What is the value of  $y$  in the triangle above?

- A. 36
- B. 40
- C. 44
- D. 48

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

Use the figure below to answer the question that follows.



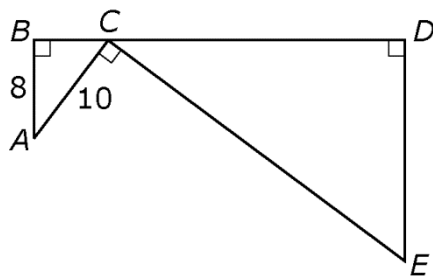
Note: Figure not drawn to scale.

22. A wheel with center  $O$  and radius 25 cm is immersed in a vat of cleaning solution, as shown in the figure above. The chord of length 48 cm indicates the solution level after the wheel was immersed. The dashed line indicates the solution level before the wheel was immersed. What is the level of the solution in the vat after the wheel has been immersed?
- A. 32 cm
  - B. 33 cm
  - C. 35 cm
  - D. 37 cm

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

Use the figure below to answer the question that follows.



23. In the figure above,  $C$  is a point on  $\overline{BD}$ . Triangles  $ABC$  and  $CDE$  are right triangles, and  $\overline{AC} \perp \overline{CE}$ . If the length of  $\overline{BD}$  is 30, what is the length of  $\overline{DE}$  ?

- A. 18
- B. 20
- C. 24
- D. 32

Answer and Rationale

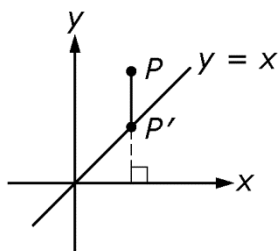
NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

Use the matrix equation below to answer the question that follows.

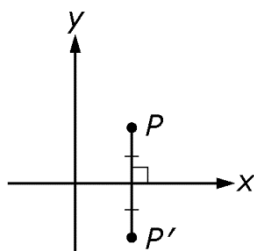
$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}$$

24. The matrix equation above defines a transformation of the  $xy$ -plane. Which of the following shows a point  $P$  and its image  $P'$  under this transformation?

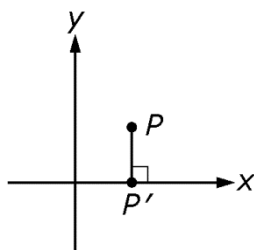
A.



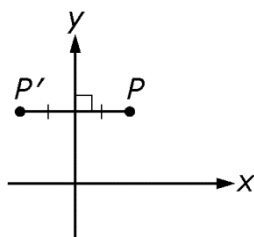
B.



C.



D.



Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

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COMPETENCY 015

**Use the definition below to answer the question that follows.**

For a set of data, a data point is an outlier if it is more than 1.5 times the interquartile range of the data set either above the third quartile or below the first quartile.

The bulb life, in hours, for 27 lightbulbs of the same brand is recorded below.

275	400	431	465	480	495
350	400	436	465	481	595
360	420	450	470	483	
380	425	452	473	490	
395	428	460	474	492	

25. Based on the definition above, which of the numbers 275 and 595 is an outlier?

- A. Neither 275 nor 595
- B. 275 only
- C. 595 only
- D. Both 275 and 595

Answer and Rationale

COMPETENCY 016

26. A computer company employs over 4000 employees, of whom 45% are women. If a focus group of 20 randomly selected employees is to be formed, what is the expected number of men in the focus group?

- A. 8
- B. 9
- C. 11
- D. 13

Answer and Rationale

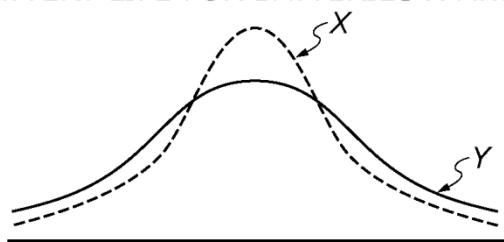
NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

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COMPETENCY 017

Use the graph below to answer the question that follows.

BATTERY LIFE FOR BATTERIES X AND Y



27. The battery life, in years, for each of two brands of car batteries,  $X$  and  $Y$ , is approximately normally distributed, as shown above. Which of the following statements about the mean and standard deviation of battery life for the two distributions is true?
- A. The mean battery life for  $X$  is less than the mean battery life for  $Y$ .
  - B. The mean battery life for  $X$  is greater than the mean battery life for  $Y$ .
  - C. The standard deviation of battery life for  $X$  is less than the standard deviation of battery life for  $Y$ .
  - D. The standard deviation of battery life for  $X$  is greater than the standard deviation of battery life for  $Y$ .

Answer and Rationale

COMPETENCY 017

28. To evaluate a new medication that was developed to reduce the occurrence of headaches, a randomized controlled experiment is conducted. One-third of the patients are given the new medication, one-third are given a placebo, and one-third are given nothing. Which of the following is the best example of the placebo effect for this study?
- A. People taking the placebo report more headaches than people taking the new medication.
  - B. People taking the placebo report fewer headaches than people taking the new medication.
  - C. People taking the placebo report more headaches than people taking nothing.
  - D. People taking the placebo report fewer headaches than people taking nothing.

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

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COMPETENCY 018

**Use the statement below to answer the question that follows.**

If  $x^2$  is even, then  $x$  is even.

29. A student is trying to prove that the statement above is true for all integers  $x$  by proving its contrapositive. Which of the following procedures should the student follow in order to use this method of proof?
- A. Assume that  $x^2$  is even, and then deduce that  $x$  is even
  - B. Assume that  $x^2$  is not even, and then deduce that  $x$  is not even
  - C. Assume that  $x^2$  is even, and then deduce that  $x$  is not even
  - D. Assume that  $x$  is not even, and then deduce that  $x^2$  is not even

Answer and Rationale

COMPETENCY 019

**Use the problem below to answer the question that follows.**

Working together at their constant rates, hoses  $A$  and  $B$  can fill an empty pool in 10 hours. Working alone, it takes hose  $B$  twice as many hours as hose  $A$  to fill the pool. How many hours would it take hose  $A$ , working alone at its constant rate, to fill the pool?

30. In the problem above, if  $x$  represents the number of hours it takes hose  $A$  to fill the pool working alone, which of the following equations correctly models the situation?
- A.  $\frac{1}{x} + \frac{1}{2x} = \frac{1}{10}$
  - B.  $\frac{1}{x} + \frac{2}{x} = \frac{1}{10}$
  - C.  $\frac{1}{x} + \frac{1}{2x} = 10$
  - D.  $x + 2x = 10$

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.



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## COMPETENCY 020

31. Of the following activities involving the quadratic expression  $ax^2 + bx + c$ , which best exemplifies inquiry-based learning?
- A. Students predict how the graph of  $y = ax^2 + bx + c$  will be affected by changing the value of  $a$ , and check their predictions using a graphing calculator.
  - B. Students solve an equation of the form  $ax^2 + bx + c = 0$  by graphing the equation on a graphing calculator.
  - C. Students derive the quadratic formula by completing the square on the left side of the equation  $ax^2 + bx + c = 0$ .
  - D. Students use a function of the form  $f(x) = ax^2 + bx + c$  to model a problem involving falling bodies.

Answer and Rationale

## COMPETENCY 021

32. If a student mistakenly states that  $-\frac{1}{2}\left(-\frac{2}{3}x + \frac{1}{2}\right) = \frac{1}{3}x + \frac{1}{2}$ , it is most likely that the mistake results from a misunderstanding of which of the following?
- A. Multiplication of fractions
  - B. Arithmetic of negative numbers
  - C. Associative property of multiplication
  - D. Distributive property of multiplication over addition

Answer and Rationale

NOTE: After clicking on a link, right click and select "Previous View" to go back to original text.

## Answer Key and Rationales

Question Number	Competency Number	Correct Answer	Rationales
1	001	C	<p><b>Option C is correct</b> because the area of hexagon <math>ABCDEF</math> is equal to 4 square units, and 4 is an integer. Each unit square has area 1, and the hexagon is composed of 2 full squares and 4 half-squares, for a total area of <math>2(1) + 4(0.5) = 2 + 2 = 4</math>.</p> <p><b>Option A is incorrect</b> because, by the Pythagorean theorem, the length of <math>\overline{BD}</math> is <math>\sqrt{2^2 + 1^2} = \sqrt{5}</math>, which is not an integer. <b>Option B is incorrect</b> because the area of triangle <math>BCE</math> is <math>\frac{1}{2}bh = \frac{1}{2}(3)(1) = 1.5</math>, which is not an integer. <b>Option D is incorrect</b> because the distance from <math>B</math> to the midpoint of <math>\overline{BE}</math> is <math>\frac{3}{2} = 1.5</math>, which is not an integer.</p> <p><a href="#">Back to Question</a></p>
2	001	D	<p><b>Option D is correct</b> because if <math>a</math> and <math>b</math> are two numbers in <math>S</math>, then <math>a = 2^j \cdot 3^k</math> and <math>b = 2^s \cdot 3^t</math>, where <math>j</math>, <math>k</math>, <math>s</math>, and <math>t</math> are positive integers. So <math>ab = (2^j \cdot 3^k)(2^s \cdot 3^t) = 2^{j+s} \cdot 3^{k+t}</math>, and <math>j+s</math> and <math>k+t</math> are both positive integers; thus <math>ab</math> is in <math>S</math>. <b>Option A is incorrect</b> because, for example, if <math>a = 18</math> and <math>b = 12</math>, then <math>a + b = 30</math>, which is not in <math>S</math>. <b>Options B and C are incorrect</b> because, for example, if <math>a = 6</math> and <math>b = 12</math>, then <math>\sqrt{ab} = \sqrt{72} = 6\sqrt{2}</math> and <math>\frac{a}{b} = \frac{6}{12} = \frac{1}{2}</math>, neither of which is in <math>S</math>.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
3	002	C	<p><b>Option C is correct</b> because the multiplicative inverse of a complex number <math>a + bi</math> is <math>\frac{a}{a^2 + b^2} - \frac{b}{a^2 + b^2}i</math>.</p> <p>Point <math>P</math> represents the complex number <math>-0.4 + 0.3i</math>, which has multiplicative inverse</p> $\frac{-0.4}{(-0.4)^2 + (0.3)^2} - \frac{0.3}{(-0.4)^2 + (0.3)^2}i, \text{ or } -1.6 - 1.2i.$ <p>This number is represented by the point with coordinates <math>(-1.6, -1.2)</math>, which can only be point C. <b>Options A and D are incorrect</b> because the multiplicative inverse cannot be obtained by reflecting <math>P</math> across the <math>y</math>-axis and the origin, respectively. <b>Option B is incorrect</b> because the inverse does not lie on the ray <math>OP</math>.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
4	003	A	<p><b>Option A is correct.</b> Olivia traveled at an average speed of 36 mph for 50 minutes, or <math>\frac{5}{6}</math> of an hour. This gives a total distance of <math>36\left(\frac{5}{6}\right) = 30</math> miles. She traveled 25 miles in the first 30 minutes, leaving only 5 miles in the last 20 minutes. A rate of 5 miles in 20 minutes is equivalent to a rate of 15 miles in an hour.</p> <p><b>Option B is incorrect</b> because if Olivia had traveled at a rate of 20 mph for the last 20 minutes, her average speed for the entire trip would have been <math>\frac{25 + \frac{20}{3}}{\frac{5}{6}} = 38</math> mph.</p> <p><b>Option C is incorrect</b> because if Olivia had traveled at a rate of 25 mph for the last 20 minutes, her average speed for the entire trip would have been <math>\frac{25 + \frac{25}{3}}{\frac{5}{6}} = 40</math> mph.</p> <p><b>Option D is incorrect</b> because if Olivia had traveled at a rate of 30 mph for the last 20 minutes, her average speed for the entire trip would have been <math>\frac{25 + \frac{30}{3}}{\frac{5}{6}} = 42</math> mph.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
5	003	B	<p><b>Option B is correct.</b> Because <math>0.00001 = 10^{-5} = 10 \times 10^{-6}</math>, <math>k</math> can be of the form <math>a \times 10^b</math> for <math>1 \leq a &lt; 10</math> and <math>b &lt; -1</math>. For example, if <math>k = 0.000002</math>, then <math>a = 2</math> and <math>b = -6</math>. <b>Option A is incorrect</b> because if <math>a</math> is negative, then the value of <math>a \times 10^b</math> will also be negative and thus cannot represent a distance. <b>Option C is incorrect</b> because if <math>1 \leq a &lt; 10</math> and <math>b &gt; 1</math>, then <math>a \times 10^b &gt; 10</math>. <b>Option D is incorrect</b> because if <math>\frac{1}{2} \leq a &lt; 1</math> and <math>b &gt; 1</math>, then <math>a \times 10^b &gt; 5</math>.</p> <p><a href="#">Back to Question</a></p>
6	004	A	<p><b>Option A is correct</b> because by the given formula, <math>a_3 = \frac{a_1}{a_2} = \frac{1}{3}</math> and</p> $a_4 = \frac{a_2}{a_3} = \frac{3}{\left(\frac{1}{3}\right)} = 9.$ <p><b>Option B is incorrect</b> because 7 is the result obtained by adding the two previous terms each time, instead of taking the quotient. <b>Option C is incorrect</b> because 1 is the result obtained by using <math>a_{n+3} = \frac{a_{n+2}}{a_{n+1}}</math>. <b>Option D is incorrect</b> because <math>\frac{1}{3}</math> is the value of <math>a_3</math> instead of <math>a_4</math>.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
7	004	D	<p><b>Option D is correct.</b> Because the terms of the sequence described are the consecutive integers starting at <math>-13</math>, the first 27 terms of the sequence, from <math>-13</math> to <math>13</math>, have a sum of 0. The next 3 terms, 14, 15 and 16, have a sum of 45, which is the given sum. So there are 30 terms in the sequence. <b>Option A is incorrect</b> because the first 27 terms in the sequence have a sum of 0. <b>Option B is incorrect</b> because the first 28 terms in the sequence have a sum of 14. <b>Option C is incorrect</b> because the first 29 terms in the sequence have a sum of 29.</p> <p><a href="#">Back to Question</a></p>
8	005	D	<p><b>Option D is correct.</b> The value of <math>f(2)</math> is a little greater than 1, and so is the value of <math>g(4)</math>. For the other options, the value of <math>g(x)</math> is not as close to the value of <math>f(2)</math> as is the value of <math>g(4)</math>. <b>Option A is incorrect</b> because <math>g(1)</math> is greater than 3. <b>Option B is incorrect</b> because <math>g(2)</math> is greater than 2. <b>Option C is incorrect</b> because <math>g(3)</math> is about 2.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
9	005	A	<p><b>Option A is correct</b> because if <math>f(x) = -x + \frac{1}{x}</math>, then</p> $f(-x) = -(-x) + \frac{1}{-x} = x - \frac{1}{x} \text{ and}$ $-f(x) = -\left(-x + \frac{1}{x}\right) = x - \frac{1}{x}. \text{ These two}$ <p>functions are equivalent. <b>Option B is incorrect</b> because</p> $f(-x) = -(-x) + \frac{1}{-x} = x - \frac{1}{x} \text{ is not}$ <p>equivalent to <math>f(x)</math>. <b>Option C is incorrect</b> because</p> $f\left(\frac{1}{x}\right) = -\left(\frac{1}{x}\right) + \frac{1}{\left(\frac{1}{x}\right)} = -\frac{1}{x} + x \text{ is not}$ <p>equivalent to <math>f(x)</math>. <b>Option D is incorrect</b> because</p> $f\left(\frac{1}{x}\right) = -\left(\frac{1}{x}\right) + \frac{1}{\left(\frac{1}{x}\right)} = -\frac{1}{x} + x \text{ is not}$ <p>equivalent to</p> $-\frac{1}{f(x)} = -\frac{1}{-x + \frac{1}{x}} = -\frac{1}{\frac{-x^2 + 1}{x}} = \frac{-x}{-x^2 + 1} = \frac{x}{x^2 - 1}.$ <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
10	006	C	<p><b>Option C is correct</b> because when a line is represented as an equation in the form <math>y = mx + b</math>, <math>m</math> represents the slope and <math>b</math> represents the y-intercept. The line containing the three points shown has a negative slope and a positive y-intercept, so <math>m &lt; 0</math> and <math>b &gt; 0</math>. <b>Option A is incorrect</b> because if <math>m &gt; 0</math> and <math>b &gt; 0</math>, the line would have a positive slope. <b>Option B is incorrect</b> because if <math>m &gt; 0</math> and <math>b &lt; 0</math>, the line would have a positive slope and a negative y-intercept. <b>Option D is incorrect</b> because if <math>m &lt; 0</math> and <math>b &lt; 0</math>, the line would have a negative y-intercept.</p> <p><a href="#">Back to Question</a></p>
11	006	D	<p><b>Option D is correct.</b> For a function to be one-to-one on an interval there must be exactly one x-value for each y-value. The graph of the function given is a parabola with vertex at <math>(a, b)</math>, where <math>0 &lt; a &lt; b</math>. On the interval <math>-b &lt; x &lt; a</math>, the parabola consists of points on the left side of the axis of symmetry. For these points, there is exactly one x-value for each y-value, so the function is one-to-one on this interval.</p> <p><b>Options A, B and C are incorrect</b> because the portion of the parabola on each of these intervals includes points on both sides of the axis of symmetry. This means that on each interval there are at least 2 points on the parabola with the same y-value but with different x-values.</p> <p><a href="#">Back to Question</a></p>



Question Number	Competency Number	Correct Answer	Rationales
12	007	B	<p><b>Option B is correct</b> because the fact that the graph intersects the <math>x</math>-axis in four places indicates that the function has 4 real zeros, and the fact that the graph has 5 local extrema indicates that the function has degree at least 6. <b>Option A is incorrect</b> because the graph cannot intersect the <math>x</math>-axis at more than the four places shown given the conditions on <math>f</math> for <math>x &lt; a</math> and <math>x &gt; b</math>. <b>Options C and D are incorrect</b> because the function must have degree at least 6.</p> <p><a href="#">Back to Question</a></p>
13	007	D	<p><b>Option D is correct</b> because as <math>x</math> approaches <math>\infty</math> or <math>-\infty</math>, the value of <math>\frac{1}{x-2}</math> approaches 0; therefore, the value of <math>y = x + 3 + \frac{1}{x-2}</math> approaches <math>y = x + 3</math>. <b>Options A and B are incorrect</b> because the only vertical asymptote of the graph occurs at <math>x = 2</math>. <b>Option C is incorrect</b> because <math>y = x - 2</math> is not an asymptote of the graph.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
14	008	D	<p><b>Option D is correct.</b> Based on the relationship shown in the graph, <math>\ln y = 2 - x</math>, so <math>y = e^{2-x}</math>. <b>Option A is incorrect</b> because if <math>y = e^x + 2</math>, then the relationship between <math>\ln y</math> and <math>x</math> would be <math>\ln y = \ln(e^x + 2)</math>, which is not represented on the graph. <b>Option B is incorrect</b> because if <math>y = 2e^x</math>, then the relationship between <math>\ln y</math> and <math>x</math> would be <math>\ln y = \ln(2e^x)</math>, which is not represented on the graph. <b>Option C is incorrect</b> because if <math>y = e^{x+2}</math>, then the relationship between <math>\ln y</math> and <math>x</math> would be <math>\ln y = x + 2</math>, which is not represented on the graph.</p> <p><a href="#">Back to Question</a></p>
15	008	A	<p><b>Option A is correct.</b> Based on the formula and information given, <math>2500 = 2000e^{0.025t}</math>. Solving for <math>t</math> yields <math>t = \frac{\ln 1.25}{0.025} \approx 8.9</math>, which to the nearest whole number is 9. <b>Options B, C and D are incorrect</b> because they are greater than the number of years it takes for the value of the account to reach \$2500.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
16	009	D	<p><b>Option D is correct</b> because when a vertical segment is drawn from point <math>P</math> to the <math>x</math>-axis, a right triangle is formed such that the vertical leg has length 0.6 and the horizontal leg has length 0.8. Thus, <math>\tan \theta = \frac{0.6}{0.8} = 0.75</math>, and <math>\theta = \tan^{-1} 0.75</math>. <b>Option A is incorrect</b> because in the right triangle described above, the length of the hypotenuse is <math>\sqrt{0.6^2 + 0.8^2} = 1</math>, so <math>\cos \theta = \frac{0.8}{1} = 0.8</math> and <math>\theta = \cos^{-1} 0.8</math>.</p> <p><b>Options B and C are incorrect</b> because in the right triangle described, <math>\sin \theta = \frac{0.6}{1} = 0.6</math> and <math>\theta = \sin^{-1} 0.6</math>.</p> <p><a href="#">Back to Question</a></p>
17	009	A	<p><b>Option A is correct</b> because one formula for the area of a triangle is <math>A = \frac{1}{2} ab \sin C</math>. (Note that <math>b</math> and <math>a \sin C</math> are the lengths of the base and corresponding altitude.) Applying this formula to the given figure yields <math>A = \frac{1}{2}(8)(15) \sin 42^\circ = 60 \sin 42^\circ</math>.</p> <p><b>Option B is incorrect</b> because cosine is used instead of sine.</p> <p><b>Option C is incorrect</b> because the <math>\frac{1}{2}</math> in the formula was not used.</p> <p><b>Option D is incorrect</b> because the <math>\frac{1}{2}</math> in the formula was not used and tangent was used instead of sine.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
18	010	B	<p><b>Option B is correct</b> because at <math>x = b</math> the limit of the function exists but does not equal the value of the function. <b>Option A is incorrect</b> because the function is continuous at <math>x = a</math>. <b>Option C is incorrect</b> because the limit of the function does not exist at <math>x = c</math>. <b>Option D is incorrect</b> because the function has a vertical asymptote at <math>x = d</math>.</p> <p><a href="#">Back to Question</a></p>
19	011	A	<p><b>Option A is correct</b> because the total area of the roof is <math>(2)(15)(60)</math> <math>\text{ft}^2</math>. The total cost of the shingles for the roof can be found with the following unit analysis:</p> $(2)(15)(60) \text{ ft}^2 \times \frac{1 \text{ square of shingles}}{100 \text{ ft}^2} \times \frac{3 \text{ bundles}}{1 \text{ square of shingles}} \times \frac{\$28.99}{1 \text{ bundle}}$ <p>So by canceling the units, the total cost is <math>\frac{(2)(15)(60)(3)(\\$28.99)}{100}</math>, which is equivalent to option A. <b>Options B, C and D are incorrect</b> because they are not equivalent to <math>\frac{(2)(15)(60)(3)(\\$28.99)}{100}</math>.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
20	011	D	<p><b>Option D is correct</b> because the circumference of the wheel is <math>9\pi</math> cm and each cog and space requires a total length of <math>\frac{\pi}{4}</math> cm. The number of cogs that will fit around the wheel with spaces in between is <math>\frac{9\pi}{\left(\frac{\pi}{4}\right)} = 36</math> cogs. <b>Option A is incorrect</b> because 12 cogs and spaces would require a circumference of only <math>3\pi</math> cm. <b>Option B is incorrect</b> because 18 cogs and spaces would require a circumference of only <math>4.5\pi</math> cm. <b>Option C is incorrect</b> because 24 cogs and spaces would require a circumference of only <math>6\pi</math> cm.</p> <p><a href="#">Back to Question</a></p>
21	012	C	<p><b>Option C is correct.</b> Based on the lower triangle, <math>x + 64 = 100</math>, so <math>x = 36</math>. Then, based on the upper triangle, <math>y + 36 + 100 = 180</math>, so <math>y = 44</math>. <b>Option A is incorrect</b> because 36 is the value of <math>x</math>, not <math>y</math>. <b>Option B is incorrect</b> because if <math>y = 40</math>, then <math>x = 40</math>. <b>Option D is incorrect</b> because if <math>y = 48</math>, then <math>x = 32</math>.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
22	013	A	<p><b>Option A is correct.</b> The level of the solution before immersion is the same as the height of the center of the wheel, which is equal to the radius of the wheel, 25 cm. The height of the solution above the center of the wheel can be found by connecting the center of the wheel to the midpoint and to one endpoint of the chord, forming a right triangle with hypotenuse of length 25 cm and one leg of length 24 cm. The length of the third leg can be found to be 7 cm by the Pythagorean theorem and is equal to the height of the solution above the center of the wheel. So the total height of the water after immersion is <math>25 + 7 = 32</math> cm.</p> <p><b>Options B, C and D are incorrect</b> because the level of the solution after immersion has been shown to be 32 cm.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
23	013	A	<p><b>Option A is correct.</b> Angle <math>ACB</math> must be complementary to both <math>\angle DCE</math> and <math>\angle BAC</math>, so <math>\angle BAC \cong \angle DCE</math>, and <math>\triangle ABC</math> is similar to <math>\triangle CDE</math> by the AA similarity criterion. Because the triangles are similar, <math>\frac{AB}{BC} = \frac{CD}{DE}</math>. By applying the Pythagorean theorem to <math>\triangle ABC</math>, <math>BC = 6</math>. Then <math>CD = 24</math>, because <math>BD = 30</math> and <math>BC = 6</math>. Substituting the known lengths into the proportion yields <math>\frac{8}{6} = \frac{24}{DE}</math>, which can be solved to show <math>DE = 18</math>.</p> <p><b>Option B is incorrect</b> because doubling the length of segment <math>\overline{AC}</math> does not equal 18. <b>Option C is incorrect</b> because 24 is the length of segment <math>\overline{CD}</math>, not the length of segment <math>\overline{DE}</math>. <b>Option D is incorrect</b> because <math>32 = AB + CD</math>, which is much greater than the length of segment <math>\overline{DE}</math>.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
24	014	A	<p><b>Option A is correct</b> because multiplying the left side of the given matrix equation gives <math>\begin{pmatrix} x \\ x \end{pmatrix} = \begin{pmatrix} x' \\ y' \end{pmatrix}</math>. This corresponds to the transformation of a point <math>(x, y)</math> to the point <math>(x, x)</math>, as shown in option A. <b>Option B is incorrect</b> because the graph corresponds to the transformation of a point <math>(x, y)</math> to the point <math>(x, -y)</math>. <b>Option C is incorrect</b> because the graph corresponds to the transformation of a point <math>(x, y)</math> to the point <math>(x, 0)</math>. <b>Option D is incorrect</b> because the graph corresponds to the transformation of a point <math>(x, y)</math> to the point <math>(-x, y)</math>.</p> <p><a href="#">Back to Question</a></p>
25	015	B	<p><b>Option B is correct</b> because the first quartile is 400 and the third quartile is 480, so the interquartile range is 80. By the definition given, any data point that is greater than <math>1.5(80) = 120</math> above the third quartile or below the first quartile is considered an outlier. So any data point greater than 600 or less than 280 is an outlier. Thus, 275 is an outlier and 595 is not. <b>Option A is incorrect</b> because 275 is an outlier. <b>Options C and D are incorrect</b> because 595 is not an outlier.</p> <p><a href="#">Back to Question</a></p>



Question Number	Competency Number	Correct Answer	Rationales
26	016	C	<p><b>Option C is correct.</b> If 45% of the employees are women, then 55% of the employees are men. So in a random sample of 20 employees, the expected number of men is <math>0.55(20) = 11</math>. <b>Options A, B and D are incorrect</b> because it has been shown that the expected number of men must be 11.</p> <p><a href="#">Back to Question</a></p>
27	017	C	<p><b>Option C is correct</b> because the curve for battery <i>X</i> is steeper and less spread out than the curve for battery <i>Y</i>, indicating that the standard deviation for battery <i>X</i> is less than that for battery <i>Y</i>. <b>Options A and B are incorrect</b> because both curves peak at the same value, indicating the same mean. <b>Option D is incorrect</b> because the standard deviation for battery <i>X</i> is less than that for battery <i>Y</i>.</p> <p><a href="#">Back to Question</a></p>
28	017	D	<p><b>Option D is correct</b> because the placebo effect refers to a perceived or actual improvement by the group receiving the placebo compared to the group receiving no treatment. <b>Options A and B are incorrect</b> because each compares the group receiving the placebo to the group receiving the treatment, not to the group receiving no treatment. <b>Option C is incorrect</b> because the placebo effect should show an improvement in the group receiving the placebo.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
29	018	D	<p><b>Option D is correct</b> because the contrapositive of the given statement is "If <math>x</math> is not even, then <math>x^2</math> is not even." This statement can be proven by assuming that <math>x</math> is not even and deducing that <math>x^2</math> is not even.</p> <p><b>Option A is incorrect</b> because it describes a method for proving the original statement, but it does not describe the contrapositive. <b>Option B is incorrect</b> because it describes a method for proving the inverse of the original statement. <b>Option C is incorrect</b> because it does not describe the contrapositive.</p> <p><a href="#">Back to Question</a></p>
30	019	A	<p><b>Option A is correct</b> because if hose <math>A</math> can fill the empty pool in <math>x</math> hours, then hose <math>B</math> can fill the empty pool in <math>2x</math> hours. The fractions of the pool that hoses <math>A</math> and <math>B</math> can each fill in 1 hour are <math>\frac{1}{x}</math> and <math>\frac{1}{2x}</math>, respectively. Working together, it takes the two hoses 10 hours to fill the empty pool, so <math>\frac{1}{10}</math> of the pool can be filled in 1 hour. Thus, <math>\frac{1}{x} + \frac{1}{2x} = \frac{1}{10}</math>. <b>Option B is incorrect</b> because it takes <math>2x</math> hours for hose <math>B</math> to fill the pool, not <math>\frac{x}{2}</math> hours. <b>Option C is incorrect</b> because the combined hourly rate equals <math>\frac{1}{10}</math> not 10. <b>Option D is incorrect</b> because the total combined time is not equal to the sum of the individual times.</p> <p><a href="#">Back to Question</a></p>

Question Number	Competency Number	Correct Answer	Rationales
31	020	A	<p><b>Option A is correct</b> because inquiry-based learning refers to the practice of allowing students to explore an idea or question on their own. In the described activity, the students use their calculators to explore the effect on the graph of changing the value of <math>a</math>. <b>Options B, C and D are incorrect</b> because they do not describe an activity in which students explore an idea or question on their own.</p> <p><a href="#">Back to Question</a></p>
32	021	D	<p><b>Option D is correct</b> because the student multiplied only the first term in the parenthesis by <math>-\frac{1}{2}</math>, thus making a mistake in the use of the distributive property. <b>Options A and B are incorrect</b> because the student multiplied <math>-\frac{1}{2}</math> by <math>-\frac{2}{3}</math> correctly.</p> <p><b>Option C is incorrect</b> because the work does not show an error in the application of the associative property of multiplication.</p> <p><a href="#">Back to Question</a></p>

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## Study Plan Sheet

STUDY PLAN						
Content covered on test	How well do I know the content?	What material do I have for studying this content?	What material do I need for studying this content?	Where can I find the materials I need?	Dates planned for study of content	Date Completed

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## Preparation Resources

The resources listed below may help you prepare for the TExES test in this field. These preparation resources have been identified by content experts in the field to provide up-to-date information that relates to the field in general. You may wish to use current issues or editions to obtain information on specific topics for study and review.

### JOURNALS

*American Mathematical Monthly*, Mathematical Association of America.

*Journal for Research in Mathematics Education*, National Council of Teachers of Mathematics.

*Mathematics Magazine*, Mathematical Association of America.

*Mathematics Teacher*, National Council of Teachers of Mathematics.

### OTHER RESOURCES

Bittenger, M. L., and Ellenbogen, D. (2005). *Elementary Algebra: Concepts and Applications*, Seventh Edition. Menlo Park, Calif.: Addison-Wesley.

Bock, D., Velleman, P., and De Veaux, R. (2009). *Stats: Modeling the World*, Third Edition. Boston, Mass.: Pearson Education, Inc.

Brahier, D. J. (2009). *Teaching Secondary and Middle School Mathematics*, Fourth Edition. Needham Heights, Mass.: Allyn & Bacon.

Brumbaugh, D. K., and Rock, D. (2006). *Teaching Secondary Mathematics*, Third Edition. Mahwah, N.J.: Lawrence Erlbaum Associates.

Burger, E., and Starbird, M. (2005). *The Heart of Mathematics: An Invitation to Effective Thinking*, Second Edition. Emeryville, Calif.: Key Curriculum Press.

COMAP (2008). *For All Practical Purposes: Mathematical Literacy in Today's World*, Eighth Edition. New York, N.Y.: W. H. Freeman and Company.

Connally, E., Hughes-Hallett, D., Gleason, A., et al. (2007). *Functions Modeling Change: A Preparation for Calculus*, Third Edition. Hoboken, N.J.: John Wiley & Sons, Inc.

Demana, F., Waits, B. K., Foley, G. D., and Kennedy D. (2010). *Precalculus: Graphical, Numerical, Algebraic*, Eighth Edition. Pearson.

Foerster, P. A. (2005). *Calculus Concepts and Applications*, Second Edition. Berkeley, Calif.: Key Curriculum Press.

Greenburg, M. (2008). *Euclidean and Non-Euclidean Geometries: Development and History*, Fourth Edition. New York, N.Y.: W. H. Freeman and Company.

Hughes-Hallett, D., Gleason, A., McCallum, W., et al. (2008). *Calculus: Single Variable*, Fifth Edition. Hoboken, N.J.: John Wiley & Sons, Inc.

Hungerford, T. W. (2004). *Contemporary College Algebra and Trigonometry: A Graphing Approach*, Second Edition. Philadelphia, Pa.: Harcourt College Publishers.

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- Johnson, K., Herr, T., Kysh, J. (2004). *Problem Solving Strategies: Crossing the River with Dogs*, Third Edition. Emeryville, Calif.: Key College Publishing.
- Kilpatrick, J., Swafford, J., and Finell, B. (Eds.) (2001). *Adding It Up: Helping Children Learn Mathematics*. Washington, D.C.: National Academy Press.
- Kinsey, L., and Moore, T. (2002). *Symmetry, Shape, and Space: An Introduction to Mathematics Through Geometry*. Emeryville, Calif.: Key College Publishing.
- Larson, R., Hostetler, R., and Edwards, B. (2005). *Calculus of a Single Variable*, Eighth Edition. Boston, Mass.: Houghton Mifflin Harcourt.
- Lay, S. (2005). *Analysis: With an Introduction to Proof*, Fourth Edition. Upper Saddle River, N.J.: Pearson Education, Inc.
- National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*. Reston, Va.: The National Council of Teachers of Mathematics, Inc.
- National Council of Teachers of Mathematics. (2009). *Focus in High School Mathematics: Reasoning and Sense Making*. Reston, Va.: The National Council of Teachers of Mathematics, Inc.
- Northey, S. (2005). *Handbook on Differentiated Instruction for Middle and High Schools*. Larchmont, N.Y.: Eye on Education.
- Robbins, N. (2006). *Beginning Number Theory*, Second Edition. Sudbury, Mass.: Jones and Bartlett Publishers.
- Rosen, K. (2006). *Discrete Mathematics and Its Applications*, Sixth Edition. Boston, Mass.: McGraw-Hill Higher Education.
- Serra, M. (2007). *Discovering Geometry: An Investigative Approach*, Fourth Edition. Emeryville, Calif.: Key Curriculum Press.
- Shaughnessy, J., Chance, B., and Kranendonk, H. (2009). *Focus in High School Mathematics: Reasoning and Sense Making in Statistics and Probability*. Reston, Va.: The National Council of Teachers of Mathematics, Inc.
- Strang, G. (2005). *Linear Algebra and Its Applications*, Fourth Edition. Pacific Grove, Calif.: Brooks Cole Publishers.
- Texas Education Agency. (2012). *Texas Essential Knowledge and Skills (TEKS)*.
- Triola, M. F. (2008). *Elementary Statistics*, Eighth Edition. Boston, Mass.: Addison Wesley Longman, Inc.
- Usiskin, Z., Peressini, A., Marchisotto, E., and Stanley, D. (2003). *Mathematics for High School Teachers: An Advanced Perspective*. Upper Saddle River, N.J.: Pearson Education, Inc.
- Williams, G. (2004). *Applied College Algebra: A Graphing Approach*, Second Edition. Philadelphia, Pa.: Harcourt College Publishers.
- Wright, D. (1999). *Introduction to Linear Algebra*. Boston, Mass.: WCB McGraw-Hill.

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**ONLINE RESOURCES**

American Mathematical Society — [www.ams.org](http://www.ams.org)

American Statistical Association — [www.amstat.org](http://www.amstat.org)

Association for Women in Mathematics — [www.awm-math.org](http://www.awm-math.org)

Internet4Classrooms — [www.internet4classrooms.com](http://www.internet4classrooms.com)

The Mathematical Association of America — [www.maa.org](http://www.maa.org)

National Association of Mathematicians — [www.nam-math.org](http://www.nam-math.org)

National Council of Teachers of Mathematics — [www.nctm.org](http://www.nctm.org)

Texas Council of Teachers of Mathematics — [www.tctmonline.org](http://www.tctmonline.org)

Texas Section of the MAA — <http://sections.maa.org/texas>