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# Fuzzy Modeling for Control



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# FUZZY MODELING FOR CONTROL

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to Dana, Michaela, and *Markéta*



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## Preface

Since its introduction in 1965, fuzzy set theory has found applications in a wide variety of disciplines. Modeling and control of dynamic systems belong to the fields in which fuzzy set techniques have received considerable attention, not only from the scientific community but also from industry. Many systems are not amenable to conventional modeling approaches due to the lack of precise, formal knowledge about the system, due to strongly nonlinear behavior, due to the high degree of uncertainty, or due to the time varying characteristics. Fuzzy modeling along with other related techniques such as neural networks have been recognized as powerful tools which can facilitate the effective development of models. One of the reasons for this is the capability of fuzzy systems to integrate information from different sources, such as physical laws, empirical models, or measurements and heuristics.

Fuzzy models can be seen as logical models which use “if-then” rules to establish qualitative relationships among the variables in the model. Fuzzy sets serve as a smooth interface between the qualitative variables involved in the rules and the numerical data at the inputs and outputs of the model. The rule-based nature of fuzzy models allows the use of information expressed in the form of natural language statements and consequently makes the models transparent to interpretation and analysis. At the computational level, fuzzy models can be regarded as flexible mathematical structures, similar to neural networks, that can approximate a large class of complex nonlinear systems to a desired degree of accuracy.

Recently, a great deal of research activity has focused on the development of methods to build or update fuzzy models from numerical data. Most approaches are based on neuro-fuzzy systems, which exploit the functional similarity between fuzzy reasoning systems and neural networks. This “marriage” of fuzzy systems and neural networks enables a more effective use of optimization techniques for building fuzzy systems, especially with regard to their approximation accuracy. However, the aspects related to the transparency and interpretation tend to receive considerably less attention. Consequently, most neuro-fuzzy models can be regarded as black-box models which provide little insight to help understand the underlying process.

The approach adopted in this book aims at the development of transparent rule-based fuzzy models which can accurately predict the quantities of interest, and at the

same time provide insight into the system that generated the data. Attention is paid to the selection of appropriate model structures in terms of the dynamic properties, as well as the internal structure of the fuzzy rules (linguistic, relational, or Takagi–Sugeno type). From the system identification point of view, a fuzzy model is regarded as a composition of local submodels. Fuzzy sets naturally provide smooth transitions between the submodels, and enable the integration of various types of knowledge within a common framework.

In order to automatically generate fuzzy models from measurements, a comprehensive methodology is developed. It employs fuzzy clustering techniques to partition the available data into subsets characterized by a linear behavior. The relationships between the presented identification method and linear regression are exploited, allowing for the combination of fuzzy logic techniques with standard system identification tools. Attention is paid to the aspects of accuracy and transparency of the obtained fuzzy models.

Using the concepts of model-based predictive control and internal model control with an inverted fuzzy model, the control design based on a fuzzy model of a nonlinear dynamic process is addressed. To this end, methods which exactly invert specific types of fuzzy models are presented. In the context of predictive control, branch-and-bound optimization is applied. Attention is paid to algorithmic solutions of the control problem, mainly with regard to real-time control aspects.

The orientation of the book is towards methodologies that in the author's experience proved to be practically useful. The presentation reflects theoretical and practical issues in a balanced way, aiming at readership from the academic world and also from industrial practice. Simulation examples are given throughout the text and three selected real-world applications are presented in detail. In addition, an implementation in a MATLAB toolbox of the techniques presented is described. This toolbox can be obtained from the author.

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