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Optimal solution of full fuzzy transportation problems using total integral ranking

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Optimal solution of full fuzzy transportation problems using total integral ranking

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Abstract. Full fuzzy transportation problem (FFTP) is a transportation problem where transport costs, demand, supply and decision variables are expressed in form of fuzzy numbers. To solve fuzzy transportation problem, fuzzy number parameter must be converted to a crisp number called defuzzyfication method. In this new total integral ranking method with fuzzy numbers from conversion of trapezoidal fuzzy numbers to hexagonal fuzzy numbers obtained result of consistency defuzzyfication on symmetrical fuzzy hexagonal and non symmetrical type 2 numbers with fuzzy triangular numbers. To calculate of optimum solution FTP used fuzzy transportation algorithm with least cost method. From this optimum solution, it is found that use of fuzzy number form total integral ranking with index of optimism gives different optimum value. In addition, total integral ranking value using hexagonal fuzzy numbers has an optimal value better than the total integral ranking value using trapezoidal fuzzy numbers.

1. Introduction

Application of transportation model in real life is used in distributing a product from supplier to demand. The problem to be solved by transportation model is determination of item distribution of that will minimize distribution total cost with item quantity to be produced does not exceed limit capacity of production and still meet demand minimum number on each destination. Some parameters can be used in the transport model, ie transportation costs, demand and inventory value (both product and supply capacity). There are transportation problems, namely value unpredictability of transportation cost coefficient, inventory amount and demand number caused by uncontrollable factors of erorr situations in real life. In addition, in some cases supply amount is not equal to demand number. Therefore, to handle these uncertainties in making decisions, Bellman and Zadeh introduce fuzziness concept. The fuzzy transportation problem (FTP) is transportation problem where transportation cost, supply quantity and demand quantity that is fuzzyness [1].

The purpose of FTP is determine delivery schedules that minimize total fuzzy costs while still meeting inventory quantities and fuzzy supply limits [2]. To solve FTP, fuzzy number parameter previously, must be converted to a number parameter in crisp number or called defuzzyfication. Total integral ranking is one of defuzzyfication method with a convex combination based on right and left integral values through an optimism index [3]. The left integral value is used to reflect the pessimistic viewpoint and the right integral value is used to reflect the optimistic viewpoint of the decision maker [3]. Previous discussion of the fuzzy number defuzzyfication method has been found that total integral ranking is superior to previously proposed defuzzyfication methods, such as Gupta and Kumar [4], Pandian and Natarajan [5], Hatami-Marbini and Tavana [6], Chakraborty and Chakraborty [7], Basirzadeh [8], Saati et al. [9] and Kumar et al. [10]. Ebrahimnejad [3] have used trapezoidal numbers for defuzzyfication of fuzzy numbers using total integral rankings with results indicating consistency in the ranking between the two fuzzy numbers. Therefore in this case it will discuss to use of total integral ranking with other fuzzy number.

Base on graphic membership function of trapezoidal numbers, can still be done addition of point parameters in the interval fuzzy number. Therefore, in this peper shows the use of new fuzzy numbers on total integral ranking by adding parameter point of trapezoidal fuzzy number which has 4 parameter points to 6 point parameters known as hexagonal fuzzy number. In the use of a total integral ranking with hexagonal fuzzy numbers on FTP is expected to produce better optimum value fuzzy triangular and trapezoid numbers. Also in this paper, to determine the optimal solution of FTP using fuzzy transportation algorithm least cost method.

2. Preliminaries

This section presents the total integral ranking.

2.1. Total Integral Ranking

Difinition 2 [3]. A is a fuzzy number with left membership function f_A^L and right membership function f_A^R . Suppose that g_A^L is the inverse function of f_A^L and g_A^R is the inverse function of f_A^R, then the left integral value of and the right integral value of A, respectively defined as,

$$I_L(A) = \int_0^1 g_A^L(y)dy \text{ and } I_R(A) = \int_0^1 g_A^R(y)dy.$$

Difinition 3 [3]. If A is fuzzy number with membership function f_A, difined as in (Eq. 1), then the total integral ranking value with index of optimism α is difined as,

$$I_T^\alpha(A) = \alpha I_R(A) + (1-\alpha) I_L(A) \quad (2)$$

where $I_R(A)$ and $I_L(A)$ are the right and left integral value of A, respectively, and $\alpha \in [0,1]$.

3. The Total Integral Rangking Value of Triangular, Trapezoidal and Hexagonal Fuzzy Number

Difinition 7 [8]. The fuzzy number is a hexagonal fuzzy number, if its membership function f_A is given by,

$$f_A = \begin{cases} \frac{1}{2}\left(\frac{x-a}{b-a}\right), & a \le x \le b \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-b}{c-b}\right), & b \le x \le c \\ 1, & c \le x \le d \\ 1 - \frac{1}{2}\left(\frac{x-d}{e-d}\right), & d \le x \le e \\ \frac{1}{2}\left(\frac{f-x}{f-e}\right), & e \le x \le f \end{cases}$$

Where $a \le b \le c \le d \le e \le f$ is real number which satisfy $b-a \le c-a$ and $e-d \le f-e$.

Suppose on introduction that has been described above that in this discussion using hexagonal fuzzy number. The hexagonal fuzzy number is generated by adding two parameter points obtained from middle value between fuzzy numbers a and b along fuzzy number c and e. The graph of membership functions can be illustrated in Figure 1.

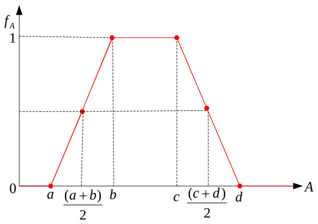


Figure 1. Conversion of trapezoidal fuzzy numbers to hexagonal

Therefore from the concept, then suppose on definitions 6 and 7 produced the following remark.
Remark 1. Fuzzy trapezoidal number A denoted by $(a,b,c,d;1)$ to be fuzzy hexagonal number denoted by $(a,(a+b)/2,b,c,(c+d)/2,d;1)$ with membership function f_A expressed as follows:

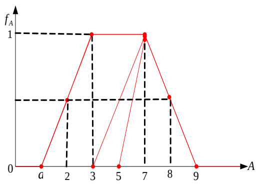
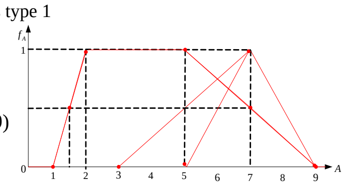
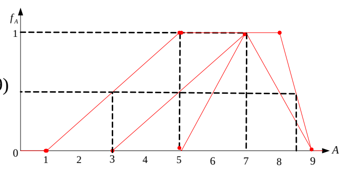
$$f_A = \begin{cases} \frac{1}{4}\left(\frac{x-a}{b-a}\right), & a \le x \le (a+b)/2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{2x-(a+b)}{b-a}\right), & (a+b)/2 \le x \le b \\ 1, & b \le x \le c \\ 1-\frac{1}{4}\left(\frac{x-c}{d-c}\right), & c \le x \le (c+d)/2 \\ \frac{1}{4}\left(\frac{d-x}{d-c}\right), & (c+d)/2 \le x \le d \end{cases}$$

fuzzy hexagonal number A denoted by $(a,(a+b)/2,b,c,(c+d)/2,d;1)$ then $f_A^L(x)=(x-a)/(b-a)+1/2+1(2x-(b+a)/2(b-a)$ and $f_A^R(x)=1-(x-d)/4(d-c)+(d-x)/(d-c)$, so invers function from f_A^L and f_A^R can be expressed by $g_A^L(y)=4y(b-a)+a+1/2[(2y-1)(b-a)+(a+b)]$ and $g_A^R(y)=(4y+4)(d-c)+d+d-4y(d-c)$, where $y\in[0,1]$. so $I_L(A)=7b+a/8$ and $I_R(A)=4(d-c)$. Thus based on (2) with $\alpha\in[0,1]$, the total integral ranking value of fuzzy hexagonal numbers denoted by $A=(a,(a+b)/2,b,c,(c+d)/2,d;1)$ is $I_T^\alpha(A)=1/8[38\alpha(d-c)+(1-\alpha)(7b+a)]$ (5)

4. Comparison Consistency Defuzzyfication on Total Integral Ranking Based on Fuzzy Number Numbers Type

Defuzzyfication of fuzzy numbers using total Integral Ranking with hexagonal fuzzy number has a different consistency with fuzzy triangular number when viewed based on fuzzy number type. Therefore, the following consistency comparison of defuzzyfication based on hexagonal type is symmetrical, non symmetrical type 1 and non symmetrical type 2 with fuzzy triangular number showed in Table 1.

Table 1. The total integral rank value

Fuzzy number of type	Optimism index		
	$\alpha=0$	$\alpha=0.5$	$\alpha=1$
Symetris $A_1=(5,7,9)$ $A_2=(3,7,9)$ $A_3=(1,3,7,9)$ 	$A_1=6$ $A_2=5$ $A_3=2.75$	$A_1=7$ $A_2=6.5$ $A_3=5.37$	$A_1=8$ $A_2=8$ $A_3=8$
Non symetris type 1 $A_1=(5,7,9)$ $A_2=(3,7,9)$ $A_3=(1,2,5,9)$ 	$A_1=6$ $A_2=5$ $A_3=1.87$	$A_1=7$ $A_2=6.5$ $A_3=8.93$	$A_1=8$ $A_2=8$ $A_3=16$
Non symetris type 2 $A_1=(5,7,9)$ $A_2=(3,7,9)$ $A_3=(1,5,8,9)$ 	$A_1=6$ $A_2=5$ $A_3=4.5$	$A_1=7$ $A_2=6.5$ $A_3=4.25$	$A_1=8$ $A_2=8$ $A_3=8$

From Table 1 it can be seen that hexagonal fuzzy number type of symmetrical and non symmetrical type 2 shows the same consistency of defuzzyfication with optimism index value that is ,

$\alpha=0$

$\alpha = 0.5$ and $\alpha = 1$. This is in according with discussions previously made by Ebrahimnejad [3] who use fuzzy trapezoidal numbers. As for comparison of defuzzyfication consistency can be seen in Table 2. From the table shows that fuzzy hexagonal type of symmetrical and non-symmetrical type 2 can be used for defuzzyfication process in solving fuzzy transportation problems that shown in Table 2.

Table 2. Comparison of defuzzyfication consistency with total integral ranking

Fuzzy number of type	New Total Integral ranking (Eq.5)			Ebrahimnejad [3]		
	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$	$\alpha = 0$	$\alpha = 0.5$	$\alpha = 1$
Symetris	$A_1 > A_2 > A_3$	$A_1 > A_2 > A_3$	$A_1 = A_2 = A_3$	$A_1 > A_2 > A_3$	$A_1 > A_2 > A_3$	$A_1 > A_2 > A_3$
Non symetris type 1	$A_1 > A_2 > A_3$	$A_3 > A_1 > A_2$	$A_1 = A_2 = 1/2\, A_3$	$A_1 > A_2 > A_3$	$A_1 > A_2 > A_3$	$A_1 > A_2 > A_3$
Non symetris type 2	$A_1 > A_2 > A_3$	$A_1 > A_2 > A_3$	$A_1 = A_2 = A_3$	$A_1 > A_2 > A_3$	$A_1 > A_2 > A_3$	$A_1 > A_2 > A_3$

5. Fuzzy Transportation problem

On the transportation problem of optimization theory, parameters S_i, D_j, C_{ij} expressed in the form crisp number. However, if parameters S_i, D_j, C_{ij} and x_{ij} expressed in form of fuzzy numbers $\tilde{S}_i, \tilde{D}_j, \tilde{C}_{ij}$ and $\tilde{X}_{ij}$ Then the problem of crisp transportation becomes FTP which can be formulated as follows:

$$\text{min: } \tilde{z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{C}_{ij}\tilde{x}_{ij} \quad (6)$$

subject to:
$$\sum_{j=1}^{n} \tilde{X}_{ij} \le \tilde{S}_i, \quad i = 1,2,\ldots,m \quad (7)$$

$$\sum_{i=1}^{m} \tilde{X}_{ij} \le \tilde{D}_j, \quad i = 1,2,\ldots,n \quad (8)$$

$$\tilde{X}_{ij} \ge 0, \quad \forall i, j .$$

where:
$$\tilde{X}_{ij} = (x_{1ij}, x_{2ij}, \ldots, x_{pij}), \tilde{C}_{ij} = (c_{1ij}, c_{2ij}, \ldots, c_{pij}), \tilde{S}_i = (s_{1i}, s_{2i}, \ldots, s_{pi}), \tilde{D}_j = (d_{1j}, d_{2j}, \ldots, d_{pj}), p = 1,2,\ldots,k$$

It will be given about the existence of Fuzzy Transportation Algorithm. Given a FTP at (6) - (8) denoted by FTP 1. Next, fuzzy transportation problem is denoted by FTP 2 as follows:

$$\text{min: } \tilde{z} = \sum_{i=1}^{m}\sum_{j=1}^{n} I_T^{\alpha}(\tilde{c}_{ij})\tilde{x}_{ij},$$

subject to:
$$\sum_{j=1}^{n} \tilde{X}_{ij} \le \tilde{S}_i, \quad i = 1,2,\ldots,m$$
$$\sum_{i=1}^{m} \tilde{X}_{ij} \le \tilde{D}_j, \quad i = 1,2,\ldots,n$$
$$\tilde{X}_{ij} \ge 0, \quad \forall i, j .$$

Existence of Fuzzy Transportation Algorithm can be shown by the following two theorems.

Theorem 1. If $(\tilde{x}_{11}^*, \tilde{x}_{12}^*, \ldots, \tilde{x}_{ij}^*, \ldots, \tilde{x}_{mn}^*)$ is a visible solution of FTP 2 and $\lambda_{ij} = X_{ij} - (v_i + w_j) \ge 0, \forall i, j$ then $(\tilde{x}_{11}^*, \tilde{x}_{12}^*, \ldots, \tilde{x}_{ij}^*, \ldots, \tilde{x}_{mn}^*)$ is optimal solution of FTP 2 with $X_{ij} = I_T^{\alpha}(\tilde{C}_{ij})$, v_i and w_j is real number

Theorem 4 explains that any FTP 2 compatible solution with $\lambda_{ij} \ge 0, \forall i, j$ is optimal solution of FTP 2. This theorem ensures that determination of optimal criteria in Step 6 in Fuzzy Transportation Algorithm is correct. Furthermore, the optimal solution obtained in Step 6 is an optimal solution of FTP 1 that can be assured by following theorem.

Theorem 2. If $(\tilde{x}_{11}^*, \tilde{x}_{12}^*, \ldots, \tilde{x}_{ij}^*, \ldots, \tilde{x}_{mn}^*)$ is optimal solution of FTP 2 then $(\tilde{x}_{11}^*, \tilde{x}_{12}^*, \ldots, \tilde{x}_{ij}^*, \ldots, \tilde{x}_{mn}^*)$ is optimal solution of FTP 1.

FTP 2 is more easily resolved than FTP 1 because FTP 2 is a transportation problem with cost transport shaped crisp transportation. Therefore, in Fuzzy Transportation Algorithm to determine optimal solution of FTP 1, first FTP 1 is changed to FTP 2, then FTP 2 is determined the optimal solution. By using Theorem 5 it is found that optimal solution of FTP 2 is optimal solution from FTP

6. Numerical Example

A motorcycle company has three factories S_1, S_2, S_3 which every week doing production activity with capacity of factory respectively that is $(40,50,60,70)$, $(80,90,60,70)$ and $(80,100,120,140)$ motorcyle unit. The company supplies motorcyle to showrooms located three destinations D_1, D_2, D_3 which has successively supply i.e $(30,40,50,60)$, $(90,95,100,105)$ and $(55,65,75,85)$ motorcycle unit. Transportation costs per motor unit are presented in Table 3. Determine minimum transportation costs of motor shipment from factory to showroom.

Table 3. Full fuzzy transportation

Company	Destination			s_i
	D1	D2	D3	
S1	(6,8,14,16)	(8,9,10,11)	(5,7,11,13)	(40,50,60,70)
S2	(1,3,7,9)	(2,4,12,14)	(7,9,1,13)	(80,90,60,70)
S3	(16,18,22,24)	(6,7,8,9)	(6,7,8,9)	(80,100,120,140)
d_j	(30,40,50,60)	(90,95,100,105)	(55,65,75,85)	

Solution: Supply quantity $\sum_{i=1}^{4} \tilde{S}_i = (175,200,225,250)$ and demand quantity $\sum_{j=1}^{4} \tilde{D}_j = (200,240,280,320)$.
So $\sum_{i=1}^{m} \tilde{S}_i \neq \sum_{j=1}^{n} \tilde{D}_j$. Therefore, added demand of fictitious $\tilde{D}_4 = (25,40,55,70)$.
Next step, defuzzyfication using the total integral ranking of formula (Eq.5) and obtained in Table 4.

Table 4. Fuzzy transportation of defuzzyfication

Company	Destination			
	D1	D2	D3	D4
S1	8	6	7	0
S2	5	6	8	0
S3	13	5	5	0

Then using fuzzy transportation algorithm from Sudhagar and Ganesan [2] with least cost method, obtained table visible solution shown in Table 6. Because $\lambda_{ij} \geq 0, \quad \forall i, j$. Then Table 5 shows the visible solution of it, then it determines value of fuzzy objective and compares the fuzzy objective value with the form of trapezoidal fuzzy number presented in Table 7. So $Z = (875,1280,2195,2820), I_T^{0.5}(Z) = 1865$

Table 5. Visible solution of fuzzy transportation

Company	Destination			
	D1	D2	D3	D4
S1	(15,20,25,30)	(0,0,0,0)	(0,0,0,0)	(25,30,35,40)
S2	(15,20,25,30)	(35,40,45,50)	(30,30,30,30)	(0,0,0,0)
S3	(0,0,0,0)	(55,55,55,55)	(25,35,45,55)	(0,0,0,0)

Table 6. Comparison of total integral ranking value fuzzy number

Fuzzy number	$I_T^{0.5}(Z)$	Rank
Trapezoidal (Eq.4)	179	2
Hexagonal (Eq.5)	1865	1

Based on Table 6 it can be seen that use of fuzzy numbers obtained from conversion of trapezoidal fuzzy numbers to hexagonal fuzzy numbers gives different optimum values on the total integral rranking value with optimism index $\alpha = 0.5$. Furthermore, given the problem of crisp transport shown in Table 8 with the optimal value that is $Z = 2315$

Table 7. Comparison of optimal value of crisp transportation and fuzzy transportation

Transportation	Fuzzy number	Optimal Value
Fuzzy	Trapezoidal (Eq.4)	(875,1280,2195,2820)=1792
	Hexagonal (Eq.5)	(875,1280,2195,2820)=1865
Crisp	-	2315

Table 8. Balance of crips transportation

Company	Destination				s_i
	D1	D2	D3	D4	
S1	14	10	11	0	60
S2	7	12	11	0	60
S3	22	8	8	0	120
d_j	50	100	75	15	

Based on Table 8 it is seen that transportation problem with the total integral ranking value by using hexagonal fuzzy number (Eq. 5) has optimal value which is closer than optimal value of crisp transportation problem compared to the total integral ranking value by using trapezoidal fuzzy number (Eq. 4). Thus, the total integral ranking value using the hexagonal fuzzy number has an optimal value better than the total integral ranking value using trapezoidal fuzzy number.

7. Conclusion

This paper provides the latest form of fuzzy membership function derived from conversion of trapezoidal fuzzy numbers to fuzzy hexagonal numbers that can be used for defuzzyfication using the total integral ranking method. In addition, in the case example a symmetric fuzzy number is generated which results in an optimal solution different from the total integral ranking value using trapezoidal fuzzy numbers. Result of the total integral ranking that uses hexagonal fuzzy number has an optimal value better than the total integral ranking value using trapezoidal fuzzy number.

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