Course: Inverse Problems in signal and image processing: Statistical Regularization, Deconvolution and Segmentation

This course will incorporate both the fundamentals of statistical regularization and introduce the utilization of methods for edge detection from both spatial and Fourier data. An objective of the course is enhancement of the mathematical understanding of the consequences of modern data collection strategies used in magnetic resonance imaging (MRI) with respect to generating high fidelity images. Examples for restoring images and signals from other modalities are also relevant.

The course topics will include:

1. Some basics of numerical linear algebra, singular value decomposition, generalized singular value decomposition. Basic iterative methods (LSQR) for solving the least squares problem.
2. The mathematical model of spatially invariant blur
3. Solution of the ill-conditioned systems of equations - regularization.
4. Discussing of the impact of noise on parameter estimation techniques.
5. How noise is transferred through the iterative process.
6. Regularization parameter estimation - statistical motivation.
7. Image segmentation - edge maps
8. Determining edges from Fourier data using the concentration enhancement of scales
9. Edge maps for noisy and blurred data
10. Semi-blind deconvolution, estimating the point spread function.
11. Reconstruction from non-harmonic data
12. Extending the Generalized Singular Value Decomposition as a generalized singular value expansion.

The course will be illustrated by readings from recent and relevant literature [2, 5, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19] augmented by portions from background texts as needed [1, 4, 6, 7, 20]. A good overview of the edge detection approaches is found in the two dissertations of Viswanathan [17, 19].

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