Pythagorean Theorem Unit

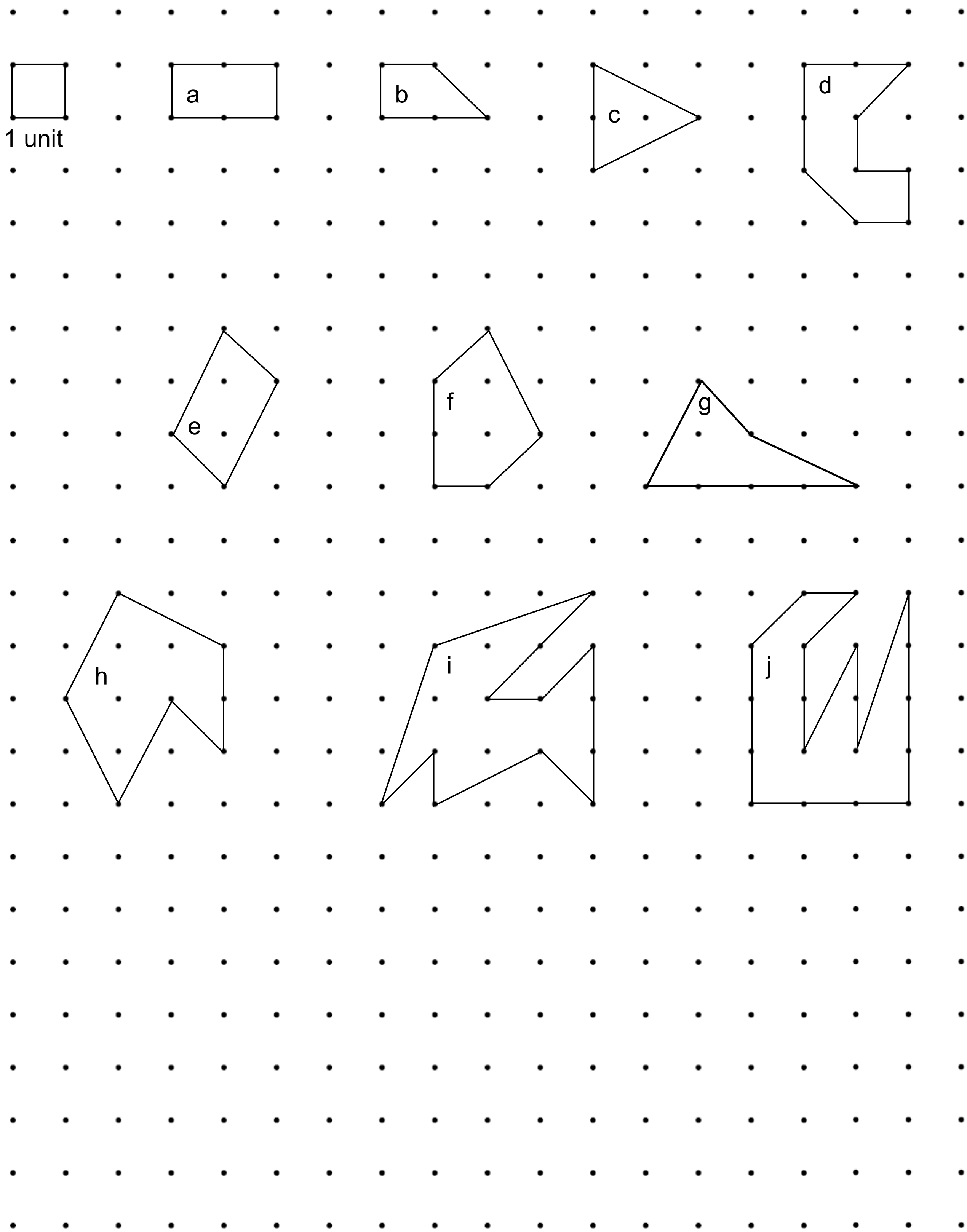
TEKS covered:
~ Square roots and modeling square roots, 8.1(C); 7.1(C)
~ Real number system, 8.1(A), 8.1(C); 7.1(A)
~ Pythagorean Theorem and Pythagorean Theorem Applications, 8.7(C), 8.9(A), 8.15(A)
~ Distance on Coordinate Plane, 8.1(C)

In this unit, you will explore an important relationship among the side lengths of a right triangle. The unit should help you to:
~ Relate the area of a square to the length of a side of the square
~ Develop strategies for finding the distance between two points on a coordinate grid
~ Understand and apply the Pythagorean Theorem
~ Locate square roots of whole numbers on the number line
~ Use the Pythagorean Theorem to solve everyday problems

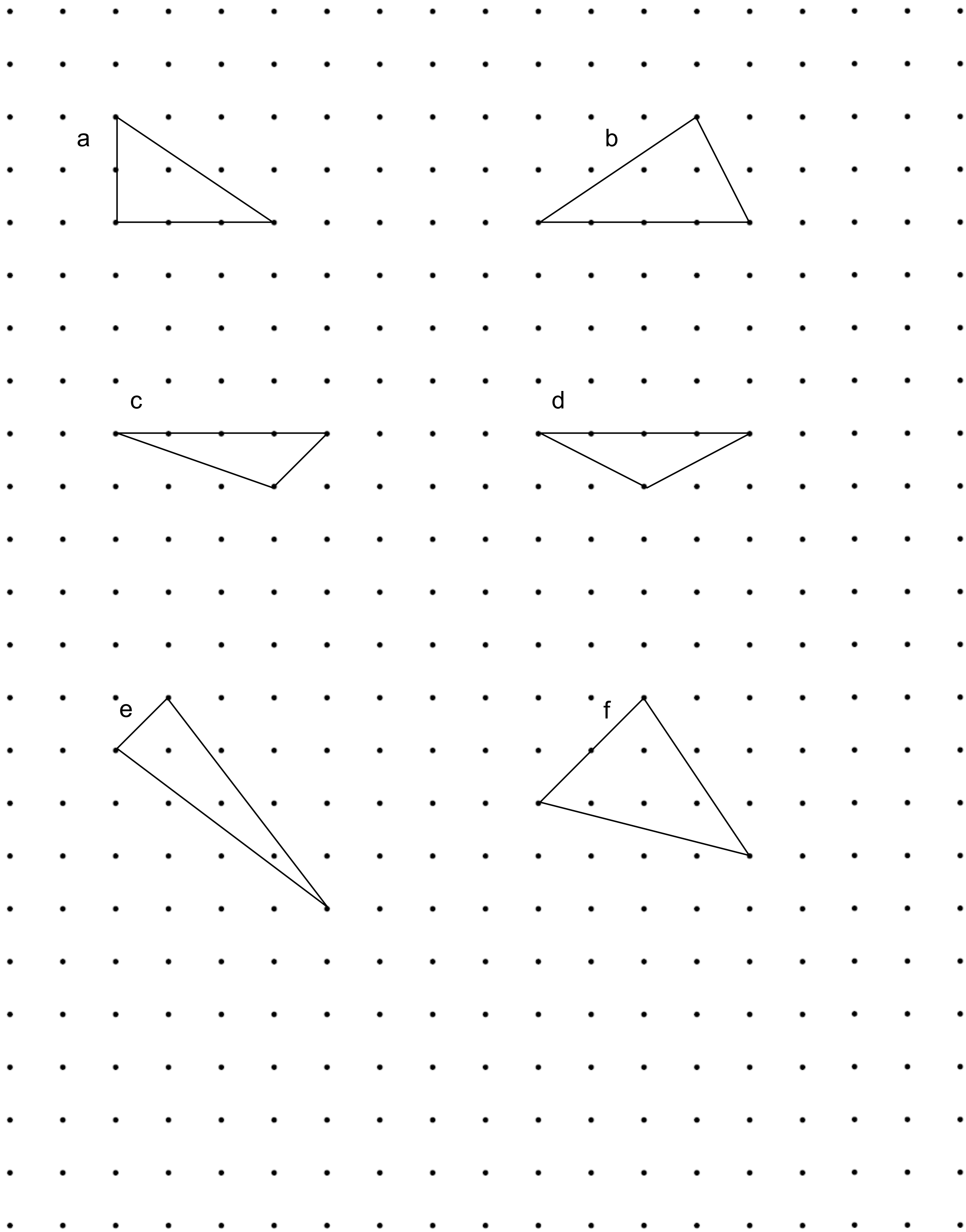
As you work problems in this unit, make it a habit to ask questions about problem situations involving the Pythagorean Theorem.
~ Is this a situation where it is appropriate to use the Pythagorean Theorem?
~ How do I know this?
~ Do I need to find the distance between two points?
~ What quantities are in the problems?
~ How are irrational numbers and areas of square related?
~ How can I estimate the square root of a number?

Part 1: Finding Area

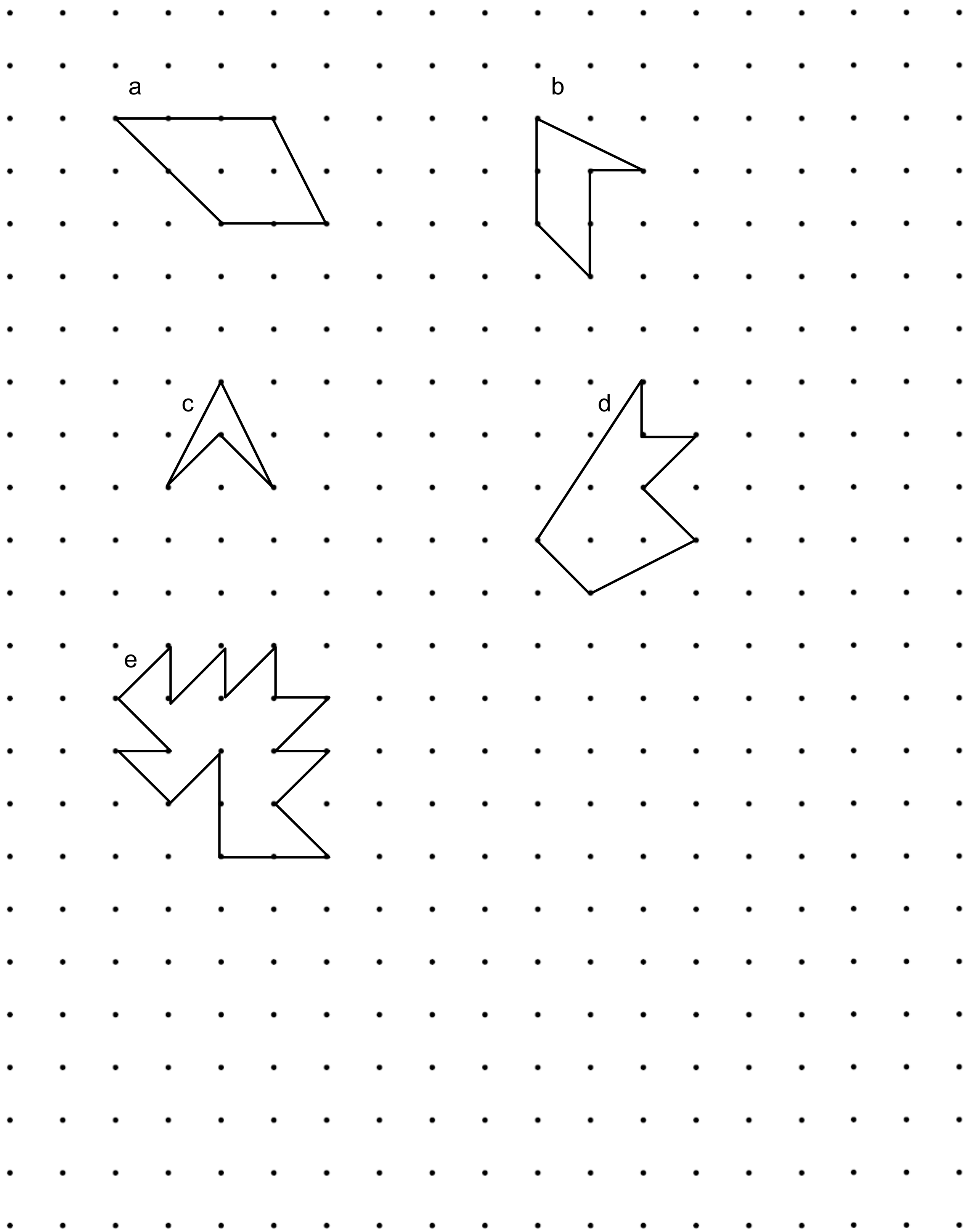
The horizontal and vertical distance between any two adjacent dots on the dot grid below is one unit. Find the area of the shapes on the grid below.



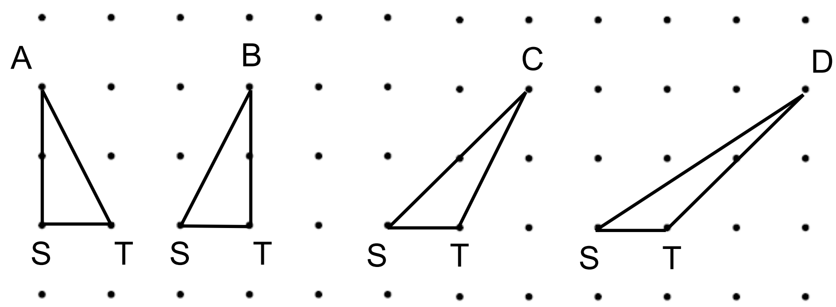
Find the area of the triangles on the grid below. Remember the horizontal and vertical distance between any two adjacent dots is one unit.



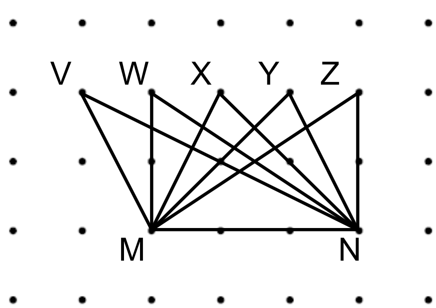
The horizontal and vertical distance between any two adjacent dots on the dot grid below is one unit. Find the area of the shapes on the grid below.



1. The horizontal and vertical distance between any two adjacent dots on the dot grid below is one unit. Find the areas of triangles *AST*, *BST, CST,* and *DST*. How do the areas compare? Why do you think this is true?



2. The horizontal and vertical distance between any two adjacent dots on the dot grid below is one unit. Find the areas of triangles *VMN, WMN, XMN, YMN,* and *ZMN*. How do the areas compare? Why do you think this is true?



Real Number System

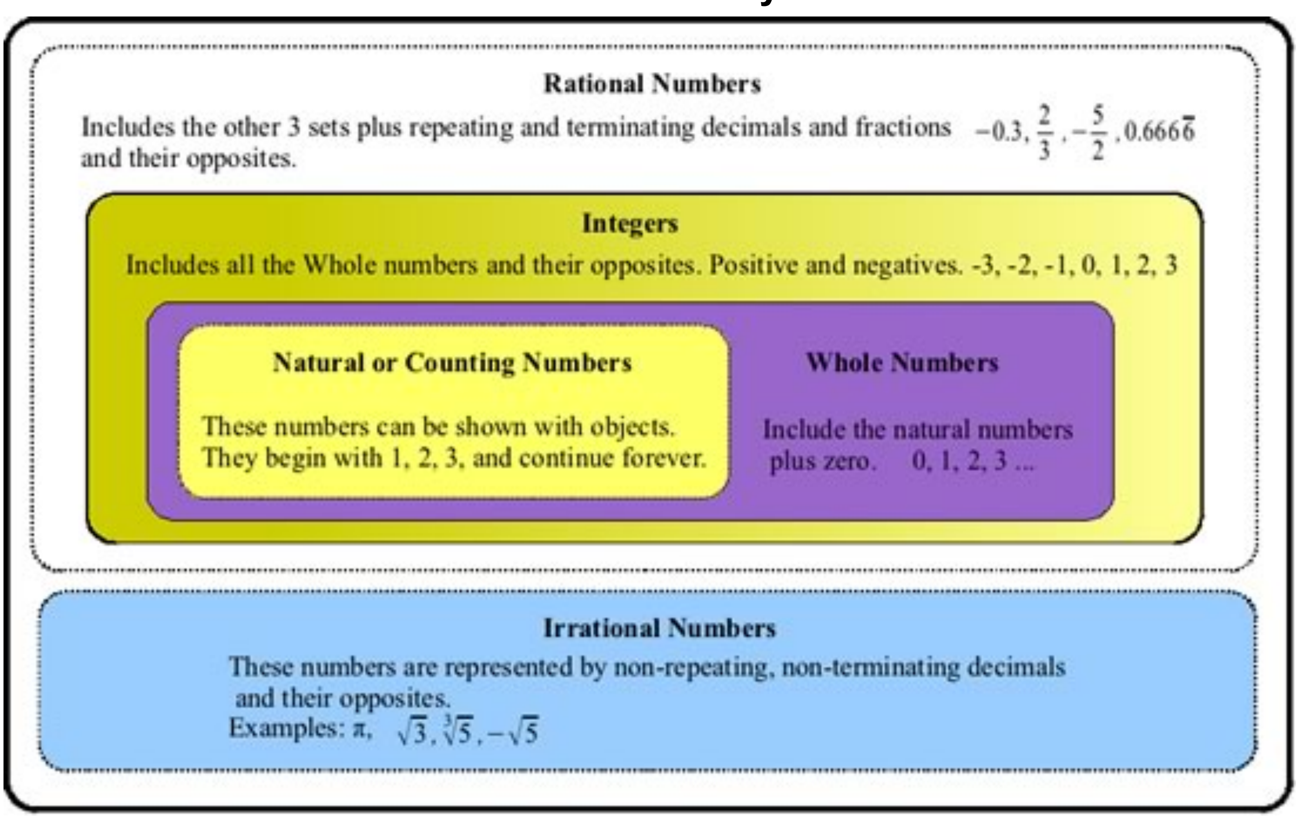
As a 7th grade student, you have worked with many different kinds of numbers. Did you know that those numbers belong to a large group called the **real number system**? The real number system is the set of all rational numbers (including its subsets) and irrational numbers. Let's take a closer look at these two subsets that form the real number system.

Decimal representation of some numbers **terminate**, that is, they have only a limited number of digits in their decimal part, such as 0.5, $\left(\frac{1}{2}\right)$, or 0.125, $\left(\frac{1}{8}\right)$. On the other hand, some numbers have decimal representations with a repeating pattern that never ends, such as 0.3333333…, $\left(\frac{1}{3}\right)$, or 0.09090909…, $\left(\frac{1}{11}\right)$. These are called **repeating decimals.** The numbers that can be represented by terminating or repeating decimals are called **rational numbers** because they can be written as a *ratio* of integers.

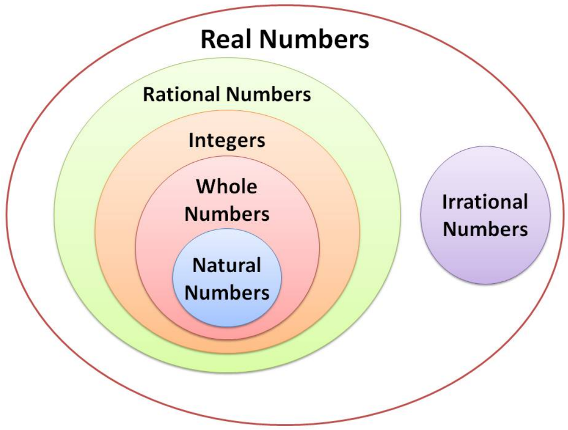
There are also numbers called **irrational numbers** that cannot be written as a ratio of integers. For example, 1.41421356237… , where the decimal part goes forever without any pattern of fixed length that repeats, is an irrational number. The number $\sqrt{2}$ is an irrational number because an exact decimal representation does not exist! Other examples of irrational numbers are $\sqrt{3}, \sqrt{5},$ and $\sqrt{11}$. In fact, $\sqrt{n}$ is an irrational number for any value of *n* that is not a square number.

An amazing fact about irrational numbers is that there are an infinite number of them between any two fractions!

The Real Number System



Real Number System



Set	Description
Natural numbers	{1, 2, 3, 4, ... }
Whole numbers	{0, 1, 2, 3, 4, ...}
Integers	{ ..., -3, -2, -1, 0, 1, 2, 3, ... }
Rational numbers	All numbers that can be written as $\frac{a}{b}$, where a and b are both integers, and b is not equal to 0.
Irrational numbers	Numbers such as $\sqrt{2}, \sqrt[3]{7}, -\pi, e$
Real numbers	The union of the sets of rational numbers and irrational numbers

The Real Number System

Classify these numbers as rational or irrational and give your reason.

1. a. 7329 b. $\sqrt{4}$

2. a. 0.95832758941... b. $\sqrt{188}$

3. Give an example of a number that is: real, rational, whole, an integer, and natural

4. Give an example of a number that is: real and irrational

5. Give an example of a number that is: real, rational, an integer

Classify each number using all sets to which it belongs: real, rational, irrational, whole, natural, and/or integer.

6. a. 3/4 b. -12/4

7. a. 0.345 345 345 … b. -0. 6473490424

8. Give examples of rational numbers that fit between the following sets numbers.

   a. -0.56 and -0.65 -5.76 b. -5.77 3.64 and 3.46

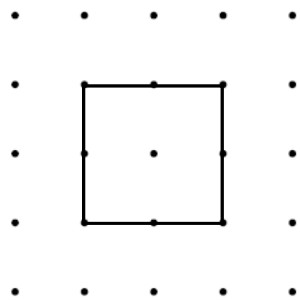
17. Choose the set of irrational numbers below that is in order from least to greatest.

   A. $\sqrt{2}, \sqrt{5}, \sqrt{11}, \pi$ B. $\sqrt{2}, \sqrt{5}, \pi, \sqrt{11}$

   C. $\sqrt{2}, \pi, \sqrt{5}, \sqrt{11}$ D. $\pi, \sqrt{2}, \sqrt{5}, \sqrt{11}$

Square Roots

If you know the area of a square, you can work backwards to find the length of a side. For example, suppose a square has an area of 4 square units. To find the length of a side, you need to figure out what positive number multiplied by itself equals 4. Since 2 x 2 = 4, the side length is 2 units. We call 2 a **square root** of 4.



This square has an area of 4 square units. The length of a side is $\sqrt{4}$, or 2.

In general, if $A = l^2$, then l is called a **square root** of A. **The square root of the area of a square equals the length of the side.**

Every positive number has two square roots. For example, $\sqrt{4}$ means what number multiplied by itself equals 4. Since 2 x 2 = 4 and (-2) x (-2) = 4, then $\sqrt{4}$ = 2 and -2, or ± 2. Since we are focusing on geometry here, we will only be concerned with the positive square roots of a number.

All positive numbers have square roots, but not all square roots are rational numbers. Think about this.

1. Does $\sqrt{2}$ equal a rational number?

2. Is 1.5 a good estimate for $\sqrt{2}$? Why?

3. Can you find a better estimate for $\sqrt{2}$?

Below is a list of the first 50 positive integers. The square of each number is given along with the square root of each number to the nearest thousandth.

Number n	Square n^2	Square Root $\sqrt{n}$
1	1	1
2	4	1.414
3	9	1.732
4	16	2
5	25	2.236
6	36	2.449
7	49	2.646
8	64	2.828
9	81	3
10	100	3.162

Number n	Square n^2	Square Root $\sqrt{n}$
11	121	3.317
12	144	3.464
13	169	3.606
14	196	3.742
15	225	3.873
16	256	4
17	289	4.123
18	324	4.243
19	361	4.359
20	400	4.472

Number n	Square n^2	Square Root $\sqrt{n}$
21	441	4.583
22	484	4.690
23	529	4.796
24	576	4.899
25	625	5
26	676	5.099
27	729	5.196
28	784	5.292
29	841	5.385
30	900	5.477

Number n	Square n^2	Square Root $\sqrt{n}$
31	961	5.568
32	1024	5.657
33	1089	5.745
34	1156	5.831
35	1225	5.916
36	1296	6
37	1369	6.083
38	1444	6.164
39	1521	6.245
40	1600	6.325

Number n	Square n^2	Square Root $\sqrt{n}$
41	1681	6.403
42	1764	6.481
43	1849	6.557
44	1936	6.633
45	2025	6.708
46	2116	6.782
47	2209	6.856
48	2304	6.928
49	2401	7
50	2500	7.071

1. Find the lengths of the sides of squares with an area of: 1, 4, 9, 16, and 25.

2. Find the values of $\sqrt{1}, \sqrt{9}, \sqrt{16}$, and $\sqrt{25}$.

3. What is the area of a square with a side length of 12 units, or $\sqrt{?} = 12$?

4. What is the area of a square with a side length of 2.5 units, or $\sqrt{?} = 2.5$?

5. $\sqrt{5}$ is between which two consecutive whole numbers? Which one is it closer to? Why?

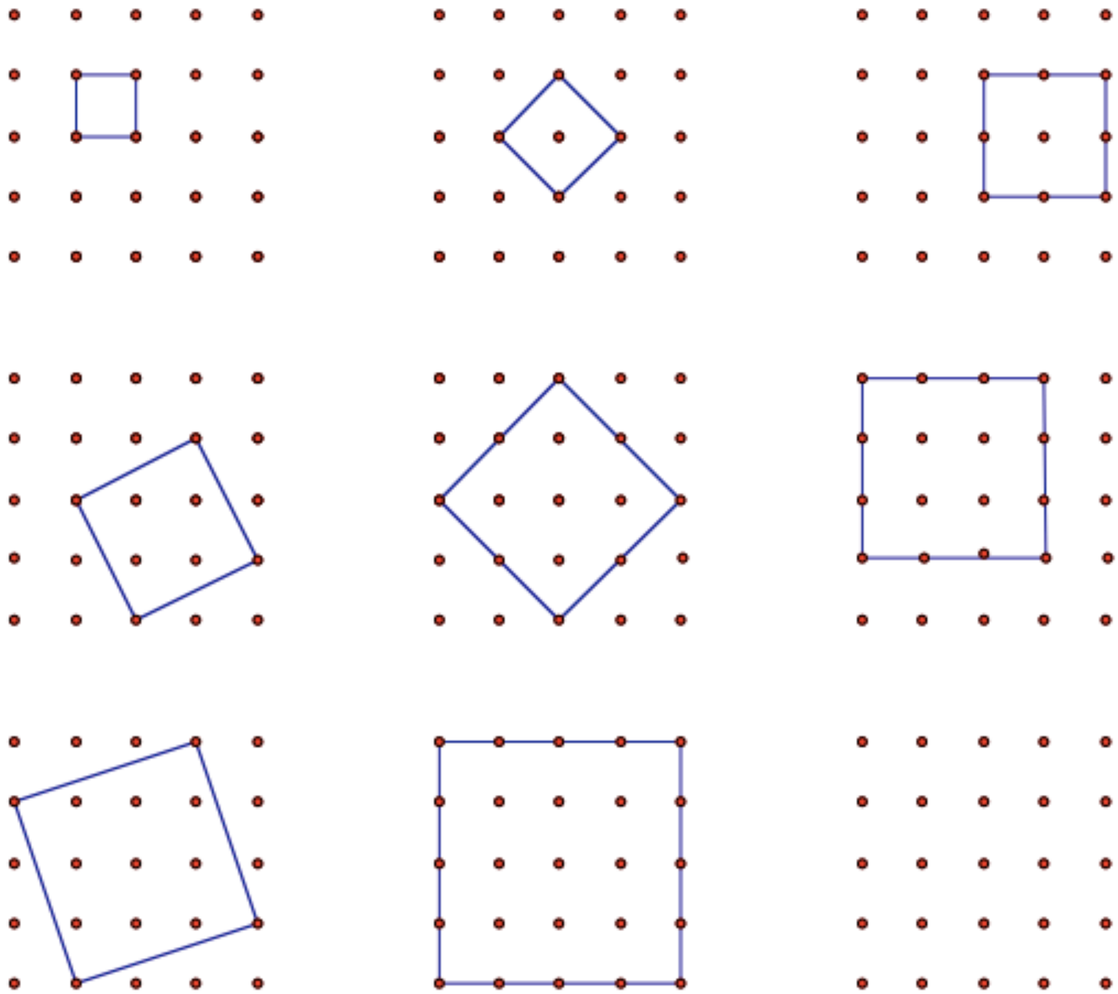
6. Put the following set of numbers in order from least to greatest:

2.3 $\quad 2\frac{1}{4}$ $\quad \sqrt{5}$ $\quad \sqrt{2}$ $\quad \frac{5}{2}$ $\quad \sqrt{4}$

7. Dalida claims that $\sqrt{8} + \sqrt{8}$ should be equal to $\sqrt{16}$ since 8 plus 8 is 16.
Is she right? Explain your thinking.

Square Roots and Finding Length

Label each square with its area and each segment with its length. Use the √ symbol to express lengths that are not whole numbers.



From least to greatest, record the area of each square with its corresponding side length in the table below. Classify the length of each side as rational or irrational. Include an equation that shows how the side lengths are used to find the area.

Area of the square	**Length of the side**	**Side Length: Rational or Irrational?**	**Area= side x side**

Approximate each square root to one decimal place. Then, classify as rational or irrational.

1. $\sqrt{144}$ 2. $\sqrt{0.36}$ 3. $\sqrt{500}$

Without using a calculator, find the two consecutive whole numbers the square root is between and explain how you found your answer.

4. $\sqrt{27}$ 5. $\sqrt{600}$

Tell whether the statement is true.

6. $6 = \sqrt{36}$ 7. $1.5 = \sqrt{2.25}$ 8. $11 = \sqrt{101}$

Find the missing numbers.

9. $\sqrt{?} = 81$ 10. $14 = \sqrt{?}$ 11. $? = \sqrt{289}$

12. $\sqrt{?} = 3.2$ 13. $\sqrt{?} = \frac{1}{4}$ 14. $\sqrt{\frac{1}{9}} = ?$

Find the value of the following products.

15. $\sqrt{2} \times \sqrt{2}$ 16. $\sqrt{3} \times \sqrt{3}$ 17. $\sqrt{4} \times \sqrt{4}$ 18. $\sqrt{5} \times \sqrt{5}$

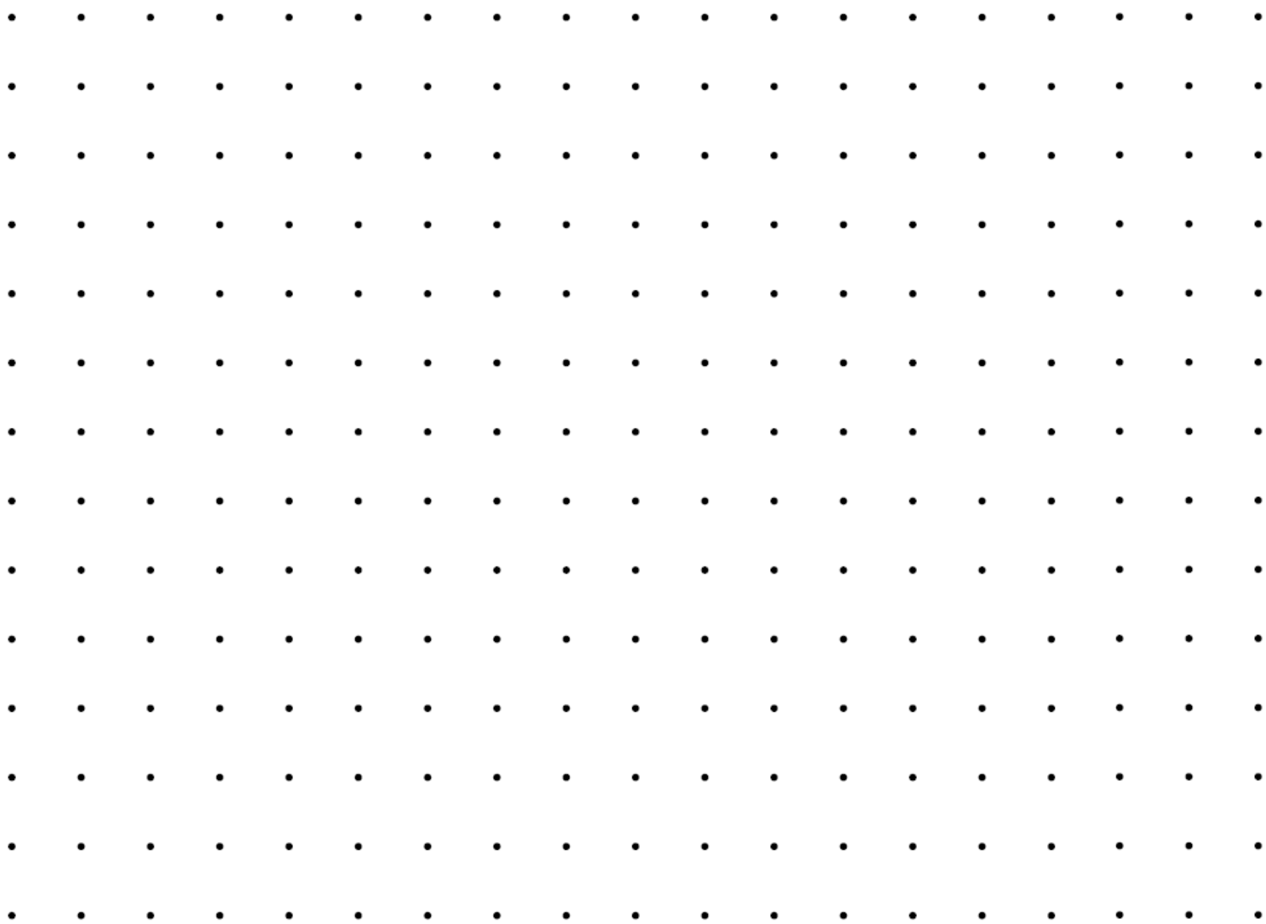
Give both positive and negative square roots of each number.

19. 1 20. 4 21. 2

22. Describe the strategies you used to find areas of figures drawn on dot paper. Give examples if it helps you to explain your thinking.

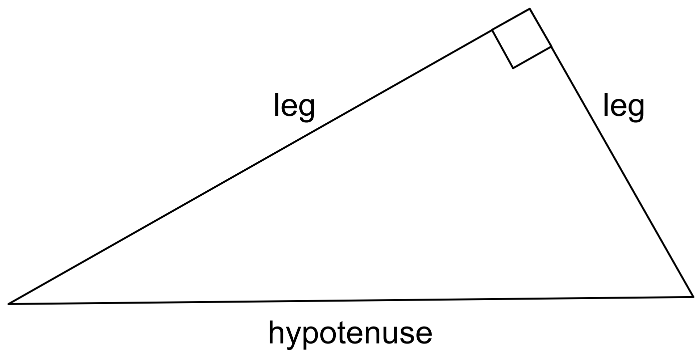
23. Describe how you would find the side length of a square drawn on dot paper without using a ruler. Consider both upright and tilted squares.

24. Bobby claims that only upright squares can have rational numbers for side lengths. Is he correct? Explain.



Part 3: The Pythagorean Theorem

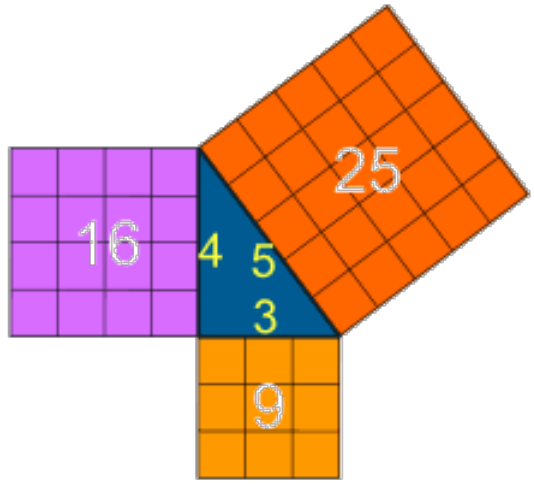
Finding areas of squares may have involved a method that used right triangles. Recall that the longest side of a right triangle is the side opposite the right angle. We call this side the **hypotenuse** of the triangle. The two sides that form the right angle are called **legs**. The right angle of a right triangle is often marked with a square.



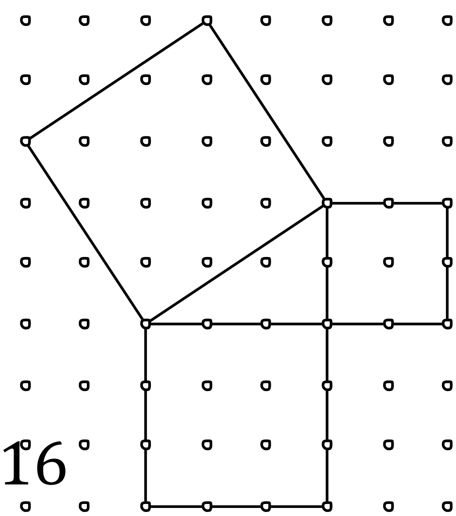
In a right triangle the legs and hypotenuse are given variables to represent their lengths. The legs of a right triangle are called *a* and *b*, though it does not matter which is *a* and which is *b.* The hypotenuse, however, is always *c.* These variable names are used in what you will discover as the Pythagorean Theorem.

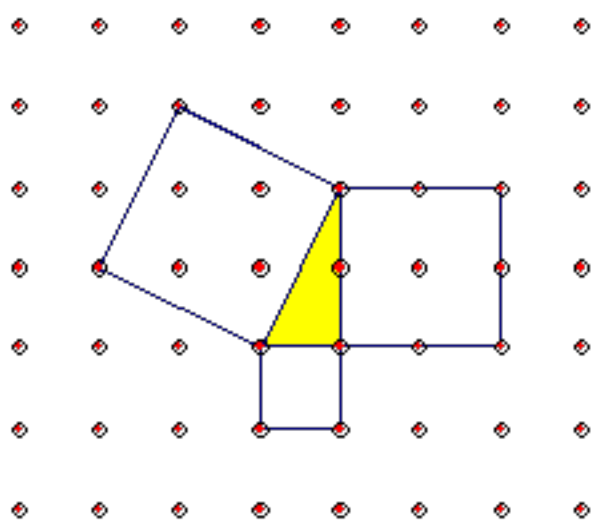
Discovering the Pythagorean Theorem

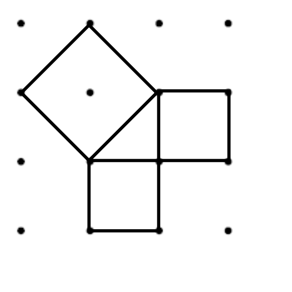
To find the length of a line segment, a square can be drawn using the segment as one of its sides. Below you will look at squares drawn on each segment of a right triangle to determine how the areas of the squares are related.



Length of Leg 1	Length of Leg 2	Area of Square On Leg 1	Area of Square On Leg 2	Area of Square on Hypotenuse
3	4	5	9	16
1	1			
1	2			
2	3			

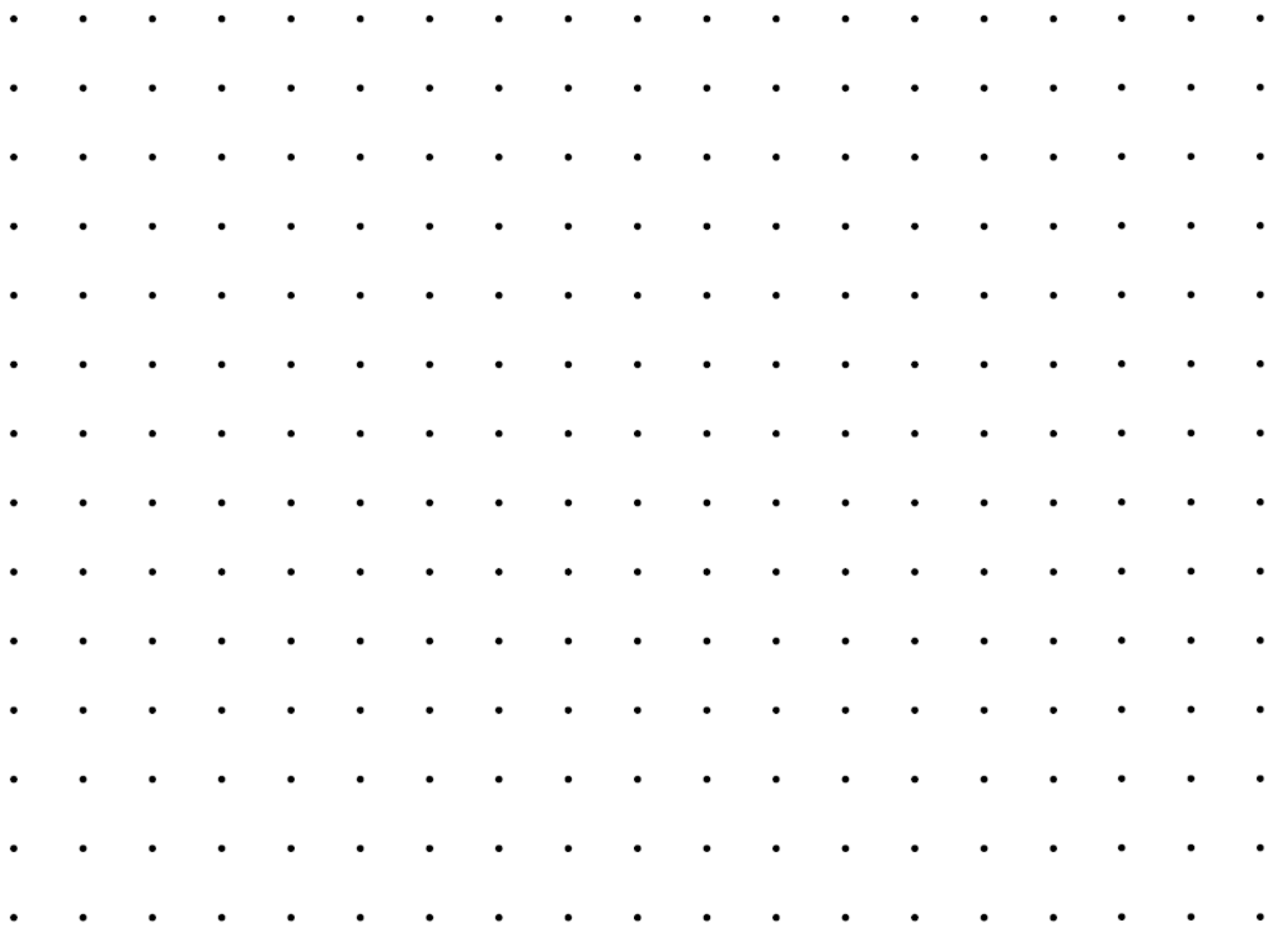




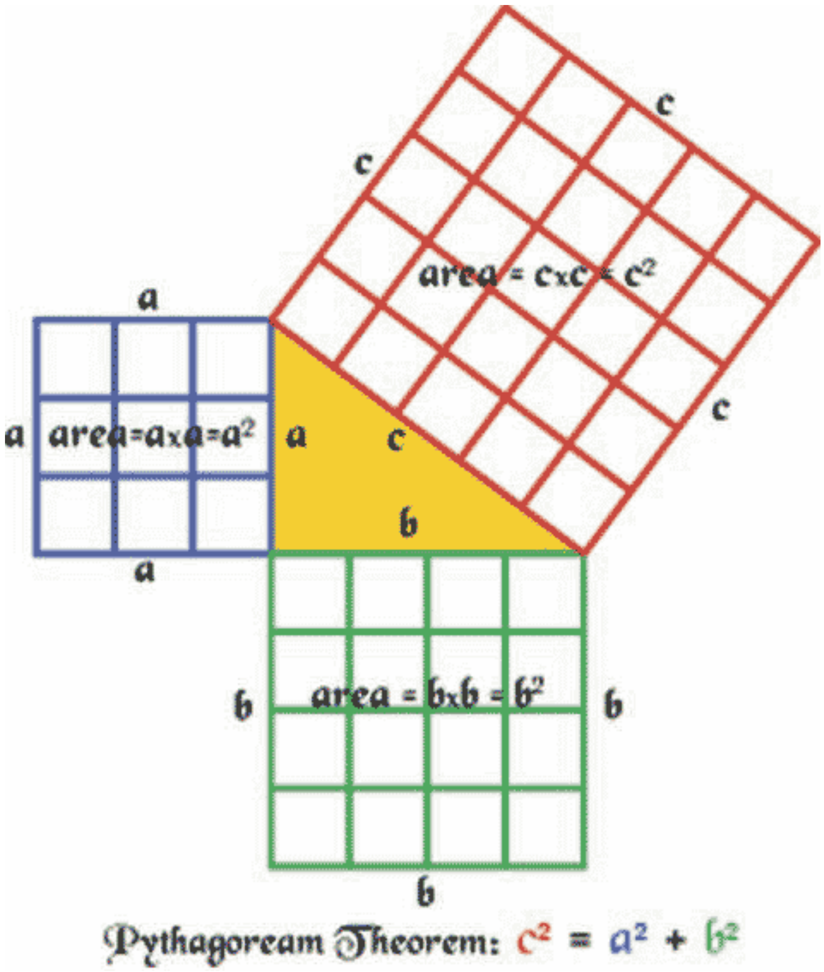


Look for a pattern in the relationship among the areas of the three squares drawn for each triangle. Use the pattern you discover to make a conjecture about the relationship among the areas.

Draw a right triangle with side lengths that are different than those given in the table. Use your triangle to test your conjecture. Is it true?



The **Pythagorean Theorem**, $a^2 + b^2 = c^2$, states that for a right triangle with legs, *a* and *b,* and hypotenuse, *c*, that the area of the square drawn on leg *a* plus the area of the square drawn on leg *b* equals the area of the square drawn on the hypotenuse *c*.



Let’s Practice!

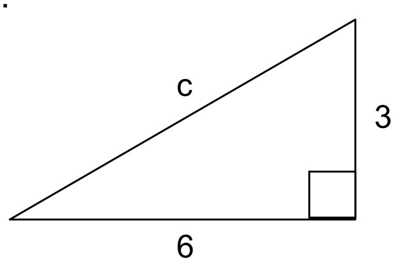
A right triangle has legs of length 5 inches and 12 inches.

1. Find the area of a square drawn on the hypotenuse of the triangle.

2. What is the length of the hypotenuse?

Find the missing length or lengths on the triangles below. (Figures are not drawn to scale.)

3.



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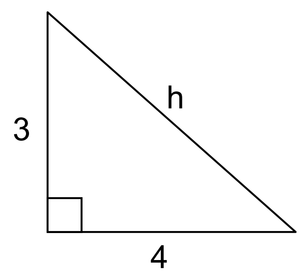

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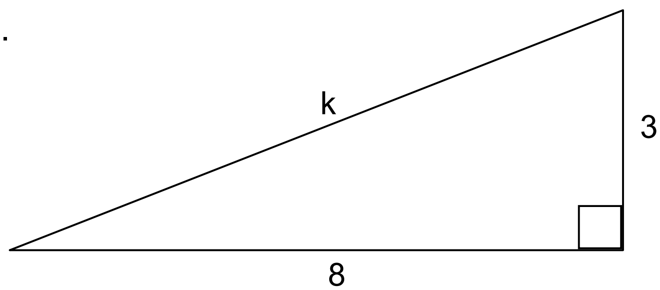

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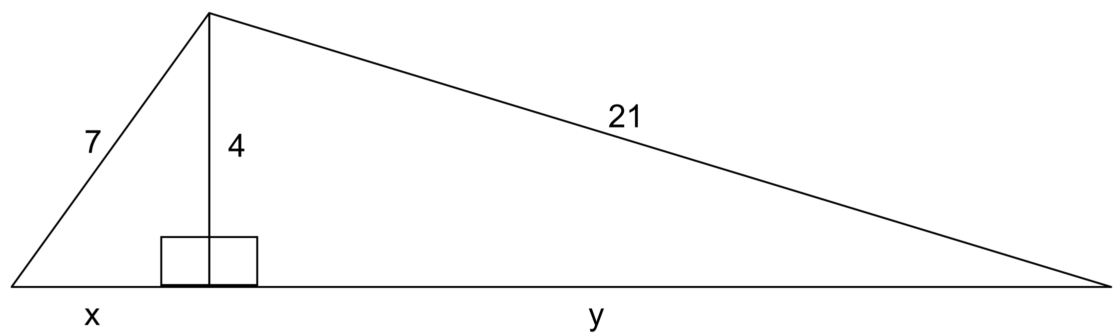
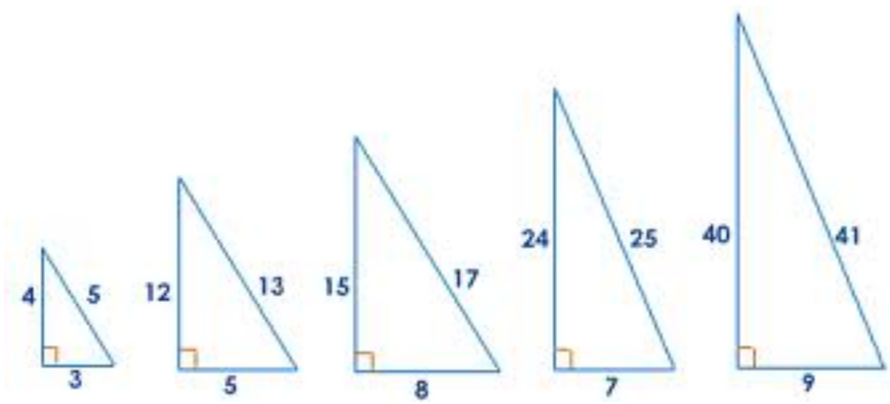


7. A 12-foot ladder is leaning against the side of a building. The top of the ladder reaches 10 feet up the side of the building. Approximately how far is the bottom of the ladder from the base of the building?

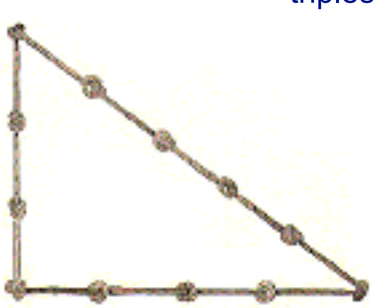
Pythagorean Triples

Pythagorean triples are integers which exactly fit the Pythagorean theorem formula. The number of possible Pythagorean triples is infinite. Below is a list of many of the initial triples. Multiples of these triples are also Pythagorean triples.

Pythagorean Triples			
(3, 4, 5)	(9, 40, 41)	(16, 63, 65)	(36, 77, 85)
(5, 12, 13)	(11, 60, 61)	(20, 21, 29)	(39, 80, 89)
(7, 24, 25)	(12, 35, 37)	(28, 45, 53)	(48, 55, 73)
(8, 15, 17)	(13, 84, 85)	(33, 56, 65)	(65, 72, 97)

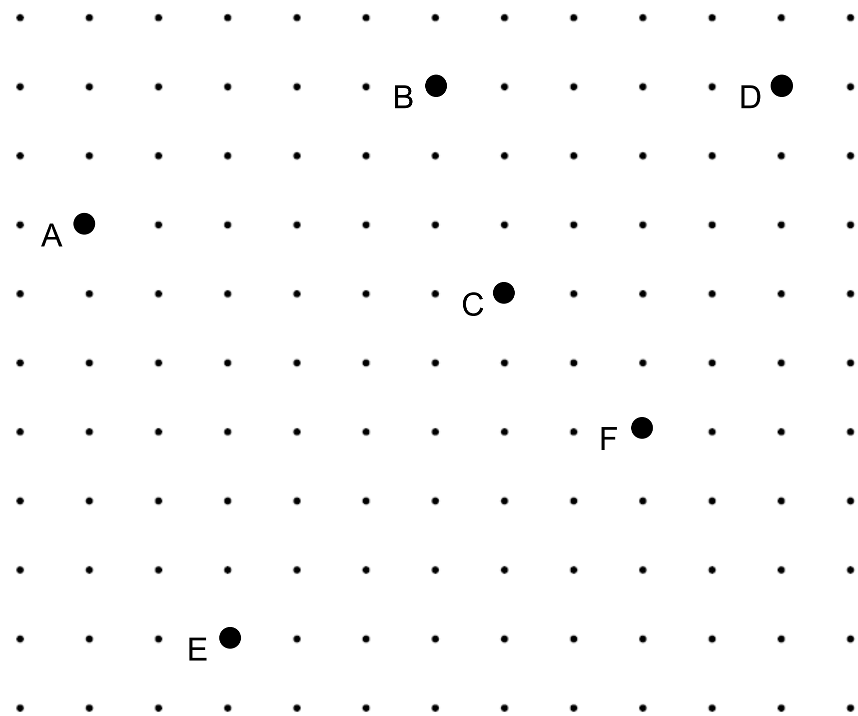


Some 1200 years before Pythagoras was born, the Egyptians measured their fields with lengths of knotted rope. This knotted rope, similar to those used by the Egyptians, indicates a triangle with sides of length 3, 4 and 5 units, one of the Pythagorean triples!



Finding Distances

Several points on the grid below have been labeled A-F.



1. Connect point A to point B. Draw a right triangle with segment AB as its hypotenuse.

2. Find the lengths of the legs of the triangle. Record above.

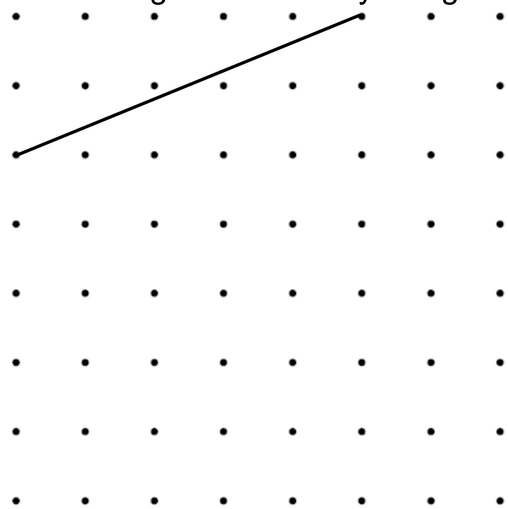
3. Use the Pythagorean Theorem to find the length of the hypotenuse of the triangle.

4. Draw a line segment from point C to point D to help you find the distance between points C and D.

5. Find the distance between points E and F.

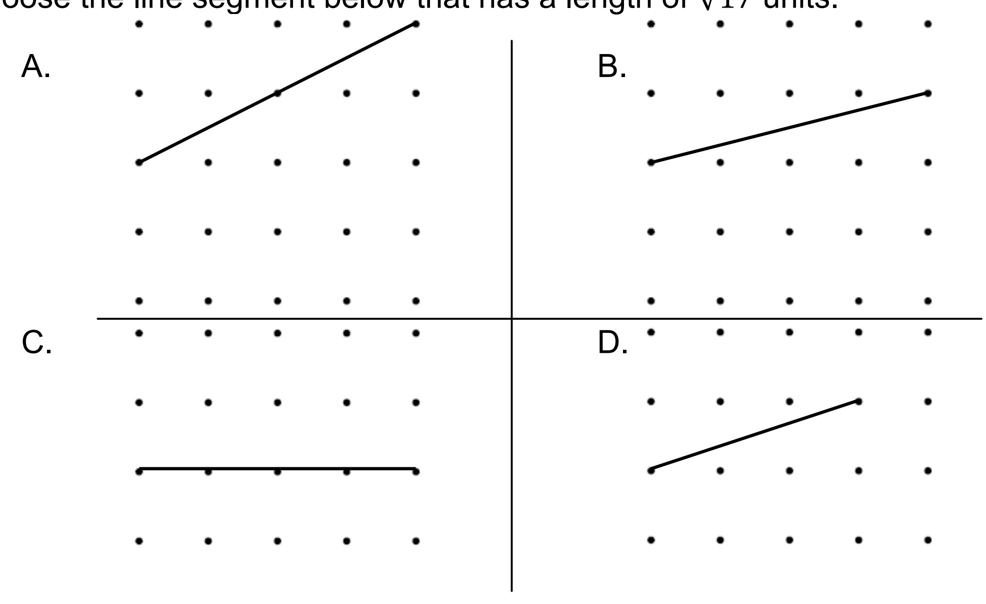
Let’s Practice

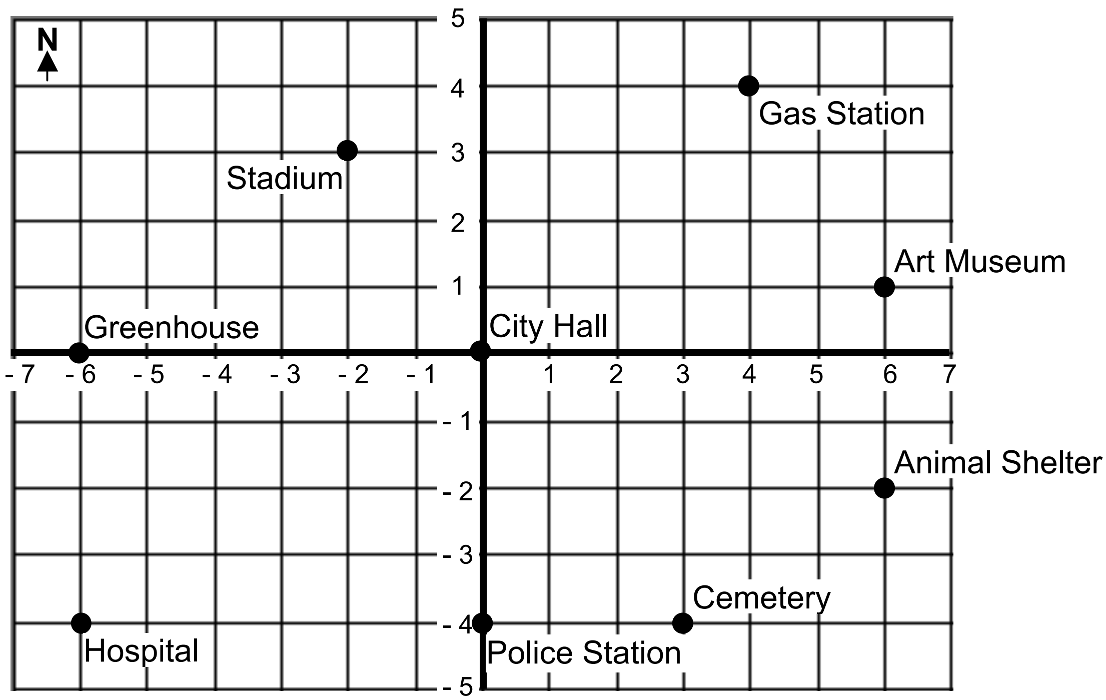
1. Express the length of the line segment below by using the √ symbol.



2. Between what two consecutive whole numbers is the length of the line segment above? Explain.

3. Choose the line segment below that has a length of $\sqrt{17}$ units.





4. Choose the pair of locations that are $\sqrt{40}$ units apart.

A. Greenhouse and Stadium

B. City Hall and Gas Station

C. Hospital and Art Museum

D. Animal Shelter and Police Station

Find the helicopter distance between the two locations in the town of Euclid in blocks by connecting the two locations with a straight line. Use the Pythagorean Theorem if necessary.

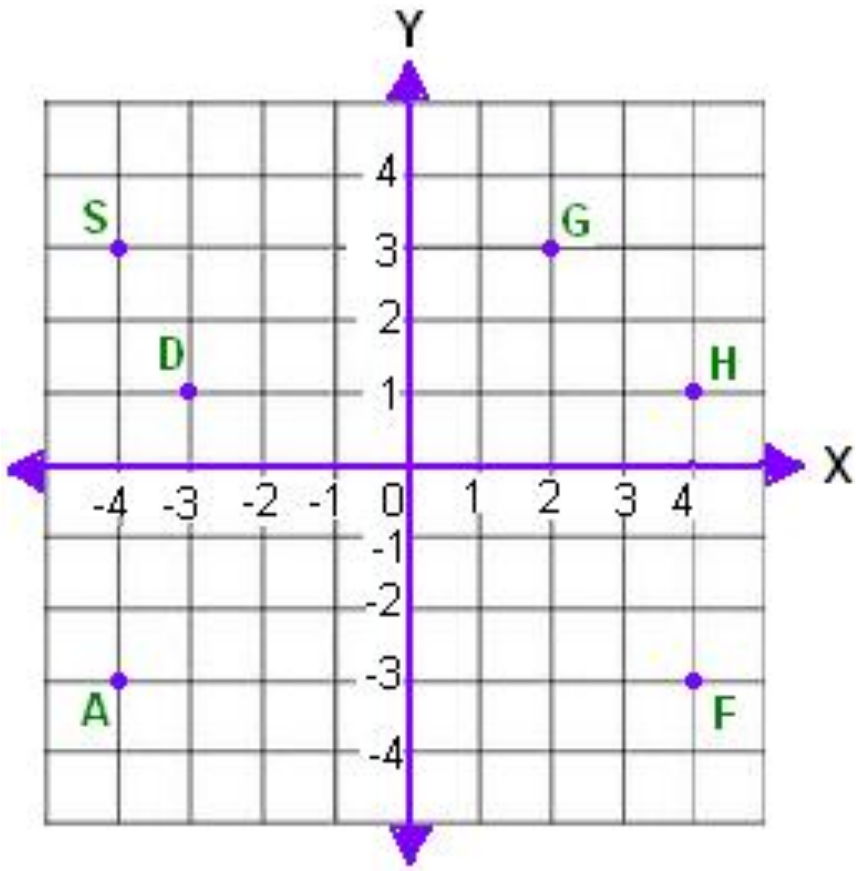
10. Greenhouse and Stadium

11. Police Station and Art Museum

12. Greenhouse and Hospital

13. City Hall and Gas Station

14. Which labeled point(s) is the same distance from the origin, (0, 0), as point *A* is from the origin? Use the Pythagorean Theorem to justify your answer.



Looking for Pythagoras

We have already seen right triangles have side lengths that satisy the Pythagorean Theorem, $\boldsymbol{a^2 + b^2 = c^2}$**.** What do you think is true for side lengths that do not fit the theorem? Make observations as you complete the table below. Show work to justify your answer.

	Lengths of Sides	Do the side lengths satisfy the relationship $\boldsymbol{a^2 + b^2 = c^2}$?	Is the Triangle a Right Triangle?
1.	3, 4, 5		
2.	5, 12, 13		
3.	2, 6, $\sqrt{10}$		
4.	6, 8, 10		
5.	3, $\sqrt{6}$, 9		
6.	$\sqrt{1}$, $\sqrt{2}$, 2		

Make a conjecture about triangles whose side lengths do not satisfy the relationship $a^2 + b^2 = c^2$.

Determine whether the given side lengths forms a right triangle. Show your work.

1. 12, 16, 20

4. 8, 9, $\sqrt{145}$

2. 8, 15, 17

5. $\sqrt{20}$, 5, 45

3. 12, 9, 16

6. $\sqrt{8}, \sqrt{10}, \sqrt{18}$

7. Choose the set(s) of side lengths that could make a right triangle.

A. 10, 24, 26 B. 4, 6, 10 C. 5, 10, $\sqrt{50}$ D. 8, 9, 15

Tell whether the triangle with the given side lengths can be a right triangle.

8. 10, 10, $\sqrt{200}$

9. 9, 16, 25

10. 5, 7, $\sqrt{74}$

11. $\sqrt{2}, \sqrt{7}, 3$

Mathematical Reflections

In this unit, you worked with a very important mathematical relationship called the Pythagorean Theorem. These questions will help you summarize what you have learned.

Think about your answers to these questions, discuss your ideas with other students and your teacher, and then write a summary of your findings in your notebook.

1. Suppose you are given the lengths of two sides of a right triangle. Describe how you can find the length of the third side.

   a. If given the length of both legs

   b. If given the length of one leg and the hypotenuse

2. How can you determine whether a triangle is a right triangle if you know only the lengths of its three sides?