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New Operations On Fuzzy Neutrosophic Soft Matrices

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I.R.Sumathi
Department of Mathematics, Nirmala College for Women, Coimbatore, Tamilnadu, India.
I.Arockiarani
Department of Mathematics, Nirmala College for Women, Coimbatore, Tamilnadu, India.

Abstract: *In this paper we define some new operations on fuzzy neutrosophic soft matrices and their properties which are more functional to make theoretical studies. We finally construct a decision making method which can be applied to the problems that contain uncertainties.*

Keywords: *Soft sets, Fuzzy Neutrosophic soft set and Fuzzy Neutrosophic soft matrices.*

1. Introduction:

In 1999, [9] Molodtsov initiated the novel concept of soft set theory which is a completely new approach for modeling vagueness and uncertainty. In [4] Maji et al. initiated the concept of fuzzy soft sets with some properties regarding fuzzy soft union , intersection, complement of fuzzy soft set. Moreover in [5],[6] Maji et al extended soft sets to intuitionistic fuzzy soft sets and Neutrosophic soft sets and the concept of Neutrosophic set was initiated by Smarandache [11] which is a generalization of fuzzy logic and several related systems. Neutrosophic sets and logic are the foundations for many theories which are more general than their classical counterparts in fuzzy, intuitionistic fuzzy and interval valued frameworks.

One of the important theory of mathematics which has a vast application in science and engineering is the theory of matrices. In [12] Yong et al. initiated a matrix representation of fuzzy soft set and applied it in decision making problems. In [8] Manoj Bora et al introduced the intuitionistic fuzzy soft matrices and applied in the application of medical diagnosis. In this paper we have introduced some new operations on fuzzy neutrosophic soft matrices and some related properties have been established.

2. Preliminaries:

Definition 2.1: [9]

Suppose U is an universal set and E is a set of parameters, Let P(U) denote the power set of U. A pair (F, E) is called a soft set over U where F is a mapping given by F: E→P(U). Clearly, a soft set is a mapping from parameters to P(U) and it is not a set, but a parameterized family of subsets of the universe.

Definition 2.2:[1]

A Fuzzy Neutrosophic set A on the universe of discourse X is defined as

A=$\langle x, T_A(x), I_A(x), F_A(x)\rangle, x \in X$ where $T, I, F: X \rightarrow$ [0, 1] and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$

Definition 2.3:[1]

Let U be the initial universe set and E be a set of parameters. Consider a non-empty set A, A $\subset$ E. Let P (U) denote the set of all fuzzy neutrosophic sets of U. The collection (F, A) is termed to be the fuzzy neutrosophic soft set over U, where F is a mapping given by

F: A $\rightarrow$ P(U).

Definition 2.4:[2]

Let U = { c_1, c_2, ………… c_m} be the universal set and E be the set of parameters given by E = {e_1, e_2, ……. e_n}. Let A $\subseteq$ E. A pair (F, A) be a Fuzzy Neutrosophic soft set over U. Then the subset of U x E is defined by R_A = { (u, e) ; e$\in$A, u $\in$ $f_A(e)$} which is called a relation form of (f_A ,E). The membership function , indeterminacy membership function and non – membership function are written by $T_{R_A} : U \times E \rightarrow [0,1]$, $I_{R_A} : U \times E \rightarrow [0,1]$ and $F_{R_A} : U \times E \rightarrow [0,1]$ where $T_{R_A}(u,e) \in [0,1]$, $I_{R_A}(u,e) \in [0,1]$ and $F_{R_A}(u,e) \in [0,1]$ are the membership value, indeterminacy value and non membership value respectively of u $\in$ U for each e$\in$E.

If $[(T_{ij}, I_{ij}, F_{ij})] = (T_{ij}(u_i, e_j), I_{ij}(u_i, e_j), F_{ij}(u_i, e_j))$ we can define a matrix

$$[(T_{ij}, I_{ij}, F_{ij})]_{mxn} = \begin{bmatrix} (T_{11}, I_{11}, F_{11}) & (T_{12}, I_{12}, F_{12}) & \cdots\cdots & (T_{1n}, I_{1n}, F_{1n}) \\ (T_{21}, I_{21}, F_{21}) & (T_{22}, I_{22}, F_{22}) & \cdots\cdots & (T_{2n}, I_{2n}, F_{2n}) \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ (T_{m1}, I_{m1}, F_{m1}) & (T_{m2}, I_{m2}, F_{m2}) & \cdots\cdots & (T_{mn}, I_{mn}, F_{mn}) \end{bmatrix}$$

which is called an m x n Fuzzy Neutrosophic Soft Matrix of the FNSS (f_A , E) over U.

Definition 2.5:[2]

Let U = { c_1, c_2, ………… c_m} be the universal set and E be the set of parameters given by E = {e_1, e_2, ……. e_n}. Let A $\subseteq$ E. A pair (F, A) be a fuzzy neutrosophic soft set. Then fuzzy neutrosophic soft set (F, A) in a matrix form as $A_{m \times n} = [a_{ij}]_{m \times n}$ or A = $[a_{ij}]$, i=1,2,….m, j = 1,2, ….. n where

$$a_{ij} = \begin{cases} \left(T_j(c_i), I_j(c_i), F_j(c_i)\right) & if \quad e_j \in A \\ (0,0,1) & if \quad e_j \notin A \end{cases},$$

where $T_j(c_i)$ represent the membership of (c_i), $I_j(c_i)$ represent the indeterminacy of (c_i) and $F_j(c_i)$ represent the non-membership of (c_i) in the fuzzy neutrosophic set $F(e_j)$.

Note: Fuzzy Neutrosophic soft matrix is denoted by FNSM

3. Fuzzy Neutrosophic Soft Matrices:

Definition 3.1:

Let $\tilde{A} = \left[T_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right]$, $\tilde{B} = \left[T_{ij}^{\tilde{B}}, I_{ij}^{\tilde{B}}, F_{ij}^{\tilde{B}}\right]$ $\in$ FNSM$_{m \times n}$. Then

a) $\tilde{A}$ is Fuzzy neutrosophic soft sub matrix of $\tilde{B}$ denoted by $\tilde{A} \subseteq \tilde{B}$ if $T_{ij}^{\tilde{A}} \leq T_{ij}^{\tilde{B}}$, $I_{ij}^{\tilde{A}} \leq I_{ij}^{\tilde{B}}$ and $F_{ij}^{\tilde{A}} \geq F_{ij}^{\tilde{B}}$, $\forall$ i and j.

b) $\tilde{A}$ is Fuzzy neutrosophic soft super matrix of $\tilde{B}$ denoted by $\tilde{A} \supseteq \tilde{B}$ if $T_{ij}^{\tilde{A}} \geq T_{ij}^{\tilde{B}}$, $I_{ij}^{\tilde{A}} \geq I_{ij}^{\tilde{B}}$ and $F_{ij}^{\tilde{A}} \leq F_{ij}^{\tilde{B}}$, $\forall$ i and j.

c) $\tilde{A}$ and $\tilde{B}$ are said to be Fuzzy neutrosophic soft equal matrices denoted by $\tilde{A} = \tilde{B}$ if $T_{ij}^{\tilde{A}} = T_{ij}^{\tilde{B}}$, $I_{ij}^{\tilde{A}} = I_{ij}^{\tilde{B}}$ and $F_{ij}^{\tilde{A}} = F_{ij}^{\tilde{B}}$, $\forall$ i and j.

Definition 3.2:

If $\tilde{A}$ = [a$_{ij}$] $\in$FNSM$_{m \times n}$, where [a$_{ij}$] = $\left(T_j(c_i), I_j(c_i), F_j(c_i)\right)$ then $\tilde{A}^C$ is called fuzzy neutrosophic soft complement matrix if $\tilde{A}^C$ = [b$_{ij}$]$_{m \times n}$ where

$$[b_{ij}] = \left(F_j(c_i), 1 - I_j(c_i), T_j(c_i) \right)$$

Definition 3.3:

Let $\tilde{A} = \left[T_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right] \in$ FNSM$_{m \times n}$. Then $\tilde{A}$ is called

d) a $T_{ij}^{\tilde{A}}$ -universal FNSM denoted by U_T= [1,0,0] if $T_{ij}^{\tilde{A}}$ = 1 , $I_{ij}^{\tilde{A}}$ = 0 and $F_{ij}^{\tilde{A}}$ = 0, $\forall$ i and j.

e) a $I_{ij}^{\tilde{A}}$ -universal FNSM denoted by U_I = [0,1,0] if $T_{ij}^{\tilde{A}}$ = 0 , $I_{ij}^{\tilde{A}}$ = 1 and $F_{ij}^{\tilde{A}}$ = 0, $\forall$ i and j.

f) a $F_{ij}^{\tilde{A}}$ -universal FNSM denoted by U_F= [0,0,1] if $T_{ij}^{\tilde{A}}$ = 0 , $I_{ij}^{\tilde{A}}$ = 0 and $F_{ij}^{\tilde{A}}$ = 1,$\forall$ i and j.

Definition 3.4:

Let $\tilde{A} = \left[T_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right]$, $\tilde{B} = \left[T_{ij}^{\tilde{B}}, I_{ij}^{\tilde{B}}, F_{ij}^{\tilde{B}}\right] \in$ FNSM$_{m \times n.}$ Then FSNM $\tilde{C} = \left[T_{ij}^{\tilde{C}}, I_{ij}^{\tilde{C}}, F_{ij}^{\tilde{C}}\right]$ is called

a) Union of $\tilde{A}$ and $\tilde{B}$ denoted by $\tilde{A} \cup \tilde{B}$ if $T_{ij}^{\tilde{C}}$ = max ($T_{ij}^{\tilde{A}}$, $T_{ij}^{\tilde{B}}$) , $I_{ij}^{\tilde{C}}$ = max ($I_{ij}^{\tilde{A}}$, $I_{ij}^{\tilde{B}}$)

and $F_{ij}^{\tilde{C}}$ = min ($F_{ij}^{\tilde{A}}$, $F_{ij}^{\tilde{B}}$) $\forall$ i and j.

b) Intersection of $\tilde{A}$ and $\tilde{B}$ denoted by $\tilde{A} \cap \tilde{B}$ if $T_{ij}^{\tilde{C}}$ = min ($T_{ij}^{\tilde{A}}$, $T_{ij}^{\tilde{B}}$) ,

$I_{ij}^{\tilde{C}}$ = min ($I_{ij}^{\tilde{A}}$, $I_{ij}^{\tilde{B}}$) and $F_{ij}^{\tilde{C}}$ = max ($F_{ij}^{\tilde{A}}$, $F_{ij}^{\tilde{B}}$) $\forall$ i and j.

Definition 3.5:

Let $\tilde{A} = \left[T_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right]$, $\tilde{B} = \left[T_{ij}^{\tilde{B}}, I_{ij}^{\tilde{B}}, F_{ij}^{\tilde{B}}\right] \in$ FNSM$_{m \times n.}$ Then FSNM $\tilde{C} = \left[T_{ij}^{\tilde{C}}, I_{ij}^{\tilde{C}}, F_{ij}^{\tilde{C}}\right]$ is called

a) The "*" (product) operation of $\tilde{A}$ and $\tilde{B}$ denoted by $\tilde{C} = \tilde{A} * \tilde{B}$ if $T_{ij}^{\tilde{C}} = T_{ij}^{\tilde{A}} . T_{ij}^{\tilde{B}}$,

$I_{ij}^{\tilde{C}} = I_{ij}^{\tilde{A}} . I_{ij}^{\tilde{B}}$ and $F_{ij}^{\tilde{C}} = F_{ij}^{\tilde{A}} + F_{ij}^{\tilde{B}} - F_{ij}^{\tilde{A}} . F_{ij}^{\tilde{B}}$ $\forall$ i and j.

b) The "$\tilde{+}$" (probabilistic sum) operation of $\tilde{A}$ and $\tilde{B}$ denoted by $\tilde{C} = \tilde{A} \tilde{+} \tilde{B}$ if

$T_{ij}^{\tilde{C}} = T_{ij}^{\tilde{A}} + T_{ij}^{\tilde{B}} - T_{ij}^{\tilde{A}} . T_{ij}^{\tilde{B}}$, $I_{ij}^{\tilde{C}} = I_{ij}^{\tilde{A}} + I_{ij}^{\tilde{B}} - I_{ij}^{\tilde{A}} . I_{ij}^{\tilde{B}}$ and $F_{ij}^{\tilde{C}} = F_{ij}^{\tilde{A}} . F_{ij}^{\tilde{B}}$ $\forall$ i and j.

c) The "@" (Arithmetic mean) operation of $\tilde{A}$ and $\tilde{B}$ denoted by $\tilde{C} = \tilde{A}$ @ $\tilde{B}$ if

$T_{ij}^{\tilde{C}} = \frac{T_{ij}^{\tilde{A}} + T_{ij}^{\tilde{B}}}{2}$, $I_{ij}^{\tilde{C}} = \frac{I_{ij}^{\tilde{A}} + I_{ij}^{\tilde{B}}}{2}$ and $F_{ij}^{\tilde{C}} = \frac{F_{ij}^{\tilde{A}} + F_{ij}^{\tilde{B}}}{2}$ $\forall$ i and j.

d) The "@^{W}" (weighted Arithmetic mean) operation of $\tilde{A}$ and $\tilde{B}$ denoted by

$\tilde{C} = \tilde{A}$ @^{W} $\tilde{B}$ if $T_{ij}^{\tilde{C}} = \frac{w_1 T_{ij}^{\tilde{A}} + w_2 T_{ij}^{\tilde{B}}}{w_1 + w_2}$, $I_{ij}^{\tilde{C}} = \frac{w_1 I_{ij}^{\tilde{A}} + w_2 I_{ij}^{\tilde{B}}}{w_1 + w_2}$ and

$F_{ij}^{\tilde{C}} = \frac{w_1 F_{ij}^{\tilde{A}} + w_2 F_{ij}^{\tilde{B}}}{w_1 + w_2}$ $\forall$ i and j.

e) The "\$" (Geometric mean) operation of $\tilde{A}$ and $\tilde{B}$ denoted by $\tilde{C} = \tilde{A}$ \$ $\tilde{B}$ if

$T_{ij}^{\tilde{C}} = \sqrt{T_{ij}^{\tilde{A}} \cdot T_{ij}^{\tilde{B}}}$, $I_{ij}^{\tilde{C}} = \sqrt{I_{ij}^{\tilde{A}} \cdot I_{ij}^{\tilde{B}}}$ and $F_{ij}^{\tilde{C}} = \sqrt{F_{ij}^{\tilde{A}} \cdot F_{ij}^{\tilde{B}}}$ $\forall$ i and j.

f) The "\$^{W}" (Weighted geometric mean) operation of $\tilde{A}$ and $\tilde{B}$ denoted by $\tilde{C} = \tilde{A}$ \$^{W} $\tilde{B}$ if

$T_{ij}^{\tilde{C}} = \left[\left(T_{ij}^{\tilde{A}}\right)^{w_1} \cdot \left(T_{ij}^{\tilde{B}}\right)^{w_2} \right]^{\frac{1}{w_1 + w_2}}$, $I_{ij}^{\tilde{C}} = \left[\left(I_{ij}^{\tilde{A}}\right)^{w_1} \cdot \left(I_{ij}^{\tilde{B}}\right)^{w_2} \right]^{\frac{1}{w_1 + w_2}}$ and $F_{ij}^{\tilde{C}} = \left[\left(F_{ij}^{\tilde{A}}\right)^{w_1} \cdot \left(F_{ij}^{\tilde{B}}\right)^{w_2} \right]^{\frac{1}{w_1 + w_2}}$

$\forall$ i and j.

g) The "&" (Harmonic mean) operation of $\tilde{A}$ and $\tilde{B}$ denoted by $\tilde{C} = \tilde{A}$ & $\tilde{B}$ if

$T_{ij}^{\tilde{C}} = 2 \cdot \frac{T_{ij}^{\tilde{A}} \cdot T_{ij}^{\tilde{B}}}{T_{ij}^{\tilde{A}} + T_{ij}^{\tilde{B}}}$, $I_{ij}^{\tilde{C}} = 2 \cdot \frac{I_{ij}^{\tilde{A}} \cdot I_{ij}^{\tilde{B}}}{I_{ij}^{\tilde{A}} + I_{ij}^{\tilde{B}}}$ and $F_{ij}^{\tilde{C}} = 2 \cdot \frac{F_{ij}^{\tilde{A}} \cdot F_{ij}^{\tilde{B}}}{F_{ij}^{\tilde{A}} + F_{ij}^{\tilde{B}}}$ $\forall$ i and j.

h) The "&^{W}" (Weighted Harmonic mean) operation of $\tilde{A}$ and $\tilde{B}$ denoted by $\tilde{C} = \tilde{A}$ &^{W} $\tilde{B}$

if $T_{ij}^{\tilde{C}} = \dfrac{w_1 + w_2}{\dfrac{w_1}{T_{ij}^{\tilde{A}}} + \dfrac{w_2}{T_{ij}^{\tilde{B}}}}$, $I_{ij}^{\tilde{C}} = \dfrac{w_1 + w_2}{\dfrac{w_1}{I_{ij}^{\tilde{A}}} + \dfrac{w_2}{I_{ij}^{\tilde{B}}}}$ and $F_{ij}^{\tilde{C}} = \dfrac{w_1 + w_2}{\dfrac{w_1}{F_{ij}^{\tilde{A}}} + \dfrac{w_2}{F_{ij}^{\tilde{B}}}}$ $\forall$ i and j.

Proposition 3.6:

Let $\tilde{A} = \left[T_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right]$, $\tilde{B} = \left[T_{ij}^{\tilde{B}}, I_{ij}^{\tilde{B}}, F_{ij}^{\tilde{B}}\right] \in$ FNSM$_{m \times n}$, then

(i) $(\tilde{A} \cap \tilde{B}) \tilde{+} (\tilde{A} \cup \tilde{B}) = \tilde{A} \tilde{+} \tilde{B}$

(ii) $(\tilde{A} \cap \tilde{B}) * (\tilde{A} \cup \tilde{B}) = \tilde{A} * \tilde{B}$

(iii) $(\tilde{A} \cap \tilde{B})$ @ $(\tilde{A} \cup \tilde{B}) = \tilde{A}$ @ $\tilde{B}$

(iv) $(\tilde{A} \cap \tilde{B}) \$ (\tilde{A} \cup \tilde{B}) = \tilde{A} \$ \tilde{B}$

(v) $(\tilde{A} \cap \tilde{B}) \& (\tilde{A} \cup \tilde{B}) = \tilde{A} \& \tilde{B}$

(vi) $(\tilde{A} \tilde{+} \tilde{B})$ @ $(\tilde{A} * \tilde{B}) = \tilde{A}$ @ $\tilde{B}$

Proof:

(i) $(\tilde{A} \cap \tilde{B}) \tilde{+} (\tilde{A} \cup \tilde{B})$ = [min {$T_{ij}^{\tilde{A}}, T_{ij}^{\tilde{B}}$} , min {$I_{ij}^{\tilde{A}}, I_{ij}^{\tilde{B}}$}, max {$F_{ij}^{\tilde{A}}, F_{ij}^{\tilde{B}}$}] $\tilde{+}$

[max {$T_{ij}^{\tilde{A}}, T_{ij}^{\tilde{B}}$} , max {$I_{ij}^{\tilde{A}}, I_{ij}^{\tilde{B}}$}, min {$F_{ij}^{\tilde{A}}, F_{ij}^{\tilde{B}}$}]

= {[min {$T_{ij}^{\tilde{A}}, T_{ij}^{\tilde{B}}$}+ max {$T_{ij}^{\tilde{A}}, T_{ij}^{\tilde{B}}$}– min {$T_{ij}^{\tilde{A}}, T_{ij}^{\tilde{B}}$}. max {$T_{ij}^{\tilde{A}}, T_{ij}^{\tilde{B}}$}] ,

[min {$I_{ij}^{\tilde{A}}, I_{ij}^{\tilde{B}}$} + max {$I_{ij}^{\tilde{A}}, I_{ij}^{\tilde{B}}$}– min {$I_{ij}^{\tilde{A}}, I_{ij}^{\tilde{B}}$} . max {$I_{ij}^{\tilde{A}}, I_{ij}^{\tilde{B}}$}],

[max {$F_{ij}^{\tilde{A}}, F_{ij}^{\tilde{B}}$}. min {$F_{ij}^{\tilde{A}}, F_{ij}^{\tilde{B}}$}]}

= {($T_{ij}^{\tilde{A}} + T_{ij}^{\tilde{B}} - T_{ij}^{\tilde{A}} . T_{ij}^{\tilde{B}}$, $I_{ij}^{\tilde{A}} + I_{ij}^{\tilde{B}} - I_{ij}^{\tilde{A}} . I_{ij}^{\tilde{B}}$ $F_{ij}^{\tilde{A}} . F_{ij}^{\tilde{B}}$)} [since min (a,b) + max(a,b) = a+ b

& min(a,b) . max(a,b) = a.b]

= $\tilde{A} \tilde{+} \tilde{B}$

Similarly we can prove the other results.

Example 3.7:

Let $\tilde{A}=\left[T_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right]$, $\tilde{B}=\left[T_{ij}^{\tilde{B}}, I_{ij}^{\tilde{B}}, F_{ij}^{\tilde{B}}\right] \in$ FNSM$_{m \times n}$ where

$$\tilde{A}=\begin{bmatrix}(0.3,0.4,0.2) & (0.4,0.6,0.5)\\(0.5,0.7,0.3) & (0.7,0.4,0.2)\end{bmatrix}_{2x2} \text{ and } \tilde{B}=\begin{bmatrix}(0.4,0.5,0.3) & (0.6,0.5,0.2)\\(0.7,0.6,0.2) & (0.3,0.5,0.4)\end{bmatrix}_{2x2}$$

$$\tilde{A}\cap\tilde{B}=\begin{bmatrix}(0.3,0.4,0.3) & (0.4,0.5,0.5)\\(0.5,0.6,0.3) & (0.3,0.4,0.4)\end{bmatrix}_{2x2} \text{ and } \tilde{A}\cup\tilde{B}=\begin{bmatrix}(0.4,0.5,0.2) & (0.6,0.6,0.2)\\(0.7,0.7,0.2) & (0.7,0.5,0.2)\end{bmatrix}_{2x2}$$

$$\left(\tilde{A}\cap\tilde{B}\right)\tilde{+}\left(\tilde{A}\cup\tilde{B}\right)=\begin{bmatrix}(0.58,0.7,0.06) & (0.76,0.8,0.1)\\(0.85,0.88,0.06) & (0.79,0.7,0.08)\end{bmatrix}_{2x2} = \tilde{A}\ \tilde{+}\ \tilde{B}$$

Definition 3.8:

Let $\tilde{A}=\left[T_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right] \in$ FNSM$_{m \times n}$

a) The necessity operation of $\tilde{A}$ denoted by $\Box\tilde{A}$ and is defined as

$$\Box\tilde{A}=\left[T_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, 1-T_{ij}^{\tilde{A}}\right] \ \forall \text{ i and j.}$$

b) The possibility operation of $\tilde{A}$ denoted by o$\tilde{A}$ and is defined as

$$o\tilde{A}=\left[T_{ij}^{\tilde{A}}, 1-I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right] \ \forall \text{ i and j.}$$

a) The falsity operation of $\tilde{A}$ denoted by $\Diamond\tilde{A}$ and is defined as

$$\Diamond\tilde{A}=\left[1-F_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right] \ \forall \text{ i and j.}$$

Proposition 3.9:

Let $\tilde{A}=\left[T_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right] \in$ FNSM$_{m \times n}$, then

(i) $[\Box(\tilde{A}^C)]^C = \Diamond\tilde{A}$

(ii) $[o(\tilde{A}^C)]^C = \tilde{A}$

(iii) $[\Diamond(\tilde{A}^C)]^C = \Box\tilde{A}$

(iv) $\Box\Diamond\tilde{A} = \Diamond\tilde{A}$

(v) $\Diamond\Box\tilde{A} = \Box\tilde{A}$

(vi) $o\,\Box\tilde{A} = \Box\,o\,\tilde{A}$

(vii) $o\,\Diamond\tilde{A} = \Diamond\,o\,\tilde{A}$

(viii) $\Box\Box\tilde{A} = \Box\tilde{A}$

(ix) $\Diamond\Diamond\tilde{A} = \Diamond\tilde{A}$

(x) $o\,o\,\tilde{A} = \tilde{A}$

Proof:

(i) $[\Box(\tilde{A}^C)]^C = [\Box\left[F_{ij}^{\tilde{A}}, 1-I_{ij}^{\tilde{A}}, T_{ij}^{\tilde{A}}\right]]^C = \left[F_{ij}^{\tilde{A}}, 1-I_{ij}^{\tilde{A}}, 1-F_{ij}^{\tilde{A}}\right]^C = \left[1-F_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right] = \Diamond\tilde{A}$

Similarly we can prove the other results.

Example 3.10:

Let $\tilde{A} = \left[T_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right] \in FNSM_{m\,x\,n}$ where

$$\tilde{A} = \begin{bmatrix} (0.3,0.4,0.2) & (0.4,0.7,0.5) \\ (0.5,0.3,0.3) & (0.7,0.9,0.2) \end{bmatrix}_{2x2} \text{ then } \tilde{A}^C = \begin{bmatrix} (0.2,0.6,0.3) & (0.5,0.3,0.4) \\ (0.3,0.7,0.5) & (0.2,0.1,0.7) \end{bmatrix}_{2x2}$$

$$\Box\tilde{A}^C = \begin{bmatrix} (0.2,0.6,0.8) & (0.5,0.3,0.5) \\ (0.3,0.7,0.7) & (0.2,0.1,0.8) \end{bmatrix}_{2x2}$$

$$[\Box[\tilde{A}^C]]^C = \begin{bmatrix} (0.8,0.4,0.2) & (0.5,0.7,0.5) \\ (0.7,0.3,0.3) & (0.8,0.9,0.2) \end{bmatrix}_{2x2} = \Diamond\tilde{A}.$$

Proposition 3.11:

Let $\tilde{A} = \left[T_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right]$, $\tilde{B} = \left[T_{ij}^{\tilde{B}}, I_{ij}^{\tilde{B}}, F_{ij}^{\tilde{B}}\right] \in$ FNSM$_{m \times n}$, then

(i) $\Box[\tilde{A} \cap \tilde{B}] = \Box\tilde{A} \cap \Box\tilde{B}$

(ii) $\Box[\tilde{A} \cup \tilde{B}] = \Box\tilde{A} \cup \Box\tilde{B}$

(iii) $[\Box[\tilde{A}^C \tilde{+} \tilde{B}^C]]^C = \Diamond\tilde{A} * \Diamond\tilde{B}$

(iv) $[\Box[\tilde{A}^C * \tilde{B}^C]]^C = \Diamond\tilde{A} \tilde{+} \Diamond\tilde{B}$

(v) $[o\,[\tilde{A}^C \tilde{+} \tilde{B}^C]]^C = o(\tilde{A} * \tilde{B})$

(vi) $[o\,[\tilde{A}^C * \tilde{B}^C]]^C = o(\tilde{A} \tilde{+} \tilde{B})$

(vii) $[\Diamond\,[\tilde{A}^C \tilde{+} \tilde{B}^C]]^C = \Box\tilde{A} * \Box\tilde{B}$

(viii) $[\Diamond\,[\tilde{A}^C * \tilde{B}^C]]^C = \Box\tilde{A} \tilde{+} \Box\tilde{B}$

(ix) $\Box[\tilde{A} \,@\, \tilde{B}] = \Box\tilde{A} \,@\, \Box\tilde{B}$

(x) $\Diamond\,[\tilde{A} \,@\, \tilde{B}] = \Diamond\tilde{A} \,@\, \Diamond\tilde{B}$

(xi) $o\,[\tilde{A} \,@\, \tilde{B}] = o\tilde{A} \,@\, o\tilde{B}$

Proof:

(i) $\Box[\tilde{A} \cap \tilde{B}] = \Box[\min\{T_{ij}^{\tilde{A}}, T_{ij}^{\tilde{B}}\}, \min\{I_{ij}^{\tilde{A}}, I_{ij}^{\tilde{B}}\}, \max\{F_{ij}^{\tilde{A}}, F_{ij}^{\tilde{B}}\}]$

$=[\min\{T_{ij}^{\tilde{A}}, T_{ij}^{\tilde{B}}\}, \min\{I_{ij}^{\tilde{A}}, I_{ij}^{\tilde{B}}\}, 1 - \min\{T_{ij}^{\tilde{A}}, T_{ij}^{\tilde{B}}\}]$ ---------(1)

$\Box\tilde{A} = \left[T_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, 1 - T_{ij}^{\tilde{A}}\right]$, $\Box\tilde{B} = \left[T_{ij}^{\tilde{B}}, I_{ij}^{\tilde{B}}, 1 - T_{ij}^{\tilde{B}}\right]$

$\Box\tilde{A} \cap \Box\tilde{B} = [\min\{T_{ij}^{\tilde{A}}, T_{ij}^{\tilde{B}}\}, \min\{I_{ij}^{\tilde{A}}, I_{ij}^{\tilde{B}}\}, \max\{1 - T_{ij}^{\tilde{A}}, 1 - T_{ij}^{\tilde{B}}\}]$

$=[\min\{T_{ij}^{\tilde{A}}, T_{ij}^{\tilde{B}}\}, \min\{I_{ij}^{\tilde{A}}, I_{ij}^{\tilde{B}}\}, 1 - \min\{T_{ij}^{\tilde{A}}, T_{ij}^{\tilde{B}}\}]$ ----------(2)

From (1) & (2) we have □[$\tilde{A} \cap \tilde{B}$] = □$\tilde{A} \cap$ □$\tilde{B}$. Similarly we can prove the other results.

Definition 3.12:

Let $\tilde{A} = \left[T_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right]$, $\tilde{B} = \left[T_{ij}^{\tilde{B}}, I_{ij}^{\tilde{B}}, F_{ij}^{\tilde{B}}\right] \in$ FNSM$_{m \times n}$. Then the operation '→'(Implication) denoted by $\tilde{A} \to \tilde{B}$ is defined as

$\tilde{A} \to \tilde{B}$ = [(max ($F_{ij}^{\tilde{A}}$, $T_{ij}^{\tilde{B}}$) , max($1 - I_{ij}^{\tilde{A}}$, $I_{ij}^{\tilde{B}}$), min ($T_{ij}^{\tilde{A}}$, $F_{ij}^{\tilde{B}}$)].

Proposition 3.13:

Let $\tilde{A} = \left[T_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right]$, $\tilde{B} = \left[T_{ij}^{\tilde{B}}, I_{ij}^{\tilde{B}}, F_{ij}^{\tilde{B}}\right]$, $\tilde{C} = \left[T_{ij}^{\tilde{C}}, I_{ij}^{\tilde{C}}, F_{ij}^{\tilde{C}}\right] \in$ FNSM$_{m \times n}$. Then

(i) $[\tilde{A} \cap \tilde{B}] \to \tilde{C} = [\tilde{A} \to \tilde{C}] \cup [\tilde{B} \to \tilde{C}]$

(ii) $\tilde{A} \to \tilde{A}^C = \tilde{A}^C$

(iii) $[\tilde{A} \cap \tilde{B}] \to \tilde{C} \supseteq [\tilde{A} \to \tilde{C}] \cap [\tilde{B} \to \tilde{C}]$

(iv) $[\tilde{A} \cup \tilde{B}] \to \tilde{C} \subseteq [\tilde{A} \to \tilde{C}] \cup [\tilde{B} \to \tilde{C}]$

(v) $[\tilde{A} \tilde{+} \tilde{B}] \to \tilde{C} \subseteq [\tilde{A} \to \tilde{C}] \tilde{+} [\tilde{B} \to \tilde{C}]$

(vi) $[\tilde{A} * \tilde{B}] \to \tilde{C} \supseteq [\tilde{A} \to \tilde{C}] * [\tilde{B} \to \tilde{C}]$

(vii) $\tilde{A} \to [\tilde{B} \tilde{+} \tilde{C}] \subseteq [\tilde{A} \to \tilde{B}] \tilde{+} [\tilde{A} \to \tilde{C}]$

(viii) $\tilde{A} \to [\tilde{B} * \tilde{C}] \supseteq [\tilde{A} \to \tilde{B}] * [\tilde{A} \to \tilde{C}]$

Proof:

$[\tilde{A} \cap \tilde{B}]$ = [min {$T_{ij}^{\tilde{A}}$, $T_{ij}^{\tilde{B}}$} , min {$I_{ij}^{\tilde{A}}$, $I_{ij}^{\tilde{B}}$}, max {$F_{ij}^{\tilde{A}}$, $F_{ij}^{\tilde{B}}$}]

$[\tilde{A} \cap \tilde{B}] \to \tilde{C}$ = [max((max ($F_{ij}^{\tilde{A}}$, $F_{ij}^{\tilde{B}}$)), $T_{ij}^{\tilde{C}}$) , max(($1 - \min(I_{ij}^{\tilde{A}}$, $I_{ij}^{\tilde{B}}$)), $I_{ij}^{\tilde{C}}$),

min((min ($T_{ij}^{\tilde{A}}$, $T_{ij}^{\tilde{B}}$)) $F_{ij}^{\tilde{C}}$)]. -------(1)

$\tilde{A} \to \tilde{C}$ = [(max ($F_{ij}^{\tilde{A}}$, $T_{ij}^{\tilde{C}}$), max($1 - I_{ij}^{\tilde{A}}$, $I_{ij}^{\tilde{C}}$), min ($T_{ij}^{\tilde{A}}$, $F_{ij}^{\tilde{C}}$)].

$\tilde{B} \to \tilde{C}$ = [(max ($F_{ij}^{\tilde{B}}$, $T_{ij}^{\tilde{C}}$), max($1 - I_{ij}^{\tilde{B}}$, $I_{ij}^{\tilde{C}}$), min ($T_{ij}^{\tilde{B}}$, $F_{ij}^{\tilde{C}}$)].

[$\tilde{A} \to \tilde{C}$] $\cup$ [$\tilde{B} \to \tilde{C}$] = [max{(max ($F_{ij}^{\tilde{A}}$, $T_{ij}^{\tilde{C}}$),(max ($F_{ij}^{\tilde{B}}$, $T_{ij}^{\tilde{C}}$)},

max { max($1 - I_{ij}^{\tilde{A}}$, $I_{ij}^{\tilde{C}}$), max($1 - I_{ij}^{\tilde{B}}$, $I_{ij}^{\tilde{C}}$)},

min {min ($T_{ij}^{\tilde{A}}$, $F_{ij}^{\tilde{C}}$), min ($T_{ij}^{\tilde{B}}$, $F_{ij}^{\tilde{C}}$)}]

= [max((max ($F_{ij}^{\tilde{A}}$, $F_{ij}^{\tilde{B}}$)), $T_{ij}^{\tilde{C}}$), max(($1 - \min(I_{ij}^{\tilde{A}}$, $I_{ij}^{\tilde{B}}$)), $I_{ij}^{\tilde{C}}$),

min((min ($T_{ij}^{\tilde{A}}$, $T_{ij}^{\tilde{B}}$)) $F_{ij}^{\tilde{C}}$)]. -------(2)

From (1) & (2) [$\tilde{A} \cap \tilde{B}$] $\to \tilde{C}$ = [$\tilde{A} \to \tilde{C}$] $\cup$ [$\tilde{B} \to \tilde{C}$]

Similarly we can prove the other results.

Example 3.14:

Let $\tilde{A} = \left[T_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right]$, $\tilde{B} = \left[T_{ij}^{\tilde{B}}, I_{ij}^{\tilde{B}}, F_{ij}^{\tilde{B}}\right]$, $\tilde{C} = \left[T_{ij}^{\tilde{C}}, I_{ij}^{\tilde{C}}, F_{ij}^{\tilde{C}}\right] \in$ $FNSM_{m \times n}$ where

$\tilde{A} = \begin{bmatrix} (0.3,0.4,0.2) & (0.4,0.6,0.5) \\ (0.5,0.7,0.3) & (0.7,0.4,0.2) \end{bmatrix}_{2x2}$, $\tilde{B} = \begin{bmatrix} (0.4,0.5,0.3) & (0.6,0.5,0.2) \\ (0.7,0.6,0.2) & (0.3,0.5,0.4) \end{bmatrix}_{2x2}$ and

$\tilde{C} = \begin{bmatrix} (0.3,0.4,0.6) & (0.4,0.5,0.6) \\ (0.6,0.7,0.3) & (0.5,0.4,0.4) \end{bmatrix}_{2x2}$

[$\tilde{A} \tilde{+} \tilde{B}$] $\to \tilde{C}$ = $\begin{bmatrix} (0.3,0.4,0.58) & (0.4,0.5,0.6) \\ (0.6,0.7,0.3) & (0.5,0.4,0.4) \end{bmatrix}_{2x2}$ .... (1)

[$\tilde{A} \to \tilde{C}$] $\tilde{+}$ [$\tilde{B} \to \tilde{C}$] $= \begin{bmatrix} (0.51,0.8,0.12) & (0.7,0.75,0.24) \\ (0.84,0.91,0.09) & (0.75,0.8,0.12) \end{bmatrix}_{2x2}$ .... (2)

From (1) and (2) $[\tilde{A} \tilde{+} \tilde{B}] \to \tilde{C} \subseteq [\tilde{A} \to \tilde{C}] \tilde{+} [\tilde{B} \to \tilde{C}]$

4. Application Of Weighted Arithmetic Mean ($\tilde{A}^{W}_{AM}$) Of Fuzzy Neutrosophic Soft Matrix In Decision Making:

In this section we define arithmetic mean($\tilde{A}_{AM}$) and weighted arithmetic mean($\tilde{A}^{W}_{AM}$) of fuzzy neutrosophic soft matrix.

Definition 4.1:

Let $\tilde{A} = \left[T_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right] \in$ FNSM$_{m \times n}$, then the weighted arithmetic mean of fuzzy neutrosophic soft matrix $\tilde{A}$ denoted by $\tilde{A}^{W}_{AM}$ is defined as,

$$\tilde{A}^{W}_{AM} = \left[\frac{\sum_{j=1}^{n} w_j T_{ij}^{\tilde{A}}}{\sum_{j=1}^{n} w_j}, \frac{\sum_{j=1}^{n} w_j I_{ij}^{\tilde{A}}}{\sum_{j=1}^{n} w_j}, \frac{\sum_{j=1}^{n} w_j F_{ij}^{\tilde{A}}}{\sum_{j=1}^{n} w_j}\right]$$

and w_j for j = 1, 2, …n.

Definition 4.2:

Let $\tilde{A} = \left[T_{ij}^{\tilde{A}}, I_{ij}^{\tilde{A}}, F_{ij}^{\tilde{A}}\right] \in$ FNSM$_{m \times n}$, then the arithmetic mean of fuzzy neutrosophic soft matrix $\tilde{A}$ denoted by $\tilde{A}_{AM}$ is defined as,

$$\tilde{A}_{AM} = \left[\frac{\sum_{j=1}^{n} T_{ij}^{\tilde{A}}}{n}, \frac{\sum_{j=1}^{n} I_{ij}^{\tilde{A}}}{n}, \frac{\sum_{j=1}^{n} F_{ij}^{\tilde{A}}}{n}\right]$$

when weights are equal.

Definition 4.3:

The score value of S_i for $u_i \in U$ is defined as $S_i = T_i - I_i . F_i$.

Algorithm:

1. Construct an FNSS (F,A) over U
2. Construct the Fuzzy Neutrosophic soft matrix of (F,A)
3. Compute arithmetic mean $\tilde{A}_{AM}$ and weighted arithmetic mean $\tilde{A}^{W}_{AM}$

4. Compute the score value of S_i for $\tilde{A}_{AM}$ and $\tilde{A}_{AM}^{W}$

5. Find $S_j = \max(S_i)$, then we conclude that u_i is suitable.

Example:

Suppose there are five suppliers U={u_1, u_2, u_3, u_4, u_5} whose core competencies are evaluated by means of parameters E = A = {e_1, e_2, e_3} where e_1 = level of technology innovation, e_2 = the ability of the management and e_3 = level of service. Suppose a manufacturing company which wants to select the best supplier according to the core competencies. A fuzzy neutrosophic soft set is given by

(F,A) =

{{F(e_1) = {(u_1,0.5,0.6,0.4), (u_2,0.9,0.4,0.1), (u_3,0.6,0.4,0.2), (u_4,0.6,0.4,0.2), (u_5,0.4,0.5,0.3)}

{F(e_2) = {(u_1,0.6,0.7,0.2), (u_2,0.5,0.7,0.1), (u_3,0.5,0.4,0.4), (u_4,0.8,0.6,0.2), (u_5,0.6,0.4,0.2)}

{F(e_3) = {(u_1,0.5,0.6,0.2), (u_2,0.4,0.8,0.3), (u_3,0.7,0.6,0.2), (u_4,0.6,0.4,0.4), (u_5,0.5,0.5,0.1)}}

The matrix representation for the above is given as

$$\tilde{A} = \begin{bmatrix} (0.5,0.6,0.4) & (0.6,0.7,0.2) & (0.5,0.6,0.2) \\ (0.9,0.4,0.1) & (0.5,0.7,0.1) & (0.4,0.8,0.3) \\ (0.6,0.4,0.2) & (0.5,0.4,0.4) & (0.7,0.6,0.2) \\ (0.6,0.4,0.2) & (0.8,0.6,0.2) & (0.6,0.4,0.4) \\ (0.4,0.5,0.3) & (0.6,0.4,0.2) & (0.5,0.5,0.1) \end{bmatrix}$$

$$\tilde{A}_{AM} = \begin{bmatrix} (0.5333,0.6333,0.2667) \\ (0.6,0.6333,0.1667) \\ (0.6,0.4667,0.2667) \\ (0.6667,0.4667,0.2667) \\ (0.5,0.4667,0.2) \end{bmatrix}$$

If we prefer to give weights 0.8, 0.1 and 0.1 on the parameters e_1, e_2 and e_3 respectively then the weighted arithmetic mean is given by

$$\tilde{A}_{AM}^{W} = \begin{bmatrix} (0.51,0.61,0.36) \\ (0.81,0.47,0.12) \\ (0.6,0.42,0.22) \\ (0.62,0.42,0.22) \\ (0.43,0.49,0.27 \end{bmatrix}$$

The score of $\tilde{A}_{AM} = \begin{bmatrix} 0.3664 \\ 0.4944 \\ 0.4755 \\ \mathbf{0.5422} \\ 0.4067 \end{bmatrix}$ and the score of $\tilde{A}_{AM}^{W} = \begin{bmatrix} 0.2904 \\ \mathbf{0.7536} \\ 0.5076 \\ 0.5276 \\ 0.2977 \end{bmatrix}$.

Form the above results , it is clear that if we give equal preference for the parameters , we have u_4 is the best alternative and when we give more preference on level of technology innovation, then we have same u_2 is the best alternative.

5. Conclusion:

Soft set theory is a general method for solving problems of uncertainty. In this paper, we define Fuzzy Neutrosophic soft matrices and their operations which are more functional to improve several new results. Finally we have given one application for decision making based on arithmetic mean.

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