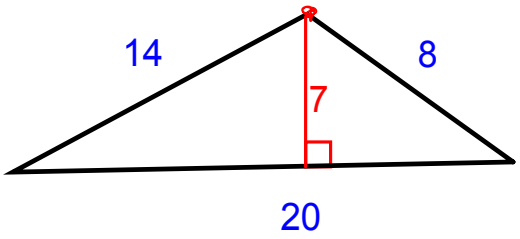
5-6: Area of a Triangle

9/15/16



Standard Area Formula

$$\text{Area} = \frac{1}{2}bh$$

any side of the △

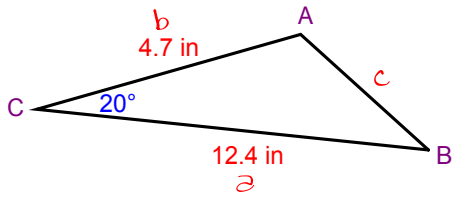
perpendicular to that side from the opposite vertex

1. Find the Area

$A = \frac{1}{2}(20)(7)$

$A = 70 \text{ units}^2$

2. Find the Area



What are we given? SAS

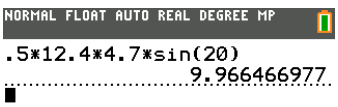
$A = \frac{1}{2}bh$

What are we missing? height How can we find it? SOH CAH TOA

$4.7 \cdot \sin 20 = \frac{h}{4.7} \cdot 4.7$

$h = 4.7 \cdot \sin 20$

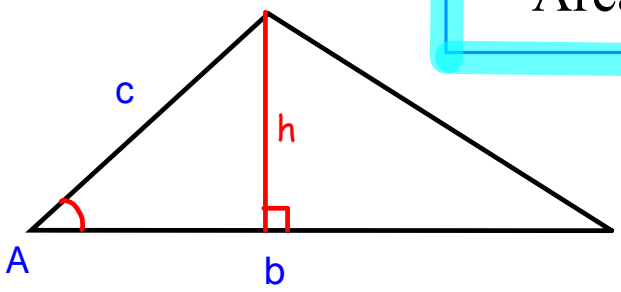
$A = \frac{1}{2}bh = \frac{1}{2}(12.4)(4.7 \cdot \sin 20)$ (a, b, sin C)



$A = 9.97 \text{ units}^2$

Side-Angle-Side Area Formula

$$\text{Area} = \frac{1}{2}bc \cdot \sin(A)$$



How we came up with this formula:

$A = 1/2bh$

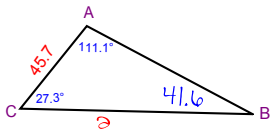
find h:

$\sin(A) = h/c$

$c\sin(A) = h$

$A = 1/2\ bc\sin(A)$

3. Find the Area



What are we given?

ASA/AAS

To use our SAS Area

What are we missing? Side How can we find it?

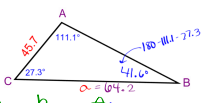
To solve for side a use law of sines

$$\frac{\sin 41.6}{45.7} = \frac{\sin 111.1}{a}$$

$$\frac{a \cdot \sin 41.6}{\sin 41.6} = \frac{45.7 \cdot \sin 111.1}{\sin 41.6}$$

$$a = \frac{45.7 \cdot \sin 111.1}{\sin 41.6}$$

Now that we have SAS, we can use our formula from the previous slide...



SAS Area

$A = \frac{1}{2}bc\sin A$

$\text{Area} = \frac{1}{2}(45.7)\left(\frac{45.7 \cdot \sin 111.1}{\sin 41.6}\right)\sin 27.3 = 673.01 \text{ units}^2$

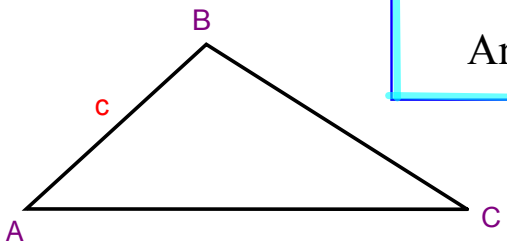
(b, b, A, B, C)

How can we streamline this process by making up an equation?

$$\text{Area} = \frac{1}{2}b^2\frac{\sin A \cdot \sin C}{\sin B}$$

Angle-Angle-Side (or Angle-Side-Angle) Area Formula

$$\text{Area} = \frac{1}{2} \cdot c^2 \cdot \frac{\sin(A)\cdot\sin(B)}{\sin(C)}$$



How we came up with this formula:

A = 1/2bh

sinA = h/c
csinA = h

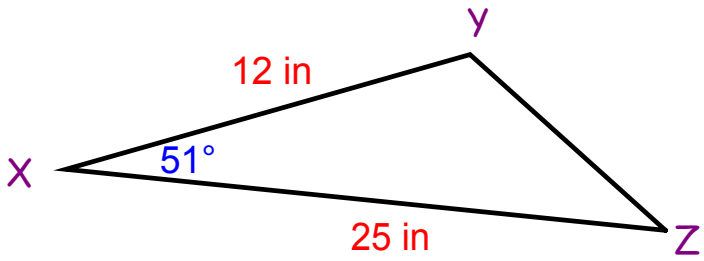
A = 1/2 bcsinA

sinB = sinC
b c
bsinC = csinB
b = csinB
sinC

A = 1/2 c csinB sinA
sinC

A = 1/2c^{2}sinBsinA
sinC

4. Find the Area



What are we given?

SAS

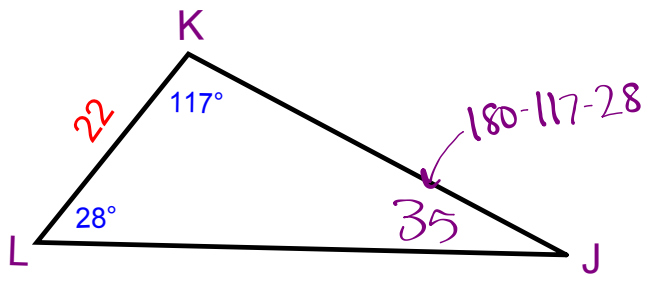
Which equation should we use?

Area = $\frac{1}{2}$bcsin A

Area = $\frac{1}{2}(12)(25)\sin(51)$

= 116.57 in^2

5. Find the Area



What are we given?

ASA

Which equation should we use?

$\text{Area} = \frac{1}{2}c^2 \cdot \frac{\sin A \cdot \sin B}{\sin C}$

$\text{Area} = \frac{1}{2}(22)^2 \cdot \frac{\sin(28)\sin(117)}{\sin 35}$

$= 176.49 \text{ units}^2$

3 Formulas for finding Area of a Triangle

Standard Area Formula

$$\text{Area} = \frac{1}{2}bh$$

Side-Angle-Side Area Formula

$$\text{Area} = \frac{1}{2}bc \cdot \sin(A)$$

Angle-Angle-Side (or Angle-Side-Angle) Area Formula

$$\text{Area} = \frac{1}{2} \cdot c^2 \cdot \frac{\sin(A) \cdot \sin(B)}{\sin(C)}$$