LESSON 24.4

Conditions for Rectangles, Rhombuses, and Squares

Common Core Math Standards

The student is expected to:

CA CC **G-CO.11**

Prove theorems about parallelograms. Also G-SRT.5

Mathematical Practices

CA CC **MP.7 Using Structure**

Language Objective

Explain to a partner how to distinguish between a condition for a quadrilateral to be a rectangle, rhombus, or square, and a property of a rectangle, rhombus, or square.

ENGAGE

Essential Question: How can you use given conditions to show that a quadrilateral is a rectangle, rhombus, or square?

You can use the converses of the theorems in the previous lesson to prove that a quadrilateral is a rectangle, rhombus, or square.

PREVIEW: LESSON PERFORMANCE TASK

View the Engage section online. Discuss the photo. Ask students to name some other animals that are related to tigers, and to explain how they are related. Then preview the Lesson Performance Task.

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Name ______________________ Class ____________ Date ________

24.4 Conditions for Rectangles, Rhombuses, and Squares

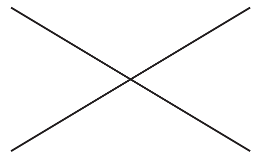
Essential Question: How can you use given conditions to show that a quadrilateral is a rectangle, a rhombus, or a square?

Resource Locker

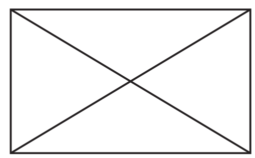
Explore Properties of Rectangles, Rhombuses, and Squares

In this lesson we will start with given properties and use them to prove which special parallelogram it could be.

(A) Start by drawing two line segments of the same length that bisect each other but are not perpendicular. They will form an X shape, as shown.

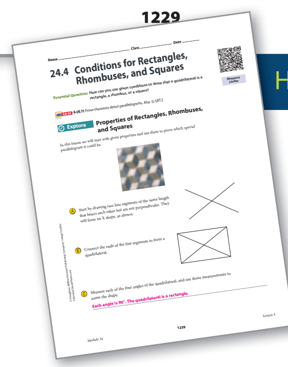


(B) Connect the ends of the line segments to form a quadrilateral.



(C) Measure each of the four angles of the quadrilateral, and use those measurements to name the shape.

Each angle is 90°. The quadrilateral is a rectangle.



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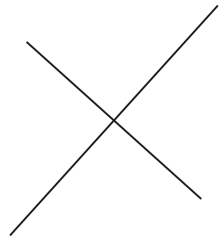
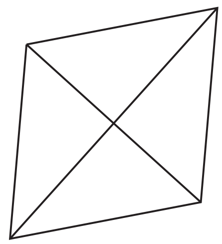
Turn to these pages to find this lesson in the hardcover student edition.

Ⓓ Now, draw two line segments that are perpendicular and bisect each other but that are not the same length.



Ⓔ Connect the ends of the line segments to form a quadrilateral.



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Ⓕ Measure each side length of the quadrilateral. Then use those measurements to name the shape.

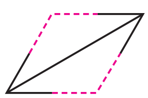
The side lengths are all equal. The quadrilateral is a rhombus.

Reflect

1. **Discussion** How are the diagonals of your rectangle in Step B different from the diagonals of your rhombus in Step E?

The diagonals of the rectangle have the same lengths, but are not perpendicular bisectors of each other. The diagonals of the rhombus are perpendicular bisectors of each other, but do not necessarily have the same lengths.

2. Draw a line segment. At each endpoint draw line segments so that four congruent angles are formed as shown. Then extend the segments so that they intersect to form a quadrilateral. Measure the sides. What do you notice? What kind of quadrilateral is it? How does the line segment relate to the angles drawn on either end of it?



The side lengths are equal. The quadrilateral is a rhombus. The line segment bisects both angles.

EXPLORE

Properties of Rectangles, Rhombuses, and Squares

INTEGRATE TECHNOLOGY

Students have the option of doing the special parallelograms activity either in the book or online.

QUESTIONING STRATEGIES

? If you draw a quadrilateral with congruent diagonals, what shape is the quadrilateral? **rectangle**

? If you draw two congruent segments that are perpendicular bisectors of one another and then connect the ends to form a quadrilateral, which shape is the quadrilateral? **rhombus**

PROFESSIONAL DEVELOPMENT

Integrate Mathematical Practices

This lesson provides an opportunity to address Mathematical Practice **MP.7**, which calls for students to "look for and make use of structure." Students are already familiar with the properties of rectangles, rhombuses, and squares, but in this lesson they must analyze the conditions that would be sufficient to make a parallelogram a more special figure. Each theorem in the lesson presents a single condition that leads to a broader conclusion that a figure is a special quadrilateral. For example, it is sufficient for one angle of a parallelogram to be a right angle to conclude that the parallelogram has four right angles (it is a rectangle).

EXPLAIN 1

Proving that Congruent Diagonals Is a Condition for Rectangles

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Communication

MP.3 Point out to students that in the previous lesson they were introduced to the properties of rectangles, rhombuses, and squares. Explain that in this lesson, they will be given a quadrilateral and will learn what conditions can be used to classify it as a rectangle, rhombus, or square. You may want to call on students to read each theorem aloud. Then ask them to explain the theorem in their own words. Challenge students to come up with unique ways to explain the theorems.

QUESTIONING STRATEGIES

? How does knowing that the diagonals of a parallelogram are congruent allow you to prove that the parallelogram is a rectangle? **If the diagonals of a parallelogram are congruent, then they form congruent triangles. That makes the corresponding angles of the congruent triangles congruent. Since the largest angles are also supplementary, each must be a right angle. A quadrilateral with four right angles is a rectangle.**

AVOID COMMON ERRORS

Some students may have trouble identifying a piece of additional information that is sufficient to make a conclusion valid. Suggest that once they have an answer, they write a complete statement of the given information, sketch the figure, mark it with this information, and then re-check their work.

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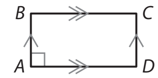
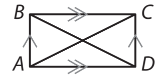
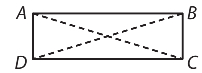
Explain 1 Proving that Congruent Diagonals Is a Condition for Rectangles

When you are given a parallelogram with certain properties, you can use the properties to determine whether the parallelogram is a rectangle.

Theorems: Conditions for Rectangles	
If one angle of a parallelogram is a right angle, then the parallelogram is a rectangle.	
If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.	 $\overline{AC} \cong \overline{BD}$

Example 1 **Prove that if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.**



Given: *ABCD* is a parallelogram; $\overline{AC} \cong \overline{BD}$.

Prove: *ABCD* is a rectangle.

Because **opposite sides of a parallelogram are congruent**, $\overline{AB} \cong \overline{CD}$.

It is given that $\overline{AC} \cong \overline{BD}$, and **$\overline{AD} \cong \overline{AD}$** by the Reflexive Property of Congruence.

So, **$\triangle ABD \cong \triangle DCA$** by the SSS Triangle Congruence Theorem,

and **$\angle BAD \cong \angle CDA$** by CPCTC. But these angles are **supplementary**

since $\overline{AB} \parallel$ **$\overline{DC}$**. Therefore, $m\angle BAD + m\angle CDA =$ **$180°$**. So

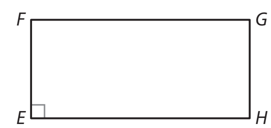
$m\angle BAD +$ **$m\angle BAD$** $=$ **$180°$** by substitution, $2 \cdot m\angle BAD = 180°$,

and $m\angle BAD = 90°$. A similar argument shows that the other angles

of *ABCD* are also **right** angles, so *ABCD* is a **rectangle**.

Reflect

3. **Discussion** Explain why this is a true condition for rectangles:
If one angle of a parallelogram is a right angle, then the parallelogram is a rectangle.

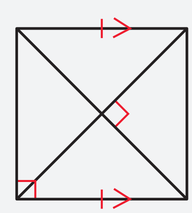


Suppose ∠E is a right angle. Opposite angles in a parallelogram are congruent, so ∠G is also a right angle. Consecutive angles in a parallelogram are supplementary. When one of two supplementary angles is a right angle, then both are right angles. So ∠F and ∠H are also right angles. Since all four angles are right angles, the parallelogram is a rectangle.

COLLABORATIVE LEARNING

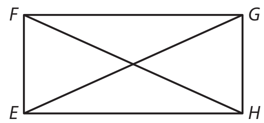
Small Group Activity

Ask students to work in small groups to classify the figure. Have each student write a conjecture about one of the *most* special figures possible: parallelogram, rectangle, rhombus, or square. Ask them to justify their conjectures to group members using the theorems they have learned in this lesson. Then have a student volunteer present the group's results to the class.



Your Turn

Use the given information to determine whether the quadrilateral is necessarily a rectangle. Explain your reasoning.



4. Given: $\overline{EF} \cong \overline{GF}$, $\overline{FG} \cong \overline{HE}$, $\overline{FH} \cong \overline{GE}$

Yes; the figure is a parallelogram because of congruent opposite sides, and it is a rectangle because it is a parallelogram with congruent diagonals.

5. Given: $m\angle FEG = 45°$, $m\angle GEH = 50°$

No; by the Angle Addition Postulate, $m\angle FEH = 45° + 50° = 95°$, so $\angle FEH$ is not a right angle and *EFGH* is not a rectangle.

Explain 2 Proving Conditions for Rhombuses

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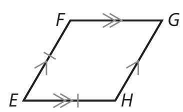
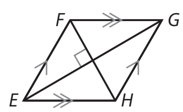
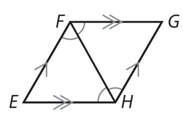
You can also use given properties of a parallelogram to determine whether the parallelogram is a rhombus.

Theorems: Conditions for Rhombuses	
If one pair of consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus.	
If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.	
If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus.	

You will prove one of the theorems about rhombuses in Example 2 and the other theorems in Your Turn Exercise 6 and Evaluate Exercise 22.

EXPLAIN 2

Proving Conditions for Rhombuses

QUESTIONING STRATEGIES

How can you use the diagonals of a parallelogram to classify a figure as a rhombus? **You can show the diagonals are perpendicular, then apply the theorem that if a parallelogram has perpendicular diagonals, it is a rhombus.**

DIFFERENTIATE INSTRUCTION

Communicating Math

Have a student say aloud four words, one of which does not fit with the other three. Have another student identify which word does not belong and explain to the class why. For example, the first student might say, “rhombus, rectangle, square, equilateral triangle.” A possible response is that the rectangle does not belong because it does not necessarily have all sides congruent.

INTEGRATE MATHEMATICAL PRACTICES

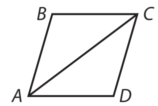
Focus on Communication

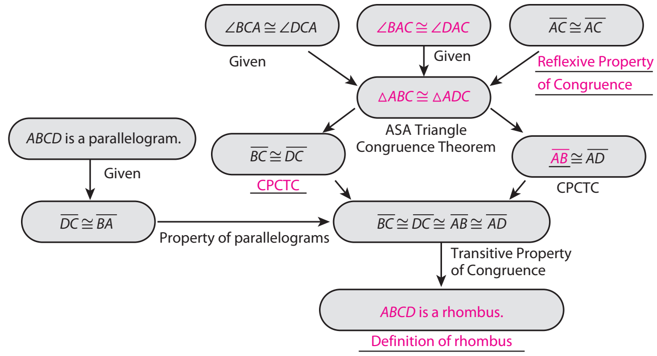
MP.3 When students make statements about what conditions prove that a parallelogram is a rhombus or any other special quadrilateral, encourage them to write a complete statement of the given information and then compare the form and content with the theorems or other statements in the lesson.

Example 2 **Complete the flow proof that if one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus.**

Given: *ABCD* is a parallelogram; $\angle BCA \cong \angle DCA$; $\angle BAC \cong \angle DAC$

Prove: *ABCD* is a rhombus.





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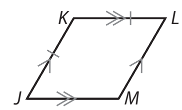
Your Turn

6. Prove that If one pair of consecutive sides of a parallelogram are congruent, then it is a rhombus.

Given: *JKLM* is a parallelogram. $\overline{JK} \cong \overline{KL}$

Prove: *JKLM* is a rhombus.



It is given that $\overline{JK} \cong \overline{KL}$. Because opposite sides of a parallelogram are congruent, $\overline{KL} \cong \overline{MJ}$ and $\overline{JK} \cong \overline{LM}$. By substituting the sides $\overline{JK}$ for $\overline{KL}$ and visa versa, $\overline{JK} \cong \overline{MJ}$ and $\overline{KL} \cong \overline{LM}$. So, $\overline{JK} \cong \overline{KL} \cong \overline{LM} \cong \overline{MJ}$, making *JKLM* a rhombus.

LANGUAGE SUPPORT EL

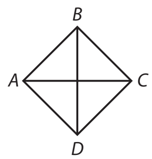
Connect Vocabulary

Students may have difficulty distinguishing the conditions for rectangles, rhombuses, and squares. Have them write the conditions on note cards and then list all the special quadrilaterals that can be further classified if those conditions are met. Have them group the note cards based on the type of figure.

Explain 3 Applying Conditions for Special Parallelograms

In Example 3, you will decide whether you are given enough information to conclude that a figure is a particular type of special parallelogram.

Example 3 **Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.**



(A) **Given:** $\overline{AB} \cong \overline{CD}$; $\overline{BC} \cong \overline{DA}$; $\overline{AD} \perp \overline{DC}$; $\overline{AC} \perp \overline{BD}$

Conclusion: *ABCD* is a square.

To prove that a given quadrilateral is a square, it is sufficient to show that the figure is both a rectangle and a rhombus.

Step 1: Determine if *ABCD* is a parallelogram.

$\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$ are given. Since a quadrilateral with opposite sides congruent is a parallelogram, we know that *ABCD* is a parallelogram.

Step 2: Determine if *ABCD* is a rectangle.

Since $\overline{AD} \perp \overline{DC}$, by definition of perpendicular lines, $\angle ADC$ is a right angle. A parallelogram with one right angle is a rectangle, so *ABCD* is a rectangle.

Step 3: Determine if *ABCD* is a rhombus.

$\overline{AC} \perp \overline{BD}$. A parallelogram with perpendicular diagonals is a rhombus. So *ABCD* is a rhombus.

Step 4: Determine if *ABCD* is a square.

Since *ABCD* is a rectangle and a rhombus, it has four right angles and four congruent sides. So *ABCD* is a square by definition.

So, the conclusion is valid.

(B) **Given:** $\overline{AB} \cong \overline{BC}$

Conclusion: *ABCD* is a rhombus.

The conclusion is not valid. It is true that if two consecutive sides of a **parallelogram** are congruent, then the **parallelogram** is a **rhombus**. To apply this theorem, however, you need to know that *ABCD* is a **parallelogram**. The given information is not sufficient to conclude that the figure is a parallelogram.

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EXPLAIN 3

Applying Conditions for Special Parallelograms

INTEGRATE MATHEMATICAL PRACTICES

Focus on Critical Thinking

MP.3 Some math textbooks define a rectangle as a parallelogram with one right angle. Point out to students that this definition is equivalent to "a quadrilateral with four right angles," because if one angle of a parallelogram is a right angle, the other three angles must also be right (opposite angles are equal; consecutive angles are supplementary).

QUESTIONING STRATEGIES

? How do you determine what additional information is needed to make a conclusion valid? **Sample answer: Make sure all parts of the hypothesis of the statement are given or established as true. Then, the conclusion is valid (by the law of detachment).**

? Can there be more than one way to demonstrate that a conclusion is valid? Explain. **Sample answer: Yes; for example, you can also prove that a given quadrilateral is a rectangle, rhombus, or square by using the definitions of the special quadrilaterals.**

ELABORATE

QUESTIONING STRATEGIES

How are these theorems different from those in the previous lesson? **They are converses; here we know the property and are trying to prove the parallelogram type.**

What is sufficient to prove that a quadrilateral is a square? **Prove that it is both a rectangle and a rhombus.**

SUMMARIZE THE LESSON

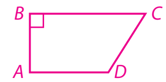
Have students fill in the blanks in the table below to summarize the conditions that lead to special parallelograms.

If a parallelogram has ______	… then the parallelogram is a ______.
one right angle	**rectangle**
congruent diagonals	rectangle
one pair of consecutive sides congruent	**rhombus**
perpendicular diagonals	rhombus
one diagonal that bisects a pair of opposite angles	**rhombus**

Reflect

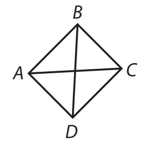
7. Draw a figure that shows why this statement is not necessarily true: If one angle of a quadrilateral is a right angle, then the quadrilateral is a rectangle.

Possible answer:



Your Turn

Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.



8. **Given:** $\angle ABC$ is a right angle.
Conclusion: *ABCD* is a rectangle.

The conclusion is not valid. You must also first be given that *ABCD* is a parallelogram.

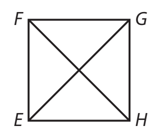
Elaborate

9. Look at the theorem boxes in Example 1 and Example 2. How do the diagrams help you remember the conditions for proving a quadrilateral is a special parallelogram?
Possible answer: The diagrams give a quick picture of the conditions stated in the theorems. The congruence marks, parallel marks, and right angles show at a glance what must be known about a figure to say it is a rectangle or a rhombus.

10. *EFGH* is a parallelogram. In *EFGH*, $\overline{EG} \cong \overline{FH}$. Which conclusion is incorrect?

A. *EFGH* is a rectangle.

B. *EFGH* is a square.



Conclusion B is incorrect. The diagonals of *EFGH* are congruent, so the parallelogram is a rectangle. However, we are given no information about how the sides are related, so we cannot conclude that it is a square.

11. **Essential Question Check-In** How are theorems about conditions for parallelograms different from the theorems regarding parallelograms used in the previous lesson?
The theorems in this lesson are the converses of the theorems in the previous lesson. In this lesson, information known about the sides, angles, or diagonals of a figure is used to prove whether the figure is a parallelogram, rectangle, rhombus, or square.

Evaluate: Homework and Practice

- Online Homework
- Hints and Help
- Extra Practice

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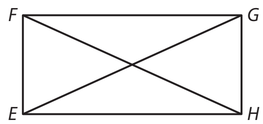
1. Suppose Anna draws two line segments, $\overline{AB}$ and $\overline{CD}$ that intersect at point E. She draws them in such a way that $\overline{AB} \cong \overline{CD}$, $\overline{AB} \perp \overline{CD}$, and $\overline{AB}$ and $\overline{CD}$ bisect each other. What is the best name to describe $ACBD$? Explain.

Square; because the diagonals of the quadrilateral bisect each other, it is a parallelogram; because the diagonals are congruent, it is a rectangle and because the diagonals are perpendicular, it is a rhombus. A figure that is both a rectangle and a rhombus must be a square.

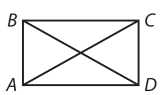
2. Write a two-column proof that if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.
Given: $EFGH$ is a parallelogram; $\overline{EG} \cong \overline{HF}$.
Prove: $EFGH$ is a rectangle.



Statements	Reasons
1. $EFGH$ is a parallelogram; $\overline{EG} \cong \overline{HF}$.	1. Given
2. $\overline{EF} \cong \overline{GH}$	2. If a quadrilateral is a parallelogram, then its opposite sides are congruent.
3. $\overline{EH} \cong \overline{EH}$	3. Reflexive Property of Congruence
4. $\triangle EFH \cong \triangle HGE$	4. SSS Triangle Congruence Theorem
5. $\angle FEH \cong \angle GHE$	5. CPCTC
6. $\angle FEH$ and $\angle GHE$ are supplementary.	6. Consecutive angles of a parallelogram are supplementary.
7. $m\angle FEH = 90°$	7. Congruent supplementary angles are right angles.
8. $EFGH$ is a rectangle.	8. Definition of rectangle

Determine whether each quadrilateral must be a rectangle. Explain.

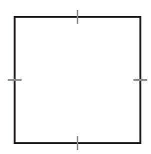
3.



Given: $BD = AC$

No information is known about its sides or angles, so it may not be a parallelogram. So, it cannot be determined if it is a rectangle.

4.



No information about the angles is known, so it cannot be determined if it is a rectangle.

EVALUATE

Personal Math Trainer

ASSIGNMENT GUIDE

Concepts and Skills	Practice
Explore Properties of Rectangles, Rhombuses, and Squares	Exercise 1
Example 1 Proving that Congruent Diagonals Is a Condition for Rectangles	Exercises 2–4
Example 2 Proving Conditions for Rhombuses	Exercises 5–7
Example 3 Applying Conditions for Special Parallelograms	Exercises 8–16

INTEGRATE MATHEMATICAL PRACTICES

Focus on Communication

MP.3 Some students may not realize how important each word is in a definition or theorem. To explain one of the theorems in this lesson, ask students to focus on exactly what they know about a given parallelogram (or quadrilateral) in order to make a conclusion about how to further classify the parallelogram. Tell them to make sure that the statement they are trying to prove contains no more and no less information than is needed to proceed deductively to the conclusion.

Exercise	Depth of Knowledge (D.O.K.)	CA CC Mathematical Practices
1–10	2 Skills/Concepts	MP.2 Reasoning
11–16	2 Skills/Concepts	MP.5 Using Tools
17–18	2 Skills/Concepts	MP.4 Modeling
19	2 Skills/Concepts	MP.2 Reasoning
20	3 Strategic Thinking H.O.T.	MP.2 Reasoning
21	3 Strategic Thinking H.O.T.	MP.3 Logic
22	3 Strategic Thinking H.O.T.	MP.3 Logic

INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1 Before doing the exercises, you may want to review the conditions for rectangles, rhombuses, and squares. In particular, if a parallelogram

- has one right angle, it is a rectangle.
- has congruent diagonals, it is a rectangle.
- has congruent consecutive sides, it is a rhombus.
- has perpendicular diagonals, it is a rhombus.
- is a rectangle and a rhombus, it is a square.

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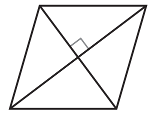
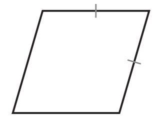
Each quadrilateral is a parallelogram. Determine whether each parallelogram is a rhombus or not.

5.



Rhombus; a parallelogram with perpendicular diagonals is a rhombus.

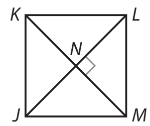
6.



Rhombus; a parallelogram with a pair of consecutive sides congruent is a rhombus.

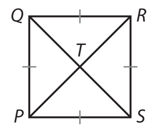
Give one characteristic about each figure that would make the conclusion valid.

7. Conclusion: *JKLM* is a rhombus.



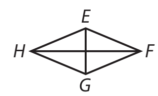
You need to know that *JKLM* is a parallelogram.

8. Conclusion: *PQRS* is a square.



Possible answer: You need to know that ∠*QPS* is a right angle.

Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.



9. Given: $\overline{EG}$ and $\overline{FH}$ bisect each other. $\overline{EG} \perp \overline{FH}$

Conclusion: *EFGH* is a rhombus.

The conclusion is valid.

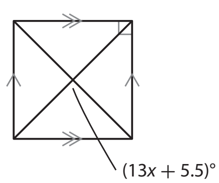
10. $\overline{FH}$ bisects ∠*EFG* and ∠*EHG*.

Conclusion: *EFGH* is a rhombus.

The conclusion is not valid. You need to know that *EFGH* is a parallelogram.

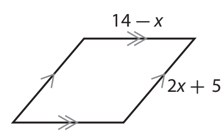
Find the value of *x* that makes each parallelogram the given type.

11. square



$x = 6.5$

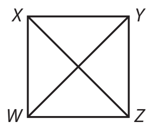
12. rhombus



$3 = x$

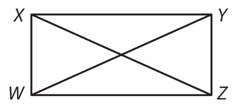
In Exercises 13–16, determine which quadrilaterals match the figure: parallelogram, rhombus, rectangle, or square? List all that apply.

13. Given: $\overline{XY} \cong \overline{ZW}$, $\overline{XY} \parallel \overline{ZW}$, $\overline{WY} \cong \overline{XZ}$, $\overline{WY} \perp \overline{XZ}$



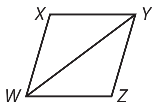
parallelogram, rhombus, rectangle, square

14. Given: $\overline{XY} \cong \overline{ZW}$, $\overline{XW} \cong \overline{ZY}$, $\overline{WY} \cong \overline{ZX}$



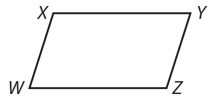
parallelogram, rectangle

15. Given: $\angle WXY \cong \angle YZW$, $\angle XWZ \cong \angle ZYX$, $\angle XWY \cong \angle YWZ$, $\angle XYW \cong \angle ZYW$



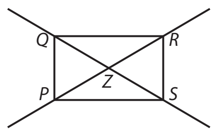
parallelogram, rhombus

16. Given: $m\angle WXY = 130°$, $m\angle XWZ = 50°$, $m\angle WZY = 130°$



parallelogram

17. **Represent Real-World Problems** A framer uses a clamp to hold together pieces of a picture frame. The pieces are cut so that $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$. The clamp is adjusted so that PZ, QZ, RZ, and SZ are all equal lengths. Why must the frame be a rectangle?



Since both pairs of opposite sides are congruent, *PQRS* is a parallelogram. Since *PZ*, *QZ*, *RZ*, and *SZ* are all equal lengths, $PZ + RZ = QZ + SZ$. So $\overline{QS} \cong \overline{PR}$. Since the diagonals are congruent, *PQRS* is a rectangle.

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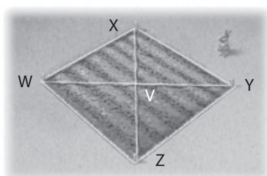
18. **Represent Real-World Problems** A city garden club is planting a square garden. They drive pegs into the ground at each corner and tie strings between each pair. The pegs are spaced so that $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$. How can the garden club use the diagonal strings to verify that the garden is a square?



Because both pairs of opposite sides of the quadrilateral garden are congruent, the garden is a parallelogram. All four sides are congruent, so it is a rhombus. The club members can measure the lengths of the diagonals to see if they are equal. Then, the parallelogram is a rectangle. If the garden is a rhombus and a rectangle, then it is a square.

19. A quadrilateral is formed by connecting the midpoints of a rectangle. Which of the following could be the resulting figure? Select all that apply.

- ● parallelogram
- ○ rectangle
- ● rhombus
- ○ square

AVOID COMMON ERRORS

Students may be confused about how to use the theorems in this lesson. Explain how some of the theorems in the lesson can be used as alternate definitions. For example, some people define a rectangle as a parallelogram with one right angle. In this case, the remaining properties and the definition as a quadrilateral with four right angles follow.

PEER-TO-PEER DISCUSSION

Ask students to work with a partner to make a physical model of a parallelogram with paper strips and brads. Ask them to manipulate the side lengths and angle measures in the parallelogram to discover the conditions necessary for a rectangle, rhombus, or square. Have students take turns making conjectures about how to get these special figures from a parallelogram.

JOURNAL

Have students explain the relationships between parallelograms, rectangles, rhombuses, and squares.

H.O.T. Focus on Higher Order Thinking

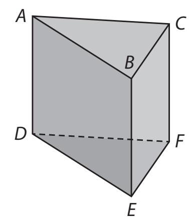
20. **Critical Thinking** The diagonals of a quadrilateral are perpendicular bisectors of each other. What is the best name for this quadrilateral? Explain your answer.

Rhombus; Since the diagonals bisect each other, the quadrilateral is a parallelogram. Since the diagonals are perpendicular, the parallelogram is a rhombus.

21. **Draw Conclusions** Think about the relationships between angles and sides in this triangular prism to decide if the given face is a rectangle.

Given: $\overline{AC} \cong \overline{DF}$, $\overline{AB} \cong \overline{DE}$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{BE} \perp \overline{EF}$, $\overline{BC} \parallel \overline{EF}$

Prove: *EBCF* is a rectangle.



It is given that $\overline{AC} \cong \overline{DF}$ and $\overline{AB} \cong \overline{DE}$. Since $\overline{AB} \perp \overline{BC}$, $\angle ABC$ is a right angle. And since $\overline{DE} \perp \overline{EF}$, $\angle DEF$ is a right angle. By the Hypotenuse-Leg (HL) Triangle Congruence Theorem, $\triangle ABC \cong \triangle DEF$. By CPCTC, $\overline{BC} \cong \overline{EF}$. Since the opposite sides of *EBCF* are parallel and congruent, it is a parallelogram. Since $\overline{BE} \perp \overline{EF}$, then $\angle BEF$ is a right angle, which makes *EBCF* a rectangle.

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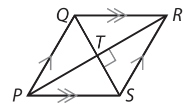
22. **Justify Reasoning** Use one of the other rhombus theorems to prove that if the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.



Given: *PQRS* is a parallelogram. $\overline{PR} \perp \overline{QS}$

Prove: *PQRS* is a rhombus.

Statements	Reasons
1. *PQRS* is a parallelogram.	1. Given
2. $\overline{PT} \cong \overline{RT}$	2. Diagonals of a parallelogram bisect each other.
3. $\overline{QT} \cong \overline{QT}$	3. Reflexive Property of Congruence
4. $\overline{PR} \perp \overline{QS}$	4. Given
5. $\angle QTP$ and $\angle QTR$ are right angles.	5. **Definition of perpendicular lines**
6. $\angle QTP \cong \angle QTR$	6. **Right angles are congruent.**
7. $\triangle QTP \cong \triangle QTR$	7. **SAS Congruence Criterion**
8. $\overline{QP} \cong \overline{QR}$	8. CPCTC
9. *PQRS* is a rhombus.	9. **If one pair of consecutive sides of a parallelogram are congruent, then it is a rhombus.**

Lesson Performance Task

The diagram shows the organizational ladder of groups to which tigers belong.

a. Use the terms below to create a similar ladder in which each term is a subset of the term above it.

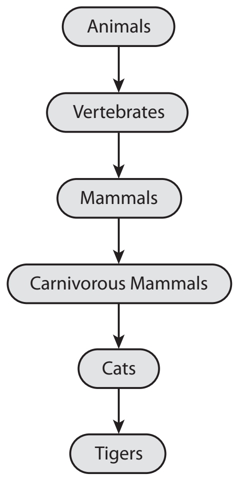
Parallelogram	Geometric figures	Squares
Quadrilaterals	Figures	Rhombuses

b. Decide which of the following statements is true. Then write three more statements like it, using terms from the list in part (a).

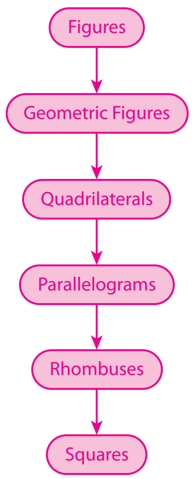
If a figure is a rhombus, then it is a parallelogram.

If a figure is a parallelogram, then it is a rhombus.

c. Explain how you can use the ladder you created above to write if-then statements involving the terms on the list.



a.



b. The true statement is "If a figure is a rhombus, then it is a parallelogram." Other statements will vary.

c. The term following "If" must be below the term following "then."

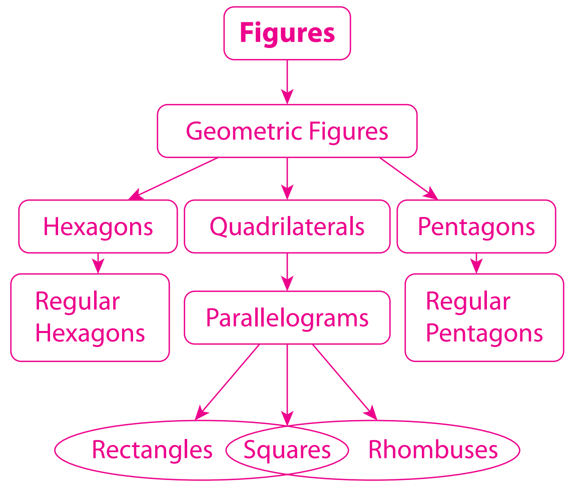
EXTENSION ACTIVITY

Have students draw the *Animals* ladder and the *Figures* ladder, using large boxes for each step. In each box, have them write information about the subject of the box, beginning by referring to the subject of the box above. For example, in the *Tigers* box they would begin, "A tiger is a cat that ...." In the *Parallelogram* box they would begin, "A parallelogram is a quadrilateral that ...." Students will likely need to research information for the *Animals* ladder. Encourage them to write precise, concise information, describing the main properties that distinguish the subject of the box and not digressing to discuss other interesting but irrelevant details.

INTEGRATE MATHEMATICAL PRACTICES

Focus on Modeling

MP.4 Have students redraw the Figures ladder, adding a box for each of these categories: Hexagons, Regular Hexagons, Pentagons, Regular Pentagons, and Rectangles.



INTEGRATE MATHEMATICAL PRACTICES

Focus on Math Connections

MP.1
- Define the term *square* using the word *regular*.

  A square is a regular rectangle.

- Define the term *rhombus* using the word *regular*.

  A rhombus is a regular parallelogram.

Scoring Rubric

2 Points: Student correctly solves the problem and explains his/her reasoning.

1 point: Student shows good understanding of the problem but does not fully solve or explain.

0 points: Student does not demonstrate understanding of the problem.