

## PHYS:1200 LECTURE 9 – MECHANICS (8)

In this lecture we will discuss three topics: conservation of energy, friction, and uniform circular motion.

**9-1. Conservation of Energy.**—Energy is one of the most fundamental and useful concepts in Physics because, as far as we can determine, it is always conserved. There are various forms of energy and we find that it can be **transformed** from one form into another. For considerations of the dynamics of objects, we define two general classes of energy: kinetic energy and potential energy. Although the reasons for introducing the concepts of kinetic and potential energy may not be initially clear, it is their connection in the conservation of energy principle that justifies them as important and useful.

*a. Kinetic energy.*—In the last lecture the concept of kinetic energy was introduced. Kinetic energy (KE) is the energy that an object has by virtue of its motion:

<b>KINETIC ENERGY</b>	$KE = \frac{1}{2}mv^2$	[1]
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KE is measured in Joules (J) when m is in kg and v is in m/s. The kinetic energy of an object does not depend on which direction the object is moving, up, down, left, right, sidewise, etc. **KE depends only on the mass and speed of the object.**

*b. Work.*—The concept of work was also introduced in terms of the applied force, F, and the distance, s, over which the force displaces the object

<b>WORK</b>	$W = F s$	[2]
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Work is also measured in J for F in N and s in m. The connection between kinetic energy and work is that the kinetic energy of an object can be used to perform work (**slide 2**).

*c. Gravitational potential energy.*—Another form of energy is **gravitational potential energy (GPE)** which is the **energy an object has because of its position relative to the earth's surface**. If an object is moving we can clearly see that it has kinetic energy, however, gravitational potential energy is a bit less obvious to see. Suppose you lift a weight 1 m up from the ground and hold it

there. Does it have energy while it is held stationary? Clearly it has no KE. However, it has GPE because it was raised above the ground. If we let it fall, the GPE is then changed into KE which can then be used to do work. If it falls on a bottle on the ground, it does work on it by smashing it to pieces. Think about the example of throwing a ball straight up into the air. To make it go up we must exert an upward force on it while it is in our hand --- we are doing work on it. Our work is then transformed into KE as it moves upward. At the very top it stops instantaneously; where did the KE go? We say that as it rises its KE is transformed into GPE. All of the KE is changed to GPE when it is at rest instantaneously at the very top. We can also say that all of the work we put into it to get it moving up is then stored as GPE at the very top. Now, as it falls back down, its GPE is then converted back to KE, and when it gets back to its starting point it all the KE that it was given initially when the work was done to make it go up. Can the amount of GPE that an object has be quantified? Clearly, the higher I lift an object, the more KE it will have when it falls back to the ground. Also the more it weighs, the more GPE it will have. This tells us that if I lift an object of weight  $w = mg$  off of the ground to a height  $h$  its GPE is

<b>GRAVITATIONAL POTENTIAL ENERGY</b>	$GPE = wh = mgh$	[3]
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and is also measured in J.

*d. Conservation of mechanical energy.*—In situations in which all forms of dissipation: friction, air resistance, etc., can be ignored, the sum of the kinetic and potential energy of an object is conserved. This is the principle of **Conservation of Mechanical Energy**. An object is given some KE, which is then converted to GPE, which is then converted back to KE. At any point in the process the sum of the **KE + GPE is constant**. If we add KE and GPE we call this mechanical energy  **$E = KE + GPE$** , and the principle says that  **$E$  is constant** (i.e., is conserved). The mechanical energy can swing back and forth between KE and GPE but the sum remains the same. (**see the examples on slides 4, 5, & 6**).

For example, suppose we throw a ball of mass  $m$  straight up with an initial speed  $v_0$ . This endows the object with a kinetic energy  $KE = \frac{1}{2} mv_0^2$ . This kinetic energy originated in the work we had to do on the ball to throw it up against the force of gravity. As the ball rises, its KE decreases – but this energy is not lost—it is converted to gravitational PE. At the highest point  $h$ ,

the ball's KE = 0 and its GPE is at its maximum value  $mgh$ . As the ball falls back to the ground, its GPE is then converted back to KE, and when it returns to its original position it has only KE.

**Example 9-1:** A mass of 10 kg is dropped from rest to the ground from a height of 5 m. (a) What kinetic energy will the mass have when it hits the ground? (b) what is its speed as it hits the ground?

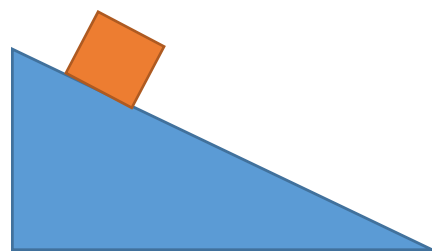
Solution: (a) At a height of 5 m above the ground, the mass has a gravitational potential energy of  $GPE = mgh = 10 \text{ kg} \times 10 \text{ m/s}^2 \times 5 \text{ m} = 500 \text{ J}$ . Since the total energy is conserved, all of this GPE will be converted to KE when the mass returns to the ground. So  $KE = 500 \text{ J}$ .

$$(b) KE = \frac{1}{2}mv^2 \rightarrow 500 \text{ J} = \frac{1}{2}(10)v^2 = 5v^2 \rightarrow v^2 = 100 \text{ m}^2/\text{s}^2 \rightarrow v = 10 \text{ m/s}.$$

*e. Other forms of potential energy.*—Gravitational potential energy is not the only form of potential energy that we know. Springs have what is called elastic potential energy. Suppose a spring is compressed and then locked in that state by some mechanism. Even though there is no motion of the compressed spring, there is energy there because when it is released its elastic potential energy can be used to make kinetic energy (**slide 7**).

**9-2. Friction.**—Friction is a force that acts between 2 surfaces that are in contact. It is a **dissipative force** that always **opposes motion**. If you give an object a push to get it moving, friction will eventually make it stop moving. All the energy that you give to the object is lost to friction. It is actually a very complicated force that acts at the microscopic level (see slide 9). When two objects are in contact, e.g., a box on a surface, the force of friction between them depends on whether the box is moving or not. There are two types of friction forces: **static friction** and **kinetic friction**.

*a. Static friction.*—Imagine an object on an inclined plane as shown on the right (**slide 13**). If the friction force is large enough, the block will remain at rest on the plane even though gravity is acting to pull it down the incline. This shows



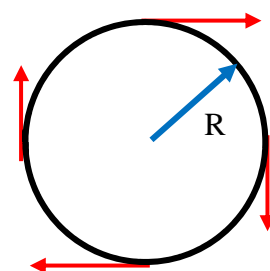
that there is a friction force acting against gravity. In this case it is called a **static friction** force because there is no motion. If we could make the incline steeper, then at some point the force

of gravity would overcome the static friction force and the block would slide down. Static friction can have any value t up to a **maximum value**.

*b. Kinetic friction.*—The friction force that acts when an object is moving is called **kinetic friction**. The **maximum force of static friction is larger than the force of kinetic friction**. (slide 12). This is why it takes more force to initially get an object moving than to keep it moving.

*c. Energy conservation and friction.*—Let's consider now the effect of friction on the kinetic energy of an object. When we push an object that is at rest, we are doing work on it. The work we do then appears as the kinetic energy of the object that is moving. If there is friction however, the block does not continue moving but slows down and stops. What became of the kinetic energy? It looks like energy was not conserved in this example. We must always look closely at every situation to keep track of the energy. Friction involves rubbing of one surface against another. You know that when two objects are rubbed together, if you feel the surfaces, they tend to get warm. This is an important clue to consider for the energy consideration. The fact that the surfaces become warm due to friction means that the kinetic energy was converted to another form of energy→ **heat energy**, and if we measure things carefully we find that no energy was actually lost it was just converted into another form of energy. The subject of heat energy is very important and we will spend an entire unit discussing it later in this course.

**9-3. Uniform Circular Motion.**—**Uniform circular motion** is the motion of an object in a **circular path** with constant **speed**. The important term here is speed, because when an object moves in a circular path its velocity (speed and direction) is **constantly changing** direction. The diagram on the right shows the path of an object moving in the clockwise direction in a circle of radius  $R$ . The red arrows show the instantaneous velocity at 4 different points. Even though the speed is not changing, **the object is in an accelerated state of motion because the direction of its velocity is constantly changing from one instant of time to the next**. A simple example of uniform circular motion occurs when you tie a string around a stone then whirl the stone around your head at a steady rate in a horizontal circle (slide 14). In the example shown on slide 14, if the string were to break at any time, the



stone would continue to move in the same direction that it was moving at the instant when the string broke.

*a. Centripetal acceleration.*—The acceleration of an object moving with speed  $v$  in a circular path of radius  $R$  can be found using simple geometry. We give this acceleration a special name because it is a very common type of motion – **centripetal acceleration**, and give it a special symbol  $a_c$ . The magnitude of the centripetal acceleration is

<b>CENTRIPETAL ACCELERATION</b>	$a_c = \frac{v^2}{R}.$	[4]
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The acceleration is higher when the speed is higher, and is higher for smaller circles. What is the direction of the centripetal acceleration? **The direction of the centripetal acceleration is always toward the center of the circle**; the term centripetal means toward the center (**slides 16, 17**). As with any acceleration,  $a_c$  is measured in  $m/s^2$ .

*b. Centripetal force.*—The inertia of an object causes it to move in a straight line with constant speed unless it is acted on by a force. Clearly, an object in circular motion is not moving in a straight line and this means that a force is required to cause it to move in a circular path. The force that, in a particular situation, causes an object to move in a circle is called **a centripetal force,  $F_c$** . The force that produces the centripetal acceleration must be identified in any particular situation. For the stone whirled around in a circle, the centripetal force is produced by the tension in the string. According to Newton's 2<sup>nd</sup> law, the net force producing an acceleration always points in the direction of the acceleration, so the **centripetal force must point toward the center of the circular path of the object, and have a magnitude**

<b>CENTRIPETAL FORCE</b>	$F_c = ma_c = m \frac{v^2}{R}.$	[5]
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If you try to whirl a stone around in a circle at high speed and the string is not strong enough to provide the required tension, it will break and the object will move initially in a straight line.

Whenever a car makes a turn it is moving in a partial circle (90 degrees) and it is accelerated and this requires a force. Where does this force come from? It is provided by an inward force of friction between the tires and the road (**slide 21**). If the road is slippery because of rain or ice, the friction force may be insufficient, and the car may fail to negotiate the turn, ending up in the ditch on the side of the road. Since the centripetal acceleration is inversely proportional to the radius of the turn, more centripetal force is required for tight turns (small radius of curvature). On very tight turns, such as on-ramps and off-ramps, often a “**maximum safe speed**” will be posted to ensure that cars will be able to safely round the curve. The turns on race tracks are banked at large angles to permit driver to maintain high speed going around the curve. The turns on the Daytona Speedway shown below are banked at 31 degrees.



c. *Centrifugal force*.—When the subject of circular motion is discussed, it is not uncommon to hear mention of the word *centrifugal*. **Centrifugal**, not to be confused with **centripetal**, means away from the center or outward. The use of or at least the familiarity with this word centrifugal, combined with the common sensation of an outward lean when experiencing circular motion,

often creates or reinforces a common student misconception. The common misconception is the notion that objects in circular motion are experiencing an outward force. "After all," a well-meaning person may say, "I can recall vividly the sensation of being thrown outward away from the center of the circle on that roller coaster ride. Therefore, circular motion must be characterized by an outward force." Centrifugal force is an **apparent force** experienced by a person who is in the roller coaster car that is going around a turn. **It is not a real force. (see slide 23).**