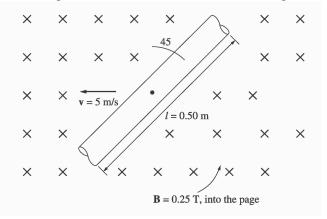
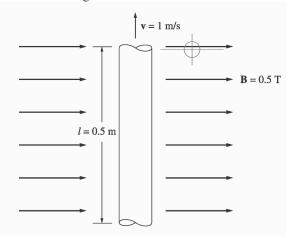
1-10. The wire is shown in Figure P1-7 is moving in the presence of a magnetic field. With the information given in the figure, determine the magnitude and direction of the induced voltage in the wire.



 $S_{OLUTION}$ The induced voltage on this wire can be calculated from the equation shown below. The voltage on the wire is positive downward because the vector quantity $\mathbf{v} \times \mathbf{B}$ points downward.

$$e_{ind} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{I} = vBl \cos 45^{\circ} = (5 \text{ m/s})(0.25 \text{ T})(0.50 \text{ m}) \cos 45^{\circ} = 0.442 \text{ V}$$
, positive down

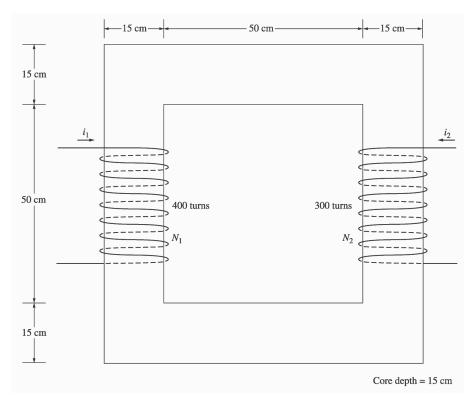
1-11. Repeat Problem 1-10 for the wire in Figure P1-8.



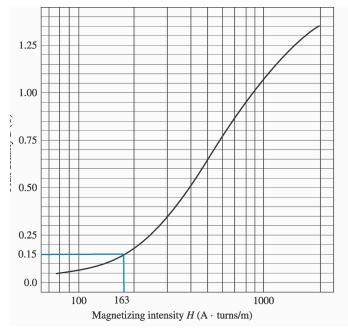
Solution The induced voltage on this wire can be calculated from the equation shown below. The total voltage is zero, because the vector quantity $\mathbf{v} \times \mathbf{B}$ points into the page, while the wire runs in the plane of the page.

$$e_{\text{ind}} = (\mathbf{v} \times \mathbf{B}) \cdot \mathbf{l} = vBl \text{ cos } 90^{\circ} = (1 \text{ m/s})(0.5 \text{ T})(0.5 \text{ m}) \text{ cos } 90^{\circ} = 0 \text{ V}$$

1-12. The core shown in Figure P1-4 is made of a steel whose magnetization curve is shown in Figure P1-9. Repeat Problem 1-7, but this time do *not* assume a constant value of μ_{Γ} . How much flux is produced in the core by the currents specified? What is the relative permeability of this core under these conditions? Was the assumption in Problem 1-7 that the relative permeability was equal to 1000 a good assumption for these conditions? Is it a good assumption in general?



Solution The magnetization curve for this core is shown below:



The two coils on this core are wound so that their magnetomotive forces are additive, so the total magnetomotive force on this core is

$$\mathfrak{I}_{\text{TOT}} = N_1 i_1 + N_2 i_2 = (400 \text{ t})(0.5 \text{ A}) + (300 \text{ t})(0.75 \text{ A}) = 425 \text{ A} \cdot \text{t}$$

Therefore, the magnetizing intensity H is

$$H = \frac{\Im}{l} = \frac{425 \text{ A} \cdot \text{t}}{2.60 \text{ m}} = 163 \text{ A} \cdot \text{t/m}$$

From the magnetization curve,

$$B = 0.15 \text{ T}$$

and the total flux in the core is

$$\phi_{\text{TOT}} = BA = (0.15 \text{ T})(0.15 \text{ m})(0.15 \text{ m}) = 0.0033 \text{ Wb}$$

The relative permeability of the core can be found from the reluctance as follows:

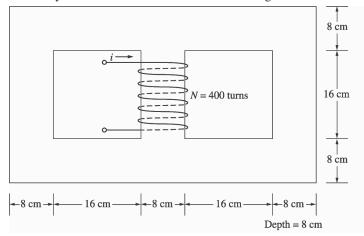
$$\mathcal{R} = \frac{\mathcal{F}_{\text{TOT}}}{\phi_{\text{TOT}}} = \frac{l}{\mu_r \mu_0 A}$$

Solving for μ_r yields

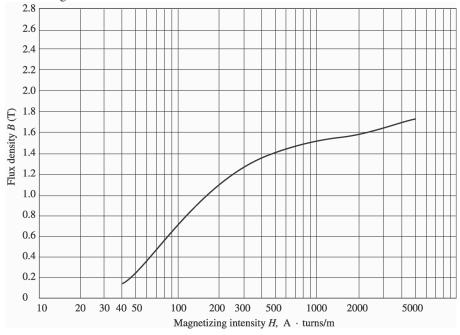
$$\mu_r = \frac{\phi_{\text{TOT}} \ l}{\mathcal{F}_{\text{TOT}} \ \mu_0 \ A} = \frac{(0.0033 \text{ Wb})(2.6 \text{ m})}{(425 \text{ A} \cdot \text{t})(4\pi \times 10^{-7} \text{ H/m})(0.15 \text{ m})(0.15 \text{ m})} = 714$$

The assumption that $\mu_r = 1000$ is not very good here. It is not very good in general.

- **1-13.** A core with three legs is shown in Figure P1-10. Its depth is 8 cm, and there are 400 turns on the center leg. The remaining dimensions are shown in the figure. The core is composed of a steel having the magnetization curve shown in Figure 1-10c. Answer the following questions about this core:
 - (a) What current is required to produce a flux density of 0.5 T in the central leg of the core?
 - (b) What current is required to produce a flux density of 1.0 T in the central leg of the core? Is it twice the current in part (a)?
 - (c) What are the reluctances of the central and right legs of the core under the conditions in part (a)?
 - (d) What are the reluctances of the central and right legs of the core under the conditions in part (b)?
 - (e) What conclusion can you make about reluctances in real magnetic cores?



Solution The magnetization curve for this core is shown below:



(a) A flux density of 0.5 T in the central core corresponds to a total flux of

$$\phi_{\text{TOT}} = BA = (0.5 \text{ T})(0.08 \text{ m})(0.08 \text{ m}) = 0.0032 \text{ Wb}$$

By symmetry, the flux in each of the two outer legs must be $\phi_1 = \phi_2 = 0.0016$ Wb, and the flux density in the other legs must be

$$B_1 = B_2 = \frac{0.0016 \text{ Wb}}{(0.08 \text{ m})(0.08 \text{ m})} = 0.25 \text{ T}$$

The magnetizing intensity H required to produce a flux density of 0.25 T can be found from Figure 1-10c. It is 50 A·t/m. Similarly, the magnetizing intensity H required to produce a flux density of 0.50 T is 70 A·t/m. Therefore, the total MMF needed is

$$\mathfrak{F}_{\text{TOT}} = H_{\text{center}} \ l_{\text{center}} + H_{\text{outer}} \ l_{\text{outer}}$$

$$\mathfrak{F}_{\text{TOT}} = (70 \text{ A} \cdot \text{t/m})(0.24 \text{ m}) + (50 \text{ A} \cdot \text{t/m})(0.72 \text{ m}) = 52.8 \text{ A} \cdot \text{t}$$

and the required current is

$$i = \frac{\mathcal{T}_{\text{TOT}}}{N} = \frac{52.8 \text{ A} \cdot \text{t}}{400 \text{ t}} = 0.13 \text{ A}$$

(b) A flux density of 1.0 T in the central core corresponds to a total flux of

$$\phi_{\text{TOT}} = BA = (1.0 \text{ T})(0.08 \text{ m})(0.08 \text{ m}) = 0.0064 \text{ Wb}$$

By symmetry, the flux in each of the two outer legs must be $\phi_1 = \phi_2 = 0.0032$ Wb, and the flux density in the other legs must be

$$B_1 = B_2 = \frac{0.0032 \text{ Wb}}{(0.08 \text{ m})(0.08 \text{ m})} = 0.50 \text{ T}$$

The magnetizing intensity H required to produce a flux density of 0.50 T can be found from Figure 1-10c. It is 70 A·t/m. Similarly, the magnetizing intensity H required to produce a flux density of 1.00 T is about 160 A·t/m. Therefore, the total MMF needed is

$$\mathfrak{F}_{\text{TOT}} = H_{\text{center}} I_{\text{center}} + H_{\text{outer}} I_{\text{outer}}$$

$$\mathfrak{F}_{\text{TOT}} = (160 \text{ A} \cdot \text{t/m}) (0.24 \text{ m}) + (70 \text{ A} \cdot \text{t/m}) (0.72 \text{ m}) = 88.8 \text{ A} \cdot \text{t}$$

and the required current is

$$i = \frac{\phi_{\text{TOT}}}{N} = \frac{88.8 \text{ A} \cdot \text{t}}{400 \text{ t}} = 0.22 \text{ A}$$

This current is less *not* twice the current in part (a).

(c) The reluctance of the central leg of the core under the conditions of part (a) is:

$$\mathcal{R}_{\text{cent}} = \frac{\mathcal{F}_{\text{TOT}}}{\phi_{\text{TOT}}} = \frac{(70 \text{ A} \cdot \text{t/m})(0.24 \text{ m})}{0.0032 \text{ Wb}} = 5.25 \text{ kA} \cdot \text{t/Wb}$$

The reluctance of the right leg of the core under the conditions of part (a) is:

$$\mathcal{R}_{\text{right}} = \frac{\mathcal{F}_{\text{TOT}}}{\phi_{\text{TOT}}} = \frac{(50 \text{ A} \cdot \text{t/m})(0.72 \text{ m})}{0.0016 \text{ Wb}} = 22.5 \text{ kA} \cdot \text{t/Wb}$$

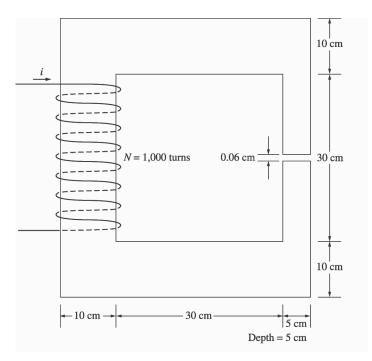
(d) The reluctance of the central leg of the core under the conditions of part (b) is:

$$\mathcal{R}_{\text{cent}} = \frac{\mathcal{F}_{\text{TOT}}}{\phi_{\text{TOT}}} = \frac{(160 \text{ A} \cdot \text{t/m})(0.24 \text{ m})}{0.0064 \text{ Wb}} = 6.0 \text{ kA} \cdot \text{t/Wb}$$

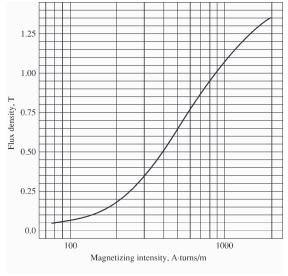
The reluctance of the right leg of the core under the conditions of part (b) is:

$$\mathcal{R}_{\text{right}} = \frac{\mathcal{F}_{\text{TOT}}}{\phi_{\text{TOT}}} = \frac{(70 \text{ A} \cdot \text{t/m})(0.72 \text{ m})}{0.0032 \text{ Wb}} = 15.75 \text{ kA} \cdot \text{t/Wb}$$

- (e) The reluctances in real magnetic cores are not constant.
- **1-14.** A two-legged magnetic core with an air gap is shown in Figure P1-11. The depth of the core is 5 cm, the length of the air gap in the core is 0.06 cm, and the number of turns on the coil is 1000. The magnetization curve of the core material is shown in Figure P1-9. Assume a 5 percent increase in effective air-gap area to account for fringing. How much current is required to produce an air-gap flux density of 0.5 T? What are the flux densities of the four sides of the core at that current? What is the total flux present in the air gap?



Solution The magnetization curve for this core is shown below:



An air-gap flux density of 0.5 T requires a total flux of

$$\phi = BA_{\text{eff}} = (0.5 \text{ T})(0.05 \text{ m})(0.05 \text{ m})(1.05) = 0.00131 \text{ Wb}$$

This flux requires a flux density in the right-hand leg of

$$B_{\text{right}} = \frac{\phi}{A} = \frac{0.00131 \text{ Wb}}{(0.05 \text{ m})(0.05 \text{ m})} = 0.524 \text{ T}$$

The flux density in the other three legs of the core is

$$B_{\text{top}} = B_{\text{left}} = B_{\text{bottom}} = \frac{\phi}{A} = \frac{0.00131 \text{ Wb}}{(0.10 \text{ m})(0.05 \text{ m})} = 0.262 \text{ T}$$

The magnetizing intensity required to produce a flux density of 0.5 T in the air gap can be found from the equation $B_{ag} = \mu_o H_{ag}$:

$$H_{\text{ag}} = \frac{B_{\text{ag}}}{\mu_0} = \frac{0.5 \text{ T}}{4\pi \times 10^{-7} \text{ H/m}} = 398 \text{ kA} \cdot \text{t/m}$$

The magnetizing intensity required to produce a flux density of 0.524 T in the right-hand leg of the core can be found from Figure P1-9 to be

$$H_{\text{right}} = 410 \text{ A} \cdot \text{t/m}$$

The magnetizing intensity required to produce a flux density of 0.262 T in the top, left, and bottom legs of the core can be found from Figure P1-9 to be

$$H_{\text{top}} = H_{\text{left}} = H_{\text{bottom}} = 240 \text{ A} \cdot \text{t/m}$$

The total MMF required to produce the flux is

$$\mathcal{F}_{TOT} = H_{ag} \ l_{ag} + H_{right} \ l_{right} + H_{top} \ l_{top} + H_{left} \ l_{left} + H_{bottom} \ l_{bottom}$$

$$\mathcal{F}_{TOT} = (398 \text{ kA} \cdot \text{t/m})(0.0006 \text{ m}) + (410 \text{ A} \cdot \text{t/m})(0.40 \text{ m}) + 3(240 \text{ A} \cdot \text{t/m})(0.40 \text{ m})$$

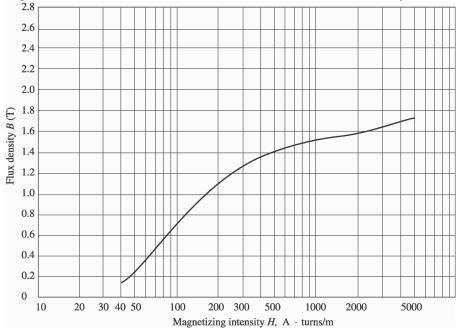
$$\mathcal{F}_{TOT} = 278.6 + 164 + 288 = 691 \text{ A} \cdot \text{t}$$

and the required current is

$$i = \frac{\mathcal{F}_{\text{TOT}}}{N} = \frac{691 \text{ A} \cdot \text{t}}{1000 \text{ t}} = 0.691 \text{ A}$$

The flux densities in the four sides of the core and the total flux present in the air gap were calculated above.

1-15. A transformer core with an effective mean path length of 10 in has a 300-turn coil wrapped around one leg. Its cross-sectional area is 0.25 in^2 , and its magnetization curve is shown in Figure 1-10c. If current of 0.25 A is flowing in the coil, what is the total flux in the core? What is the flux density?



SOLUTION The magnetizing intensity applied to this core is