Missile Guidance and Pursuit:
Kinematics, Dynamics and Control

There are three things which are too wonderful for me,
The way of an eagle in the air,
The way of a serpent on a rock,
The way of a ship in the midst of the sea.

Book of Proverbs

Armed with bows, using both the right hand and the left in hurling stones and shooting arrows with the bow.

Book of Chronicles

DR. N.A. SHNEYDOR

Neryahu A. Shneydor was born in Jerusalem, then Palestine, now Israel, in 1932. His secondary education was interrupted by the 1948 Independence War and the siege of Jerusalem. He was then 16 years old and became a junior member of the Haganah (the semi-legal military organisation) and joined the minuscule Signal Group of the city. He manned a signal station which used Morse-key operated lamps and, in daytime, heliographs, most of the equipment being of World War I vintage. This may have aroused his interest in electrical engineering. He studied Electrical Engineering in 1950-54 at the Technion - Israel Institute of Technology, Haifa, Israel, where he obtained his B.Sc. and Dipl. Ingénieur degrees. The latter was a relic from the time when Technion was under German influence: most of the founding professors being immigrants from Germany, some escapees from Nazism.

There followed four years of military service, mostly with the navy where he specialised in servo and in gunnery- and torpedo-fire control. Interest in these topics led to studies in control theory and the Master's degree. Dr Shneydor joined RAFAEL, Israel Armament Development Authority, in 1960. A few years before, it was a secret military unit called HEMED, Hebrew acronym for Science Corps. Among the early achievements of HEMED-RAFAEL were a radio-guided boat which became operational in 1955, and a sea-to-sea guided missile which evolved to the well-known Gabriel. Dr Shneydor joined the servo group of RAFAEL, which specialised in mechanisms for controlling antennas, gimbals, actuators and other devices. He later participated in the development of Shafrir and Python air-to-air missiles. For his contribution to the development of the former's guidance system, he was awarded the Security Prize, Israel's highest token of appreciation.

In 1972-75 RAFAEL enabled him to study for a Doctoral degree, again at the Haifa Technion, under the supervision of the late Professor George Zames. The thesis was in nonlinear feedback control theory. In 1970 he had started to lecture on guidance and navigation at the Technion Department of Aeronautical Engineering (now the Faculty of Aerospace Engineering) and later on nonlinear control at the Department of Electrical Engineering, as Adjunct Assistant Professor, being promoted to Adjunct Associate Professorship in 1979.

Since 1975 Dr Shneydor was occupied with research and development projects at RAFAEL. In 1982-87 and 1992-94 he was R&D Deputy Director of the Missile Division. During his professional activity he published many reports, papers and texts on guidance. Retired from RAFAEL, he continues lecturing on guidance and control and is a consulting engineer in these areas.

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Horwood Publishing
Chichester

First published in 1998 by
HORWOOD PUBLISHING LIMITED
International Publishers
Coll House, Westergate, Chichester, West Sussex, PO20 6QL
England

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British Library Cataloguing in Publication Data
A catalogue record of this book is available from the British Library

ISBN 1-898563-43-8

Printed in Great Britain by Martins Printing Group, Bodmin, Cornwall

Preface

Navigation has been with the human race from time immemorial. It is not surprising, therefore, that a very great number of books have been published on this ancient art. Guidance, on the other hand, has been first implemented, by building a remotely-guided unmanned boat for military purposes, in the beginning of this century. The technical literature on it is immense - articles, conference papers, reviews, bibliographies. However, surprisingly few books have been published that deal with guidance. If we do not count texts that are mainly descriptive, most of which appeared in the first decennary after World War II, we have less than half a dozen books in English. During my professional career as a research-and-development (R&D) engineer I also taught at a technical university and lectured for various industry and military audiences. I naturally used the existing texts, but gradually developed an approach which is different from theirs. Encouraged by colleagues and students, I eventually turned my lecture notes and transparencies into this book.

I believe this text differs from other ones in the field in several respects. Here are some of its key features.

⋆ Although it necessarilly emphasizes military applications of guidance, i.e., guided weapons, it also pays attention to guidance in nature: some real, some anecdotic, some invented by recreational mathematicians.

⋆ This book does not purport to be a history. However, it does try to give credit to pioneering scientists and to early developments and inventions.

⋆ In the theory of guidance one often has to solve differential and other equations. Wherever practical I present an analytic solution rather than resort to numerical ones: very often, the analytic solution enables one to discover interesting properties which would otherwise be obscured by lines of code and numerical data. Furthermore, most of the engineers and scientists among the readers, especially the younger ones, could easily make their own computer code wherever they wish to deepen their quantitative understanding of a problem studied.

⋆ Geometrical rules and guidance laws are stated in three-dimensional vector terms as well as the usual planar ones, and several examples have to do with three-

dimensional guidance situations.

⋆ Many graphical illustrations are given of trajectories, launch zones and intercept zones, as well as of time histories of maneuver acceleration and other important variables. This should be of practical value for many readers.

This text is intended for people — students, engineers, analysts, physicists, programmers — involved or interested in any of the various aspects of guidance systems: use, development, design, manufacture, marketing, analysis, operational research. Mathematics at a first-year university level is the only prerequisite. However, for comprehending some portions of the text, acquaintance with feedback control theory would be helpful.

Acknowledgements

I would like to thank the director of the Ecole des Mines de Paris, France, Dr. Jacques B. Levy, who made it possible for me to spend a sabbatical at the institute, when most of the work on this text was done, and Prof. Jean Lévine of the Centre Automatique et Systèmes of the Ecole des Mines for his invaluable aid during various stages of the work. I also wish to thank Prof. Aviv Rosen, Dean of the Faculty of Aerospace Engineering, Technion - Israel Institute of Technology, where I spent the last semester of the sabbatical. I was very fortunate to have the manuscript reviewed by several colleagues. The comments and suggestions made by Oded Golan and Ilan Rusnak of RAFAEL are gratefully acknowledged. I am especially indebted to Uri Reychav, also of RAFAEL, who devoted many long hours of his time during his sabbatical in the United States to going over the manuscript while it was being written in France. Particular thanks are extended to Rachel Weissbrod and Sarah Segev of RAFAEL library at Leshem and to Dr. Guy Shaviv of the Technion for their patient aid. Finally, I express my gratitude and love for my wife Na'ama, without whose support and encouragement this undertaking would not be possible.

N. A. Shneydor

Haifa, Israel
February 4, 1998

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Introduction

According to the dictionary, 'guidance' is "the process for guiding the path of an object towards a given point, which in general may be moving." If the given point, which we will call the *target*, is fixed, e.g., a sea port, or its path in the future is known with sufficient accuracy, e.g., planet Mars, then the process is usually called *navigation*. If the target moves in a way that is not quite predictable — for example, a prey escaping its predator, an aircraft evading ground-to-air missiles — then the process is *guidance* in its narrower sense, which is the sense we will give it in this text.

The guided object may be a vehicle (a car, a boat, a missile, a spacecraft), a robot or, in fact, a living being. The process of guidance is based on the position and the velocity of the target relative to the guided object. The participants in the guidance process are also referred to in the literature as the *evader* and the *pursuer*, respectively. In nature, the ways predators catch their prey and some insects rendezvous their mates are certainly guidance processes. In human history, it is said that seamen, especially those who excercised the ignoble art of sea piracy, practised the rule we now call 'parallel navigation' (the 'navigation' part of the term being of course a misnomer) or 'collision course'. Mariners in general have known the inverse rule, which they apply in order to avoid collision at sea.

Modern, i.e., analytic, approach to guidance problems dates from the eighteenth century, when several mathematicians studied what we now call 'pure pursuit' or 'hound and hare pursuit'. This pursuit follows a very straightforward *geometrical rule*: run (or fly, or sail, as the case may be) where you see your target. Both this simple rule and the aforementioned parallel-navigation are *two-point guidance* rules, called so because only the pursuer and the target are involved in their respective definitions.

A family of geometrical rules for *three-point guidance* exists as well; the name derives from the fact that a third, reference point is required for the statement of the rule. In the most basic three-point geometrical rule, the pursuer is required to be on the line between the reference point and the target. For obvious reasons, this type of guidance is called 'line-of-sight guidance'.

Most of the applications for the theory of guidance are in weaponry. History for this kind of application begins in 1870, when Werner von Siemens submitted a proposal for "the destruction of enemy vessels by guided torpedos" to the Prussian ministry of war. Although not specifically said so by Siemens, the guidance of his proposed torpedo would have been of the line-of-sight type. We shall describe this proposal briefly later on; suffice it to mention now that by 1916 it had materialized into the first operational guided-weapon system in history.

The pure-pursuit rule was first applied to weapon systems in the early 1940's, during the second world war, when most of the basic relevant theory had in fact been known for two centuries and technical means for detecting targets and for conrolling guided vehicles had been developed. Towards the end of the war, a more sophisticated type of two-point guidance, called 'proportional navigation' for historical reasons, was studied. The basic theory of proportional navigation (PN) was first formulated in the United States in 1943. Some steps towards implementing a variant of PN in missile systems were taken in 1944 or 1945 by German scientists, who presumably did not know that the theory had already been developed elsewhere. The vast majority of two-point guided weapon systems existing today utilize PN in one of its numerous variants. There are nonmilitary applications of PN, too; for example, in space travel, extraterrestrial landing, and robotics.

PN has its limitations, though. In particular one should mention sensitivity to noise and to maneuvers carried out by the evader when the pursuer is approaching it. (To 'maneuver' means here to make abrupt changes in the direction of motion, i.e., execute high-acceleration turns; in pilots' parlance, to 'jink', and in mariners' one, to 'zigzag'.) A family of so-called 'modern guidance laws' has been developing since the early 1960's that do not suffer from these limitations or suffer much less. These laws are based on several recently developed techniques, in particular optimal-control theory and optimal-estimation theory, hence the often used terms 'optimal-control guidance' or just 'optimal guidance'.

This family of laws can be regarded as the most recent stage of the evolution process that started with Siemens's proposal. It seems that in spite of the maturity of the theory and the availability of the necessary technology, mostly microelectronics and computer science, practical application is still somewhat rare, probably due to economical reasons. Needless to say, however, the secrecy that prevails over armament development issues makes up-to-date, reliable information inacessible, and therefore statements on recent developments are uncertain.

This is about as far as we go in this book. The next evolutionary stage would probably consist of laws based on differential-game theory. Although papers regarding this approach to guidance have been appearing since the 1970's, it seems that it is not ripe enough for inclusion in an introductory text like the present one.

The very fast progress of guided weaponry in the past fifty years would not be possible without advances in many technologies. One should mention internal-

combustion engines, rocket motors, inertial instrumentation (especially gyroscopes), aeronautics, electronics (especially microelectronics and radar), electro-optics, and computer engineering. These and some other technological disciplines relevant to guidance are beyond the scope of this text, except where they have direct implications regarding its main topics. There are two reasons for this exclusuion. Firstly, including even some of the relevant disciplines would have made the book much weightier than what the author had in mind; secondly, an abundant literature is available that deals with most of the said technologies.

This book regards guidance from the point of view of the pursuer, i.e., how to arrive at the target, or intercept it. The inverse problem, that of avoidance, is not dealt with. Guidance is treated from the viewpoints of kinematics, dynamics, and control. In other words, we study trajectories, zones of interception, required maneuver effort, launch envelopes, stability of the guidance process, and related topics. Furthermore, technical problems involved with implementation and mechanization are discussed when they may affect accuracy, energy expenditure, and structural limits, hence, finally, costs.

The book is organized as follows. Following Chapter 1, which presents basic definitions and terminology, Chapters 2-7 deal with what have come to be called the *classical guidance laws*, namely
⋆ Line-of-sight guidance (Chapter 2),
⋆ Pure pursuit (Chapter 3),
⋆ Parallel navigation (Chapter 4),
⋆ Proportional navigation (Chapters 5 and 6),
⋆ Several guidance laws related to proportional navigation (Chapter 7).
Chapter 8 is dedicated to optimal-control guidance.

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