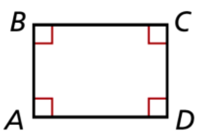
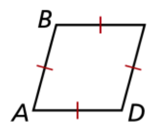
Geometry – Chapter 6 – Notes and Examples

Sections 4 and 5 – Properties and Conditions of Special Parallelograms

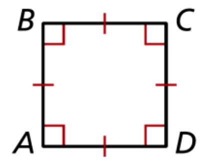
When you are given a parallelogram with certain properties, you can use those properties to determine if a parallelogram is a rectangle, rhombus or square.



Rectangle *ABCD*



Rhombus *ABCD*



Square *ABCD*

Rectangles, rhombuses, and squares are sometimes referred to as *special parallelograms*.

Use your quadrilaterals page to complete the following:

Properties of Rectangles:

If a quadrilateral is a **rectangle,** then it is a parallelogram.

If a parallelogram is a **rectangle,** then its diagonals are congruent.

Since a rectangle is a parallelogram, a rectangle "inherits" all the properties of parallelograms.

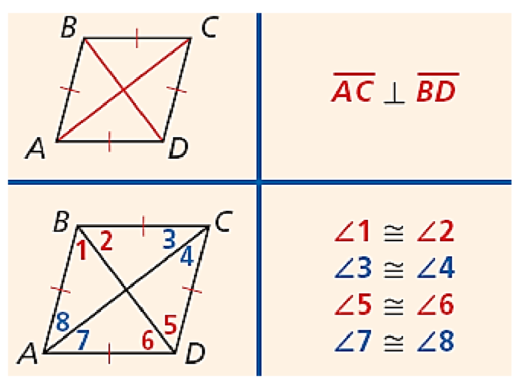
Properties of Rhombuses:

A **rhombus** is a quadrilateral with four congruent sides.

If a quadrilateral is a **rhombus,** then it is a parallelogram with one pair of consecutive sides congruent

If a parallelogram is a **rhombus,** then

- Its diagonals are perpendicular.
- Each diagonal bisects a pair of opposite angles.

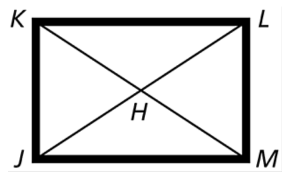


Like a rectangle, a rhombus is a parallelogram. So you can apply the properties of parallelograms to rhombuses.

Properties of Squares:

A **square** is a quadrilateral with four right angles and four congruent sides. A square is a parallelogram, a rectangle, and a rhombus. So a square has the properties of all three.

A woodworker constructs a rectangular picture frame so that JK = 50 cm and JL = 86 cm. Find HM.



$\overline{KM} \cong \overline{JL}$

$KM = JL = 86$

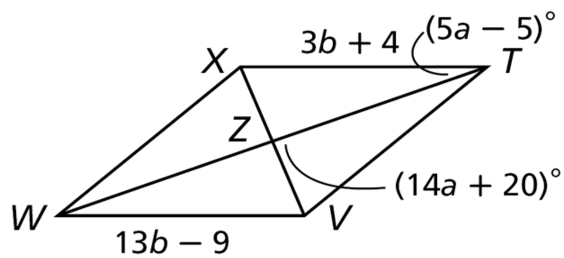
$HM = \frac{1}{2}KM$

$HM = \frac{1}{2}(86) = 43cm$

Determine whether each statement is always (A), sometimes (S) or never (N) true.

- **A rhombus is a square. ____**
- **A rectangle is a parallelogram. _____**
- **The legs of a trapezoid are congruent. ______**
- **A square is a trapezoid. ______**
- **In a rhombus, opposite angles are ≅ . ____**

***TVWX* is a rhombus. Find *TV*.**



$WV = XT$

$13b - 9 = 3b + 4$

$10b = 13$

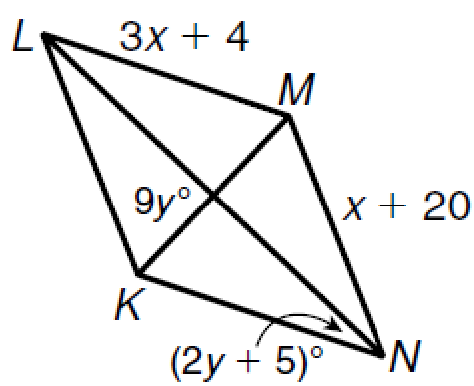
$b = 1.3$

$TV = XT$

$TV = 3b + 4$

$= 3(1.3) + 4 = 7.9$

***KLMN* is a rhombus. Find each measure.**



KL

$LM = MN$

$3x + 4 = x + 20$

$2x = 16$

$x = 8$

$KL = LM = MN = 28$

m∠MNK

$9y = 90°$

$y = 10$

$m\angle LNK = m\angle LNM$ *and*

$m\angle MNK = m\angle LNK + m\angle LNM$

$m\angle MNK = 2y + 5 + 2y + 5$

$m\angle MNK = 2(10) + 5 + 2(10) + 5 = 50$

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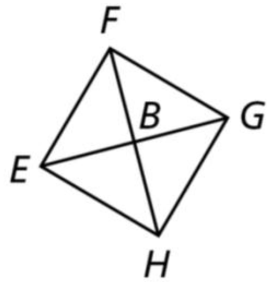
Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.
Given: EF ≅ FG and EG ⊥ FH
Conclusion: EFGH is a rhombus


The conclusion is not valid. If one pair of consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus. If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. To apply either theorem, you must first know that ABCD is a parallelogram.
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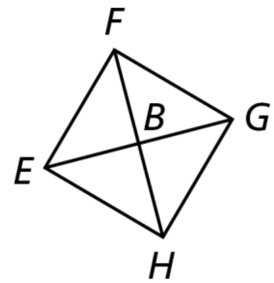
Determine if the conclusion is valid.
If not, tell what additional information is needed to make it valid.
Given: $\overline{EB} \cong \overline{BG}, \overline{FB} \cong \overline{BH}, \overline{EG} \cong \overline{FH}$, △EBF ≅ △EBH
Conclusion: EFGH is a square


• $\overline{EB} \cong \overline{BG}, \overline{FB} \cong \overline{BH}$, so EFGH is a parallelogram
• $\overline{EG} \cong \overline{FH}$ so EFGH is a rectangle
• $\overline{EF} \cong \overline{EH}$ because ΔEBF ≅ ΔEBH
• EFGH is a rhombus because it is a parallelogram with one pair of consecutive sides ≅.
• EFGH is a rectangle and a rhombus therefore it has four right angles and four congruent sides so it is a square.
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Answer the following questions:
• What do all quadrilaterals have in common?
They are all polygons, and they all have 4 sides.
• What would you have to change in a rhombus to make it a square?
All 4 angles would have to be right angles.
• What would you have to change in a rectangle to make it a square?
All 4 sides would have to be congruent.
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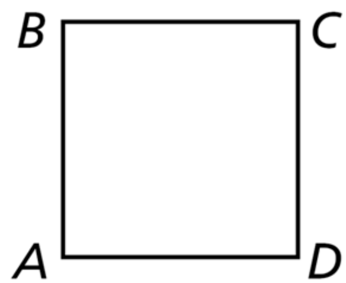
Given that AB = BC = CD = DA, what additional information is needed to conclude that ABCD is a square?


$\overline{AC} \cong \overline{BD}$
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