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# A question about the consistency of Bell's correlation formula

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**Abstract** In the paper it is demonstrated that two equally consistent but conflicting uses of sign functions in the context of a simple probability density shows that Bell's formula is based on only one consistent principle. The two conflicting principles give different result. However, according to use of powers, i.e. $3 \times (1/2) = (1/2) \times 3$, one must have the same result in both cases.

**Keywords** Inconsistency, Bell's theorem.

## 1 Introduction

In 1964, John Bell wrote an important paper [1] on the possibility of hidden variables [2] causing the entanglement correlation $E(a, b)$ between two particles. His paper was a response to the criticism of Einstein on the completeness of quantum theory. In his paper, together with Rosen and Podolsky, Einstein [2] argued that the quantum description must be supplemented with extra variables to explain the entanglement phenomenon. von Neuman [4] presented a mathematical proof that any hidden variables theory is in conflict with quantum mechanics. However, one can doubt if von Neuman's view on the matter was completely related to the physics. Bell's paper opened the possibility of experiment. In the present paper, an inconsistency in the starting formula of Bell [1] will be demonstrated. This paper is a continuation / response to [7]. In the paper Nordén already admitted that Bell's formula may perhaps be not as solid as presented in most discussions about Einstein. Here we will add to that argument and show that Bell's formula holds ambiguous elements that must be resolved before serious conclusions can be drawn from experiments.

Bell, based his hidden variable description on particle pairs with entangled spin, originally formulated by Bohm [3]. Bell used hidden variables $\lambda$ that are elements of a universal set $\Lambda$ and are distributed with a density $\rho(\lambda) \geq 0$. Suppose, $E(a, b)$ is the correlation between measurements with distant A and B that have unit-length, i.e. $||a|| = ||b|| = 1$, real 3 dim parameter vectors $a$ and $b$.

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Then with the use of the $\lambda$ we can write down the classical probability correlation between the two simultaneously measured spins of the particles. This is what we will call Bell's formula.

$$E(a,b) = \int_{\lambda\in\Lambda} \rho(\lambda)A(a,\lambda)B(b,\lambda)d\lambda \tag{1.1}$$

The spin measurement functions are, $A(a,\lambda) \in \{-1,1\}$ and $B(b,\lambda) \in \{-1,1\}$. The probability density is normalized, $\int \rho(\lambda)d\lambda = 1$.

## 2 What about the sign in Bell's formula for spin measurement?

This section contains a study on the obvious use of a sign distribution representing a (part of a) measurement function, for either $A$ or $B$ in (1.1). We refer for definition of sign to [9] and [10].

### 2.1 A sub-model that can be incorporated in any hidden variable theory

Suppose we look at a a probability density function in a single real variable $x \in \mathbb{R}$.

$$\rho(x) = \begin{cases} -x, & x \in [-1,0] \\ +x, & x \in [0,1] \\ 0, \ otherwise \end{cases} \tag{2.1}$$

It can be easily verified that $\rho(x) \geq 0$ for all $x \in [-1,1]$. Moreover, it is also easy to establish that

$$\int_{-1}^{+1} \rho(x)dx = -\int_{-1}^{0} xdx + \int_{0}^{+1} xdx = -\frac{1}{2}(0-(-1)^2) + \frac{1}{2}(1^2-0) = 1 \tag{2.2}$$

Hence, $\rho$ is a real possibility for (part of a) probablity desity in (1.1). It is subsequently noted that $\rho(x) = |x|$ for all $-1 \leq x \leq 1$. Let us look at a part of a more complete model. We have e.g.

$$E = \int_{-1}^{1} |x|\mathrm{sign}(x)dx \tag{2.3}$$

The $E$ defined previously can be the result of any model where, $|x|\rho(\lambda)$, with, $-1 \leq x \leq 1$, replaces the density $\rho(\lambda)$ and $A(a,\lambda)\mathrm{sign}(x)$ replaces the $A(a,\lambda)$. Then, $E$ will occur in the evaluation of the $E(a,b)$ from (2.1).

### 2.2 sign algebra

Our object of study will be $|x|\mathrm{sign}(x)$. We have $\mathrm{sign}(x) = 1$ when $x \geq 0$ and $\mathrm{sign}(x) = -1$ when $x < 0$.

#### 2.2.1 Exponential

A sign defined as previously can be written down as an exponetial form:

$$\mathrm{sign}(x) = \exp\left[i\pi(1-H(x))\right] \tag{2.4}$$

We use, $H(x) = 1$ for $x \geq 0$ and $H(x) = 0$ for $x < 0$. Now we know that $|x|\mathrm{sign}(x) = x\mathrm{sign}(x)\mathrm{sign}(x)$. Usually, people claim that $\mathrm{sign}(x)\mathrm{sign}(x) = 1$. Here we will show it is highly likely that $\mathrm{sign}(x)\mathrm{sign}(x)$ cannot be computed. Of course that may sound crazy but we will show an ambiguity.

### 2.2.2 Fractional exponents

The first thing we note is that there can exist no objections towards complex numbers intermediate results. Therefore we may write down the following expression

$$\mathrm{sign}(x)\mathrm{sign}(x) = \{\mathrm{sign(x)}\}^{1/2} \times \{\mathrm{sign(x)}\}^{3/2} \tag{2.5}$$

Now let us look at $\{\mathrm{sign}(x)\}^{3/2}$. In terms of (2.4) we then have

$$\{\mathrm{sign}(x)\}^{3/2} = \exp\left[\frac{3i\pi}{2}(1 - H(x))\right] \tag{2.6}$$

Moreover, note that $(1/2) \times 3 = 3 \times (1/2)$. The equality is, however, not reflected in the exponential expression. We have

$$\{\mathrm{sign}(x)\}^{3/2} = \sqrt{\exp\left[3i\pi(1 - H(x))\right]} \tag{2.7}$$

and

$$\{\mathrm{sign}(x)\}^{3/2} = \left(\exp\left[\frac{i\pi}{2}(1 - H(x))\right]\right)^3 \tag{2.8}$$

Equation (2.7) is a case of principle 1 below. If $x < 0$ then $\{\mathrm{sign}(x)\}^{3/2} = \{\{\mathrm{sign}(x)\}^3\}^{1/2} = \sqrt{(-1)} = i$. Equation (2.8) is a case of principle 2 below and for $x < 0$ we see, $\{\mathrm{sign}(x)\}^{3/2} = \{\{\mathrm{sign}(x)\}^{1/2}\}^3 = i \times i \times i = -i$. The result reflects the inequality

$$\left[\{\mathrm{sign}(x)\}^3\right]^{1/2} \not\equiv \left[\{\mathrm{sign}(x)\}^{1/2}\right]^3 \tag{2.9}$$

This also follows from comparing $x < 0$ between (2.7) and (2.8). However, with the use of exponentials it is absolutely clear that in both cases we are looking at the same $\{\mathrm{sign}(x)\}^{3/2}$. Therefore, we are looking at an ambiguity. The principles are itemized below and refer to the treatment of

$$\{\mathrm{sign(x)}\}^{3/2} = \{\mathrm{sign(x)}\}^{-1/2} \tag{2.10}$$

reflected in the exponentials above.

- *Principle 1:* In the evaluation of $\{\mathrm{sign}(x)\}^{3/2}$ of (2.10), the equation (2.7) is based on first the power 3 then the power 1/2 and this concurs with, on the right hand of (2.10), first the power $-1$ then the power 1/2.

- *Principle 2:* In the evaluation of $\{\mathrm{sign}(x)\}^{3/2}$ of (2.10), the equation (2.8) is based on first the power 1/2 then the power 3 and this concurs with, on the right hand of (2.10), first the power 1/2 then the power $-1$.

The two different breakdowns, reflecting the principles above, of $|x|\mathrm{sign}(x)$ in terms of exponential functions are, firstly,

$$|x|\mathrm{sign}(x) = x\sqrt{\exp\left[3i\pi(1 - H(x))\right]} \times \{\mathrm{sign(x)}\}^{1/2} \tag{2.11}$$

together with, secondly,

$$|x|\mathrm{sign}(x) = x\left(\exp\left[\frac{i\pi}{2}(1 - H(x))\right]\right)^3 \times \{\mathrm{sign(x)}\}^{1/2} \tag{2.12}$$

## 3 Conclusion & discussion

To the authors, the previous section represents two conflicting breakdowns of $|x|\mathrm{sign}(x)$.

Given $3 \times (1/2) = (1/2) \times 3$ the question is why $3 \times (1/2)$ gives another result compared to $(1/2) \times 3$. Again, in powers we have $(1/2) \times 3 = 3 \times (1/2)$. Despite the latter fact, principle 1 gives another result than principle 2 because of (2.9).

The claim of the authors, supported by [5], is that because of this demonstrated multi-valuedness, concrete mathematical incompleteness is the cause of the ambiguity. As it was stated previously $E$ can occur in any model as a result of supplementing such a, perhaps physically meaningful, model for a Bell formula.

Finally note, the experimentally demonstrated difference between classical and quantum mechanics does not at all need Bell methodology. This state of affairs can be found at [7] and [8].

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