Graph Theory and Optimization
Why is it useful?

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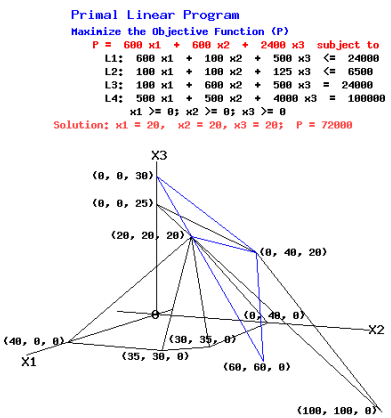
Outline

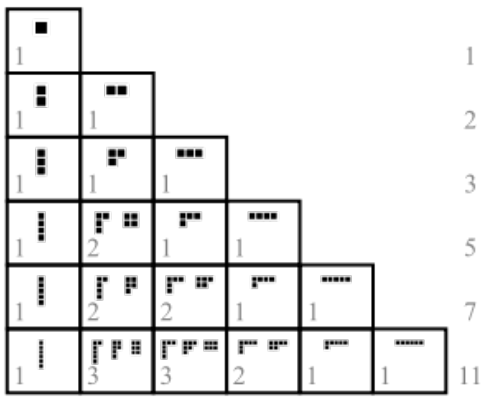
Combinatorics

Branch of mathematics concerning the study of finite or countable objects (existence, enumeration, structure).

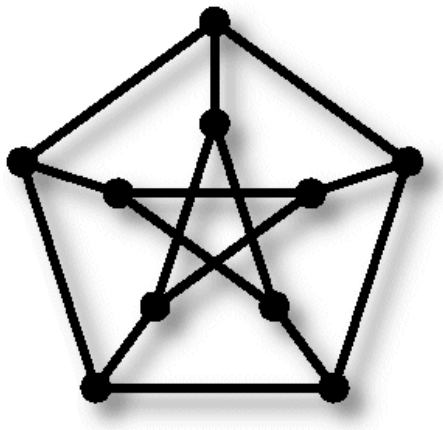
Integer Linear Programming



Integer partition



Graphs

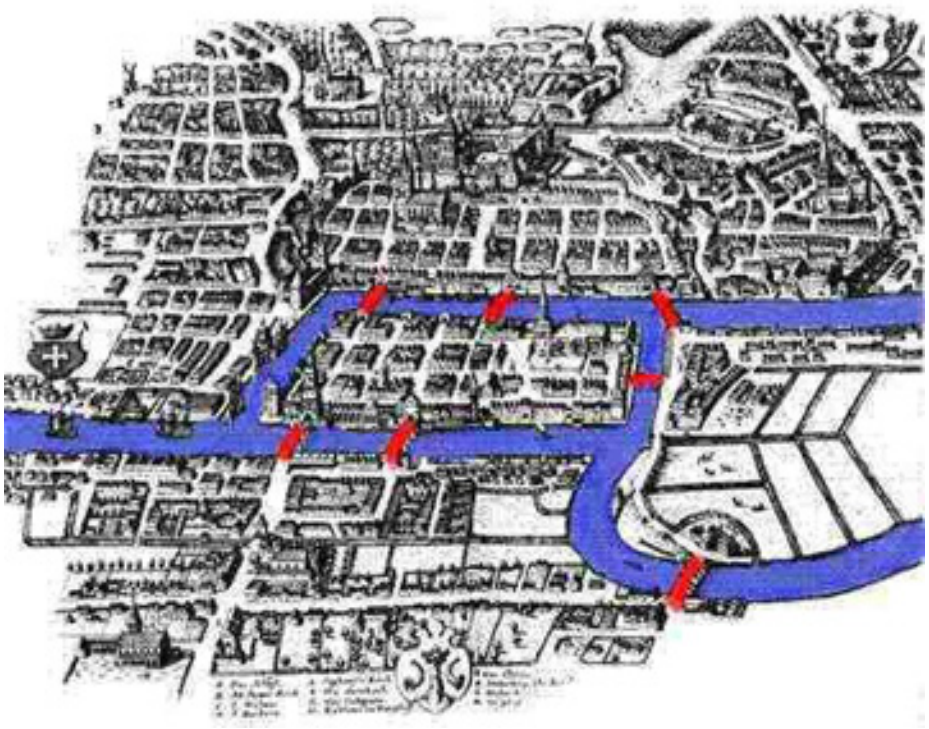


Graph Theory an old story

Euler 1735: **Koenisberg bridges**.

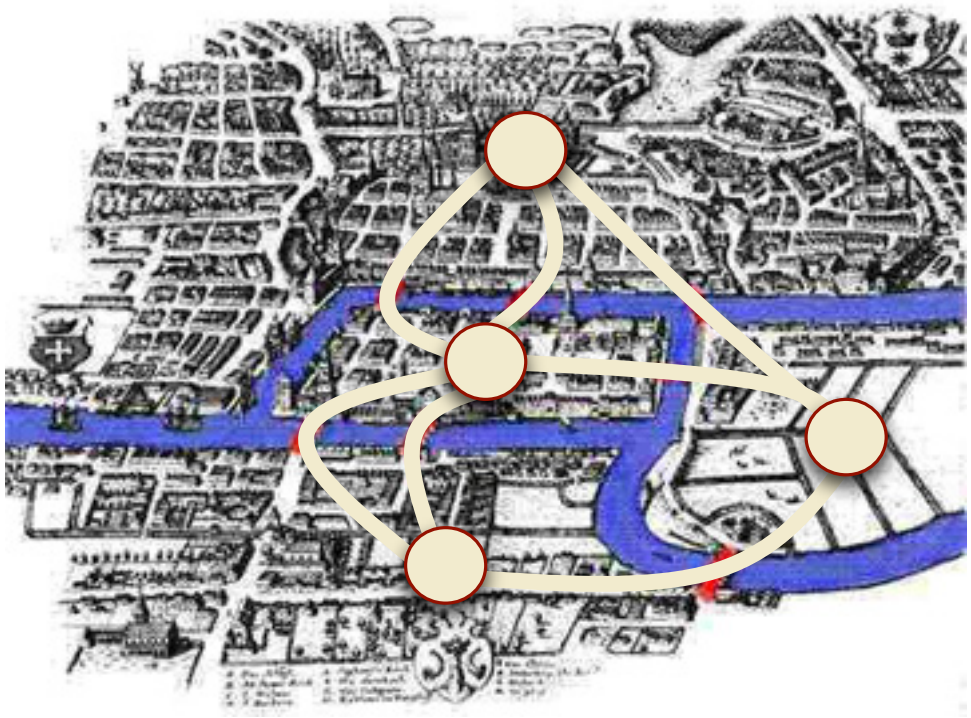
Existe-t-il un parcours empruntant tous les ponts une fois et une seule ?

Is there a cycle going through each bridge exactly once?



Graph Theory an old story

Modeling: city = graph, island = vertex, bridge = edge

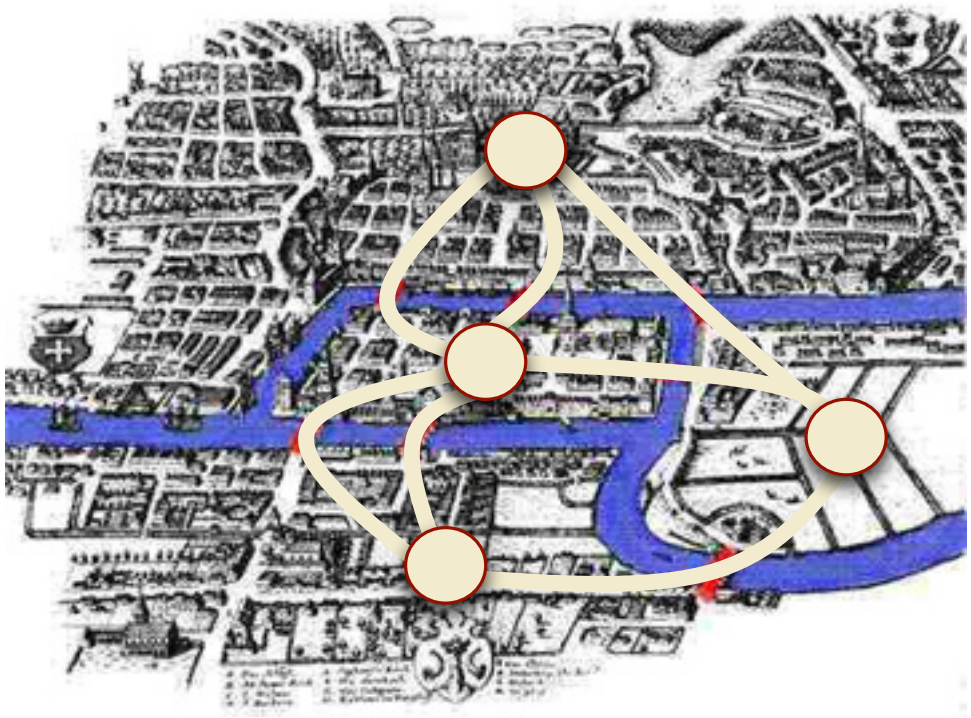


Graph Theory an old story

Modeling: city = graph, island = vertex, bridge = edge

Question: can we find an eulerian cycle in this graph?

Cycle going through all edges once and only once



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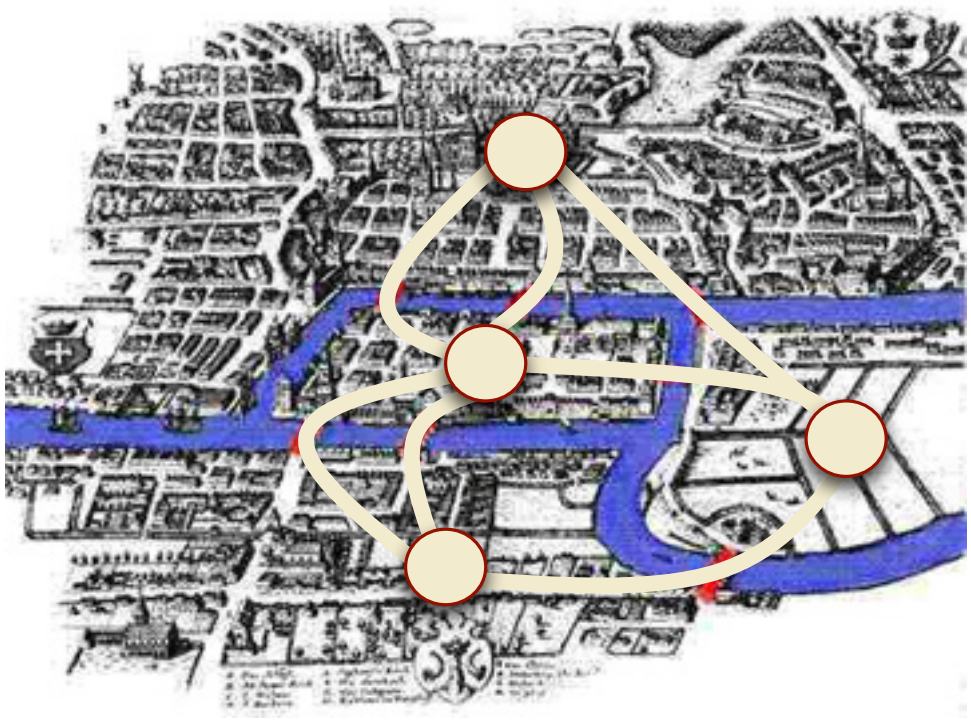
Modeling: city = graph, island = vertex, bridge = edge

Question: can we find an eulerian cycle in this graph?

Solution: Such cycle exists if and only if all nodes have even degree



Graph Theory an old story

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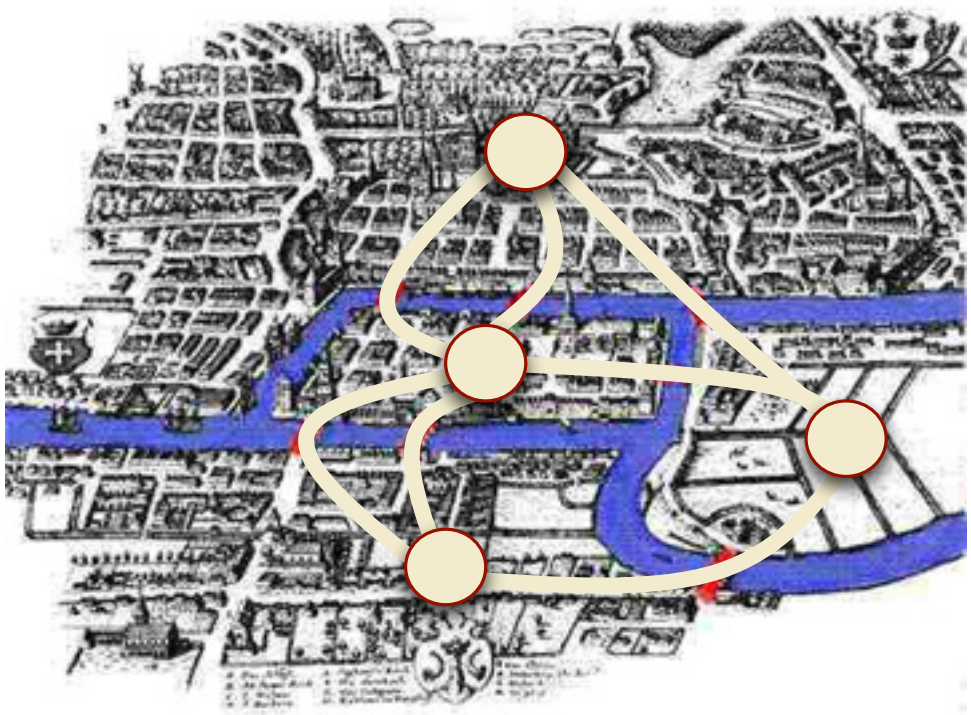
An intriguing variant: find a cycle going through all vertices once and only once (Hamiltonian cycle) is very difficult

One million dollar (Clay price) !



Outline

1st Example: roads' network

What is the "best" road for reaching Oulu from Helsinki?

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Model geographical netowrk by a graph

1st Example: roads' network

What is the "best" road for reaching Oulu from Helsinki?

Model geographical netowrk by a graph

Use powerful tools that deal with graphs

1st Example: roads' network

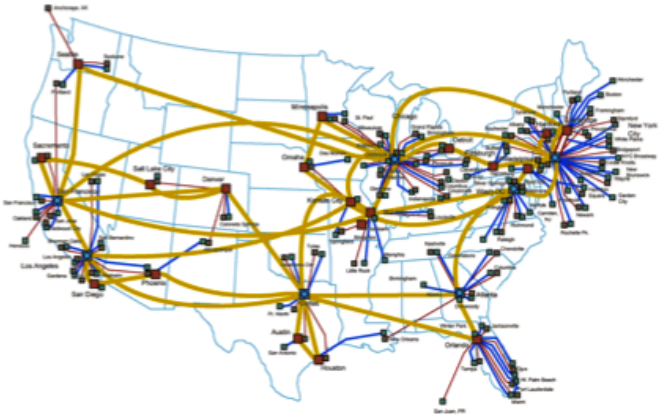
More difficult setting

- traffic jam
- bus/subway schedule
- no-left, no-right and no U-turn signs at intersections.

Again, graph algorithm tools may help

That is how your GPS work !!

2nd Example: the Internet



Internet network (Autonomous Systems)
Optical networks (WDM)

- node= IP routers
- links= optical fiber
- capacity on links

- How to compute “best" routes?
- Where to put Amplificators?
- Which links to be turned off to limit energy consumption?

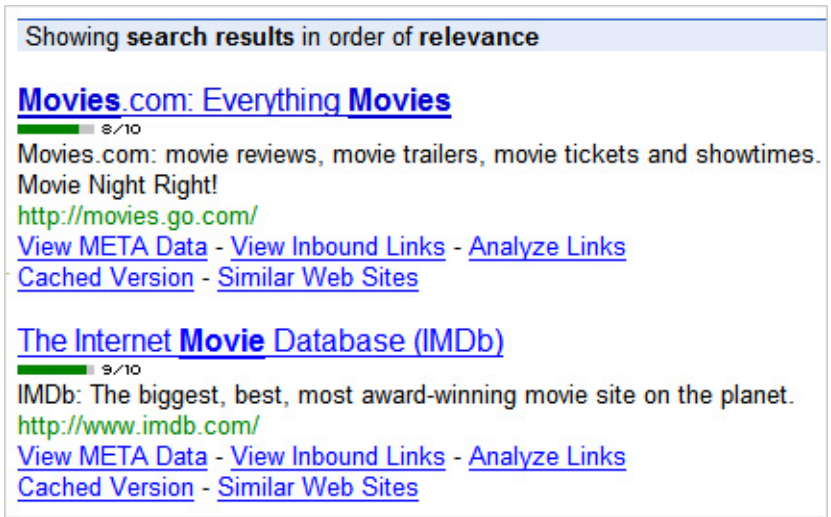
...

3rd Example: Social Network

Model of social interaction
a user = a node
two friends = an edge

- structure of social networks?
- communities?
- how to do advertisement?
- how to prevent advertisement?

More Example: Web (google)



Google PageRank:

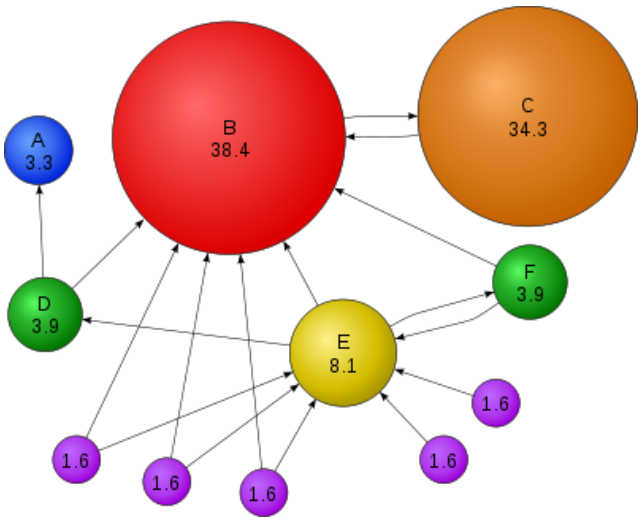
sort search results

node= web page

link = hyperlink

1. finding pages with the word movies in it
2. determining the importance of a page.

More Example: Web (google)



Google PageRank:

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node= web page

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1. finding pages with the word movies in it
2. - build the graph of the Web
   - do a random walk on a the graph or compute the eigenvector of a matrix

Outline

Objectives of this school

1. learn how to model problems (from many domains) using graphs
2. know the available tools to handle these problem
   - classical algorithms
   - Linear Programming
3. decide if a problem is “easy" or “difficult"
4. know what to do when facing a “difficult" problem
   - exact exponential algorithms
   - parameterized algorithms
   - approximation algorithms
   - heuristics