Trigonometric Functions on the Unit Circle

Given a point on the terminal side of an angle θ in standard position. Then:

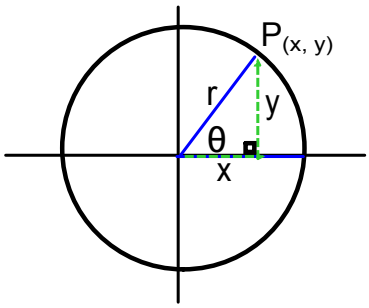
$r = \sqrt{x^2 + y^2}$

$\sin\theta = \frac{y}{r}$ $\quad \csc\theta = \frac{r}{y}$

$\cos\theta = \frac{x}{r}$ $\quad \sec\theta = \frac{r}{x}$

$\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$

use when you have ordered pairs and you need to find the trig values



x y

Ex. Find the six trig values of (8, -6) if it's a point on the terminal side of an angle in standard position.

$r = \sqrt{(8)^2 + (-6)^2}$

$r = 10$

$x = 8$

$y = -6$

$\cos\theta = \frac{8}{10} = \frac{4}{5}$ $\quad \sec\theta = \frac{5}{4}$

$\sin\theta = \frac{-6}{10} = \frac{-3}{5}$ $\quad \csc\theta = -\frac{5}{3}$

$\tan\theta = \frac{-6}{8} = \frac{-3}{4}$ $\quad \cot\theta = -\frac{4}{3}$

Ex. Find the six trig values of (4, 3) if it's a point on the terminal side of an angle in standard position.

$r = \sqrt{(4)^2 + (3)^2}$

$r = 5$

$x = 4$

$y = 3$

$\cos\theta = \frac{4}{5}$ $\quad \sec\theta = \frac{5}{4}$

$\sin\theta = \frac{3}{5}$ $\quad \csc\theta = \frac{5}{3}$

$\tan\theta = \frac{3}{4}$ $\quad \cot\theta = \frac{4}{3}$

Ex. Find the six trig values of (-2, -1) if it's a point on the terminal side of an angle in standard position.

$r = \sqrt{(-2)^2 + (-1)^2}$

$r = \sqrt{4+1}$

$r = \sqrt{5}$

$x = -2$

$y = -1$

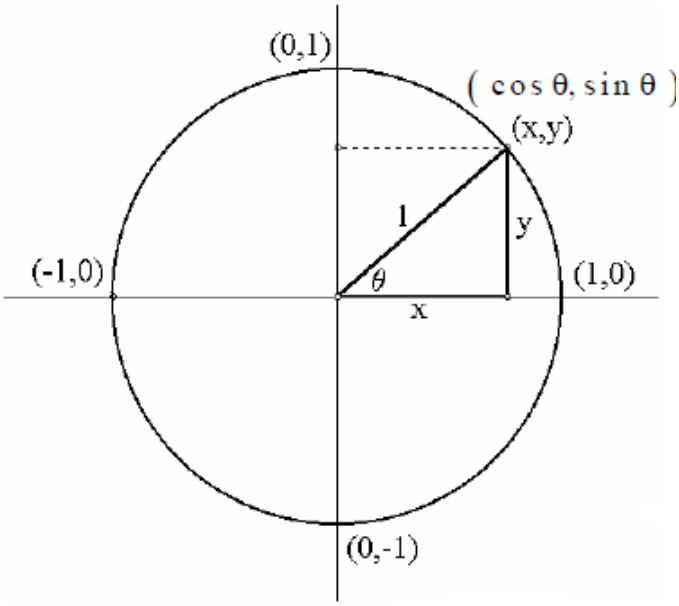
$\cos\theta = \frac{-2}{\sqrt{5}} = \frac{-2\sqrt{5}}{5}$ $\quad \sec\theta = -\frac{\sqrt{5}}{2}$

$\sin\theta = \frac{-1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$ $\quad \csc\theta = -\sqrt{5}$

$\tan\theta = \frac{-1}{-2} = \frac{1}{2}$ $\quad \cot\theta = \frac{-2}{-1} = 2$

The Six Trigonometric Functions

The Unit Circle



Unit Circle Trigonometry

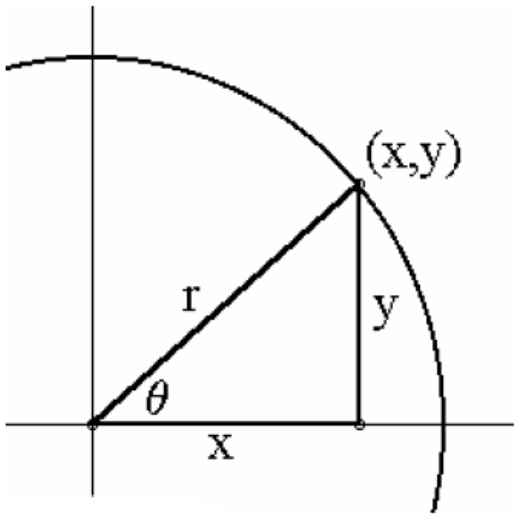
$$x^2 + y^2 = 1$$

$\cos\theta = x$ $\quad \sec\theta = \frac{1}{x}$

$\sin\theta = y$ $\quad \csc\theta = \frac{1}{y}$

$\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$

Circle of Radius **r**



Triangle / Circle Trigonometry

$$x^2 + y^2 = r^2$$

$\cos\theta = \frac{x}{r}$ $\quad \sec\theta = \frac{r}{x}$

$\sin\theta = \frac{y}{r}$ $\quad \csc\theta = \frac{r}{y}$

$\tan\theta = \frac{y}{x}$ $\quad \cot\theta = \frac{x}{y}$

Quadrantal angle: an angle whose terminal side lies on an axis

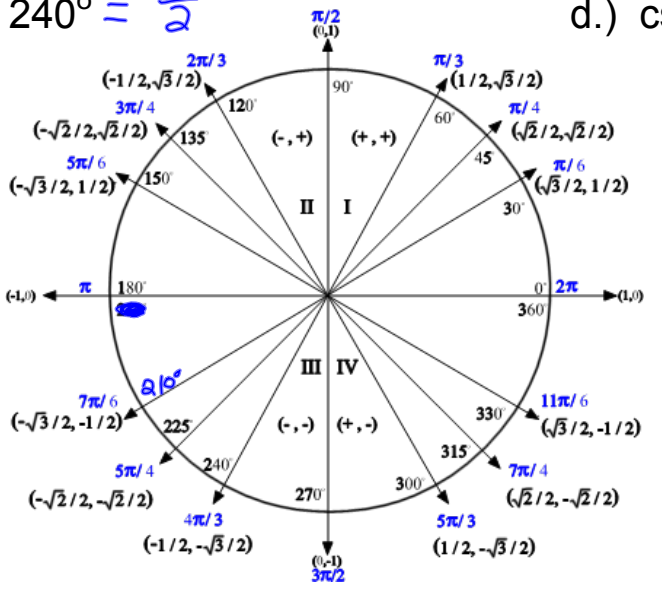
Find the exact value of each trig fct, if defined. If not defined write undefined.

Ex a) $\tan \frac{3\pi}{2}$ = undef

b.) $\sec 4\pi = 1$

c.) $\cos 240^\circ = -\frac{1}{2}$

d.) $\csc \frac{\pi}{4} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2}$

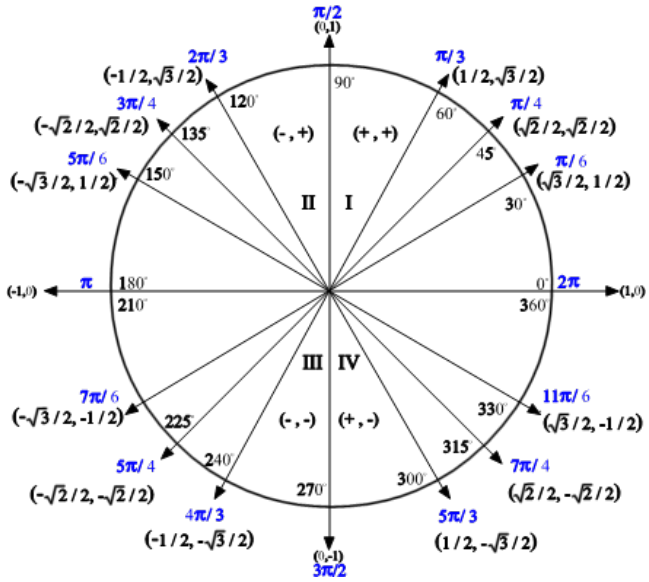


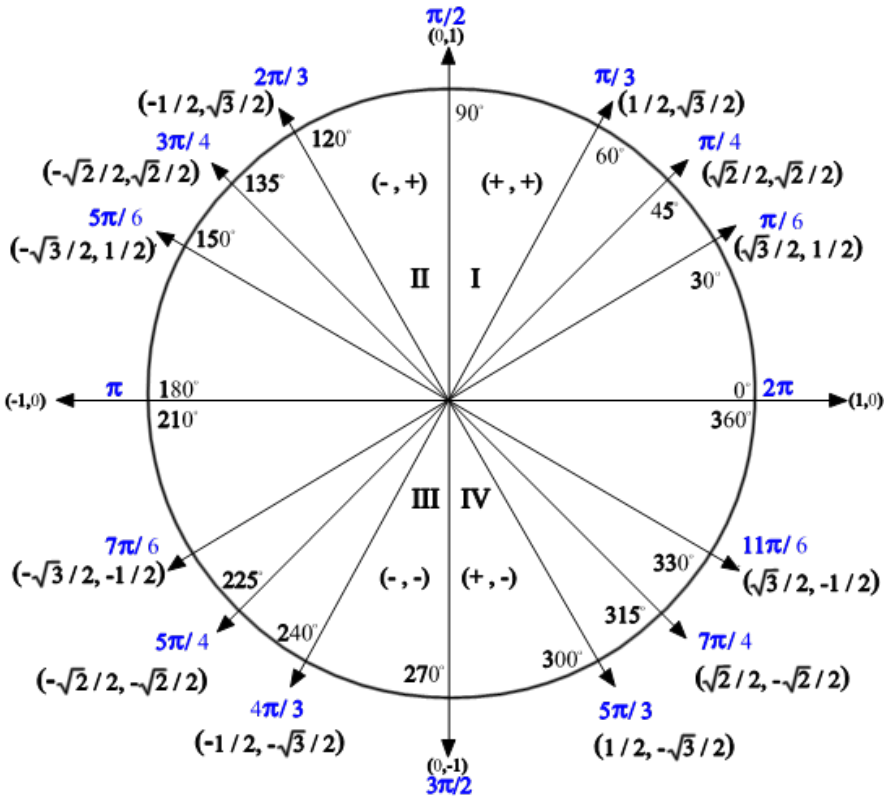
e.) $\cot(-90^\circ) = 0$

f.) $\cos -\pi = -1$

g.) $\tan 390^\circ = \frac{\sqrt{3}}{3}$

h.) $\cot \frac{7\pi}{3} = \frac{\sqrt{3}}{3}$





Ex. Find the exact value of each trig function, if defined. If not defined write undefined.

a.) 300°

$\cos 300^\circ =$

$\cot 300^\circ =$

$\csc$

b.) $-\frac{2\pi}{3}$

$\sin -\frac{2}{3}\pi =$

$\sec -\frac{2}{3}\pi =$

c.) $\frac{5\pi}{4}$

$\tan \frac{5}{4}\pi$

d.) -240°

e.) 390°

$\cot 390^\circ =$

$\sec 390^\circ =$

f.) $\frac{3\pi}{4}$

$\sin \frac{3}{4}\pi =$

$\sec \frac{3}{4}\pi =$

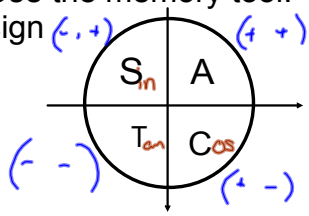
g.) -150°

$\cot -150^\circ =$

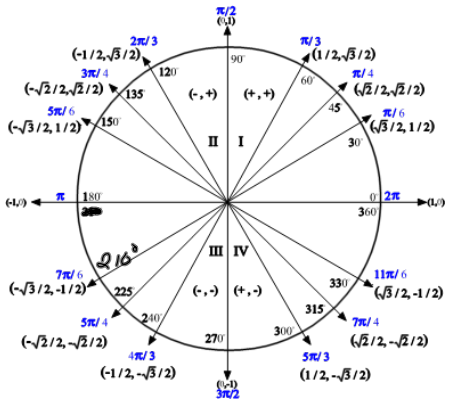
$\cos -150^\circ =$

You can use reference angles to find the trig value of any angle, BUT you must make sure to record the correct sign.

Use the memory tool: All Students Take Calculus to find the sign



QI = all trig fcts are positive
QII = sin & csc trig fcts are positive
QIII = tan & cot trig fcts are positive
QIV = cos & sec trig fcts are positive



Ex.) Find the exact value of each expression

a.) $\cos 120^\circ = -\frac{1}{2}$

b.) $\tan \frac{7\pi}{6} = \frac{\sqrt{3}}{3}$

c.) $\sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$

d.) $\tan 150^\circ \quad -\frac{\sqrt{3}}{3}$

e.) $\sec \frac{15\pi}{4} \quad \sqrt{2}$

f.) $\tan \frac{5\pi}{3} \quad -\sqrt{3}$

g.) $\sin \frac{5\pi}{6} \quad \frac{1}{2}$

h.) $\sec(-135^\circ) \quad -\sqrt{2}$

i) $\csc -315^\circ = \sqrt{2}$

j) $\cot -300^\circ \quad \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

If you know the value of one trig function AND the quadrant it's in you can use your knowledge of it's ratio to find the other trig functions.

Ex. given: $\tan\theta = \frac{-5}{-12}$ $\frac{y}{x}$ and $\sin\theta < 0$ (neg)

QIII

since $\tan\theta = y/x$ and it's positive the angle could be either in QI or QIII (because these are the only two quadrants where tan is positive); however, since $\sin\theta$ is negative then it can't be in QI b/c sin is positive in QI. Therefore, the angle must be in QIII which means: $y = -5$ and $x = -12$

$\frac{-5}{-12}$ $\frac{y}{x}$

$(-5)^2 + (-12)^2 = r^2$

$\sqrt{25 + 144} = r$

$\sqrt{169} = r$

$13 = r$

$x = -12$ $y = -5$ $r = 13$

so: $\sin\theta = -5/13$ $\csc\theta = -13/5$

$\cos\theta = -12/13$ $\sec\theta = -13/12$

$\tan\theta = 5/12$ $\cot\theta = 12/5$

Ex. a) $\sec\theta = \sqrt{3}$ $\frac{r}{x}$ and $\tan\theta < 0$ (neg)

(+ −)

I IV II III

$x^2+y^2=r^2$ $1^2+y^2=(\sqrt{3})^2$ $1+y^2=3$ $y^2=2$ $y=\sqrt{2}$

$x=1$ $y=-\sqrt{2}$ $r=\sqrt{3}$

$\cos\theta = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\sec\theta = \frac{\sqrt{3}}{1} = \sqrt{3}$

$\sin\theta = \frac{-\sqrt{2}}{\sqrt{3}} = \frac{-\sqrt{6}}{3}$ $\csc\theta = \frac{\sqrt{3}}{\sqrt{2}} = -\frac{\sqrt{6}}{2}$

$\tan\theta = \frac{-\sqrt{2}}{1} = -\sqrt{2}$ $\cot\theta = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

y r

Ex b) $\sin\theta = 5/7$ $\cot\theta > 0$

I II I III

(+ +)

$x^2+y^2=r^2$ $x^2+5^2=7^2$ $x^2+25=49$ $x^2=24$ $x=\sqrt{24}$ $x=2\sqrt{6}$

$x=2\sqrt{6}$ $y=5$ $r=7$

$\cos\theta = \frac{2\sqrt{6}}{7}$ $\sec\theta = \frac{7}{2\sqrt{6}} = \frac{7\sqrt{6}}{12}$

$\sin\theta = \frac{5}{7}$ $\csc\theta = \frac{7}{5}$

$\tan\theta = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$ $\cot\theta = \frac{2\sqrt{6}}{5}$

Ex. c) $\sec\theta = \sqrt{29}$ $\sin\theta < 0$

I IV I II (+, +)

$x^2+y^2=r^2$ $5^2+y^2=(\sqrt{29})^2$ $25+y^2=29$ $y^2=4$ $y=2$

$x=5$ $y=2$ $r=\sqrt{29}$

$\cos\theta = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{29}$

$\sin\theta = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29}$

$\tan\theta = \frac{2}{5}$

$\sec\theta = \frac{\sqrt{29}}{5}$

$\csc\theta = \frac{\sqrt{29}}{2}$

$\cot\theta = \frac{5}{2}$

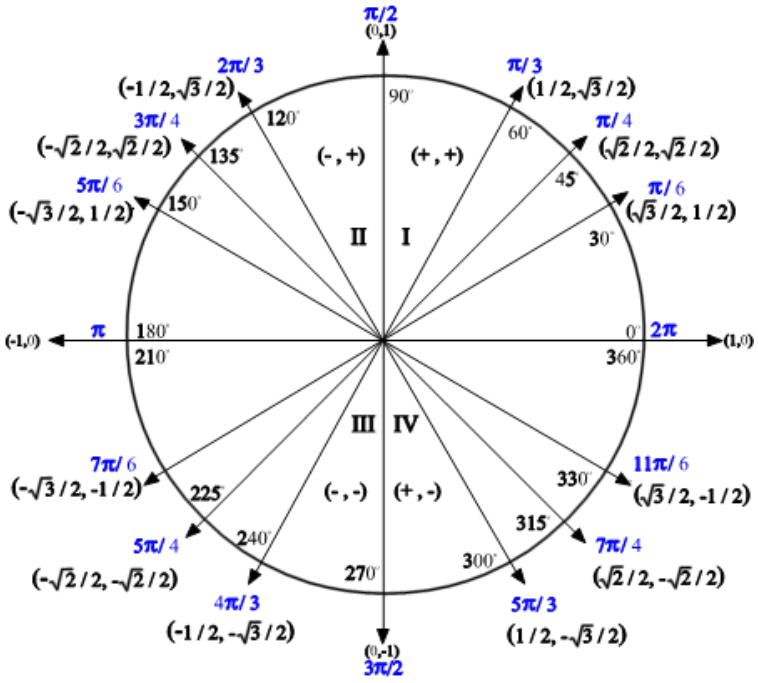
Because on a unit circle r = 1 you can simplify trig fcts:

THIS ONLY WORKS WITH THE 17 SPECIAL TRIG ANGLES WE'VE LEARNED ON THE UNIT CIRCLE!!!!!!! (and their coterminal angles)

$\sin\theta = \frac{y}{r}$ -- $\sin\theta = \frac{y}{1}$ -- $\sin\theta = y$ $\qquad \csc\theta = \frac{1}{y}$

$\cos\theta = \frac{x}{r}$ -- $\cos\theta = \frac{x}{1}$ -- $\cos\theta = x$ $\qquad \sec\theta = \frac{1}{x}$

$\tan\theta = \frac{y}{x}$ $\qquad \cot\theta = \frac{x}{y}$



Ex. $\sin\frac{\pi}{3} =$

$\cos 135^\circ =$

$\tan 270^\circ =$

$\csc\frac{11\pi}{6} =$

$\sin\frac{7\pi}{6} =$

$\sec 270^\circ =$

Periodic Functions: Functions with values that repeat at regular intervals. (The periods of sin, csc, cos, sec are 360° or 2π, the periods of tan, cot are 180° or π)

This can help you to find values of functions that aren't one of our friendly 17.

Ex. $\cos \frac{11\pi}{4} - \frac{8\pi}{4} = \frac{3\pi}{4}$ $\cos\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$

$\sin\left(\frac{-2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$

$\tan \frac{19\pi}{6} = \frac{\sqrt{3}}{3}$

$\cos \frac{9\pi}{4}$

$\sin(-300^\circ)$

$\tan \frac{29\pi}{6}$