

The Power and Pitfalls of *Mathematica* for 3D Design

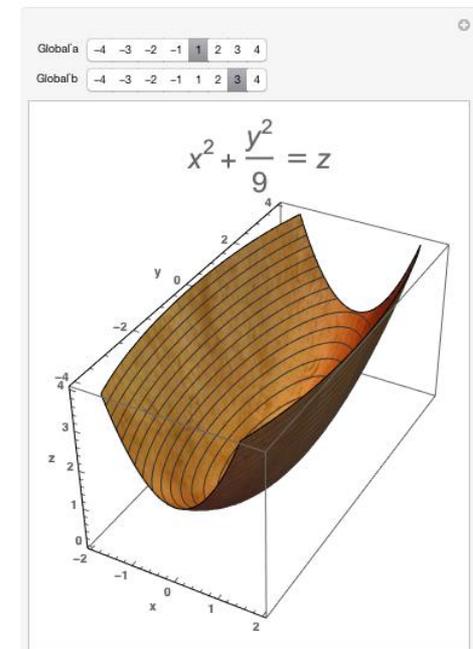
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My *Mathematica* Experience

- **Symbolic computational software**
- **Starting in 2008**
 - **Friendly syntax**
 - **Extensive documentation**
 - **Visualization capabilities**
- **Research Exploration**
 - **Experimental Math**
- **In my Teaching**
 - **Math Models, MV Calc, Math w/Mathematica**

Paraboloids

Investigate the behavior of quadric surfaces of the form $\pm \frac{x^2}{a^2} \pm \frac{y^2}{b^2} = z$



Teaching Methods

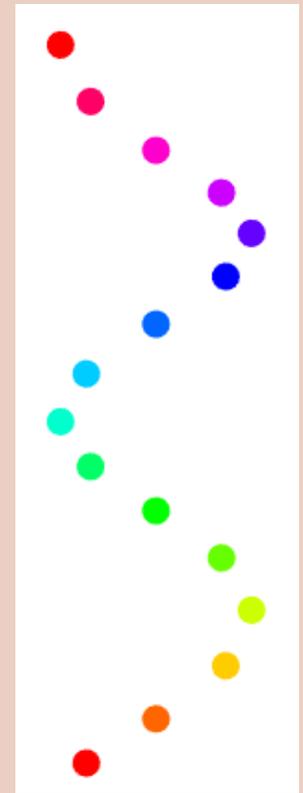
- **Give students the tools to succeed**
 - How to use the Documentation Center
 - Stand-alone tutorials
 - One-on-one help
- **Project-based learning**
 - Let them explore individually
 - Clear instructions and rubric
 - 3D printing since Spring '15



Mathematics in 3D Printing

Design in *Mathematica* requires **specifying coordinates**

- **3D Coordinate System**
- **Transformations**
 - Rotation, Translation, Scaling
- **Parametric Curves / Vector Functions**
- **Trigonometry**
 - Angles between lines, sine, arctan,
- **Geometric Objects**
 - Equations of Torus, Ellipsoid, ...
 - Polyhedra



Example: Geometry Basics

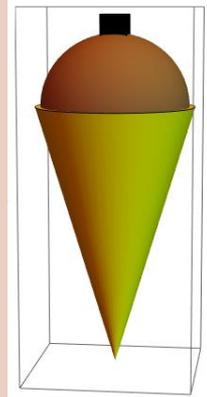
How does it work?

- **Coordinates for Primitives:**

`Sphere[{0, 0, 0}, .28]`

`Cuboid[{-.05, -.05, .26}, {.05, .05, .35}]`

`Cone[{{0, 0, 0}, {0, 0, -1}}, .3]`



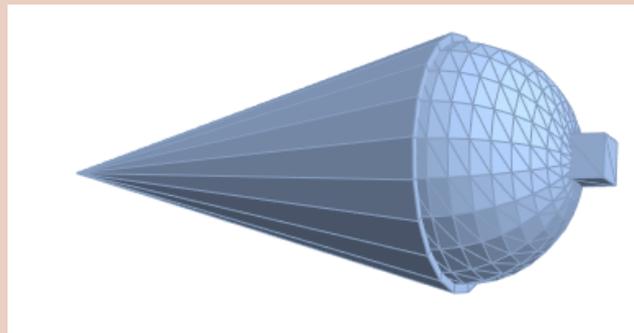
What doesn't work?

- **Certain primitives not exportable!**

- e.g. Tetrahedron, Pyramid

- **Resolution in STL file is poor**

- Sphere, Cone
- Use `ParametricPlot3D`



Example: Curves and Surfaces

How does it work?

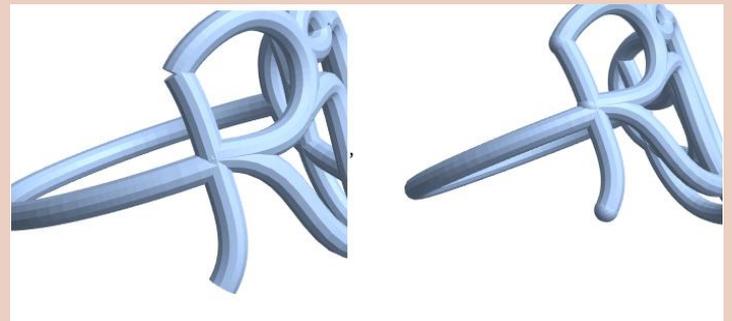
- BSplineCurve, Interpolation
- ParametricPlot3D
 - $f: \mathbb{R} \rightarrow \mathbb{R}^3$ plots a curve
 - $f: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ plots a surface

ParametricPlot3D[... , PlotStyle \rightarrow Tube[.1]]

- Secret option: Extrusion

What doesn't work?

- Can't export Tube[curve] .
 - Must add Tube in PlotStyle
- Caps of tubes are missing
 - Insert them independently and mind their resolution



Example: Polyhedra

How does it work?

- **PolyhedronData**
 - Imports curated polyhedra
 - Extract for modifications
 - Vertices, Incidences...
- **GraphicsComplex**
 - Create your own polyhedron!

What doesn't work?

- Time intensive to specify **EVERYTHING**
Is it manifold?



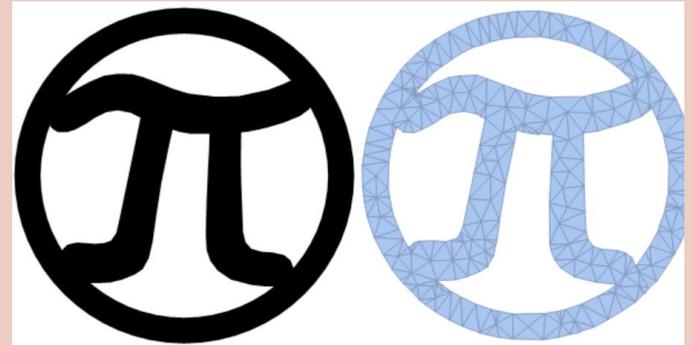
Example: MeshRegion Objects

How does it work?

- **DiscretizeRegion / DiscretizeGraphics**
 - Triangulates 2D and 3D objects
- **ImageMesh**
 - Image → MeshRegion
- **Mesh Operations**
 - RegionProduct, RegionUnion, RegionIntersection
- **Extraction for Modifications**
 - MeshCoordinates, MeshPrimitives

What doesn't work?

- Can't intersect 3D MeshRegions!

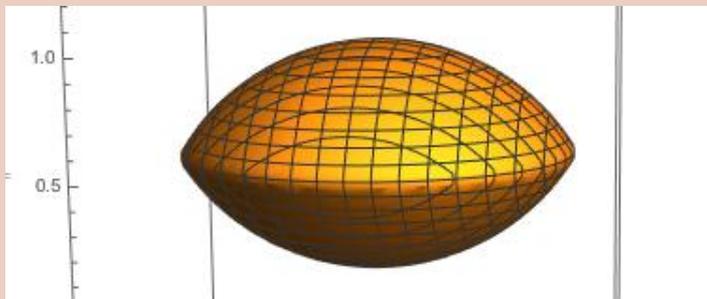


Example: RegionPlot3D

How does it work?

- Specify Boolean operations

$$x^2 + y^2 + z^2 \leq 1 \ \&\& \ x^2 + y^2 + (z - 1)^2 \leq 1$$

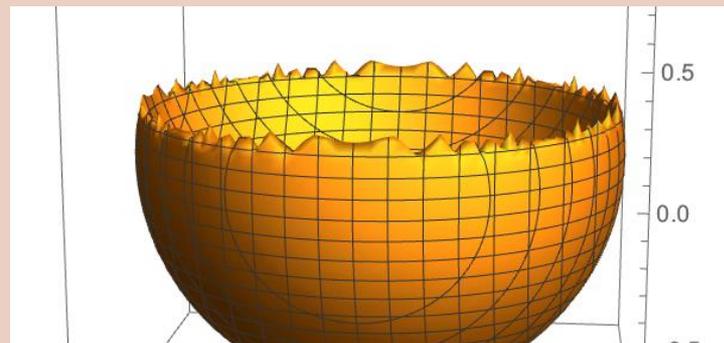


- Thicken a 2D surface

```
PlotStyle -> {Directive[Thickness[.1]]}
```

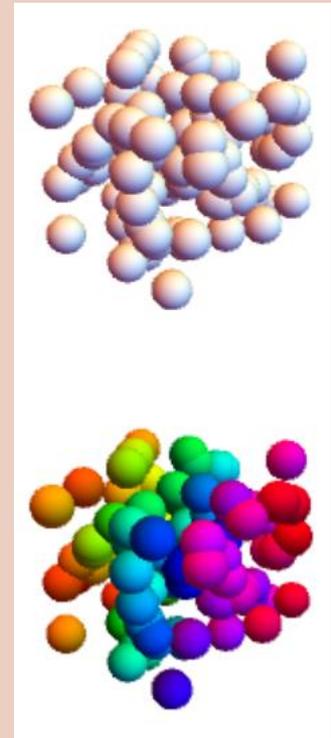
What doesn't work?

- Poor edges / smoothing
 - Improve via PlotPoints



Mathematica's Power

- **Programmable & Algorithmic approach to all**
 - **Notebook interface: No point and click!**
 - **Table** and **Map** to apply systematically
 - **Easy to add randomness.** `RandomReal[]`
 - **Easy to add color.** `Hue[]`
- **Exports to multiple file formats**
 - `Export["filename.stl",model]`
- **Visualization Capabilities**
- **Scheduled Updates**



Drawbacks

- 3D printing rather new to *Mathematica*.
- It doesn't "just work".
 - Certain commands not exportable at all
 - Never know when it will export or crash
 - Difficult to understand error messages
- Requires license (\$\$\$)
- Colleagues use Sage

```
... BoundaryMeshRegion: The boundary curves self-intersect or cross each other in  
BoundaryMeshRegion[{{(68.6021, 41.1552, 99.), {85.7527, 51.444, 99.}, {85.7527, 51.  
75.4392, 152.}, {-33.282, 94.2991, 152.}, {-33.282, 94.2991, 99.}, {32.0944, 94.  
1>>}}].  
... Join: Heads List and MeshCoordinates at positions 1 and 2 are expected to be the s  
... First: Nonatomic expression expected at position 1 in First[Fail].  
... First: Nonatomic expression expected at position 1 in First[2].  
... MeshCells: Options expected (instead of Multicells) beyond position 2 in MeshCell:  
... MeshCells: MeshCells called with 3 arguments; 2 arguments are expected.  
... Join: Heads List and MeshCells at positions 1 and 2 are expected to be the same.
```

Resources

- **Documentation Center**
- ***Mathematica* StackExchange**
- **Henry Segerman's**
3D Printing for Mathematical Visualisation
- **My class tutorials:**
 - ***213.mathzorro.com***
- **My blog: (3D design with *Mathematica* series)**
 - ***blog.mathzorro.com***
- ***blog.wolfram.com***

THANK YOU!

qc.edu/~chanusa
> Research > Talks

**Queens College
Students and Colleagues**



mathartshop.com

**Shapeways &
Lauren Slowik**

