LP Key

Algebra 2 **Standard Deviation & Normal Distribution Notes**

Last new lesson of Algebra 2!

Yahoo!

Review: WHAT IS THE *MEAN* OF A SET OF DATA? The average.

Standard Deviation is a statistical measure that shows how much data values deviate from the mean of a data set.

AKA – they tell us how spread out the data is!

For example, the more spread out the data is, the larger the standard deviation!

The formula for the ***standard deviation*** *is*: $\sigma = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}$

BUT…GOOD NEWS… the calculator will tell us the standard deviation if we enter in the data!!

Example 1: Below are the test scores of three students (Sally, Sue, Sandy).

Sally's scores: 70, 70, 70, 70, 70, 70

Sue's scores: 75, 65, 73, 67, 71, 69

Sandy's scores: 90, 50, 82, 58, 79, 61

**All three sets of data have the SAME MEAN (believe it or not)!*

Predict: a. Which of the students is going to have the highest standard deviation? Why?

Sandy because the data is the most spread out.

b. What will the standard deviation for Sally's scores be? Why do you think so?

0 because none of her scores deviate at all from the mean of 70.

Now, calculate each σ by using your calculator:

Step 1: Stat → Edit → Enter data into L_1
Step 2: Stat → Calc → 1-Var Stats → Enter

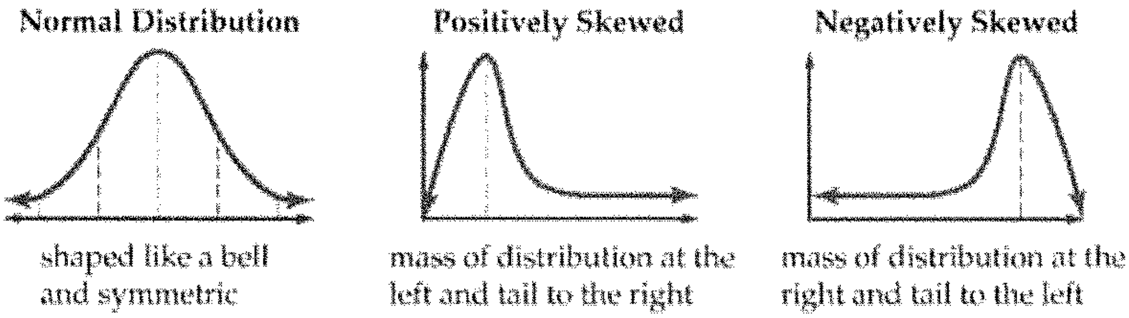
(mean: $\bar{x}$ standard deviation: σ)

Sally's σ 0

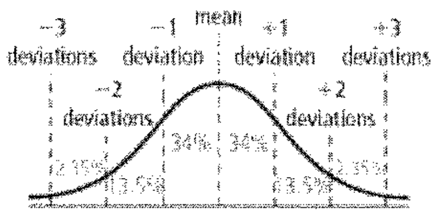
Sue's σ 3.41565

Sandy's σ 14.434

THE NORMAL DISTRIBUTION

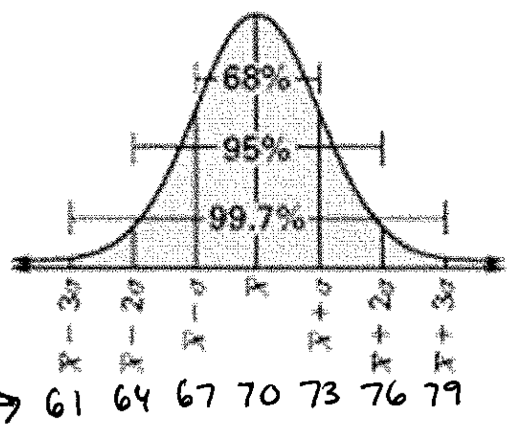


A normal distribution has data that vary randomly from the mean. The graph of a normal distribution is a normal curve.



In a normal distribution,
- 68% of data fall within one standard deviation of the mean
- 95% of data fall within two standard deviations of the mean
- 99.7% of data fall within three standard deviations of the mean

A normal distribution has a symmetric bell shape, centered at the mean.
Many common statistics such as human height, weight or blood pressure have a normal distribution about the mean.

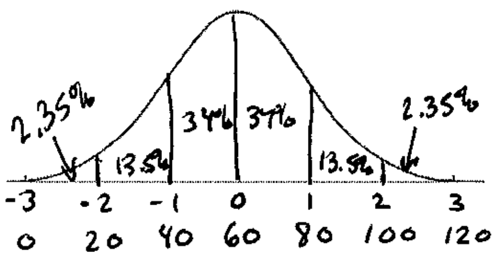


For example: Suppose the mean height for 20-year-old men is 70 inches and the standard deviation is 3 inches. This means that 68% of 20-year-old men have a height between 67 and 73 inches inclusive. Fill in the blanks below:

95% of 20-year-old men have a height between __64__ and __76__ inches inclusive.

__99.7__ % of 20-year-old men have a height between 61 and 79 inches inclusive.

Example 4) Given the quiz scores 30 50 60 70 90. Draw a normal curve.



Mean = 60

Standard Deviation = 20

What percent of the scores are above 60? 50% What percent of the scores are below 40? 16%

$34 + 50 = 84$

$100 - 84 = 16$

What percent of the scores are between 40 and 80? 68%

If 50 students took this quiz how many scored less than 40? 8

16% of 50 = $(.16)(50) = 8$

Z-Scores (standard deviations from mean)

A *z*-score reflects how many standard deviations above or below the mean a raw score is. The *z*-score is positive if the data value lies above the mean and negative if the data value lies below the mean.

$$z = \frac{x - \mu}{\sigma}$$

Where x represents an element of the data set, the mean is represented by μ and standard deviation by σ.

Example 5) Suppose SAT scores among college students are normally distributed with a mean of 500 and a standard deviation of 100. If a student scores a 700, what would be her **z**-score?

$$z = \frac{x - \mu}{\sigma} \rightarrow z = \frac{700 - 500}{100} = 2$$

Her **z**-score would be 2 which means her score is 2 standard deviations above the mean.

Example 6) In Harold's math class, a recent test has a mean of 70 and a standard deviation of 8. In Harold's English class, a recent test has a mean of 74 and a standard deviation of 16. If Harold earned a score of 78 on both tests, then in which subject is his performance better?

Find the z-score for each test:

Math:

$$z = \frac{x - \mu}{\sigma} = \frac{78 - 70}{8} = 1$$

English:

$$z = \frac{x - \mu}{\sigma} = \frac{78 - 74}{16} = 0.25$$

The Math score would have the highest standing since it is 1 standard deviations above the mean, while the English score is only 0.25 standard deviations above the mean.

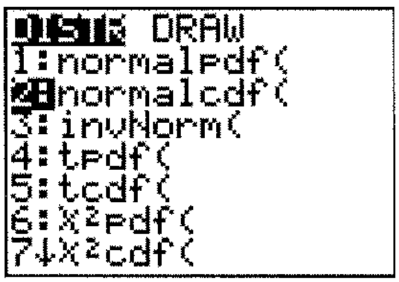
Example 7) The mean of a test is 76 with a standard deviation of 8.

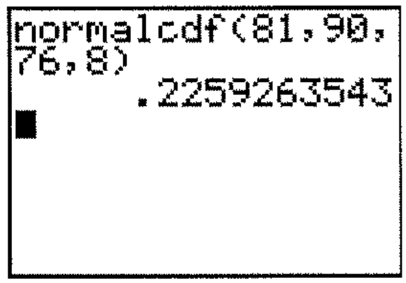
a) What percentage of scores would be between 81 and 90? $\boxed{22.6\%}$ (see below)

We can do this on our calculator!!!!

How to calculate the percentage under the normal curve

1. Pressing 2nd, VARS, and 2:normalcdf(.
2. Enter the lower end of your range, the upper end of your range, the mean (μ) and the standard deviation (σ).
3. Press ENTER.

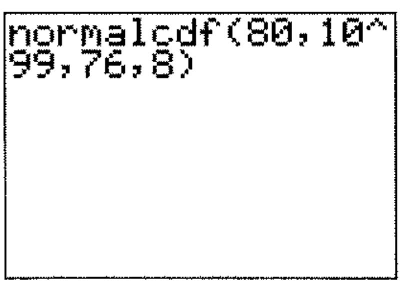




b) What percentage of scores would be between 60 and 92?

$\boxed{95.4\%}$

c) What percentage of scores would be above 80?



$\boxed{30.85\%}$

Note: Use 10^{99} as the upper end

d) What percentage of scores would be below 80? (Think: what will the lower end be?) → -10^{99}

$100 - 30.85$

$\boxed{69.15\%}$

or

normalcdf$(-10^{99}, 80, 76, 8)$

$= 0.6915$

$= \boxed{69.15\%}$

Example 8) Each year, the College Board publishes the mean SAT and the standard deviation for students taking the test. SAT scores are normally distributed. Assume for a group of students that the mean SAT score is 500 with a standard deviation of approximately 100 points.

a) Find the score that is 1.5 standard deviations above the mean.

$$z = \frac{x-\mu}{\sigma} \rightarrow 1.5 = \frac{x-500}{100}$$

$$150 = x - 500 \rightarrow \boxed{x = 650}$$

b) Find the score that is 1.5 standard deviations below the mean.

$$z = \frac{x-\mu}{\sigma} \longrightarrow -1.5 = \frac{x-500}{100}$$

$$-150 = x - 500 \rightarrow \boxed{x = 350}$$

c) Approximately what percentage would be expected to score between 350 and 650?

$$\text{normalcdf}(350, 650, 500, 100) = 0.8664$$
$$= \boxed{86.64\%}$$

d) If 2,000 students took the SAT that day how many would be expected to score between 350 and 650?

$$86.64\% \text{ of } 2000 = (0.8664)(2000)$$
$$= 1732.8$$
$$\boxed{\text{about } 1733 \text{ students}}$$

e) Approximately what percentage would be expected to score above 575?

$$\text{normalcdf}(575, 10^{99}, 500, 100) = 0.2266$$
$$= \boxed{22.66\%}$$

f) If 2,000 students took the SAT that day how many would be expected to score above 575?

$$22.66\% \text{ of } 2000 = (0.2266)(2000)$$
$$= 453.2$$
$$\boxed{\text{about } 453 \text{ students}}$$

SOL Questions on Normal Distribution

1) **A normally distributed set of 968 values has a mean of 108 and a standard deviation of 11. Which is closest to the number of values expected to be above 125 ?**

A 910

B 789

C 210

D 59

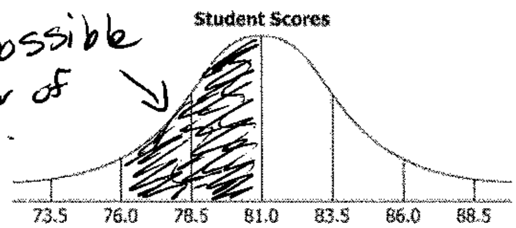
normalcdf$(125, 10^{99}, 108, 11) = 0.0611$

so 6.11% of values are above 125

6.11% of 968 $= (0.0611)(968) \approx \boxed{59}$

2) **This graph summarizes the test scores of 50,000 students. The data is normally distributed with a mean of 81 and a standard deviation of 2.5.**

one possible answer of many.



$13.5 + 34 = 47.5\%$

$\frac{23750}{50000} = 0.475 = 47.5\%$

so click on the regions that add to 47.5%.

Identify the regions under the curve where only the data for approximately 23,750 students are located.

3) **The heights of 200 kindergarten students at T.E. Wright Elementary are normally distributed with a mean of 40 inches and a standard deviation of 1.8 inches. Approximately how many students have a height between 37.3 inches and 44.5 inches?**

$\boxed{185}$ **Students**

normalcdf$(37.3, 44.5, 40, 1.8) = 0.9270 = 92.7\%$

92.7% of 200 $= (0.9270)(200) = 185.4$ so about 185 students

4) **The heights of a large population of ostriches are normally distributed. Which is closest to the percentage of these heights that is within 3 standard deviations of the mean?**

A 0.3%

B 5%

C 95%

D 99.7%

5) **A normally distributed data set has a mean of 0 and a standard deviation of 0.5. Which is the closest to the percent of values between -1 and 1?**

A 34%

B 50%

C 68%

D 95%

normalcdf$(-1, 1, 0, 0.5) = 0.9545 = 95.45\%$