Complex Analysis: Final Review

A function that is not analytic at $z = \alpha$, but *is* analytic in $D_r^*(\alpha)$ for some $r > 0$ is said to have an __________ ______________ at α.

A function that is not analytic at $z = \alpha$, but *is* analytic in $D_r^*(\alpha)$ for some $r > 0$ is said to have an ISOLATED SINGULARITY at α.

The singularity classifications at $z = 0$ of the functions

$$\frac{\sin z}{z},\ \sin\frac{1}{z},\ \text{and}\ \frac{1-\cos z}{z^3}\ \text{are}$$

____________, ____________, and ____________________.

The singularity classifications at $z = 0$ of the functions

$$\frac{\sin z}{z},\ \sin\frac{1}{z},\ \text{and}\ \frac{1-\cos z}{z^3}\ \text{are}$$

REMOVABLE, ____________, and ____________________.

The singularity classifications at $z = 0$ of the functions

$$\frac{\sin z}{z}, \ \sin\frac{1}{z}, \text{ and } \frac{1-\cos z}{z^3} \text{ are}$$

REMOVABLE, ESSENTIAL, and ___________________.

The singularity classifications at $z = 0$ of the functions

$$\frac{\sin z}{z},\ \sin\frac{1}{z},\ \text{and}\ \frac{1-\cos z}{z^3}\ \text{are}$$

REMOVABLE, ESSENTIAL, and POLE OF ORDER 1.

An expression of the form $\sum_{n=-\infty}^{\infty} c_n(z-\alpha)^n$ is known as a

__________ ________.

An expression of the form $\sum_{n=-\infty}^{\infty} c_n(z-\alpha)^n$ is known as a LAURENT SERIES.

The coefficient c_{-1} in the Laurent series
$f(z) = \sum_{n=-\infty}^{\infty} c_n(z-\alpha)^n$ is called the ________ ___ _ ___ _.

The coefficient c_{-1} in the Laurent series
$f(z) = \sum_{n=-\infty}^{\infty} c_n(z-\alpha)^n$ is called the <u>RESIDUE</u> <u>OF</u> <u>f</u> <u>AT</u> <u>α</u>.

Why did the mathematician name her dog Cauchy?

__________ __ ______ _ __________ ___ ________ ______.

Why did the mathematician name her dog Cauchy?

BECAUSE IT LEFT A RESIDUE AT EVERY POLE.

A complex number is an __________ pair of _____ __________.

A complex number is an ORDERED pair of REAL NUMBERS.

A function defined in an open subset D of the complex plane that is analytic (also called *holomorphic*) except at a set of isolated _______________ is said to be _______________.

A function defined in an open subset D of the complex plane that is analytic (also called *holomorphic*) except at a set of isolated SINGULARITIES is said to be _______________.

A function defined in an open subset D of the complex plane that is analytic (also called *holomorphic*) except at a set of isolated SINGULARITIES is said to be MEROMORPHIC.

A neat technique for evaluating real integrals of the form

$$\int_{-\infty}^{\infty} \frac{P(x)}{Q(x)}\, dx,$$

where $P(x)$ and $Q(x)$ are polynomials with

$$\deg Q(x) \geq \underline{\hspace{3cm}}$$

is to use the ________ __________ on the complex integral

$$\int_{C_R} \frac{P(z)}{Q(z)}\, dz,$$

where C_R is the positively oriented contour from ___ to _ on the x-axis, and the upper-half _____ _______ of radius _.

A neat technique for evaluating real integrals of the form

$$\int_{-\infty}^{\infty} \frac{P(x)}{Q(x)}\, dx,$$

where $P(x)$ and $Q(x)$ are polynomials with

$$\deg Q(x) \geq \underline{\deg P(x) + 2}$$

is to use the ________ __________ on the complex integral

$$\int_{C_R} \frac{P(z)}{Q(z)}\, dz,$$

where C_R is the positively oriented contour from ___ to _ on the x-axis, and the upper-half _____ _______ of radius _.

A neat technique for evaluating real integrals of the form

$$\int_{-\infty}^{\infty} \frac{P(x)}{Q(x)}\, dx,$$

where $P(x)$ and $Q(x)$ are polynomials with

$$\deg Q(x) \geq \underline{\deg P(x) + 2}$$

is to use the RESIDUE THEOREM on the complex integral

$$\int_{C_R} \frac{P(z)}{Q(z)}\, dz,$$

where C_R is the positively oriented contour from ___ to _ on the x-axis, and the upper-half _____ _______ of radius _.

A neat technique for evaluating real integrals of the form

$$\int_{-\infty}^{\infty} \frac{P(x)}{Q(x)}\, dx,$$

where $P(x)$ and $Q(x)$ are polynomials with

$$\deg Q(x) \geq \underline{\deg P(x) + 2}$$

is to use the RESIDUE THEOREM on the complex integral

$$\int_{C_R} \frac{P(z)}{Q(z)}\, dz,$$

where C_R is the positively oriented contour from $\underline{-R}$ to $\underline{R}$ on the x-axis, and the upper-half _____ _______ of radius _.

A neat technique for evaluating real integrals of the form

$$\int_{-\infty}^{\infty} \frac{P(x)}{Q(x)}\,dx,$$

where $P(x)$ and $Q(x)$ are polynomials with

$$\deg Q(x) \geq \underline{\deg P(x) + 2}$$

is to use the RESIDUE THEOREM on the complex integral

$$\int_{C_R} \frac{P(z)}{Q(z)}\,dz,$$

where C_R is the positively oriented contour from $\underline{-R}$ to $\underline{R}$ on the x-axis, and the upper-half SEMI CIRCLE of radius $\underline{R}$.

The ______-______ theorem states that the ______ of the ___________ of an analytic function are contained in the ________ _____ of the ______ of the original function.

The GAUSS-LUCAS theorem states that the ZEROS of the DERIVATIVE of an analytic function are contained in the CONVEX HULL of the ZEROS of the original function.

An easy way to evaluate $\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^{25}$ is to use

_________________ notation.

An easy way to evaluate $\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^{25}$ is to use

EXPONENTIAL notation.

An easy way to evaluate $\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)^{25}$ is to use

EXPONENTIAL notation.

Doing so gives __________ as an answer.

An easy way to evaluate $\left(\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}\right)^{25}$ is to use

EXPONENTIAL notation.

Doing so gives $\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}$ as an answer.

If f and g are meromorphic in a simply connected domain D, and Γ is a simple closed positively oriented contour contained in D, and $\_\_\_\_\_\_\_\_\_\_ < \_\_\_\_\_ + \_\_\_\_\_$ for $z \in \Gamma$, then $\_\_\_\_\_\_ = \_\_\_\_\_\_$.

If f and g are meromorphic in a simply connected domain D, and Γ is a simple closed positively oriented contour contained in D, and $\underline{|f(z) + g(z)|} < \underline{|f(z)|} + \underline{|g(z)|}$ for $z \in \Gamma$, then $\_\_\_\_\_\_ = \_\_\_\_\_\_$.

If f and g are meromorphic in a simply connected domain D, and Γ is a simple closed positively oriented contour contained in D, and $\underline{|f(z)+g(z)|} < \underline{|f(z)|} + \underline{|g(z)|}$ for $z \in \Gamma$, then $\underline{Z_f - P_f} = \underline{Z_g - P_g}$.

If f is __________ in a simply connected domain D, and Γ is a simple closed positively oriented contour in D, then the expression

____

counts the number of times $f(\Gamma)$ ______ ________ ____ _______.

If f is ANALYTIC in a simply connected domain D, and Γ is a simple closed positively oriented contour in D, then the expression

$$\underline{\frac{1}{2\pi i}\int_\Gamma \frac{f'(z)}{f(z)}\,dz}$$

counts the number of times $f(\Gamma)$ WINDS AROUND THE ORIGIN.

If f is ANALYTIC in a simply connected domain D, and Γ is a simple closed positively oriented contour in D, then the expression

$$\frac{1}{2\pi i}\int_{\Gamma}\frac{f'(z)}{f(z)}\,dz$$

counts the number of times $f(\Gamma)$ WINDS AROUND THE ORIGIN.

The expression

$$\frac{1}{2\pi i}\int_{\Gamma}\frac{f'(z)}{f(z)-\alpha}\,dz$$

counts the number of times $f(\Gamma)$ ______ ________ the point _.

If f is ANALYTIC in a simply connected domain D, and Γ is a simple closed positively oriented contour in D, then the expression

$$\underline{\frac{1}{2\pi i}\int_\Gamma \frac{f'(z)}{f(z)}\,dz}$$

counts the number of times $f(\Gamma)$ WINDS AROUND THE ORIGIN.

The expression

$$\frac{1}{2\pi i}\int_\Gamma \frac{f'(z)}{f(z)-\alpha}\,dz$$

counts the number of times $f(\Gamma)$ WINDS AROUND the point α.

The Taylor series for $f(z) = \frac{1}{z}$ valid for the disk $D_2(2)$ is

$$\underline{\hspace{6cm}}.$$

The Taylor series for $f(z) = \frac{1}{z}$ valid for the disk $D_2(2)$ is

$$f(z) = \sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}}(z-2)^n.$$

The Laurent series for $f(z) = \dfrac{3}{2+z-z^2}$ valid for the annulus $A(0,1,2)$ is

$$f(z) = \sum_{n=1}^{\infty} \underline{\hspace{4cm}} + \sum_{n=0}^{\infty} \underline{\hspace{2.5cm}}.$$

The Laurent series for $f(z) = \dfrac{3}{2 + z - z^2}$ valid for the annulus $A(0, 1, 2)$ is

$$f(z) = \sum_{n=1}^{\infty} \underline{(-1)^{n+1} z^{-n}} + \sum_{n=0}^{\infty} \underline{\frac{1}{2^{n+1}} z^n}.$$

__________ __________ ________ _____ __________ ________.

COMPLEX ANALYSIS MAKES REAL ANALYSIS SIMPLE.

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__________ __________ __ ____ _____ _________ __ _______________.

COMPLEX ANALYSIS IS THE BEST SUBJECT IN MATHEMATICS.

______ ____ _____ _________, _________ __ ___ _________ ___________.

WHEN ____ _____ _________, _________ ___ ___ _________ ___________.

WHEN LIFE _____ _________, _________ __ ___ _________ ___________.

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WHEN LIFE GETS __________, __________ ___ ___ __________ ____________.

WHEN LIFE GETS COMPLEX, __________ ___ ___ __________ ____________.

WHEN LIFE GETS COMPLEX, MULTIPLY BY ITS COMPLEX CONJUGATE.