

# **SYSTEM IDENTIFICATION:**

## **Theory for the User**

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## PREFACE

System identification is a diverse field that can be presented in many different ways. The subtitle, *Theory for the User*, reflects the attitude of the present treatment. Yes, the book is about theory, but the focus is on theory that has direct consequences for the understanding and practical use of available techniques. My goal has been to give the reader a firm grip on basic principles so that he or she can confidently approach a practical problem, as well as the rich and sometimes confusing literature on the subject.

Stressing the utilitarian aspect of theory should not, I believe, be taken as an excuse for sloppy mathematics. Therefore, I have tried to develop the theory without cheating. The more technical parts have, however, been placed in appendices or in asterisk-marked sections, so that the reluctant reader does not have to stumble through them. In fact, it is a redeeming feature of life that we are able to use many things without understanding every detail of them. This is true also of the theory of system identification. The practitioner who is looking for some quick advice should thus be able to proceed rapidly to Part III (User's Choices) by hopping through the summary sections of the earlier chapters.

The core material of the book should be suitable for a graduate-level course in system identification. As a prerequisite for such a course, it is natural, although not absolutely necessary, to require that the student should be somewhat familiar with dynamical systems and stochastic signals. The manuscript has been used as a text for system identification courses at Stanford University, the Massachusetts Institute of Technology, Yale University, the Australian National University and the Univer-

sities of Lund and Linköping. Course outlines, as well as a solutions manual for the problems, are available from the publisher.

For a course on system identification, the role of computer-based exercises should be stressed. Simulation sessions demonstrating how hidden properties of data are readily recovered by the techniques discussed in the book enhance the understanding and motivation of the material. In the problems labeled S, in Chapters 2 through 16, a basic interactive software package is outlined that should be possible to implement rather painlessly in a high-level environment. A PC-MATLAB version of this package is commercially available (see Ljung, 1986b). With such a package all basic techniques of this book can be illustrated and tested on real and simulated data.

The existing literature on system identification is indeed extensive and virtually impossible to cover in a bibliography. In this book I have tried to concentrate on recent and easily available references that I think are suitable for further study, as well as on some earlier works that reflect the roots of various techniques and results. Clearly, many other relevant references have been omitted.

Finally, some words about the structure of this book: The dependence among the different chapters is illustrated in Figure 1.13, which shows that some chapters are not necessary prerequisites for the following ones. Also, some portions contain material that is directed more toward the serious student of identification theory than to the user. These portions are put either in appendixes or in sections and subsections marked with an asterisk (\*). While occasional references to this material may be encountered, it is safe to regard it as optional reading; the continuity will not be impaired if it is skipped.

The problem sections for each chapter have been organized into six groups of different problem types:

- *G problems*: These could be of General interest and it may be worthwhile to browse through them, even without intending to solve them.
- *E problems*: These are regular pencil-and-paper Exercises to check the basic techniques of the chapter.
- *T problems*: These are Theoretically oriented problems and typically more difficult than the E problems.
- *D problems*: In these problems the reader is asked to fill in technical Details that were glossed over in the text (a way to dump straightforward technicalities from the book into the solutions manual!).
- *S problems*: These develop the basic identification Software package mentioned earlier.
- *C problems*: These require a Computer. Clearly, with the software package at hand, the C problems can be complemented with a myriad of problems experimenting with identification methods and data. Such problems are not specifically listed, but the reader is encouraged to apply those techniques in an exploratory fashion.