

ACTUATOR SATURATION CONTROL

edited by

Vikram Kapila
Polytechnic University
Brooklyn, New York

Karolos M. Grigoriadis
University of Houston
Houston, Texas



MARCEL DEKKER, INC.

NEW YORK • BASEL

ISBN: 0-8247-0751-6

This book is printed on acid-free paper.

Headquarters

Marcel Dekker, Inc.
270 Madison Avenue, New York, NY 10016
tel: 212-696-9000; fax: 212-685-4540

Eastern Hemisphere Distribution

Marcel Dekker AG
Hutgasse 4, Postfach 812, CH-4001 Basel, Switzerland
tel: 41-61-261-8482; fax: 41-61-261-8896

World Wide Web

<http://www.dekker.com>

The publisher offers discounts on this book when ordered in bulk quantities. For more information, write to Special Sales/Professional Marketing at the headquarters address above.

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Current printing (last digit):

10 9 8 7 6 5 4 3 2 1

PRINTED IN THE UNITED STATES OF AMERICA

Series Introduction

Many textbooks have been written on control engineering, describing new techniques for controlling systems, or new and better ways of mathematically formulating existing methods to solve the ever-increasing complex problems faced by practicing engineers. However, few of these books fully address the applications aspects of control engineering. It is the intention of this new series to redress this situation.

The series will stress applications issues, and not just the mathematics of control engineering. It will provide texts that present not only both new and well-established techniques, but also detailed examples of the application of these methods to the solution of real-world problems. The authors will be drawn from both the academic world and the relevant applications sectors.

There are already many exciting examples of the application of control techniques in the established fields of electrical, mechanical (including aerospace), and chemical engineering. We have only to look around in today's highly automated society to see the use of advanced robotics techniques in the manufacturing industries; the use of automated control and navigation systems in air and surface transport systems; the increasing use of intelligent control systems in the many artifacts available to the domestic consumer market; and the reliable supply of water, gas, and electrical power to the domestic consumer and to industry. However, there are currently many challenging problems that could benefit from wider exposure to the applicability of control methodologies, and the systematic systems-oriented basis inherent in the application of control techniques.

This series presents books that draw on expertise from both the academic world and the applications domains, and will be useful not only as academically recommended course texts but also as handbooks for practitioners in many applications domains. *Actuator Saturation Control* is another outstanding entry to Dekker's Control Engineering series.

Neil Munro

Preface

All real-world applications of feedback control involve control actuators with amplitude and rate limitations. In particular, any physical electro-mechanical device can provide only a limited force, torque, stroke, flow capacity, or linear/angular rate. The control design techniques that ignore these actuator limits may cause undesirable transient response, degrade the closed-loop performance, and may even cause closed-loop instability. For example, in advanced tactical fighter aircraft with high maneuverability requirements, actuator amplitude and rate saturation in the control surfaces may cause pilot-induced oscillations leading to degraded flight performance or even catastrophic failure. Thus, actuator saturation constitutes a fundamental limitation of many linear (and even nonlinear) control design techniques and has attracted the attention of numerous researchers, especially in the last decade.

In prior research, the control saturation problem has been examined via the extensions of optimal control theory, anti-windup compensation, supervisory error governor approach, Riccati and Lyapunov-based local and semi-global stabilization, and bounded-real, positive-real, and absolute stabilization frameworks. This prior research literature and the currently developing research directions provide a rich variety of techniques to account for actuator saturation. Furthermore, tremendous strides are currently being made to advance the saturation control design techniques to address important issues of performance degradation, disturbance attenuation, robustness to uncertainty/time delays, domain of attraction estimation, and control rate saturation.

The scope of this edited volume includes advanced analysis and synthesis methodologies for systems with actuator saturation, an area of intense current research activity. This volume covers some of the significant research advancements made in this field over the past decade. It emphasizes the issue of rigorous, non-conservative, mathematical formulations of actuator saturation control along with the development of efficient computational algorithms for this class of problems. The volume is intended for researchers and graduate students in engineering and applied mathematics with interest in control systems analysis and design.

This edited volume provides a unified forum to address various novel aspects of actuator saturation control. The contributors of this edited volume include some nationally and internationally recognized researchers who have

made or continue to make significant contributions to this important field of research in our discipline. Below we highlight the key issues addressed by each contributor.

Chapter 1 by Barbu *et al.* considers the design of anti-windup control for linear systems with exponentially unstable modes in the presence of input magnitude and rate saturation. The chapter builds on prior work by these authors on uniting local and global controllers. Specifically, the anti-windup design of this chapter enables exponentially unstable saturated linear systems to perform satisfactorily in a large operating region. In addition, the chapter provides sufficient conditions for this class of systems to achieve local performance and global stability. Finally, via a manual flight control example involving an unstable aircraft with saturating actuators, it illustrates the efficacy of the proposed control design methodology in facilitating aggressive maneuvers while preserving stability.

Chapter 2 by Eun *et al.* focuses on selecting the actuator saturation level for small performance degradation in linear designs. A novel application of a general stochastic linearization methodology, which approximates the saturation nonlinearity with a quasi-linear gain, is brought to bear on this problem. Specifically, to determine the allowable actuator saturation level, standard deviations of performance and control in the presence of saturation are obtained using stochastic linearization. The resulting expression for the allowable actuator saturation level is shown to be a function of performance degradation, a positive real number based on the Nyquist plot of the linear part of the system, and the standard deviation of controller output. Numerical examples show that by choosing performance degradation of 10 percent, the actuator saturation level is a weak function of a system intrinsic parameter, *viz.*, the positive real number based on the Nyquist plot of the linear part of the system.

Chapter 3 by Hu *et al.* is motivated by the issue of asymmetric actuators, a problem of considerable practical concern. In previous research, the authors studied the problem of null controllable regions and stabilizability of exponentially unstable linear systems in the presence of actuator saturation. However, this earlier attempt was restricted to symmetric actuator saturation and hence excluded a large class of real-world problems with asymmetric actuator saturation. This chapter addresses the characterization of null controllable regions and stabilization on the null controllable region, for linear, exponentially unstable systems with asymmetrically saturating actuators. First, it is shown that the trajectories produced by extremal control inputs of linear low-order systems have explicit reachable boundaries. Next, under certain conditions, a closed-trajectory is demonstrated to be the boundary of the domain of attraction under saturated

linear state feedback. Finally, it is proven that the domain of attraction of second order anti-stable systems under the influence of linear quadratic control can be enlarged arbitrarily close to the null controllability region by using high gain feedback.

Chapter 4 by Iwasaki and Fu is concerned with regional H_2 performance synthesis of dynamic output feedback controllers for linear time-invariant systems subject to known bounds on control input magnitude. In order to guarantee closed-loop stability and H_2 performance, this chapter utilizes the circle and linear analysis techniques. Whereas the circle analysis is applicable to a state space region in which the actuator may saturate, the linear analysis is restricted to a state space region in which the saturation is not activated. It is shown that the circle criterion based control design does not enhance the domain of performance for a specified performance level *vis-a-vis* the linear design. Finally, since the performance overbound is inherently conservative, it is illustrated that the circle criterion based control design can indeed lead to improved performance *vis-a-vis* the linear design. Both fixed-gain and switching control design are addressed.

Chapter 5 by Jabbari employs a linear parameter varying (LPV) approach to handle the inevitable limitations in actuator capacity in a disturbance attenuation setting. The chapter begins by converting a saturating control problem to an unconstrained LPV problem. Next, a fixed Lyapunov function based approach is considered to address an output feedback control design problem for polytopic LPV system. To overcome the conservatism of LPV control designs based on fixed Lyapunov function, a parameter-dependent LPV control methodology is presented. It is shown that the LPV control design framework is capable of handling input magnitude and rate saturation. A scheduling control design approach to deal with actuator saturation is also considered. Two numerical examples illustrate the effectiveness of the proposed control methodologies.

Chapter 6 by Pan and Kapila is focused on the control of discrete-time systems with actuator saturation. It is noted that a majority of the previous research effort in the literature has focused on the control of continuous-time systems with control signal saturation. Nevertheless, in actual practical applications of feedback control, it is the overwhelming trend to implement controllers digitally. Thus, this chapter develops linear matrix inequality (LMI) formulations for the state feedback and dynamic, output feedback control designs for discrete-time systems with simultaneous actuator amplitude and rate saturation. Furthermore, it provides a direct methodology to determine the stability multipliers that are essential for reducing the conservatism of the weighted circle criterion-based saturation control design. The chapter closes with two illustrative numerical examples which

demonstrate the efficacy of the proposed control design framework.

Chapter 7 by Pare *et al.* addresses the design of feedback controllers for local stabilization and local performance synthesis of saturated feedback systems. In particular, the chapter formulates optimal control designs for saturated feedback systems by considering three different performance objectives: region of attraction, disturbance rejection, and \mathcal{L}_2 -gain. The Popov stability theory and a sector model of the saturation nonlinearity are brought to bear on these optimal control design problems. The bilinear matrix inequality (BMI) and LMI optimization frameworks are exploited to characterize the resulting optimal control laws. Commercially available LMI software facilitates efficient numerical computation of the controller matrices. A linearized inverted pendulum example illustrates the proposed local \mathcal{L}_2 -gain design.

Chapter 8 by Saberi *et al.* focuses on output regulation of linear systems in the presence of state and input constraints. A recently developed novel nonlinear operator captures the simultaneous amplitude and rate constraints on system states and input. The notion of a constraint output is developed to handle both the state and input constraints. A taxonomy of constraints is developed to characterize conditions under which various constraint output regulation problems are solvable. Low-gain and low-high gain control designs including a scheduled low-gain control design are developed for linear systems with amplitude and rate saturating actuators. Finally, output regulation problems in the presence of right invertible and non-right-invertible constraints are also considered.

Chapter 9 by Soroush and Daoutidis begins by surveying the notions of directionality and windup and recent directionality and windup compensation schemes that account for and negate the degrading influence of constrained actuators. The principal focus of the chapter is on stability and performance issues for input-constrained multi-input multi-output (MIMO) nonlinear systems subject to directionality and integrator windup. In particular, the chapter poses the optimal directionality compensation problem as a finite-time horizon, state dependent, constrained quadratic optimization problem with an objective to minimize the distance between the output of the unsaturated plant with an ideal controller and the output of the saturated plant with directionality compensator. Simulation results for a MIMO linear time invariant system and a nonlinear bioreactor subject to input constraints illustrate that the optimal directionality compensation improves system performance *vis-a-vis* traditional clipping and direction preservation algorithms. Finally, the chapter proposes an input-output linearizing control algorithm with integral action and optimal directionality compensation to handle input-constrained MIMO nonlinear systems

affected by integrator windup. This windup compensation methodology is illustrated to be effective on a simulated nonlinear chemical reactor.

Chapter 10 by Tarbouriech and Garcia develops Riccati- and LMI-based approaches to design robust output feedback controllers for uncertain systems with position and rate bounded actuators. The proposed controllers ensure robust stability and performance in the presence of norm-bounded time-varying parametric uncertainty. In addition, this control design methodology is applicable to local stabilization of open-loop unstable systems. It is noted that in this chapter, the authors present yet another novel approach, *viz.*, polytopic representation of saturation nonlinearities, to address the actuator saturation problem. Two numerical examples illustrate the efficacy of the proposed saturation control designs.

Chapter 11 by Wu and Grigoriadis addresses the problem of feedback control design in the presence of actuator amplitude saturation. Specifically, by exploiting the LPV design framework, this chapter develops a systematic anti-windup control design methodology for systems with actuator saturation. In contrast to the conventional two-step anti-windup design approaches, the proposed scheme involving induced \mathcal{L}_2 gain control schedules the parameter-varying controller by using a saturation indicator parameter. The LPV control law is characterized via LMIs that can be solved efficiently using interior-point optimization algorithms. The resulting gain-scheduled controller is nonlinear in general and would lead to graceful performance degradation in the presence of actuator saturation nonlinearities and linear performance recovery. An aircraft longitudinal dynamics control problem with two input saturation nonlinearities is used to demonstrate the effectiveness of the proposed LPV anti-windup scheme.

We believe that this edited volume is a unique addition to the growing literature on actuator saturation control, in that it provides coverage to competing actuator saturation control methodologies in a single volume. Furthermore, it includes major new control paradigms proposed within the last two to three years for actuator saturation control. Several common themes emerge in these 11 chapters. Specifically, actuator amplitude and rate saturation control is considered in Chapters 1, 5, 6, 8, and 10. LMI-based tools for actuator saturation control are employed in Chapters 4, 5, 6, 7, 10, and 11. Furthermore, an LPV approach is used to handle input saturation in Chapters 5 and 11. Finally, scheduled/switching control designs for saturating systems are treated in Chapters 4, 5, and 8.

We thank all the authors who made this volume possible by their contributions and by providing timely revisions. We also thank the anonymous reviewers who reviewed an early version of this manuscript and provided valuable feedback. We thank B. J. Clark, Executive Acquisitions Editor,

Marcel Dekker, Inc., who encouraged this project from its inception. Last but not least, we thank Dana Bigelow, Production Editor, Marcel Dekker, Inc., who patiently worked with us to ensure timely completion of this endeavor.

Vikram Kapila

Karolos M. Grigoriadis

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Contributors

C. Barbu	University of California, Santa Barbara, California
P. Daoutidis	University of Minnesota, Minneapolis, Minnesota
Y. Eun	University of Michigan, Ann Arbor, Michigan
M. Fu	University of Newcastle, Newcastle, Australia
G. Garcia	Laboratoire d'Analyse et d'Architecture des Systèmes du C.N.R.S., Toulouse, France
C. Gökçek	University of Michigan, Ann Arbor, Michigan
K.M. Grigoriadis	University of Houston, Houston, Texas
H. Hindi	Stanford University, Stanford, California
J. How	Massachusetts Institute of Technology, Cambridge, Massachusetts
T. Hu	University of Virginia, Charlottesville, Virginia
T. Iwasaki	University of Virginia, Charlottesville, Virginia
F. Jabbari	University of California, Irvine, California
P.T. Kabamba	University of Michigan, Ann Arbor, Michigan
V. Kapila	Polytechnic University, Brooklyn, New York
Z. Lin	University of Virginia, Charlottesville, Virginia
S.M. Meerkov	University of Michigan, Ann Arbor, Michigan
H. Pan	Polytechnic University, Brooklyn, New York
T. Pare	Malibu Networks, Campbell, California
A.N. Pitsillides	University of Virginia, Charlottesville, Virginia
R. Reginatto	University of California, Santa Barbara, California
A. Saberi	Washington State University, Pullman, Washington

P. Sannuti	Rutgers University, Piscataway, New Jersey
G. Shi	Washington State University, Pullman, Washington
M. Soroush	Drexel University, Philadelphia, Pennsylvania
A.A. Stoorvogel	Eindhoven University of Technology, Eindhoven, and Delft University of Technology, Delft, the Netherlands
S. Tarbouriech	Laboratoire d'Analyse et d'Architecture des Systèmes du C.N.R.S., Toulouse, France
A.R. Teel	University of California, Santa Barbara, California
F. Wu	North Carolina State University, Raleigh, North Carolina
L. Zaccarian	University of California, Santa Barbara, California

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