

Area of a circle

Purpose:

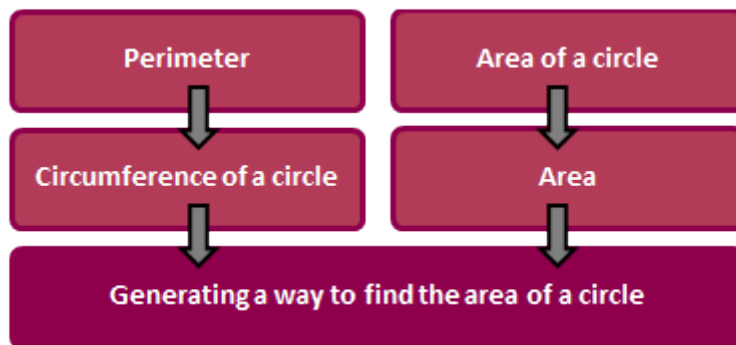
The purpose of this multi-level task is to engage students in an investigation of the area of circles.

Achievement Objectives:

GM5-4: Find the perimeters and areas of circles and composite shapes and the volumes of prisms, including cylinders.

Description of mathematics:

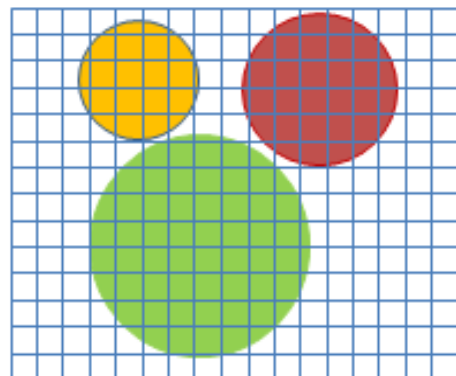
The background knowledge presumed for this task is outlined in the diagram below:



The task can be presented with graded expectations to provide appropriate challenge for individual learning needs. The word establish has been used so that students can receive much guidance to verify the given rule, or can deduce a rule, using the radius and circumference of the circle. The extension of the algebraically able student, in this task, is intended to develop the thinking needed to understand calculus. The early ideas of decreasing the size of the sectors, leading to greater accuracy and approaching a true model is a necessary concept for students to understand integration (Level 8 mathematics) though the Level 5 content of areas of a circle.

Activity:

Task: Find the area of one of these circles and use this to establish a general rule for any circle of radius r .

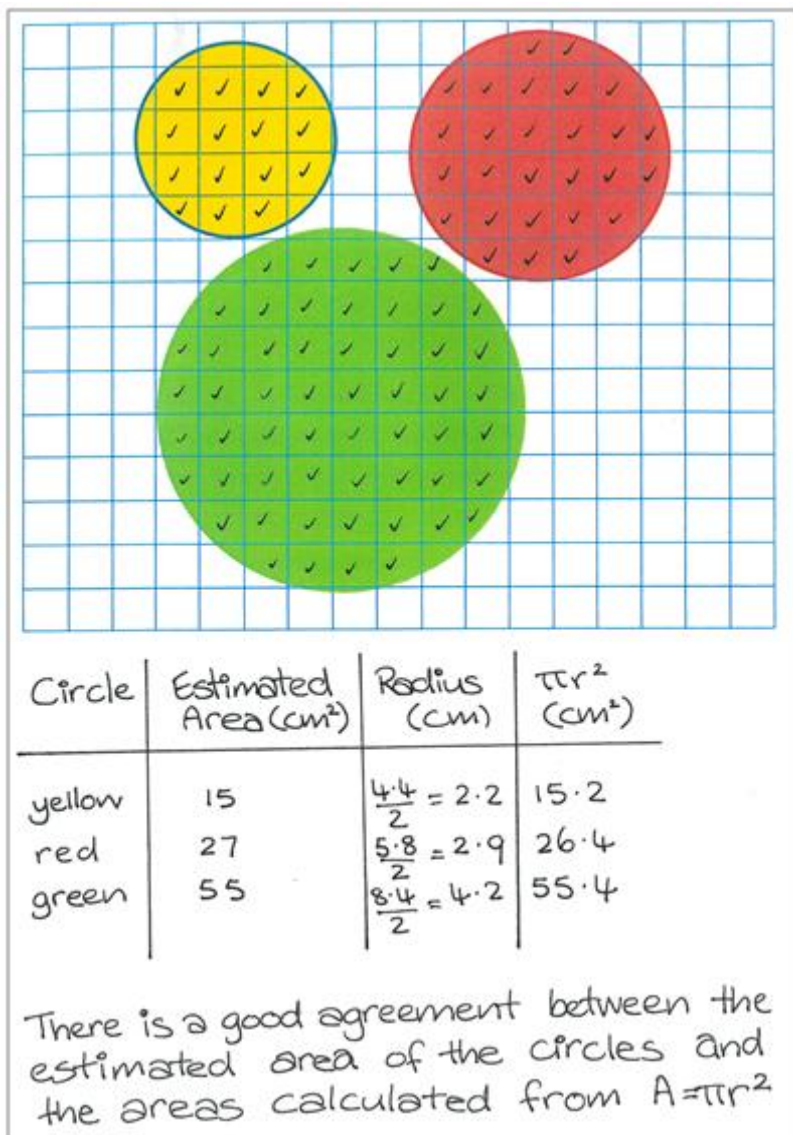


The arithmetic approach

The student carries out directed calculations that will lead them to verify the formula for the area of a circle.

Prompts from the teacher could be:

1. Estimate the area of each of the circles provided on the resource sheet, using the 1cm^2 grid lines drawn over them.
2. Measure the radius of each circle.
3. Calculate the area for each of the circles using $A = \pi r^2$.
4. Compare your results for the estimated and calculated areas.



The procedural algebraic approach

The student carries out an algebraic investigation that would allow them to verify and to show, by example, the derivation of the area of a circle formula.

Prompts from the teacher could be:

1. Mark 1cm^2 gridlines on the circle given to estimate its area.
2. Measure the radius of the circle and use the formula $A = \pi r^2$ to find the area of the circle.
3. Cut your circle into many sectors and arrange these into a shape that is roughly rectangular. Use this to find another estimate of the area of the circle.
4. Discuss the ways you found the area of the circle.

1. Estimate of Area
 54 cm^2

2. From measuring
 $D = 8.3\text{ cm}$
 $r = 4.15\text{ cm}$
 $A = \pi \times 4.15^2$
 $= 54.1\text{ cm}^2$ (1 d.p.)

3. Making a rough rectangle
 4.1 cm
 13.2 cm
 $A = 4.1 \times 13.2$
 $= 54.1\text{ cm}^2$ (1 d.p.)

T: How did you make this as accurate as possible?

S: Well, to get r for my calculation, I measured the diameter and divided by two. But the estimation was surprisingly accurate!

T: How did you make this as close to a rectangle as possible?

S: I cut 12 equal sectors, but that made a parallelogram, so I cut one of the sectors to be able to put half at each end.

T: Tell me about these three methods – which is best?

S: Well, they all gave the same area, but estimating doesn't give d.p. (decimal places). The quickest is to get r and calculate πr^2 so I'd use that most often.

The conceptual algebraic approach

The student develops a rule, using familiar processes and a chain of reasoning.

nb: The early ideas of decreasing the size of the sectors, leading to greater accuracy and approaching a true model is a necessary concept for students to understand integration (Level 8 mathematics).

Further exploration of the quadratic relationship can be encouraged, with extended questioning:

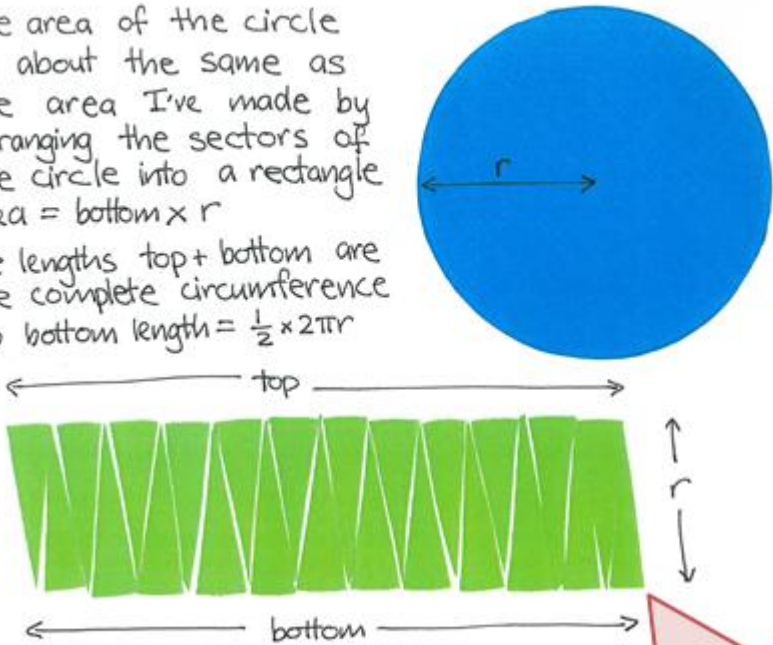
1. Cut your circle into many sectors and arrange these into a shape that allows the area to be found in terms of r , the radius.
2. Use this to find a general rule for the area of the circle. (Hint: in this case, general rule would mean a rule that is in terms of r)

The early ideas of decreasing the size of the sectors, leading to greater accuracy and approaching a true model is a necessary concept for students to understand integration (Level 8 mathematics).

The area of the circle is about the same as the area I've made by arranging the sectors of the circle into a rectangle

area = bottom \times r

The lengths top + bottom are one complete circumference
so bottom length = $\frac{1}{2} \times 2\pi r$



Area = $\frac{1}{2} \times 2\pi r \times r$
 $= \pi r^2$

T: Is this a good approximation of a rectangle?

S: Not exactly. The top and bottom are wobbly, but if I used a really big circle or lots of cuts it would be nearly flat.

T: And the height, is it r ?

S: Well, its closer to a parallelogram and the height would be less than r . But again, with a huge circle and lots of sectors, the height would be close to r .