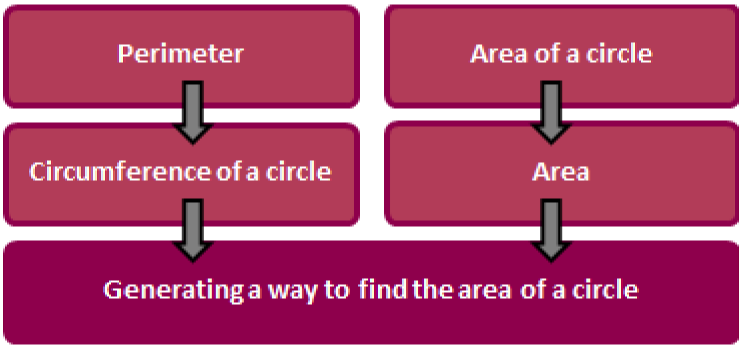
Area of a circle

Purpose:
The purpose of this multi-level task is to engage students in an investigation of the area of circles.

Achievement Objectives:
GM5-4: Find the perimeters and areas of circles and composite shapes and the volumes of prisms, including cylinders.

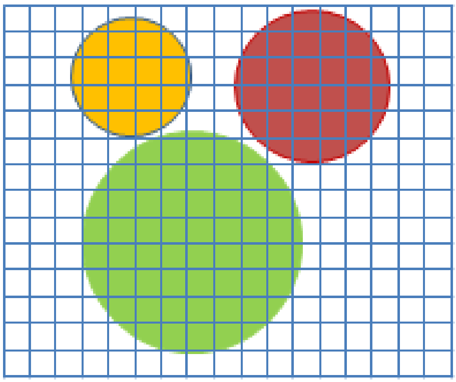
Description of mathematics:
The background knowledge presumed for this task is outlined in the diagram below:



The task can be presented with graded expectations to provide appropriate challenge for individual learning needs. The word establish has been used so that students can receive much guidance to verify the given rule, or can deduce a rule, using the radius and circumference of the circle. The extension of the algebraically able student, in this task, is intended to develop the thinking needed to understand calculus. The early ideas of decreasing the size of the sectors, leading to greater accuracy and approaching a true model is a necessary concept for students to understand integration (Level 8 mathematics) though the Level 5 content of areas of a circle.

Activity:

Task: Find the area of one of these circles and use this to establish a general rule for any circle of radius r.

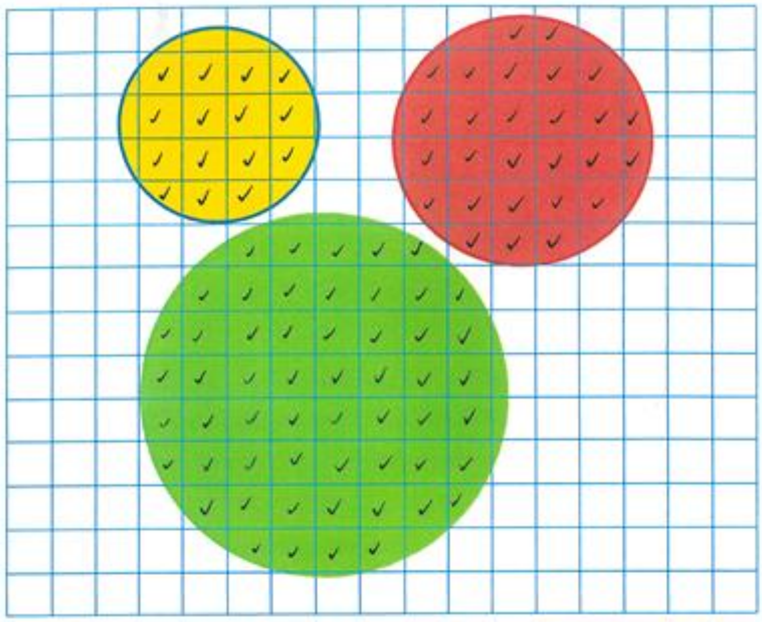


The arithmetic approach

The student carries out directed calculations that will lead them to verify the formula for the area of a circle.

Prompts from the teacher could be:

1. Estimate the area of each of the circles provided on the resource sheet, using the $1cm^2$ grid lines drawn over them.
2. Measure the radius of each circle.
3. Calculate the area for each of the circles using $A = \pi r^2$.
4. Compare your results for the estimated and calculated areas.



Circle	Estimated Area (cm^2)	Radius (cm)	πr^2 (cm^2)
yellow	15	$\frac{4.4}{2} = 2.2$	15.2
red	27	$\frac{5.8}{2} = 2.9$	26.4
green	55	$\frac{8.4}{2} = 4.2$	55.4

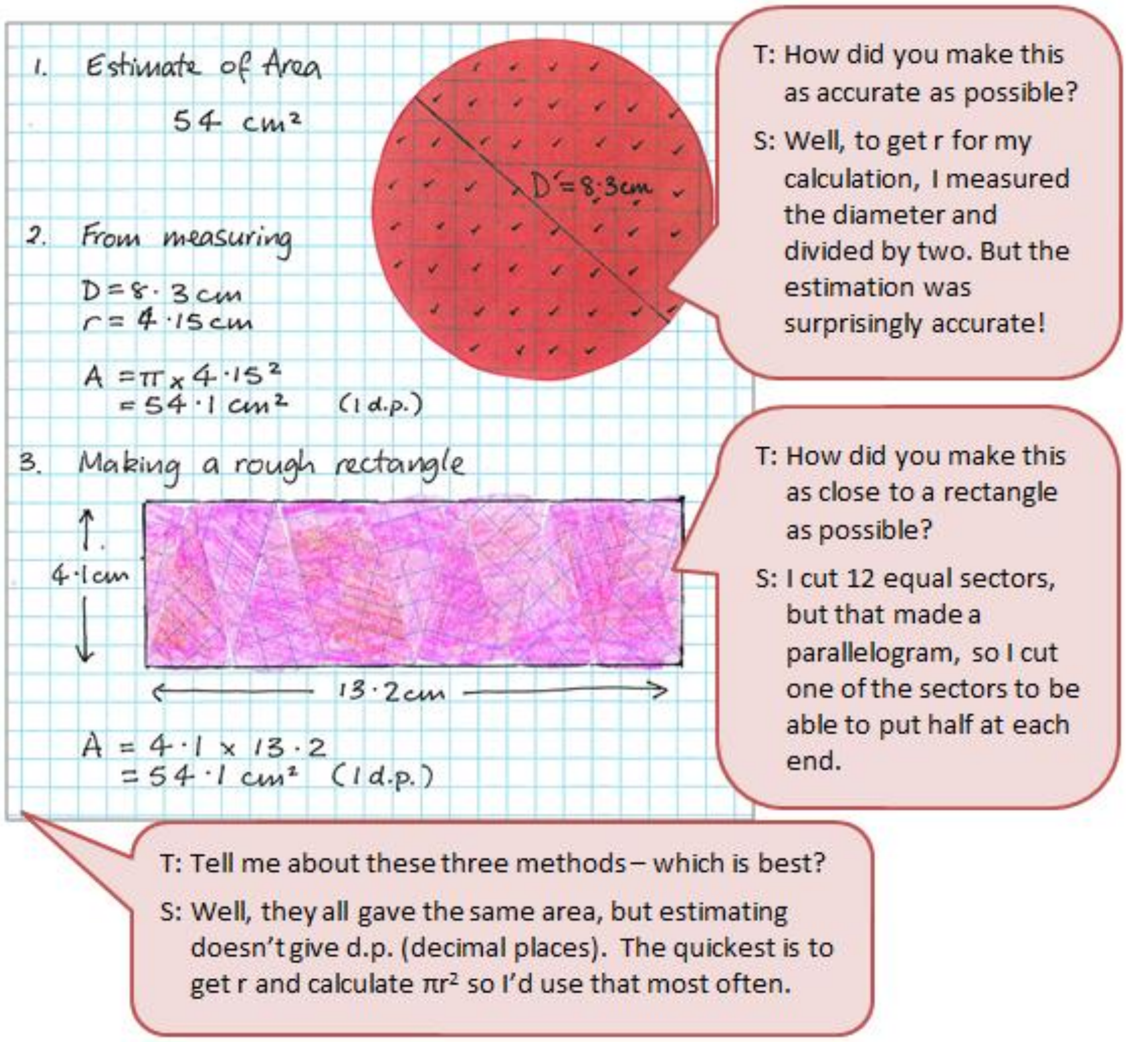
There is a good agreement between the estimated area of the circles and the areas calculated from $A = \pi r^2$

The procedural algebraic approach

The student carries out an algebraic investigation that would allow them to verify and to show, by example, the derivation of the area of a circle formula.

Prompts from the teacher could be:

1. Mark $1cm^2$ gridlines on the circle given to estimate its area.
2. Measure the radius of the circle and use the formula $A = \pi r^2$ to find the area of the circle.
3. Cut your circle into many sectors and arrange these into a shape that is roughly rectangular. Use this to find another estimate of the area of the circle.
4. Discuss the ways you found the area of the circle.



The conceptual algebraic approach

The student develops a rule, using familiar processes and a chain of reasoning.

nb: The early ideas of decreasing the size of the sectors, leading to greater accuracy and approaching a true model is a necessary concept for students to understand integration (Level 8 mathematics).

Further exploration of the quadratic relationship can be encouraged, with extended questioning:

1. Cut your circle into many sectors and arrange these into a shape that allows the area to be found in terms of r, the radius.
2. Use this to find a general rule for the area of the circle. (Hint: in this case, general rule would mean a rule that is in terms of r)

The early ideas of decreasing the size of the sectors, leading to greater accuracy and approaching a true model is a necessary concept for students to understand integration (Level 8 mathematics).