8.4 Area of Triangles

Goal

Find the area of triangles.

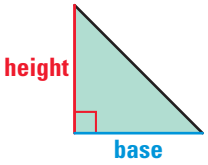
Key Words

- height of a triangle
- base of a triangle

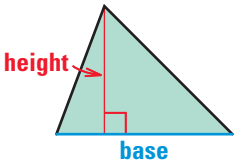
The amount of material needed to make the sail at the right is determined by the area of the triangular sail.

The *height* and *base* of a triangle are used to find its area.

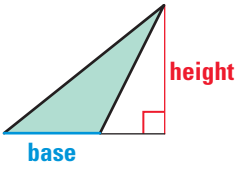
The **height of a triangle** is the perpendicular segment from a vertex to the line containing the opposite side. The opposite side is called the **base of the triangle**. The terms *height* and *base* are also used to represent the segment lengths.



In a right triangle, a leg is a height.



A height can be inside the triangle.



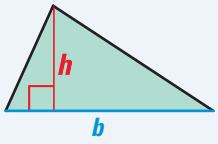
A height can be outside the triangle.

As shown in Activity 8.4, the area of a triangle is found using a base and its corresponding height.

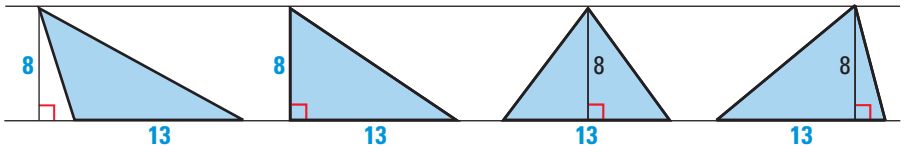
AREA OF A TRIANGLE

Words Area $= \frac{1}{2}$ (base)(height)

Symbols $A = \frac{1}{2}bh$

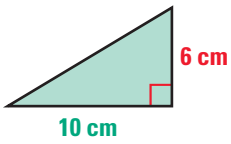


Triangles with the Same Area Triangles can have the same area without necessarily being congruent. For example, all of the triangles below have the same area but they are not congruent.



EXAMPLE 1 Find the Area of a Right Triangle

Find the area of the right triangle.



Solution

Use the formula for the area of a triangle. Substitute 10 for b and 6 for h.

$A = \frac{1}{2}bh$ Formula for the area of a triangle

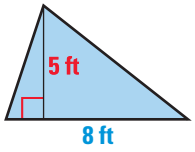
$= \frac{1}{2}(10)(6)$ Substitute 10 for b and 6 for h.

$= 30$ Simplify.

ANSWER ▶ The triangle has an area of 30 square centimeters.

EXAMPLE 2 Find the Area of a Triangle

Find the area of the triangle.



Solution

$A = \frac{1}{2}bh$ Formula for the area of a triangle

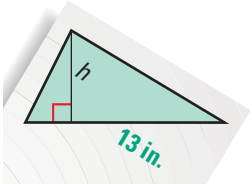
$= \frac{1}{2}(8)(5)$ Substitute 8 for b and 5 for h.

$= 20$ Simplify.

ANSWER ▶ The triangle has an area of 20 square feet.

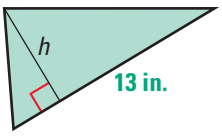
Visualize It!

To help you determine the base and height of a tilted triangle, turn your book so that the base is horizontal.



EXAMPLE 3 Find the Height of a Triangle

Find the height of the triangle, given that its area is 39 square inches.



Solution

$A = \frac{1}{2}bh$ Formula for the area of a triangle

$39 = \frac{1}{2}(13)h$ Substitute 39 for A and 13 for b.

$78 = 13h$ Multiply each side by 2.

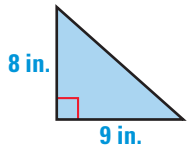
$6 = h$ Divide each side by 13.

ANSWER ▶ The triangle has a height of 6 inches.

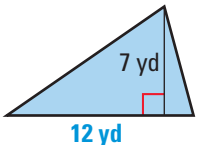
Checkpoint ✓ Area of Triangles

In Exercises 1–3, find the area of the triangle.

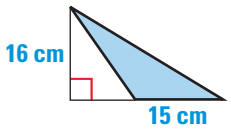
1.



2.



3.

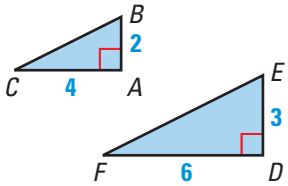


4. A triangle has an area of 84 square inches and a height of 14 inches. Find the base.

EXAMPLE 4 Areas of Similar Triangles

a. Find the ratio of the areas of the similar triangles.

b. Find the scale factor of $\triangle ABC$ to $\triangle DEF$ and compare it to the ratio of their areas.



$\triangle ABC \sim \triangle DEF$

Solution

a. Area of $\triangle ABC = \frac{1}{2}bh = \frac{1}{2}(4)(2) = 4$ square units

Area of $\triangle DEF = \frac{1}{2}bh = \frac{1}{2}(6)(3) = 9$ square units

Ratio of areas $= \frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \frac{4}{9}$

b. The scale factor of $\triangle ABC$ to $\triangle DEF$ is $\frac{2}{3}$.

The ratio of the areas is the square of the scale factor: $\frac{2^2}{3^2} = \frac{4}{9}$.

Student Help

LOOK BACK
To review scale factor, see p. 366.

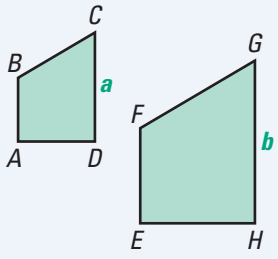
The relationship in Example 4 is generalized for all similar polygons in the following theorem.

THEOREM 8.3

Areas of Similar Polygons

Words If two polygons are similar with a scale factor of $\frac{a}{b}$, then the ratio of their areas is $\frac{a^2}{b^2}$.

Symbols If $ABCD \sim EFGH$ with a scale factor of $\frac{a}{b}$, then $\frac{\text{Area of } ABCD}{\text{Area of } EFGH} = \frac{a^2}{b^2}$.

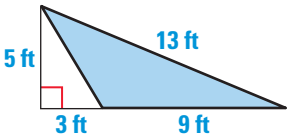


8.4 Exercises

Guided Practice

Vocabulary Check

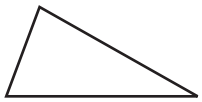
1. What are the measures of the base and the height of the shaded triangle at the right?



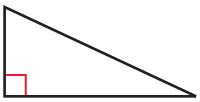
Skill Check

The triangle has a horizontal base of 15 units and a height of 7 units. Sketch the triangle and label its base and its height.

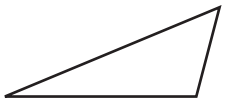
2.



3.



4.



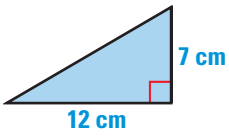
Practice and Applications

Extra Practice

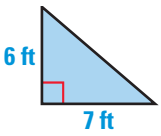
See p. 690.

Area of a Right Triangle **Find the area of the right triangle.**

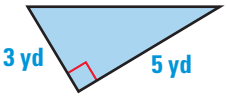
5.



6.

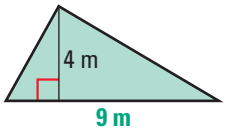


7.

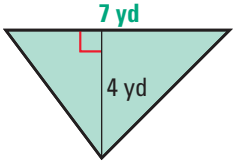


Finding Area **In Exercises 8–13, find the area of the triangle.**

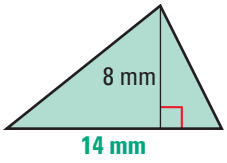
8.



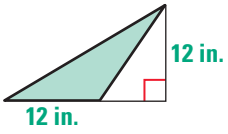
9.



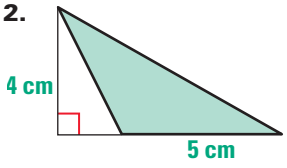
10.



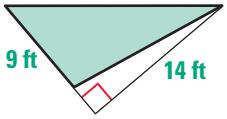
11.



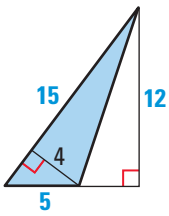
12.



13.



14. **You be the Judge** In the triangle at the right, Trisha says the base is 15 and the height is 4. Luis says that the base is 5 and the height is 12. Who is right? Explain your reasoning.



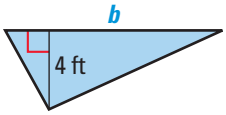
Homework Help

Example 1: Exs. 5–7
Example 2: Exs. 8–13
Example 3: Exs. 16–20
Example 4: Exs. 26, 27

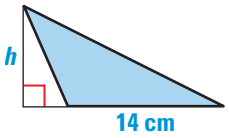
15. **Visualize It!** Draw three different triangles that each have an area of 24 square units.

Using Algebra **In Exercises 16–18, *A* gives the area of the triangle. Find the missing measure.**

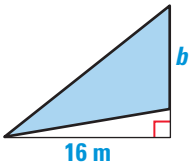
16. $A = 22 \text{ ft}^2$



17. $A = 63 \text{ cm}^2$



18. $A = 80 \text{ m}^2$



19. **Finding the Height** A triangle has an area of 78 square inches and a base of 13 inches. Find the height.

20. **Finding the Base** A triangle has an area of 135 square meters and a height of 9 meters. Find the base.

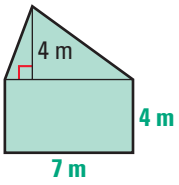
Tiles **In Exercises 21 and 22, use the diagram of the tile pattern.**

21. Find the area of one triangular tile.

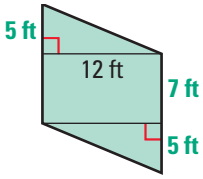
22. The tiles are being used to make a rectangular border that is 4 inches high and 48 inches long. How many tiles are needed for the border? (*Hint*: Start by finding the area of the border.)

Complex Polygons **Find the area of the polygon by using the triangles and rectangles shown.**

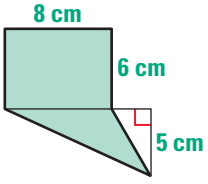
23.



24.

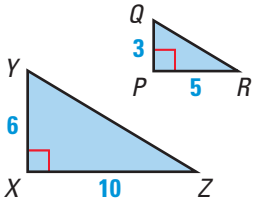


25.

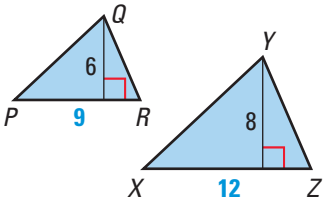


Areas of Similar Triangles **In Exercises 26 and 27, the triangles are similar. Find the scale factor of $\triangle PQR$ to $\triangle XYZ$. Then find the ratio of their areas.**

26.



27.

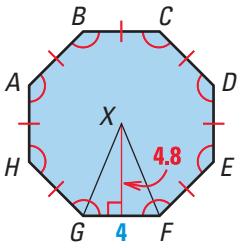


Link to Geology

ROCK FORMATIONS
Geologists learn about the structure of the earth by studying rock formations such as the basaltic columns at the Giant's Causeway in Ireland pictured above.

Application Links
CLASSZONE.COM

Area of a Regular Octagon **In Exercises 28–30, use the regular octagon at the right.**

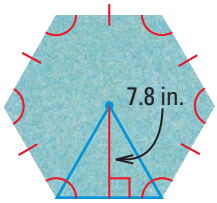


28. Find the area of $\triangle GXF$ in the octagon.

29. Copy the diagram. To form congruent triangles, connect the following pairs of vertices: A and E, B and F, C and G, D and H. How many triangles are formed?

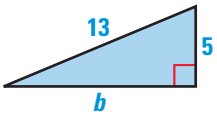
30. What is the area of the octagon? Explain.

31. Rock Formations Many basaltic columns are hexagonal. The top of one of these columns is a regular hexagon as shown below. Find its area. (Another photograph of basaltic columns is on page 408.)



EXAMPLE **Using the Pythagorean Theorem**

Find the area of the triangle.



Solution

First, find the base. Use the Pythagorean Theorem to find the value of b.

$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$ — Pythagorean Theorem

$(13)^2 = (5)^2 + (b)^2$ — Substitute.

$169 = 25 + b^2$ — Simplify.

$144 = b^2$ — Subtract 25 from each side.

$12 = b$ — Find the positive square root.

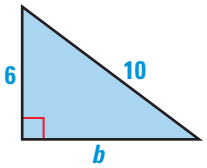
Use 12 as the base in the formula for the area of a triangle.

$A = \frac{1}{2}bh = \frac{1}{2}(12)(5) = 30$ square units

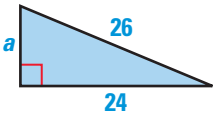
Student Help
LOOK BACK
To review the Pythagorean Theorem, see pp. 192 and 193.

Using the Pythagorean Theorem **Find the area of the triangle.**

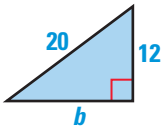
32.



33.



34.

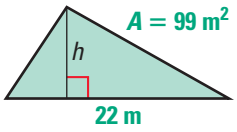


Standardized Test Practice

35. Multiple Choice Given that the area of the triangle is 99 square meters, what is the height of the triangle?

Ⓐ 4.5 m Ⓑ 9 m

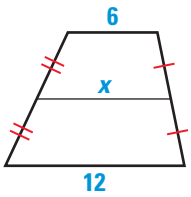
Ⓒ 11 m Ⓓ 22 m



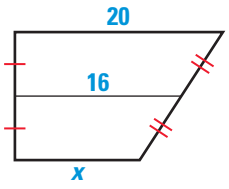
Mixed Review

Trapezoids **Find the value of *x* in the trapezoid.** *(Lesson 6.5)*

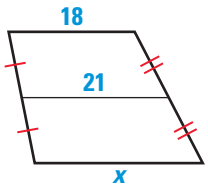
36.



37.



38.



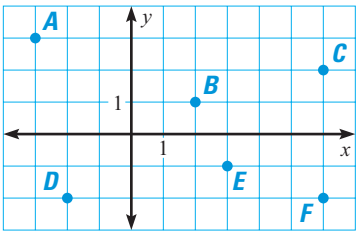
Algebra Skills

Naming Coordinates **Give the coordinates of the point.** *(Skills Review, p. 664)*

39. A **40.** B

41. C **42.** D

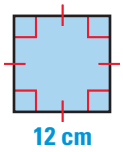
43. E **44.** F



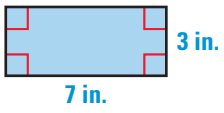
Quiz 2

Find the area of the polygon. *(Lessons 8.3, 8.4)*

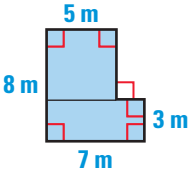
1.



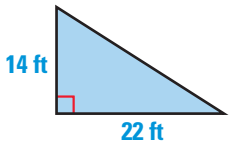
2.



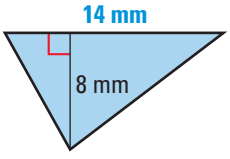
3.



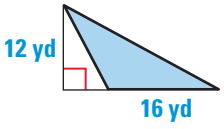
4.



5.

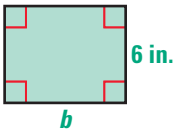


6.

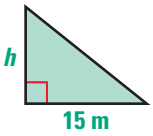


In Exercises 7–9, *A* gives the area of the polygon. Find the missing measure. *(Lessons 8.3, 8.4)*

7. $A = 48$ in.^{2}



8. $A = 90$ m^2



9. $A = 63$ cm^2