Casey Trenkamp Why Question #40 Why is the Unit Circle Important? Where did trigonometry originate from? How were the cosine and tangent functions invented?

History of Trigonometry and the Unit Circle

sine

History

- 1900 BC

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- 180 - 125 BC

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- 100 AD

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- 90 - 168 AD

Claudius Ptolemy expanded upon Hipparchus chords in a circle.

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- 830 AD

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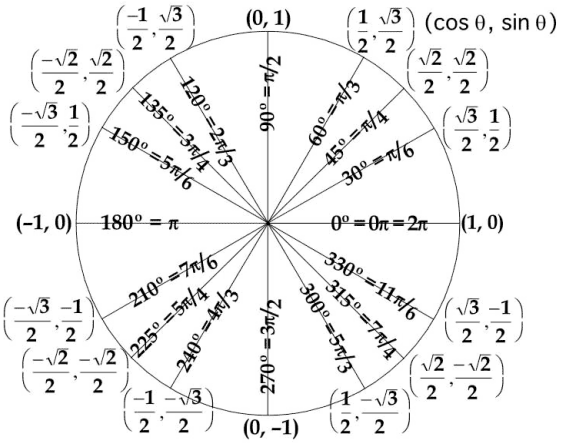

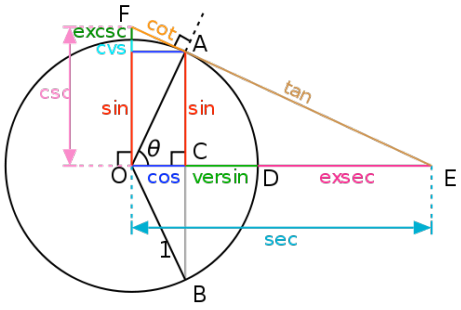

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- 853 - 929 AD

"Muhammad ibn Jabir al Hahaai discovered the reciprocal functions of secant and casecant, and produced the table of cosecants for each degree from 1˚ to 90˚. He was also responsible for establishing a number of important trigometrical relationships." (History of Trigonometry Wikipedia)

$$\tan a = \frac{\sin a}{\cos a}$$

- 10th Century

"Abu al-Wafa al-Buz jani were using all six trigonometric functions. He had sine tables in 0.25˚ increments, to 8 decimal places of accuracy, and accurate tables of tangent values." (History of Trigonometry Wikipedia) He also developed Angle Addition Identities

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}$$

Chinese Mathematics

- 718 AD

Aryabhata's tables were translanted into Chinese mathematics books.

- 960 -1279 AD

"During Song Dynasty, Chinese mathematicians began to express greater emphasis for the need of spherical trigonometry in calendrical science and astronomical calculations." (History of Trigonometry Wikipedia)

- 1231 - 1316 AD

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*No other publication of trigonometry came out for China until 1607.

European Mathematics

- 1464 AD

"Regiomontanus was the first European to treat trigonometry as a direct mathematical discipline. He wrote a book called *De triangulis omnimodus".* (History of Trigonometry Wikipedia) and (Mac Tutor History)

- 1596 AD

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Work Cited List

http://en.wikipedia.org/wiki/Law_of_cosines
Though the notion of cosine was not yet developed in his time, Euclid's *Elements*, dating back to the 3rd century BC, contains an early geometric theorem equivalent to the law of cosines. The case of obtuse triangle and acute triangle (corresponding to the two cases of negative or positive cosine) are treated separately, in Propositions 12 and 13 of Book 2. Trigonometric functions and algebra (in particular negative numbers) being absent in Euclid's time, the statement has a more geometric flavor:

It was not until the development of modern trigonometry in the Middle Ages by Muslim mathematicians, especially the discovery of the cosine, that the general law of cosines was formulated. The Persian astronomer and mathematician al-Battani generalized Euclid's result to spherical geometry at the beginning of the 10th century, which permitted him to calculate the angular distances between stars. In the 15th century, al-Kashi in Samarqand computed trigonometric tables to great accuracy and provided the first explicit statement of the law of cosines in a form suitable for triangulation. In France, the law of cosines is still referred to as the *theorem of Al-Kashi*.

The theorem was popularised in the Western world by François Viète in the 16th century. At the beginning of the 19th century, modern algebraic notation allowed the law of cosines to be written in its current symbolic form.

http://en.wikipedia.org/wiki/Islamic_mathematics#Trigonometric_functions
In the early 9th century, Muhammad ibn Mūsā al-Khwārizmī (c. 780-850) produced tables for the trigonometric functions of sines and cosine,[75] and the first tables for tangents.[76] In 830, Habash al-Hasib al-Marwazi produced the first tables of cotangents as well as tangents.[74][77] Muhammad ibn Jābir al-Harrānī al-Battānī (853-929) discovered the reciprocal functions of secant and cosecant, and produced the first table of cosecants, which he referred to as a "table of shadows" (in reference to the shadow of a gnomon), for each degree from 1° to 90°.[74] By the 10th century, in the work of Abū al-Wafā' al-Būzjānī (959-998), Muslim mathematicians were using all six trigonometric functions, and had sine tables in 0.25° increments, to 8 decimal places of accuracy, as well as accurate tables of tangent values.

Jamshīd al-Kāshī (1393-1449) gives trigonometric tables of values of the sine function to four sexagesimal digits (equivalent to 8 decimal places) for each 1° of argument with differences to be added for each 1/60 of 1°.[78] In one of his numerical approximations of π, he correctly computed 2π

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to 9 sexagesimal digits.[79] In order to determine sin 1°, al-Kashi discovered the following triple-angle formula often attributed to François Viète in the 16th century:[80]

Images
http://upload.wikimedia.org/wikipedia/commons/thumb/9/9d/Circle-trig6.svg/500px-Circle-trig6.svg.png

http://en.wikipedia.org/wiki/Muhammad_ibn_Mūsā_al-Khwārizmī#Trigonometry
In trigonometry, al-Khwārizmī (c. 780-850) produced tables for the trigonometric functions of sines and cosine in the *Zīj al-Sindhind*,[23] alongside the first tables for tangents. He was also an early pioneer in spherical trigonometry, and wrote a treatise on the subject.[19]

http://en.wikipedia.org/wiki/Al-Kashi

In French, the law of cosines is named *Théorème d'Al-Kashi* (Theorem of Al-Kashi), as al-Kashi was the first to provide an explicit statement of the law of cosines in a form suitable for triangulation.

http://en.wikipedia.org/wiki/Trigonometric_functions
In mathematics, the **trigonometric functions** (also called **circular functions**) are functions of an angle. They are used to relate the angles of a triangle to the lengths of the sides of a triangle. Trigonometric functions are important in the study of triangles and modeling periodic phenomena, among many other applications.

The most familiar trigonometric functions are the sine, cosine, and tangent. The sine function takes an angle and tells the length of the *y*-component (rise) of that triangle. The cosine function takes an angle and tells the length of *x*-component (run) of a triangle. The tangent function takes an angle and tells the slope (*y*-component divided by the *x*-component). More precise definitions are detailed below. Trigonometric functions are commonly defined as ratios of two sides of a right triangle containing the angle, and can equivalently be defined as the lengths of various line segments from a unit circle. More modern definitions express them as infinite series or as solutions of certain differential equations, allowing their extension to arbitrary positive and negative values and even to complex numbers.

Trigonometric functions have a wide range of uses including computing unknown lengths and angles in triangles (often right triangles). In this use, trigonometric functions are used for instance in navigation, engineering, and physics. A common use in elementary physics is resolving a vector

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into Cartesian coordinates. The sine and cosine functions are also commonly used to model periodic function phenomena such as sound and light waves, the position and velocity of harmonic oscillators, sunlight intensity and day length, and average temperature variations through the year.

In modern usage, there are six basic trigonometric functions, tabulated here with equations that relate them to one another. Especially with the last four, these relations are often taken as the *definitions* of those functions, but one can define them equally well geometrically, or by other means, and then derive these relations.

http://en.wikipedia.org/wiki/Unit_circle

Using the unit circle, the values of any trigonometric function for many angles other than those labeled can be calculated without the use of a calculator by using the Sum and Difference Formulas.

http://en.wikipedia.org/wiki/History_of_trigonometry

Although there is no trigonometry in the works of Euclid and Archimedes, in the strict sense of the word, there are theorems presented in a geometric way (rather than a trigonometric way) that are equivalent to specific trigonometric laws or formulas.[6] For instance, propositions twelve and thirteen of book two of the *Elements* are the laws of cosine for obtuse and acute angles, respectively. Theorems on the lengths of chords are applications of the law of sines. And Archimedes' theorem on broken chords is equivalent to formulas for sines of sums and differences of angles.

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History of trigonometry

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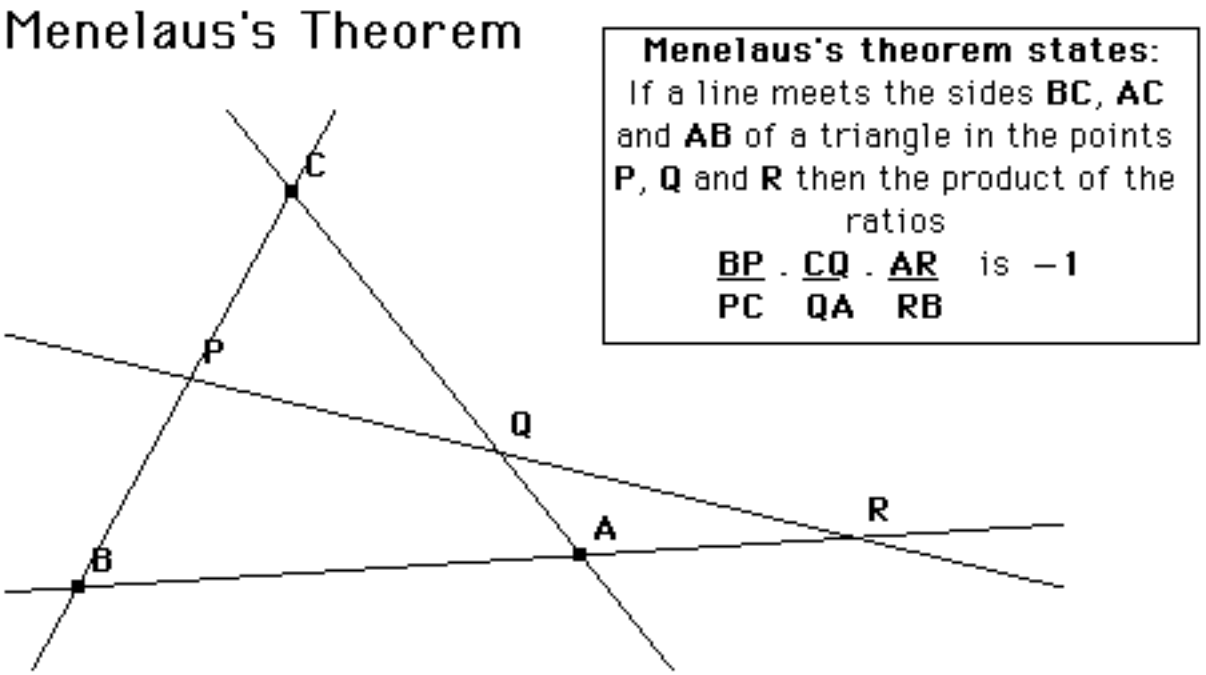
http://www-history.mcs.st-andrews.ac.uk/HistTopics/Trigonometric_functions.html

The use of trigonometric functions arises from the early connection between mathematics and astronomy. Early work with spherical triangles was as important as plane triangles.

The first work on trigonometric functions related to chords of a circle. Given a circle of fixed radius, 60 units were often used in early calculations, then the problem was to find the length of the chord subtended by a given angle. For a circle of unit radius the length of the chord subtended by the angle x was $2\sin(x/2)$. The first known table of chords was produced by the Greek mathematician Hipparchus in about 140 BC. Although these tables have not survived, it is claimed that twelve books of tables of chords were written by Hipparchus. This makes Hipparchus the founder of trigonometry.

The next Greek mathematician to produce a table of chords was Menelaus in about 100 AD. Menelaus worked in Rome producing six books of tables of chords which have been lost but his work on spherics has survived and is the earliest known work on spherical trigonometry. Menelaus proved a property of plane triangles and the corresponding spherical triangle property known the *regula sex quantitatum*.



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Ptolemy was the next author of a book of chords, showing the same Babylonian influence as Hipparchus, dividing the circle into 360° and the diameter into 120 parts. The suggestion here is that he was following earlier practice when the approximation 3 for π was used. Ptolemy, together with the earlier writers, used a form of the relation $\sin^2 x + \cos^2 x = 1$, although of course they did not actually use sines and cosines but chords.

Time line of Trigonometry

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