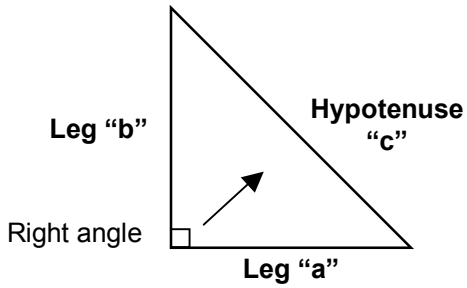
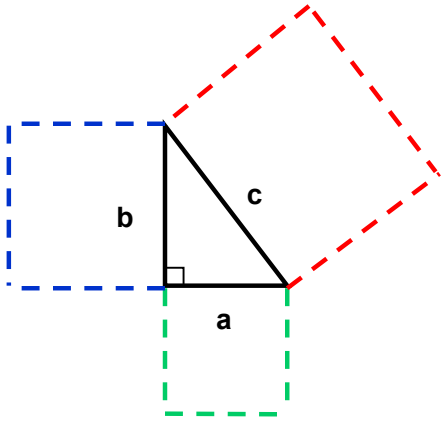
The Pythagorean Theorem

A triangle that contains a right angle (an angle that measures 90°, symbolized by a small square "□") is called a **right triangle**. The longest side of the right triangle (the side opposite the 90° angle) is called the **hypotenuse** and the other two (shorter) sides are called the **legs** of the triangle. The legs of a right triangle are commonly labeled "**a**" and "**b**," while the hypotenuse is labeled "**c**."

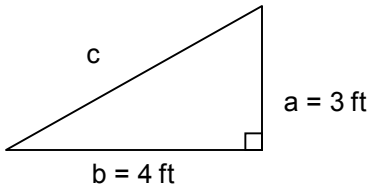


The **Pythagorean Theorem** describes the relationship among the three sides of a right triangle. In any right triangle, the sum of the areas of the squares formed on the legs of the triangle equals the area of the square formed on the hypotenuse: $\mathbf{a^2 + b^2 = c^2}$.



If you know the lengths of any two sides of a right triangle, you can always find the length of the third side with this theorem. For example, if the length of leg a = 3 feet and the length of leg b = 4 feet, we can find the length of the hypotenuse "c" by substituting the values for "a" and "b" in the theorem:

$$c^2 = a^2 + b^2$$
$$c^2 = 3^2 + 4^2$$
$$c^2 = 9 + 16$$
$$c^2 = 25$$



Since $25 = 5^2$, this means c = 5 feet. We can also find the value of "c" by taking the square root of 25:

$$c^2 = 25$$
$$c = \sqrt{25} = 5$$

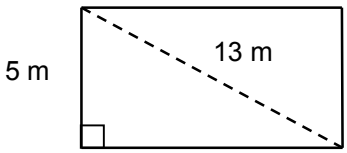
The Pythagorean Theorem is used in many applications that involve right triangles.

Examples:

1) A sail maker makes two sails by cutting a piece of sailcloth diagonally, as shown. If the width of the sailcloth is 5 meters and the diagonal is 13 meters, how many meters is the length?

   The theorem can be used to find the length of the cloth because the diagonal forms the hypotenuse of a right triangle. Let "a" equal the length of the cloth. Substituting the known values of the diagonal and the width gives us:

   $a^2 + b^2 = c^2$
   $a^2 + 5^2 = 13^2$
   $a^2 + 25 = 169$



   To find a^2, subtract b^2 from c^2:

   $a^2 = 169 - 25$
   $a^2 = 144$

   Next, take the square root of 144 to find the value of a:

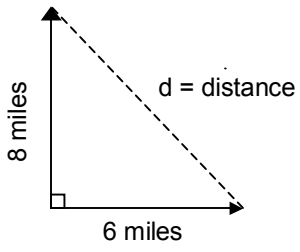
   $a = \sqrt{144}$
   $a = 12$, because $12^2 = 144$

   The length of the sail cloth is 12 meters.

2) Two cars leave town from the same parking lot. One car travels north for 8 miles. The other travels east for 6 miles. How far apart are the cars when they stop?

   A right angle is formed when one car travels north and the other car travels east. As the drawing shows, the distance "d" between the two cars is a straight line that forms the hypotenuse of a right triangle. Since the number of miles each car travels from the legs of the triangle, we can use the theorem to find the value of "d."

   $d^2 = 6^2 + 8^2$
   $d^2 = 36 + 64$
   $d^2 = 100$
   $d = \sqrt{100}$
   $d = 10$



   The cars are 10 miles apart when they stop.