HARMONIC FORM & DOUBLE ANGLE FORMULAS

A2 Unit 3: Pure Mathematics B

WJEC past paper questions: 2010 – 2017

Total marks available 93 (approximately 1 hour 50 minutes)

1.

(a) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$2\cos 2\theta = 9\cos\theta + 7.$$ [5]

(b) (i) Express $5\sin x - 12\cos x$ in the form $R\sin(x - \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.

(ii) Use your results to part (i) to find the least value of

$$\frac{1}{5\sin x - 12\cos x + 20}.$$

Write down a value for x for which this least value occurs. [6]

(Summer 10)

2.

(a) Find all values of x in the range $0° \leqslant x \leqslant 180°$ satisfying

$$\tan 2x = 4\tan x.$$ [5]

(b) Express $7\cos\theta + 24\sin\theta$ in the form $R\cos(\theta - \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.
Hence, find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$7\cos\theta + 24\sin\theta = 16.$$ [6]

(Summer 11)

3.

(a) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$8\cos 2\theta + 6 = \cos^2\theta + \cos\theta.$$ [6]

(b) Express $\sqrt{15}\cos\theta - \sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.
Hence find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$\sqrt{15}\cos\theta - \sin\theta = 3.$$ [6]

(Summer 13)

4. (a) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$4\cos 2\theta = 1 - 2\sin\theta.$$ [6]

(b) (i) Express $8\sin x + 15\cos x$ in the form $R\sin(x+\alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.

(ii) Find all values of x in the range $0° \leqslant x \leqslant 360°$ satisfying

$$8\sin x + 15\cos x = 11.$$

(iii) Find the greatest possible value for k so that

$$8\sin x + 15\cos x = k$$

has solutions. Give a reason for your answer. [7]

(Summer 12)

5. (a) Find all values of x in the range $0° \leqslant x \leqslant 180°$ satisfying

$$\tan 2x = 3\cot x.$$ [4]

(b) (i) Express $21\sin\theta - 20\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.

(ii) Use your results to part (i) to find the greatest value of

$$\frac{1}{21\sin\theta - 20\cos\theta + 31}.$$

Write down a value for θ for which this greatest value occurs. [6]

(Summer 14)

6. (a) Find all values of x in the range $0° \leqslant x \leqslant 180°$ satisfying

$$\tan(x + 45°) = 8\tan x.$$ [5]

(b) (i) Express $\sqrt{13}\sin\theta - 6\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.

(ii) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$\sqrt{13}\sin\theta - 6\cos\theta = -4.$$ [6]

(Summer 15)

7. (a) The angle x is such that $0° \leqslant x \leqslant 180°$, $x \neq 90°$.

Given that x satisfies the equation $3\tan 2x + 16\cot^2 x = 0$,

(i) show that $3\tan^3 x - 8\tan^2 x + 8 = 0$,

(ii) find all possible values of x, giving each answer in degrees, correct to one decimal place. [8]

(b) Express $24\cos\theta - 7\sin\theta$ in the form $R\cos(\theta + \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.

Hence, find the range of values of k for which the equation

$$24\cos\theta - 7\sin\theta = k$$

has no solutions. [5]

(Summer 16)

8. (a) Show that the equation

$$5\cos^2\theta + 7\sin 2\theta = 3\sin^2\theta$$

may be rewritten in the form

$$a\tan^2\theta + b\tan\theta + c = 0,$$

where a, b, c are non-zero constants whose values are to be found.
Hence, find all values of θ in the range $0° \leqslant \theta \leqslant 180°$ satisfying the equation

$$5\cos^2\theta + 7\sin 2\theta = 3\sin^2\theta.$$ [6]

(b) (i) Express $\sqrt{5}\cos\phi + \sqrt{11}\sin\phi$ in the form $R\cos(\phi - \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.

(ii) Use your result to part (i) to find the least value of

$$\frac{1}{\sqrt{5}\cos\phi + \sqrt{11}\sin\phi + 6}.$$

Write down a value for ϕ for which this least value occurs. [6]

(Summer 17)