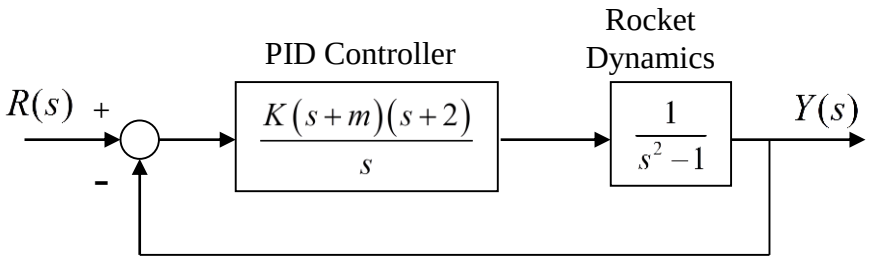
ME 3600 Control Systems
Using MATLAB for Root Locus Analysis

As an example of how to use MATLAB to perform a root locus analysis, consider design problem **DP**6.4. The block diagram of the closed-loop system is shown below. The goal is to use MATLAB to draw a root locus diagram for the parameter *K*, given the parameter $m = 4$.

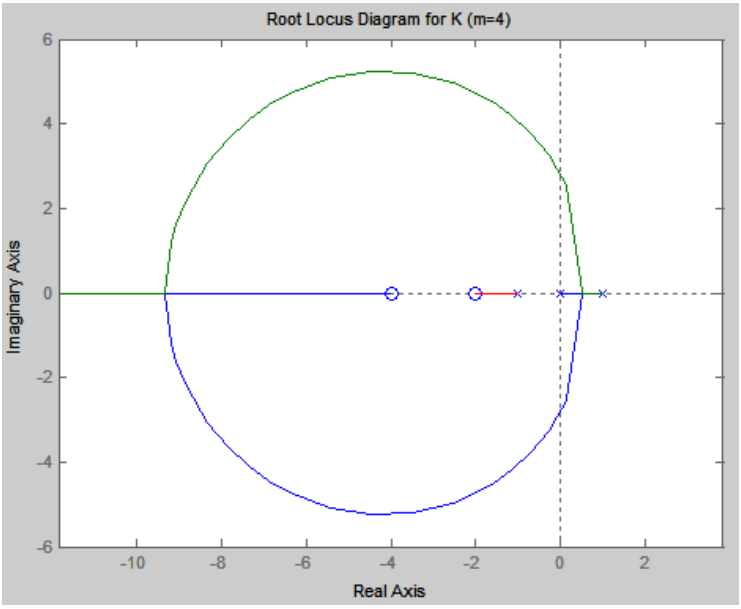


The characteristic equation of the closed-loop system is $1 + GH(s) = 0$ or $1 + KP(s) = 0$. Substituting the transfer functions from the block diagram gives

$$1 + K\left[\frac{(s+4)(s+2)}{s(s^2-1)}\right] = 1 + K\left[\frac{s^2+6s+8}{s^3-s}\right] = 0$$

The MATLAB commands that produce the root locus diagram are:

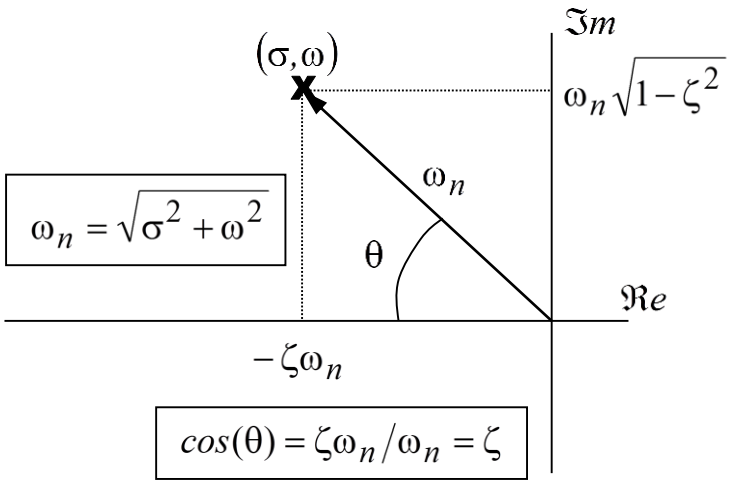
```
>> num=[1,6,8];
>> den=[1,0,-1,0];
>> sys=tf(num,den)

Transfer function:
        s^2 + 6 s + 8
        -------------
          s^3 - s

>> rlocus(sys)
>> axis('equal')
>> title('Root Locus Diagram for K (m=4)')
```



Note that MATLAB does not show the direction of the movement of the poles. It is understood that the movement is from the *poles to the zeros of P*(*s*). The "axis" command ensures that the diagram is shown in its *true shape*.

Target Regions for Poles

- The *damping ratios* and *settling times* of the poles are determined by their location on the root locus diagram.
- To ensure a settling time less than T_s, the real parts of all the poles of the system must be to the *left* of $4/T_s$.
- The damping ratio of each of the complex poles is determined by drawing a vector from the origin to the location of the pole and measuring the angle θ between this vector and the negative real axis.



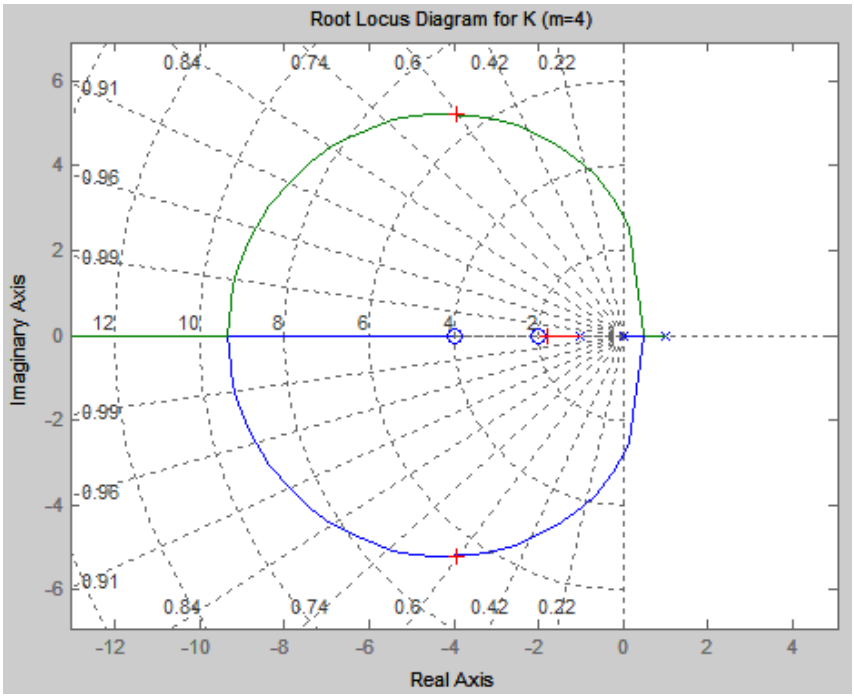
- The damping ratio is calculated as $\zeta = cos(\theta)$. For example, poles that lie below the $\theta = 45^o$ line have damping ratios $\zeta > 0.7$. See the diagram below.

Parameter Values Associated with Poles in the Target Region

To find parameter values associated with poles within the target region, use the "rlocfind" command in MATLAB. After executing the "rlocfind" command, click on a desirable pole location on one of the branches of the root locus in the root locus plot window. MATLAB automatically picks the point on the branch that is closest to your selection.

The MATLAB commands are:

```
>> grid
>> [k,poles]=rlocfind(sys)
    Select a point in graphics window

        selected_point =
          -3.9356 + 5.1761i

        k =
            9.6740


        poles =
          -3.9335 + 5.2303i
          -3.9335 -  5.2303i
          -1.8070
```



Note that MATLAB places a "+" at the location of the chosen poles.