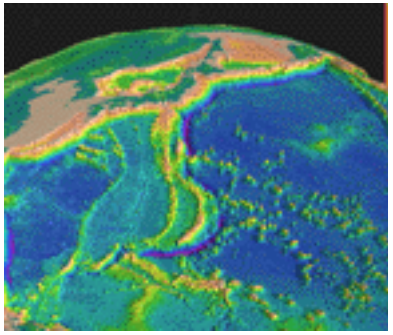
Physics of Surfing Waves

David T. Sandwell
(http://topex.ucsd.edu/ps)



- Physics of waves
- Characteristics of waves
- Generation of waves by storms
- Wave speed - shallow vs. deep ocean
- Sets - dispersion
- Refraction of waves - Why is Black's so good?

Optional Exercises:

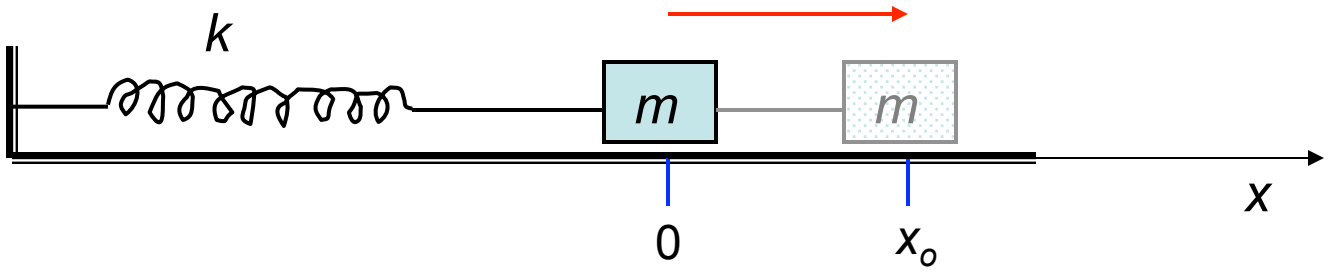
(each problem is covered in class today)

1. Derive the expression for the period of a harmonic oscillator with mass *m* and spring constant *k*.

2. Derive the expression for the speed of a deep water wave in terms of the wave period *T*.

3. What are *sinh, cosh,* and *tanh* in terms of the exponential function? What is *tanh*(10^{-6})? What is *tanh*(10)?

harmonic oscillator



$$m\frac{d^2x}{dt^2} = -kx$$

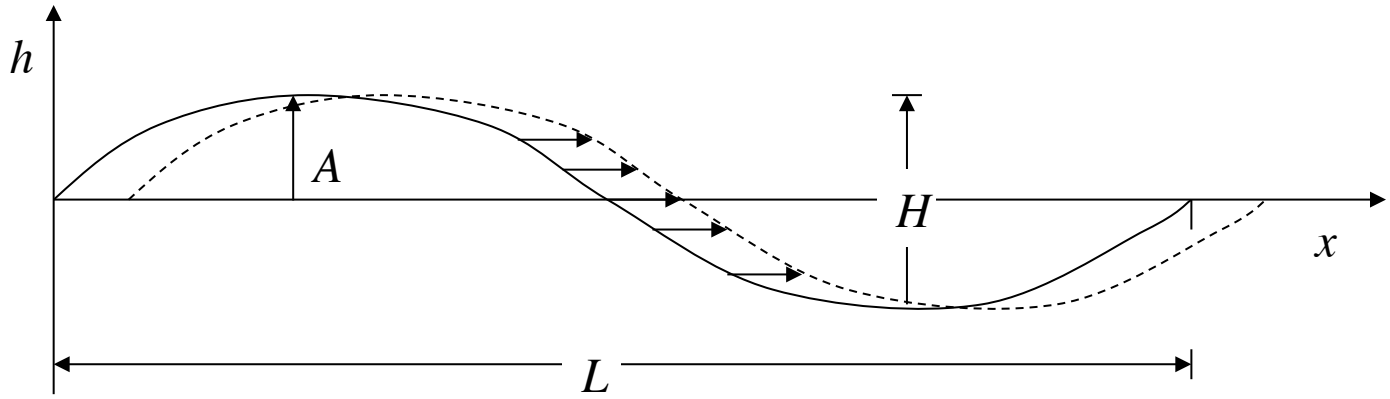
m - mass
k - spring constant

force from acceleration	=	restoring force of spring

guess solution $x(t) = x_o \cos \omega t$

How do we solve for ω ?

wave characteristics



$$h(x,t) = A\sin\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right)$$

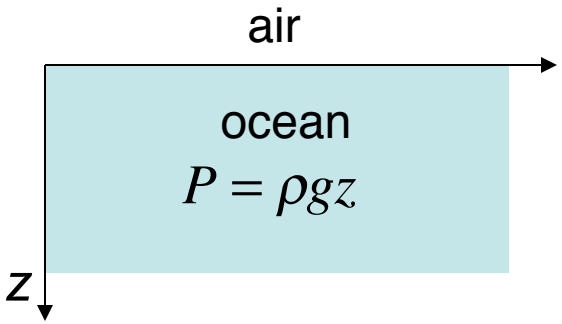
L - wavelength

A - amplitude

H - height

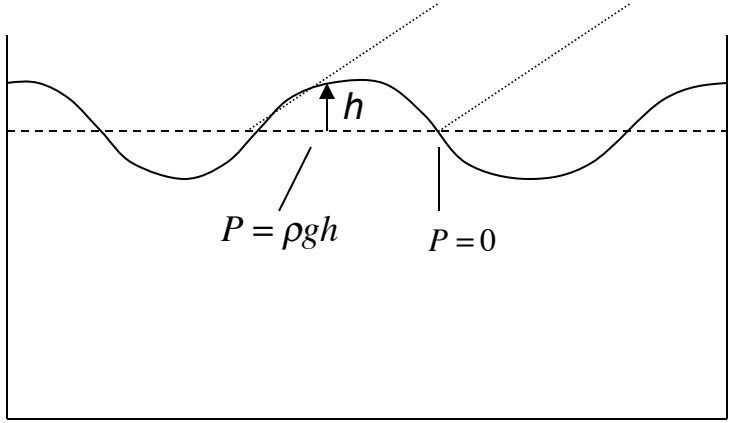
T - period (5 - 18 s)

deep ocean waves



r- density (kg m^{-3})
g - acceleration of gravity (9.8 m s^{-2})
z - depth (m)

What are the units of pressure?



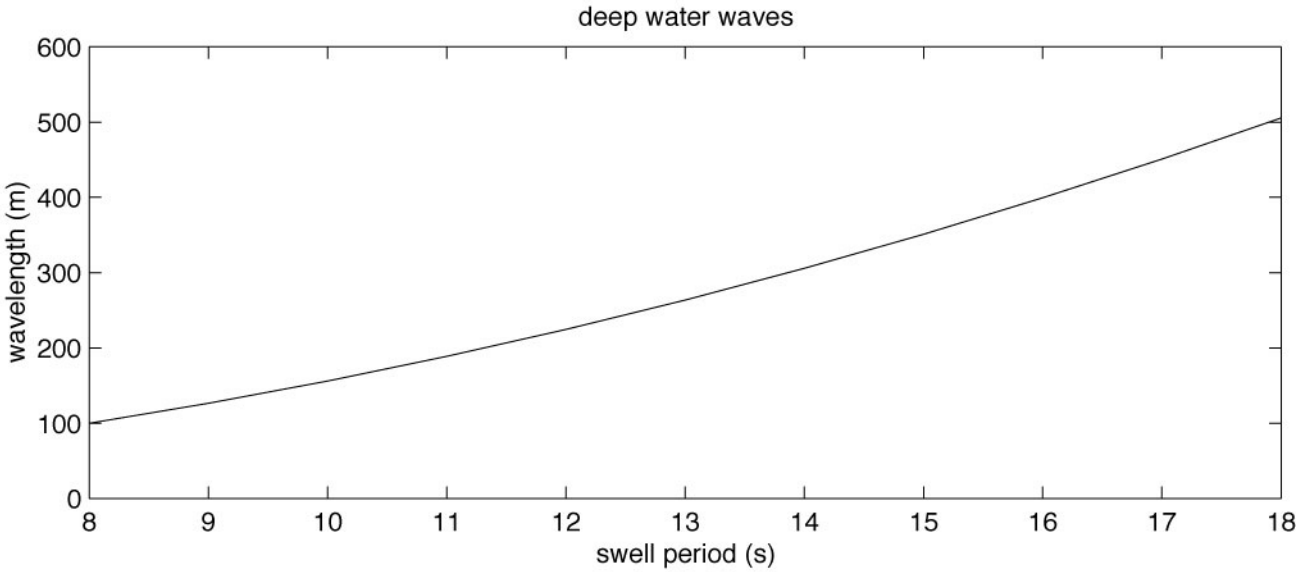
restoring force $= -\rho g h$

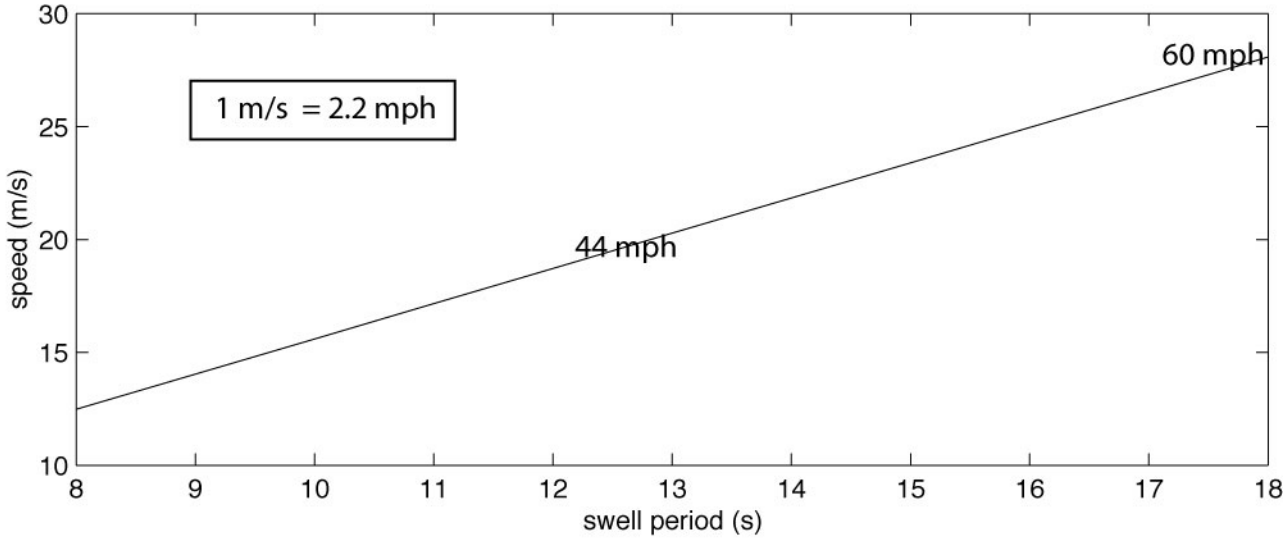
acceleration = $\rho \frac{L}{2\pi} \frac{d^2 h}{dt^2}$
force

$$\rho \frac{L}{2\pi} \frac{d^2 h}{dt^2} = -\rho g h$$

guess: $h(t) = A \cos \omega t$

What is ω?


deep water waves
wavelength (m)
swell period (s)
0
100
200
300
400
500
600
8
9
10
11
12
13
14
15
16
17
18


1 m/s = 2.2 mph
60 mph
44 mph
speed (m/s)
swell period (s)
10
15
20
25
30
8
9
10
11
12
13
14
15
16
17
18

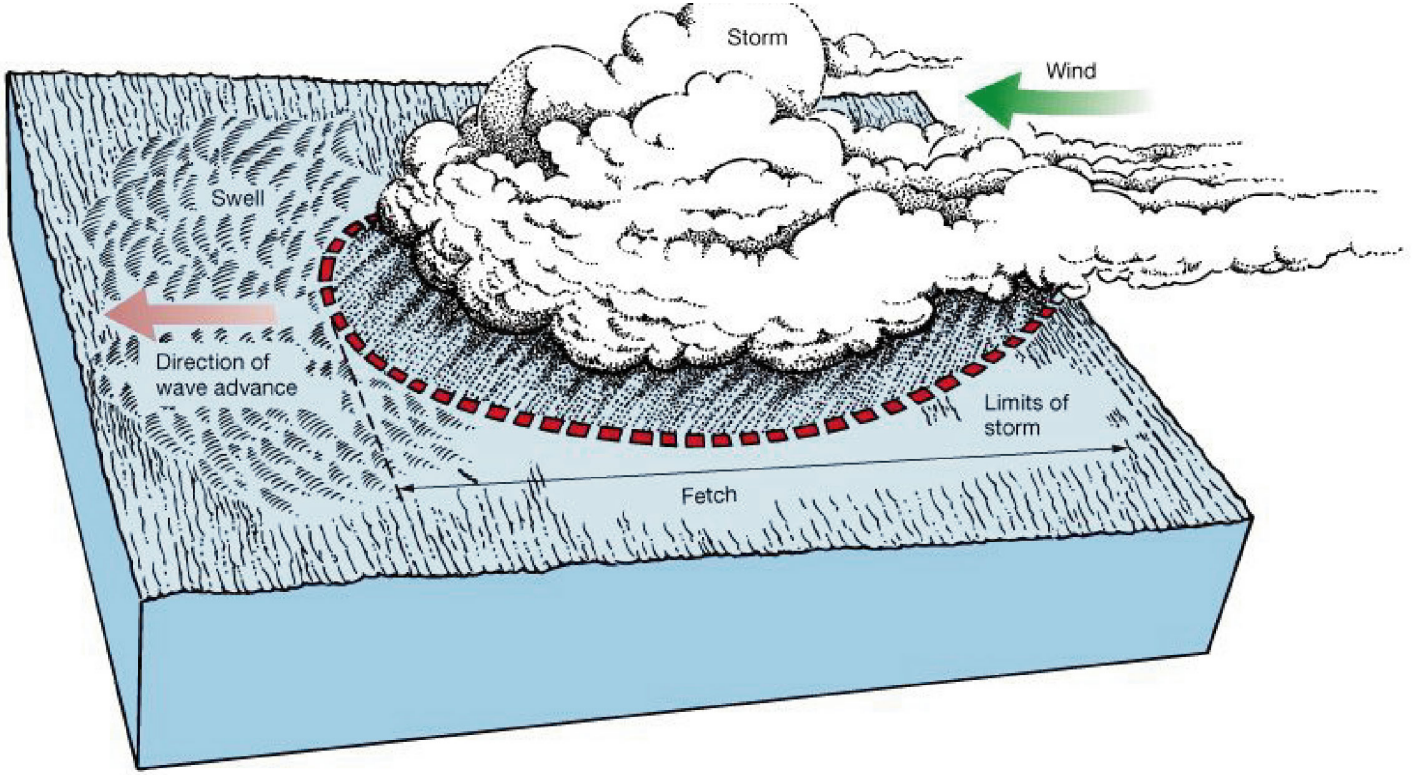
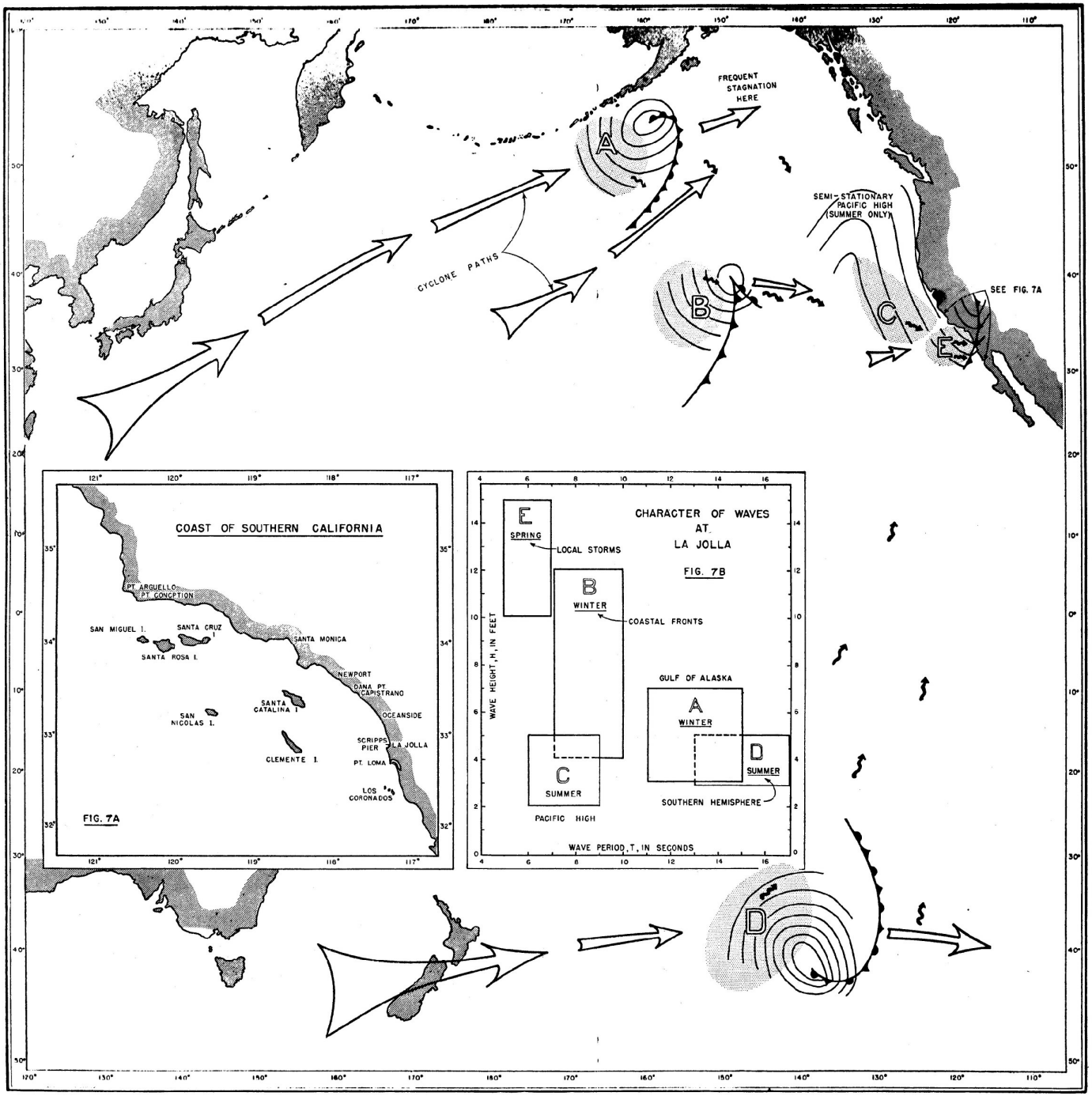


Table 8–1 Description of a fully developed sea for a given wind speed.

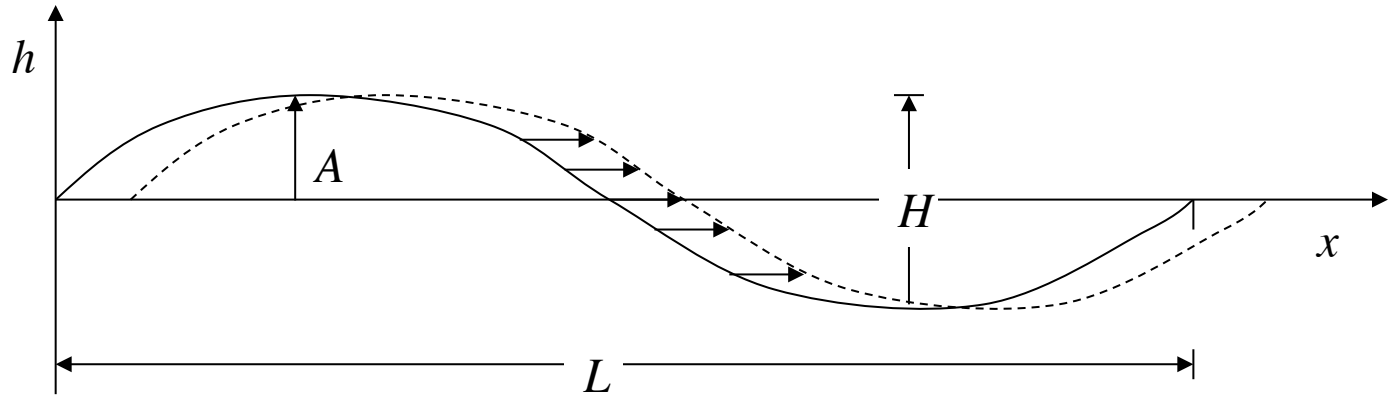
Wind speed in km/h (mi/h)	*Average height in m (ft)*	*Average length in m (ft)*	*Average period in sec*	*Highest 10% of waves in m (ft)*
20 (12)	0.33 (1.0)	10.6 (34.8)	3.2	0.75 (2.5)
30 (19)	0.88 (2.9)	22.2 (72.8)	4.6	2.1 (6.9)
40 (25)	1.8 (5.9)	39.7 (130.2)	6.2	3.9 (12.8)
50 (31)	3.2 (10.5)	61.8 (202.7)	7.7	6.8 (22.3)
60 (37)	5.1 (16.7)	89.2 (292.6)	9.1	10.5 (34.4)
70 (43)	7.4 (24.3)	121.4 (398.2)	10.8	15.3 (50.2)
80 (50)	10.3 (33.8)	158.6 (520.2)	12.4	21.4 (70.2)
90 (56)	13.9 (45.6)	201.6 (661.2)	13.9	28.4 (93.2)

Munk, W. H. and M. A. Traylor,
Refraction of Ocean Waves,
J. Geology, v. LV, No. 1, 1947



wave generation

- generated by storms at sea
- far from the storm they are sinusoidal



$$h(x,t) = A\sin\left(\frac{2\pi x}{L} - \frac{2\pi t}{T}\right)$$

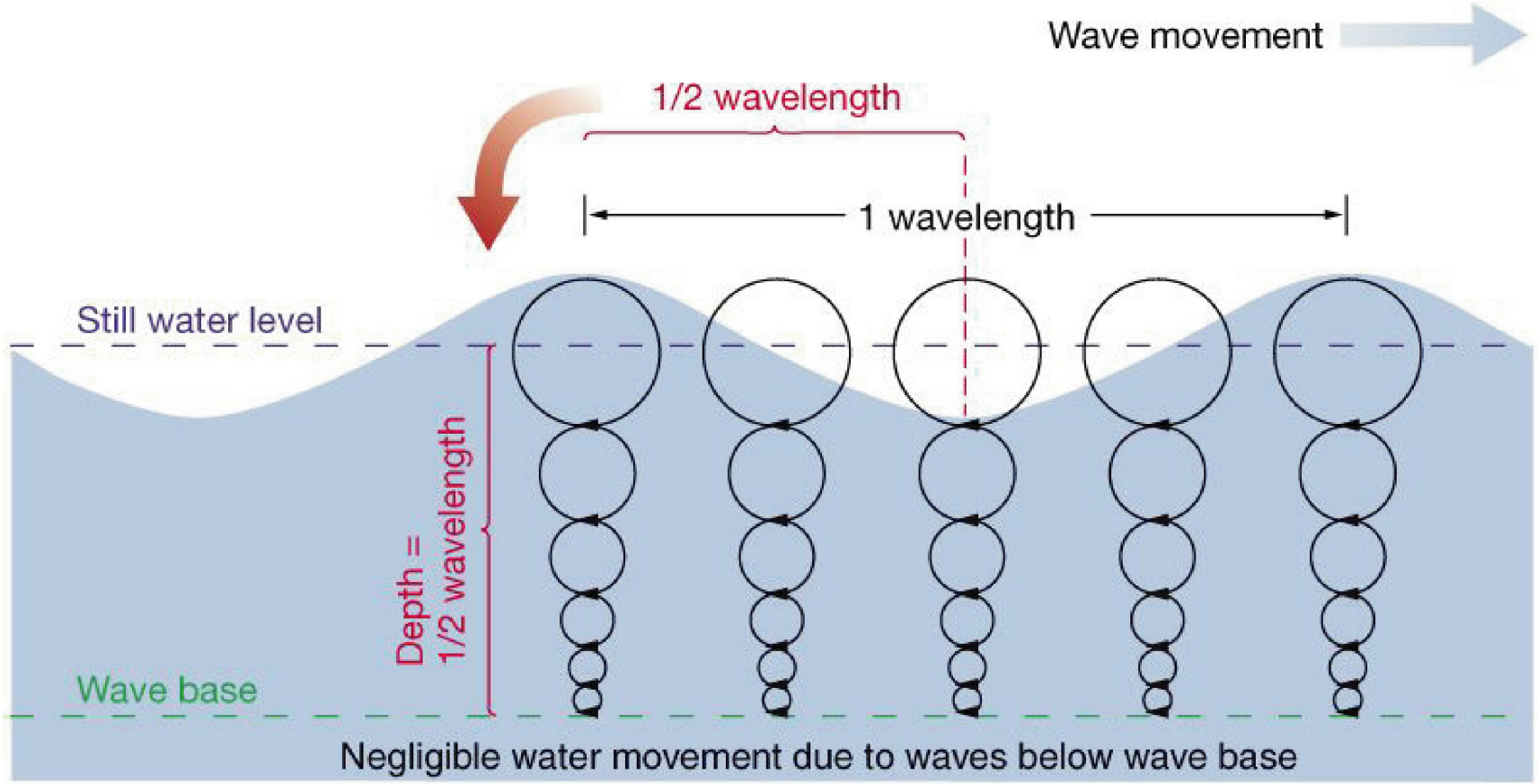
L - wavelength

A - amplitude

H - height

T - period (5 - 18 s)

$$c = \frac{L}{T}$$ phase velocity


Wave movement
1/2 wavelength
1 wavelength
Still water level
Depth = 1/2 wavelength
Wave base
Negligible water movement due to waves below wave base

Airy solution

$$c(d) = \left[\frac{gL}{2\pi} \tanh\left(\frac{2\pi d}{L} \right) \right]^{1/2}$$

L - wavelength
g - acc. gravity
d - ocean depth

deep water waves

$$d >> L/2$$

$$c_d = \sqrt{\frac{gL}{2\pi}}$$

shallow water waves

$$d << L/2$$

$$c_s = \sqrt{gd}$$

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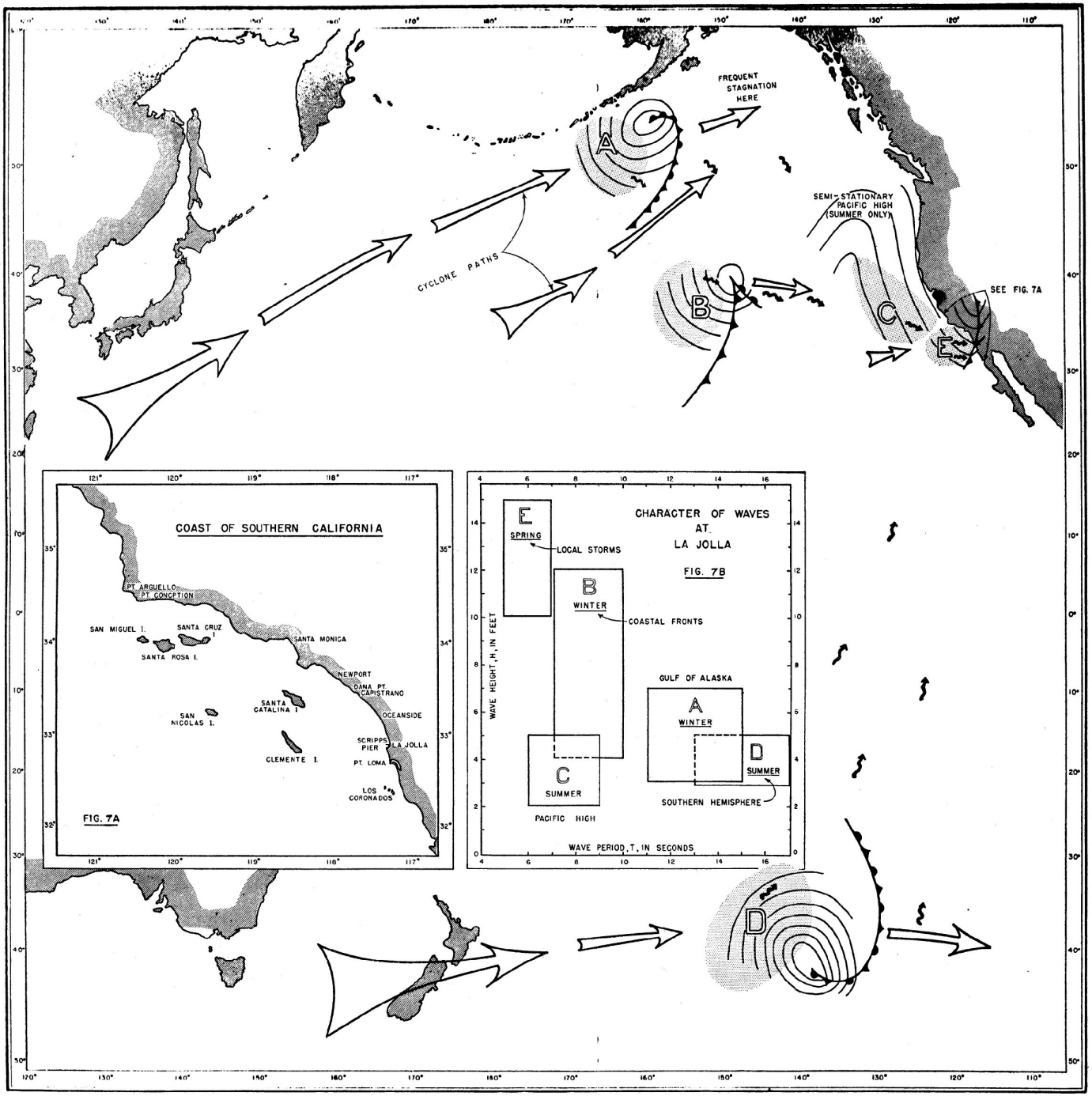
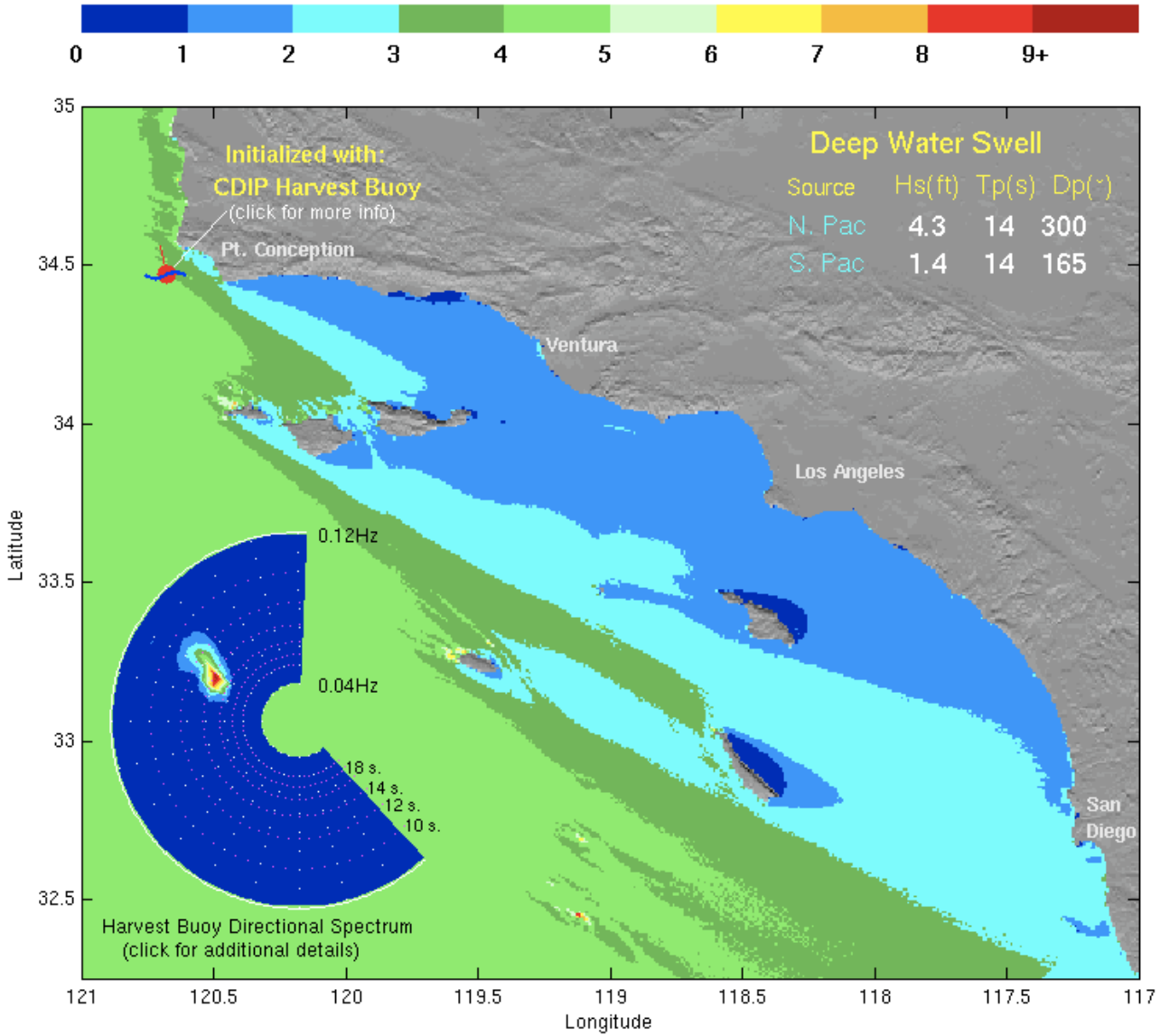


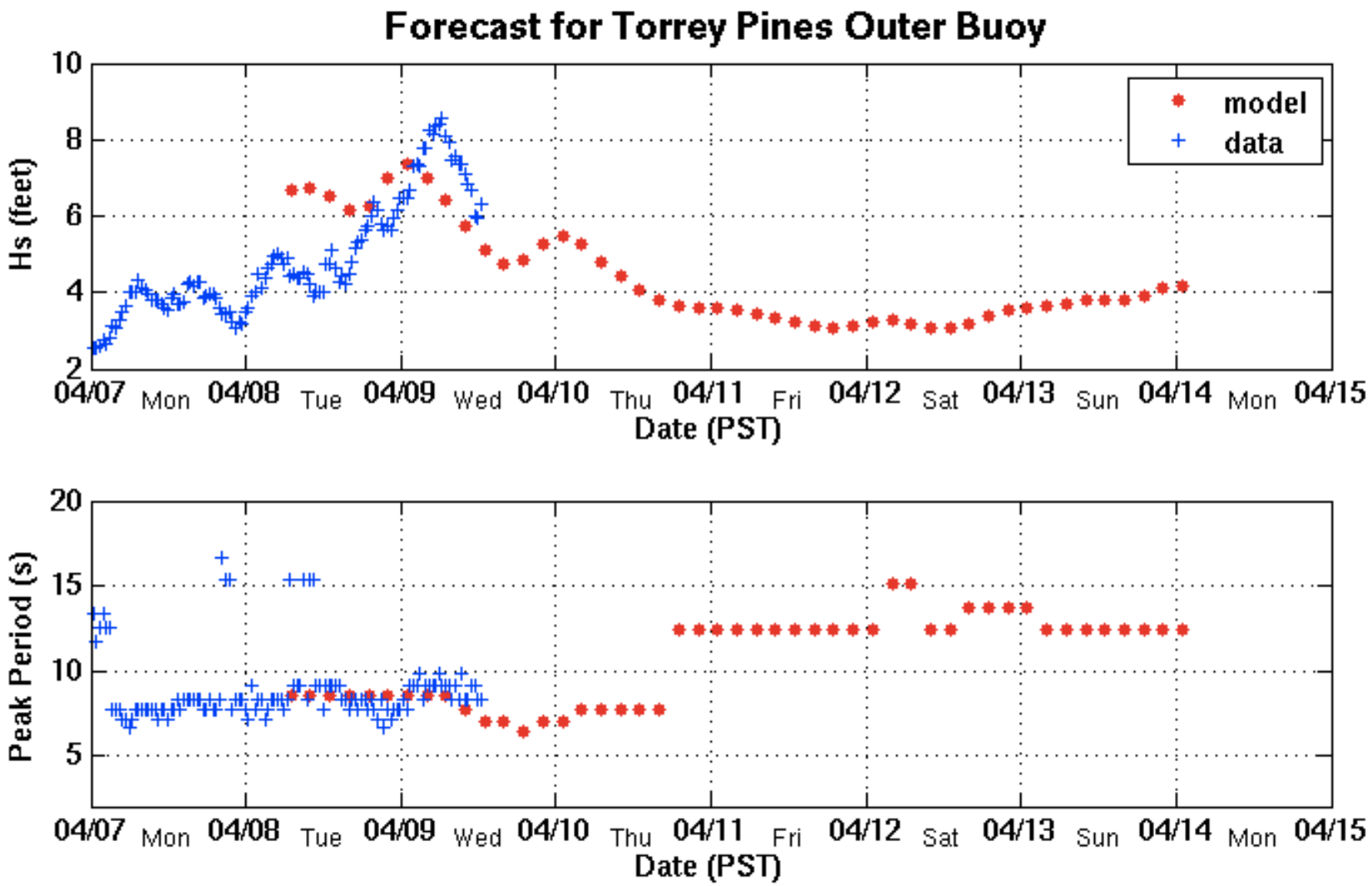
The Coastal Data Information Program
Integrative Oceanography Division
SCRIPPS Institution of Oceanography

Analysis Time – 9 APR 2008 : 1233 PST

Swell Height (ft) – Southern California Bight

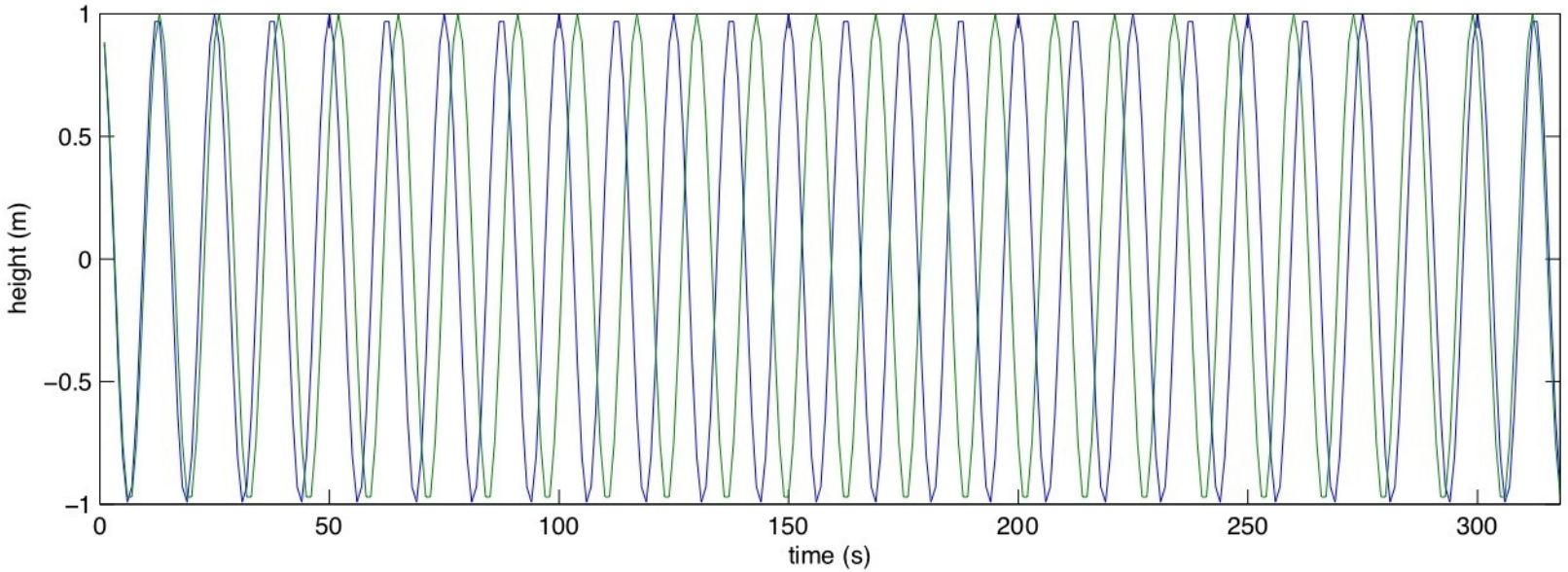


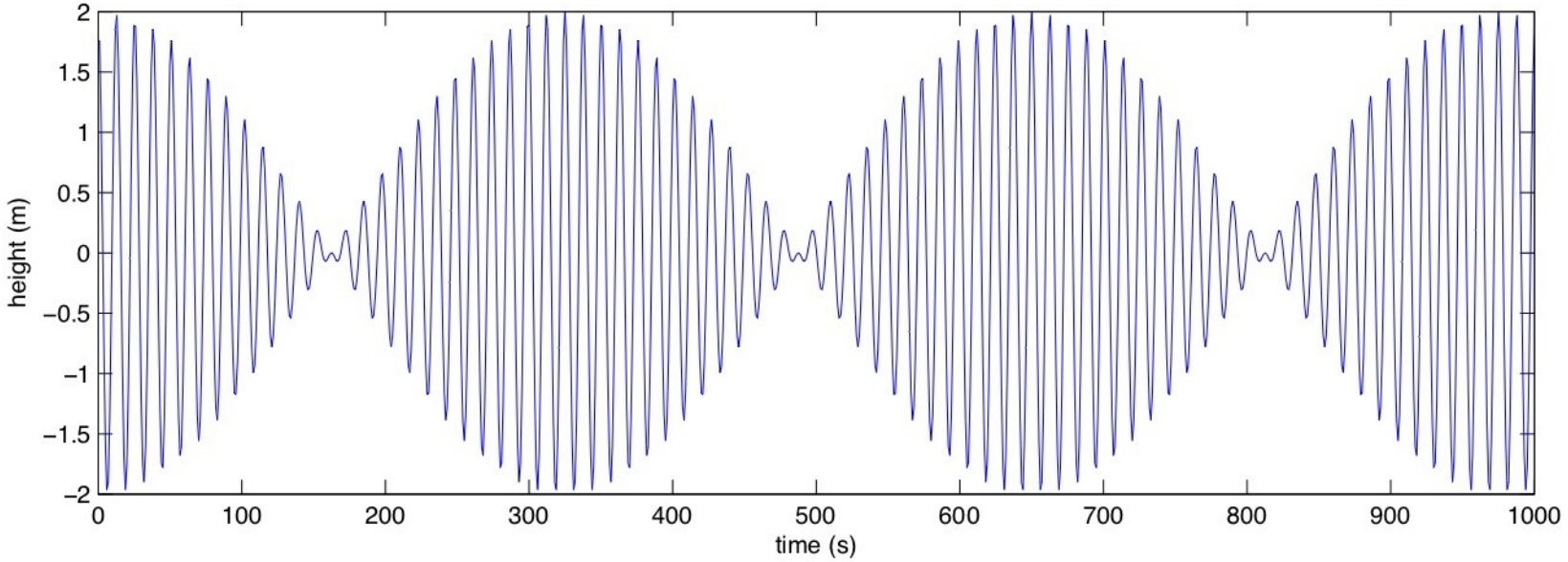
Additional Information @ http://cdip.ucsd.edu/


Forecast for Torrey Pines Outer Buoy
Hs (feet)
10
8
6
4
2
model
data
04/07 Mon
04/08 Tue
04/09 Wed
04/10 Thu
04/11 Fri
04/12 Sat
04/13 Sun
04/14 Mon
04/15
Date (PST)
Peak Period (s)
20
15
10
5
04/07 Mon
04/08 Tue
04/09 Wed
04/10 Thu
04/11 Fri
04/12 Sat
04/13 Sun
04/14 Mon
04/15
Date (PST)

Sets

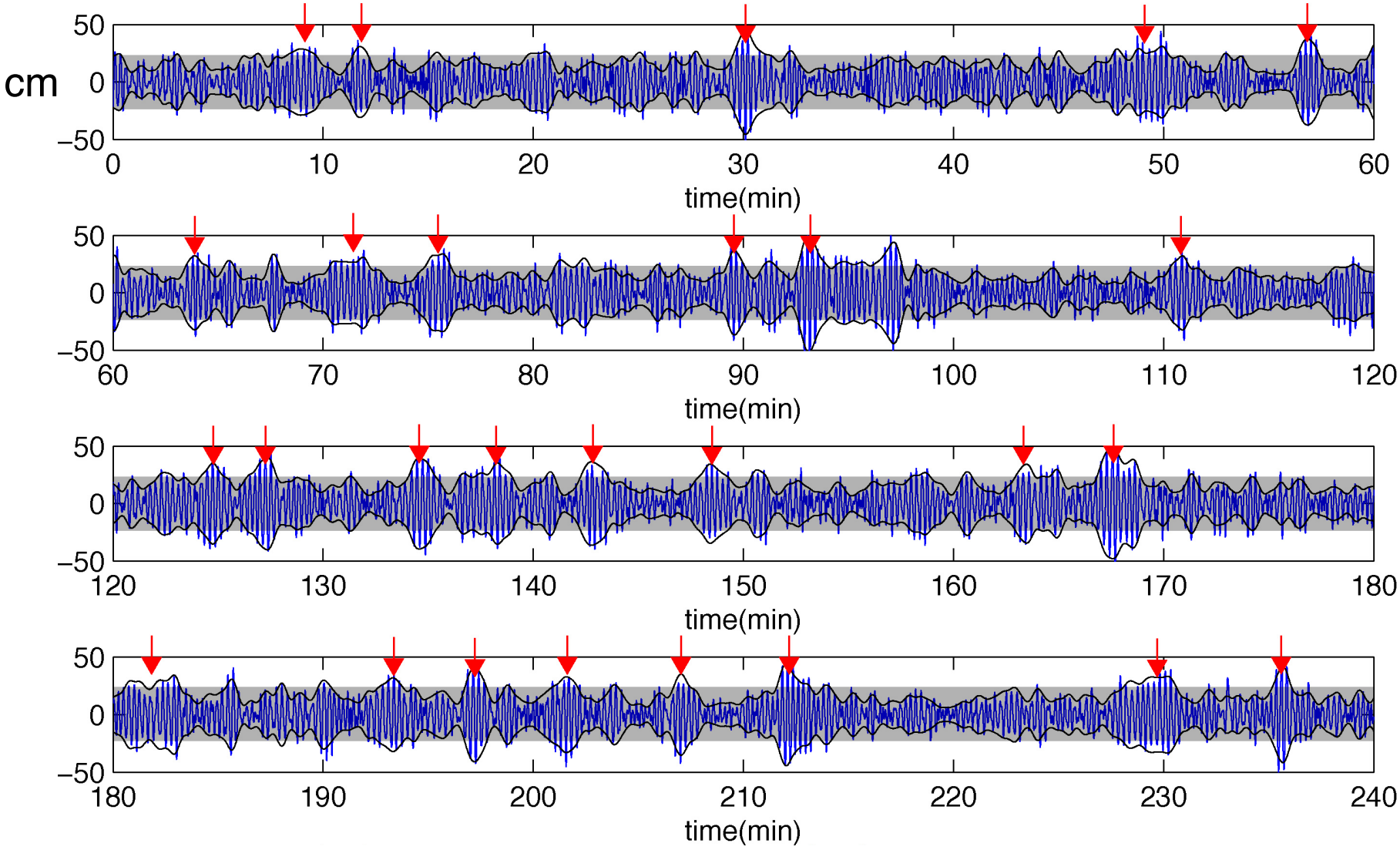
- Are sets real? How is a set defined?
- More analysis of buoy data can provide characteristics of sets.
- Why do waves come in sets?


height (m)
time (s)
1
0.5
0
−0.5
−1
0
50
100
150
200
250
300

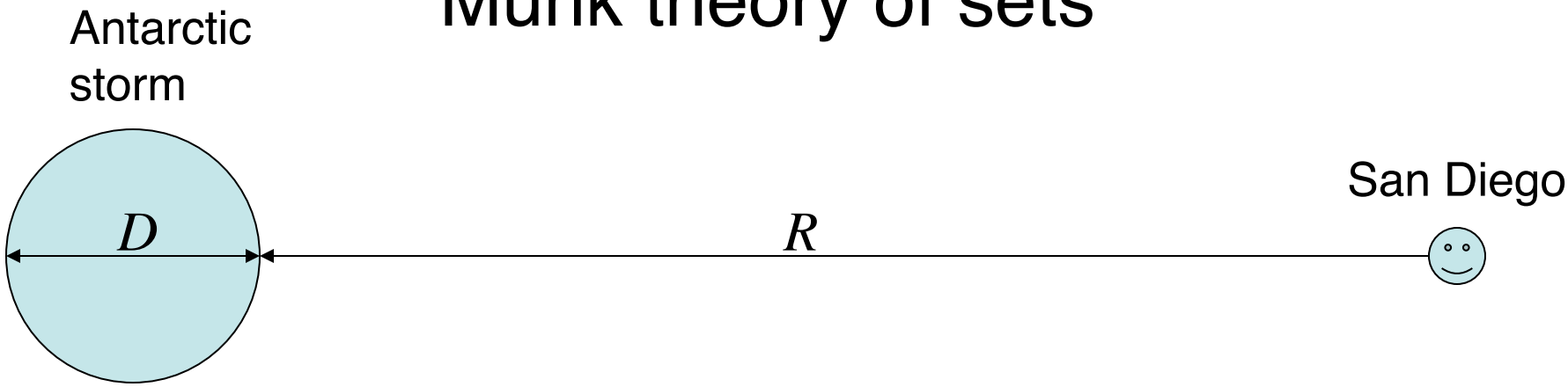

height (m)
time (s)
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
0
100
200
300
400
500
600
700
800
900
1000

OCEANSIDE OFFSHORE, CA - Station: 04501
Water depth(m): 220.00
August 3, 2007

Average time between sets
8.8 min



Munk theory of sets



Waves arrive in San Diego at the same time t_1.

Suppose the waves were generated at the same time t_o.

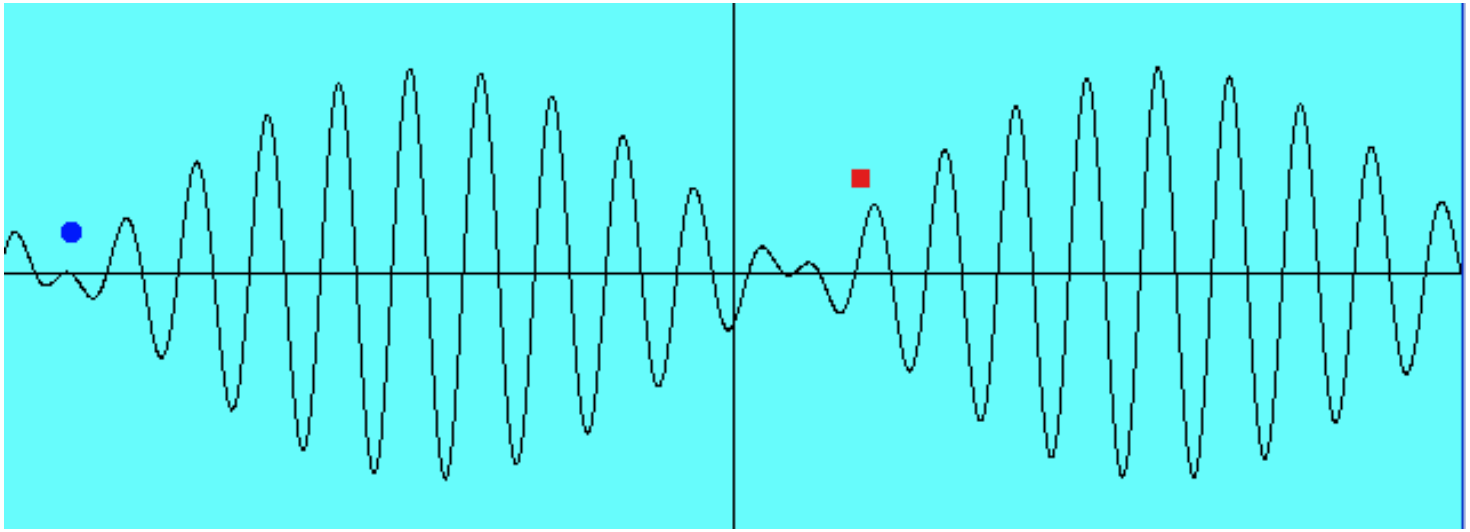
$$t_1 - t_o = 2R/c_1$$

$$t_1 - t_0 = 2(R+D)/c_2$$

$$c_2 = c_1\left(\frac{R+D}{R}\right)$$

$$c = \frac{gT}{2\pi}$$ deep water dispersion

$$T_2 = T_1\left(\frac{R+D}{R}\right)$$



$$h(t) = A\cos\left(\frac{2\pi t}{T_1}\right) + B\cos\left(\frac{2\pi t}{T_2}\right), \qquad \text{suppose} \quad B = A$$

$$h(t) = 2A\cos\left[\pi t\left(\frac{1}{T_1} + \frac{1}{T_2}\right)\right]\cos\left[\pi t\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right]$$

surf = mean period modulated by beat period

interval between sets

$$T_B = T_1\left(1 - \frac{R}{R+D}\right)^{-1}$$

$R = 7000$ km $\quad D = 400$ km $\quad T_1 = 17s,$

$$T_B = 5.5\,\text{min}$$

A long time to wait between sets!

Optional Exercises:

(each problem is covered in class)

1. Derive the expression for the period of a harmonic oscillator with mass *m* and spring constant *k*.

2. Derive the expression for the speed of a deep water wave in terms of the wave period *T*.

3. What are *sinh, cosh,* and *tanh* in terms of the exponential function? What is *tanh*(10^{-6})? What is *tanh*(10)?

4. One more problem. Consider two waves of equal height but different period (T_1=12.5s and T_2=13 s). What is the time between sets? Here is a hint: add two cosine functions $h(t) = \cos(\omega_1 t) + \cos(\omega_2 t)$ where $\omega_1 = 2\pi / T_1$,use the trigonometric formula for the sum of two cosines, then interpret or plot the results.

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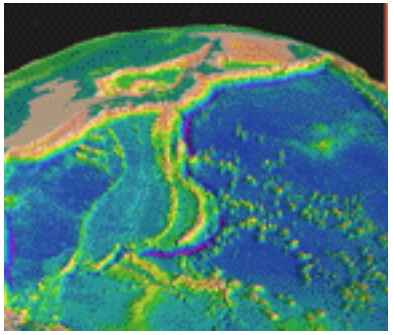
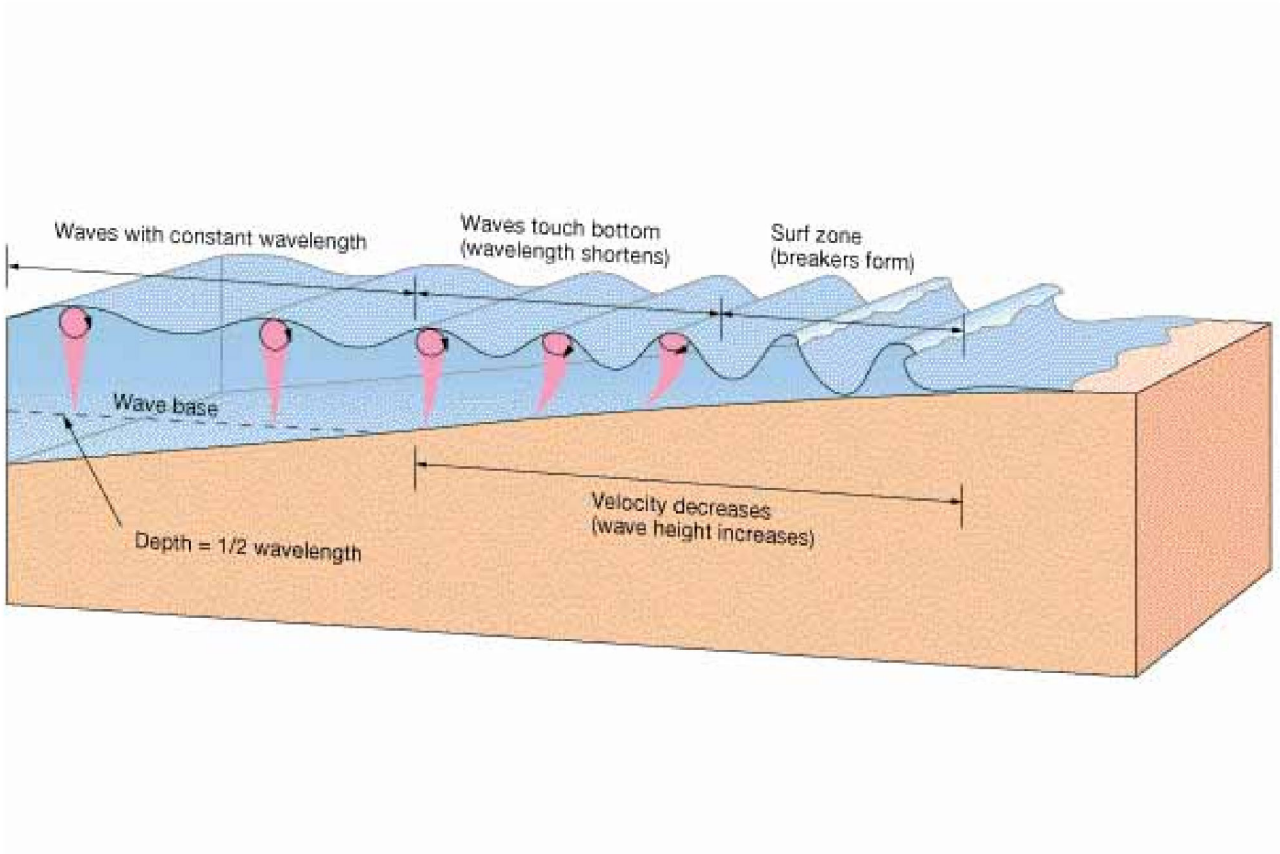


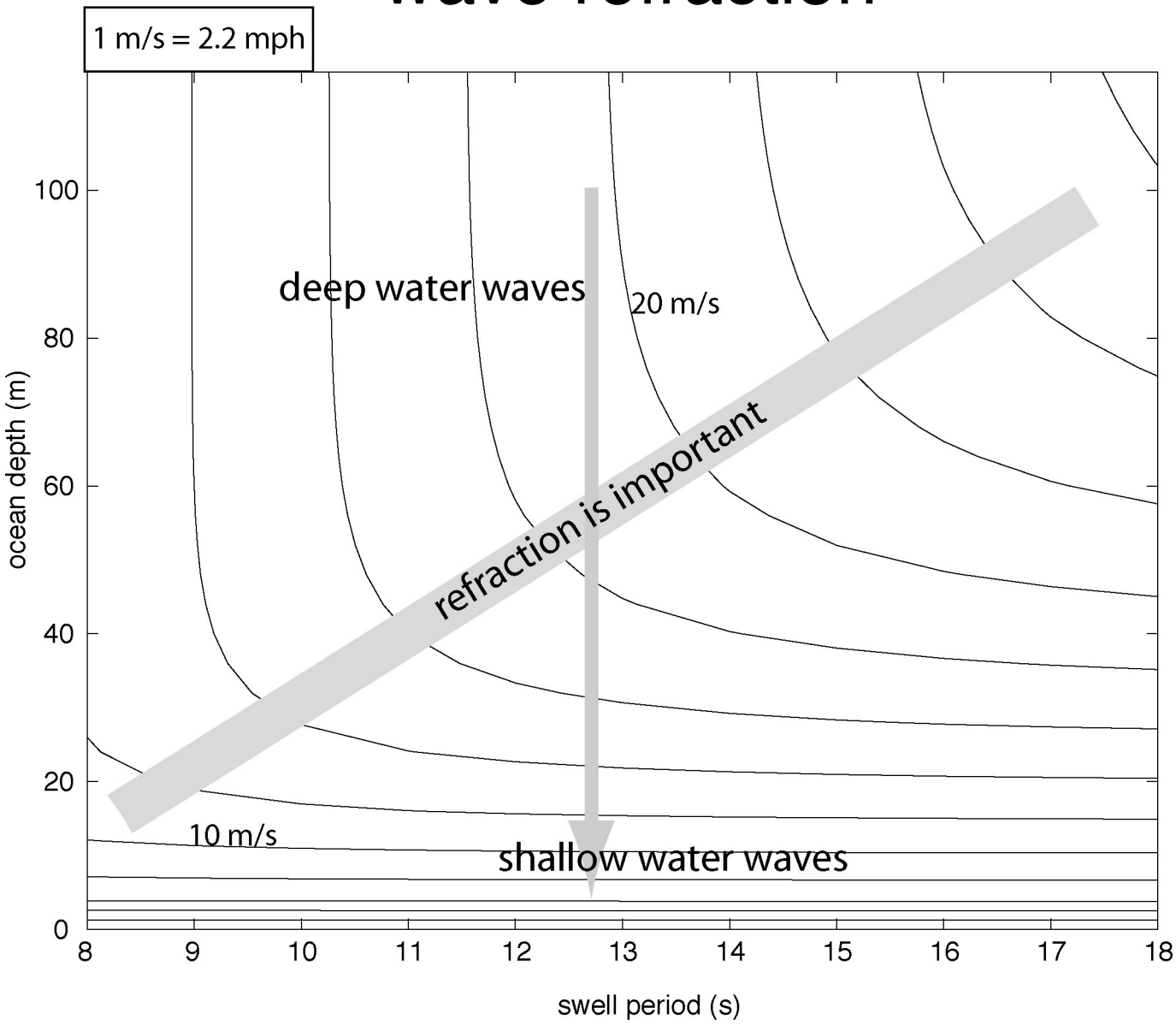
- Physics of waves
- Characteristics of waves
- Generation of waves by storms
- Wave speed - shallow vs. deep ocean
- Sets - dispersion
- Refraction of waves - Why is Black's so good?

shallow water waves

$$c_s = \sqrt{gd}$$



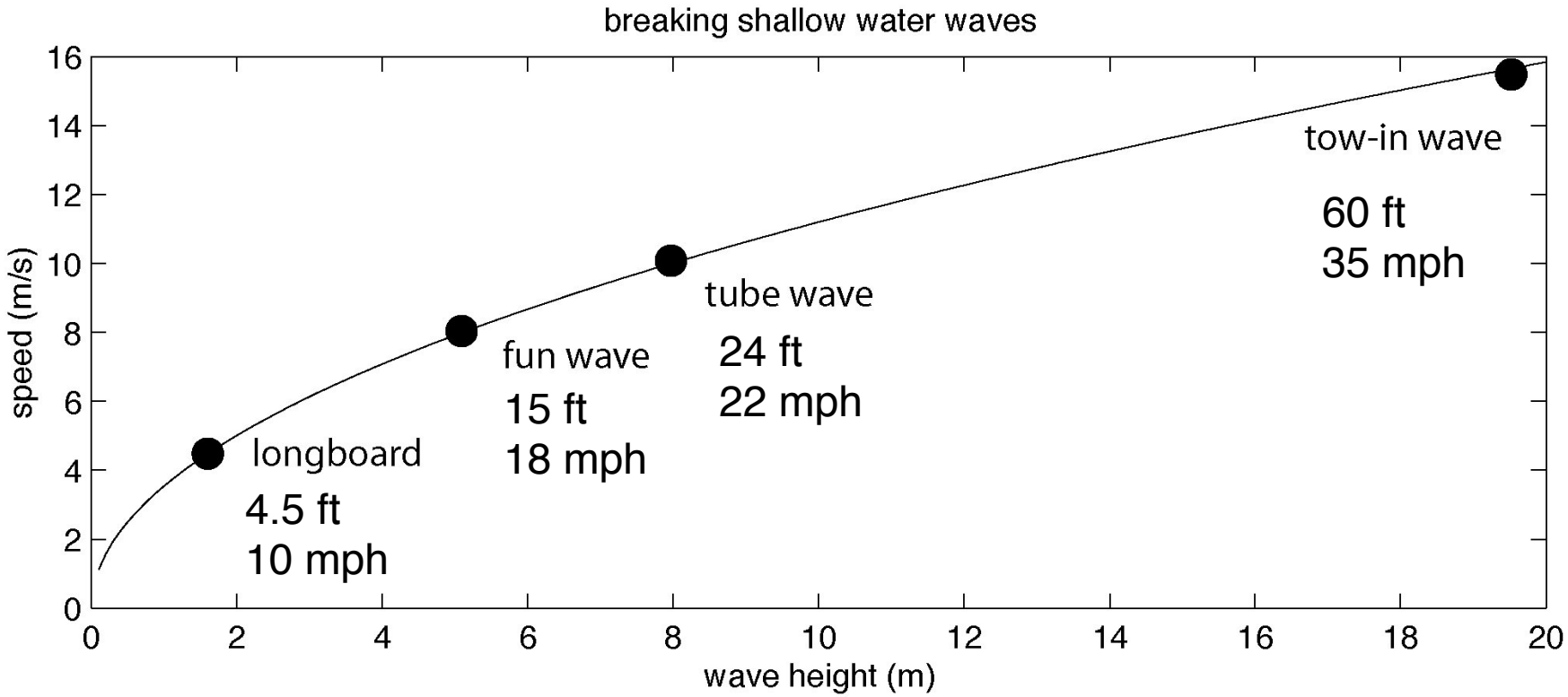
wave refraction



ocean depth and breaker height - **empirical**

$$d_b = 1.28 H_b$$

H_b - height of breaker
d_b - depth where wave breaks



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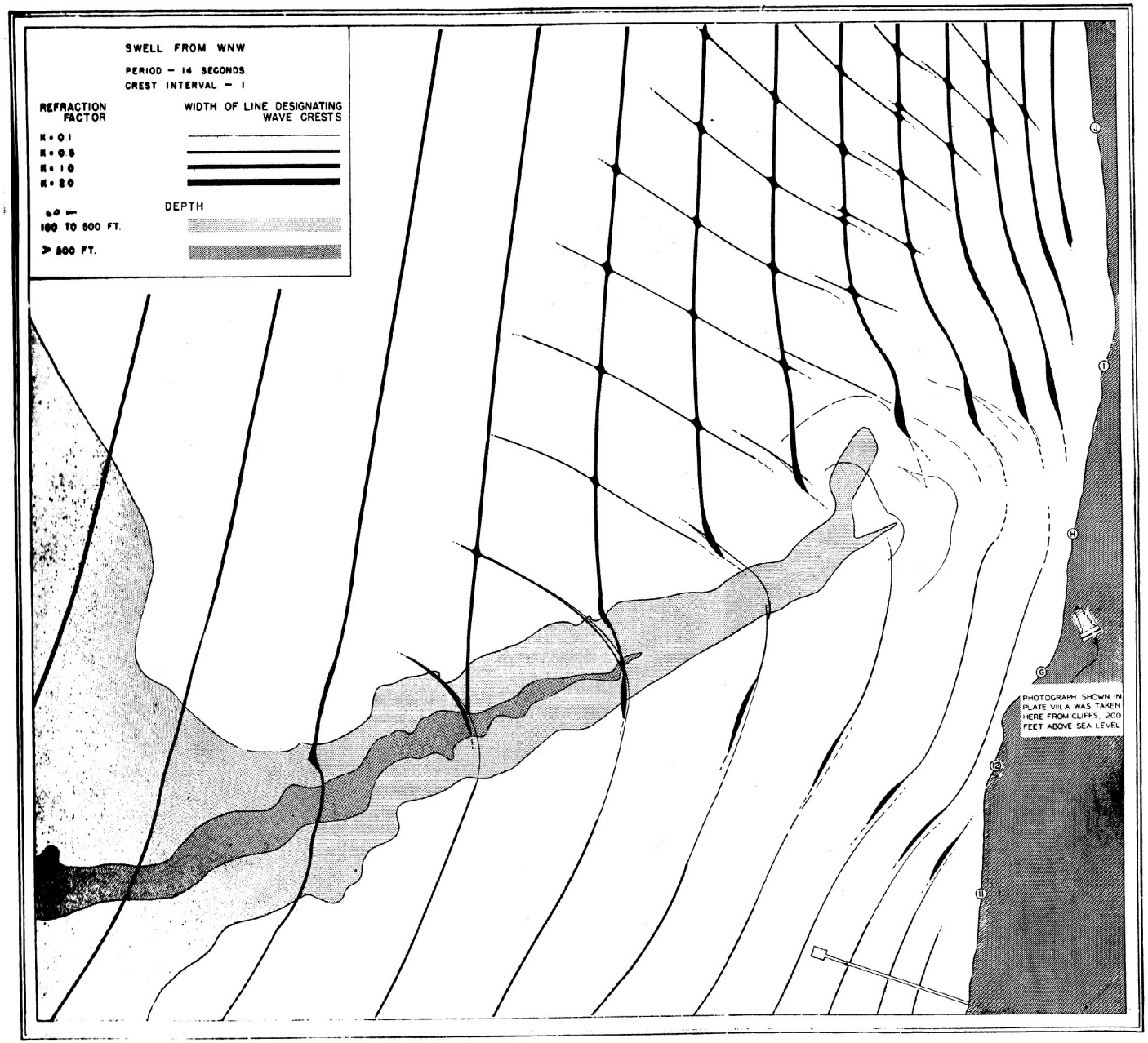
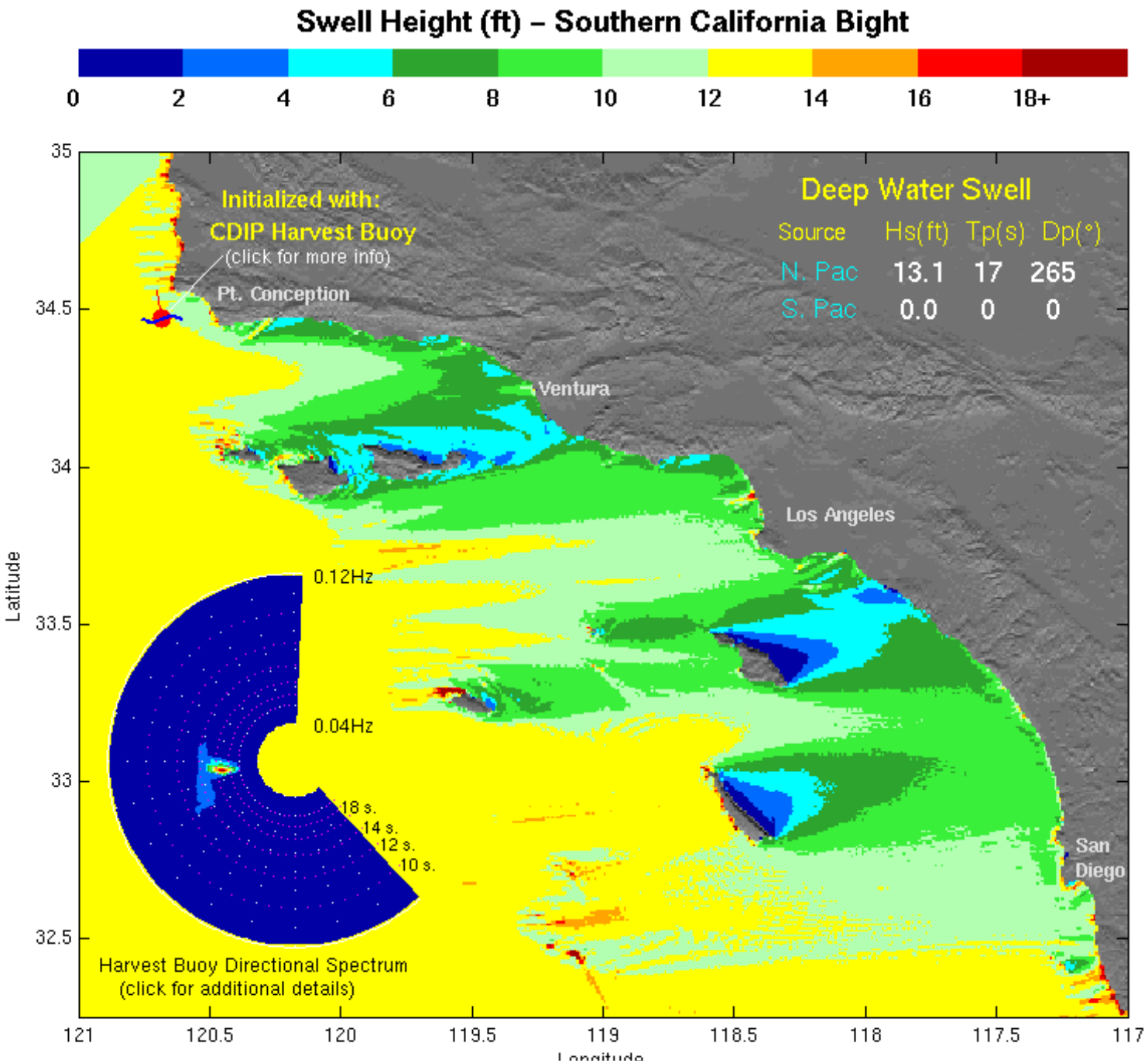


PLATE VIII

Analysis Time – 21 DEC 2005 : 1123 PST



Stn 073: 9-Band Energy

Units: metric Timezone: UTC update

Date/Time (UTC)	Hs (m)	Tp (s)	ENERGY (cm^2) - by period band (sec)								
			22+	22-18	18-16	16-14	14-12	12-10	10-8	8-6	6-
12-21-2005 19:45	1.64	9	129	26	121	292	227	176	311	203	199
12-21-2005 18:45	1.49	15	116	21	134	233	215	172	161	167	172
12-21-2005 17:45	1.51	15	87	27	209	240	153	106	209	170	231
12-21-2005 16:45	1.52	17	121	38	287	201	137	98	184	160	219
12-21-2005 15:45	1.46	9	114	37	120	195	155	122	239	176	182
12-21-2005 14:45	1.35	13	89	49	150	116	203	105	112	159	151
12-21-2005 13:45	1.09	9	48	24	66	49	121	94	126	104	108
12-21-2005 12:45	1.05	9	48	22	34	44	93	107	159	82	95
12-21-2005 11:45	1.05	9	41	20	22	37	104	87	143	122	119
12-21-2005 10:45	1.10	9	31	10	9	53	130	132	177	96	117
12-21-2005 09:45	1.12	9	27	8	7	51	121	102	229	107	133
12-21-2005 08:45	1.00	9	26	7	5	33	71	107	160	104	117
12-21-2005 07:45	1.01	9	26	4	5	35	82	104	151	92	137
12-21-2005 06:45	0.94	9	17	2	3	44	53	95	127	104	102
12-21-2005 05:45	0.86	4	13	1	3	21	45	76	107	84	115
12-21-2005 04:45	0.90	4	12	2	4	17	75	98	105	83	112
12-21-2005 03:45	0.85	9	9	1	3	19	60	77	104	75	99
12-21-2005 02:45	0.92	4	11	1	5	12	75	105	108	103	111
12-21-2005 01:45	0.97	9	11	1	6	21	48	122	170	107	108
12-21-2005 00:45	1.03	11	12	2	6	16	73	153	126	136	140
12-20-2005 23:45	1.05	9	14	2	3	16	77	165	168	112	139
12-20-2005 22:45	1.09	9	25	2	4	14	78	158	171	135	152
12-20-2005 21:45	1.12	9	21	3	3	14	134	133	211	122	150
12-20-2005 20:45	1.10	9	24	3	4	12	143	122	189	108	151

Conclusions - Waves

- Ocean waves: force of acceleration is balanced by the force of gravity.
- Wind speed >= wave speed. 17-s period waves require wind speed of 27 m/s = 60 mph.
- Wave speed:

  deep water (d >> L/2), speed depends on period (dispersive)

  shallow water (*d << L/2),* speed depends on depth (refraction)
- Refraction is important when $d < L$ or about 200 m = 650 feet
- Surfers believe sets are real but the data are not clear. Why?