International Journal for Uncertainty Quantification, 6 (5): 377–385 (2016)

AN IMPROVED SCORE FUNCTION FOR RANKING NEUTROSOPHIC SETS AND ITS APPLICATION TO DECISION-MAKING PROCESS

Nancy & Harish Garg[*]

School of Mathematics, Thapar University Patiala 147004, Punjab, India

[*]Address all correspondence to: Harish Garg, E-mail: harishg58iitr@gmail.com

Original Manuscript Submitted: 8/1/2016; Final Draft Received: 9/28/2016

The neutrosophic set (NS) is a more general platform which generalizes the concept of crisp, fuzzy, and intuitionistic fuzzy sets to describe the membership functions in terms of truth, indeterminacy, and false degree. Under this environment, the present paper proposes an improved score function for ranking the single as well as interval-valued NSs by incorporating the idea of hesitation degree between the truth and false degrees. Shortcomings of the existing function have been highlighted in it. Further, the decision-making method has been presented based on proposed function and illustrates it with a numerical example to demonstrate its practicality and effectiveness.

KEY WORDS: *score function, neutrosophic set, expert system, decision making*

1. INTRODUCTION

Decision making is one of the most widely used phenomena in our day-to-day life. Almost all decisions take several steps to reach the final destination and some of them may be vague in nature. On the other hand, with the growing complexities of the systems day-by-day, it is difficult for the decision maker to make a decision within a reasonable time by using uncertain, imprecise, and vague information. For handling this, researchers pay more attention to the fuzzy set (FS) theory [1] and corresponding extensions such as intuitionistic fuzzy set (IFS) theory [2], interval-valued IFS (IVIFS) [3] , Neutrosophic set (NS) [4], etc. To date, IFSs and IVIFSs have been widely applied by the various researchers in different decision-making problems. For instance, some authors [5–9] proposed an aggregation operator for handling the different preferences of the decision makers towards the alternatives under IFS environment. Ye [10] proposed a novel accuracy function for ranking the different alternatives of IFS or IVIFS. A ranking method on IVIFSs has been presented in Mitchell [11] and Nayagam et al. [12]. Garg [13] presented a generalized score function for ranking the IVIFSs. Xu [14] and Liu and Xie [15] developed a weighted score and accuracy function to rank the IVIFSs. Garg [16] presented some series of geometric aggregation operator under an intuitionistic multiplicative set environment. Garg [17] presented an accuracy function for interval-valued Pythagorean fuzzy sets. A novel correlation coefficient between the Pythagorean fuzzy sets has been studied by Garg [18]. Recently, Xu and Zhao [19] presented a comprehensive analysis of these methods under IFSs and/or IVIFSs and their corresponding applications in multicriteria decision-making (MCDM) problems.

Although the FSs or IFSs have been widely used by the researchers they cannot deal with indeterminate and inconsistent information. For example, if an expert takes an opinion from a certain person about a certain object, then the person may say that 0.5 is the possibility that statement is true, 0.7 say that the statement is false, and 0.2 say that he or she is not sure about it. This issue is not handled by the FSs or IFSs. To resolve this, Smarandache [4] proposed neutrosophic sets (NSs) which zre characterized by three independent components lying in $]0^+, 1^+[$ namely the "truth degree" (T), "indeterminacy degree" (I), and "falsity degree" (F), rather than only two T and F in IFS and only T in

2152–5080/16/$35.00

FS theory. In NSs, the indeterminate information is dependent on T and F whereas the inconsistent information is dependent on the degree of the belongings and nonbelongings of IFSs. However, without specification, NSs are difficult to apply in real-life problems. Thus, an extension of the NS, called a single-valued NS (SNS) and interval-valued NSs (INSs) were proposed by Wang et al. [20, 21], respectively. Majumdar and Samant [22] and Ye [23] proposed entropy and similarity measures of SVNSs and IVNSs, respectively. Ye [24] and Broumi and Smarandache [25] proposed a correlation coefficient of SVNSs and IVNSs. Ye [26] and Zhang et al. [27] proposed an aggregation operator for SVNSs and IVNSs. Later on, Peng et al. [28] showed that some operations in Ye [26] may be unrealistic and hence define the novel operations and aggregation operators for MCDM problems. Sahin [29] proposed score and accuracy function for SVNSs and IVNSs. However, the most important task for the decision maker is to rank the objects so as to get the desired decision(s). Thus, the objective of the present work is to present a novel score function for ranking the NSs and INSs by overcoming certain shortcomings of the existing functions. Furthermore, a ranking approach to MCDM problems is proposed in which preferences related to different alternatives with respect to each criterion are represented in the form of NSs.

The rest of the paper is organized as follows. Section 2 briefly introduces the concepts, operations, and operators of NSs, SVNSs, and INSs. Section 3 proposes an improved score function for SVNSs and INSs by overcoming the shortcoming of the existing functions. In Section 4, a novel approach to MCDM problems with SVNSs and INSs is developed. Then in Section 5, two examples are presented to illustrate the proposed methods and a comparative analysis is provided. Finally, Section 6 concludes the paper.

2. PRELIMINARIES

An overview of NS, SNS, and INS has been addressed here on the universal set X.

Definition 2.1. [4] A NS A in X is defined by its "truth membership function" (T_A), an "indeterminacy-membership function" $(I_A(x))$, and a "falsity membership function" $(F_A(x))$ where all are the subset of $]0^-, 1^+[$ such that $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$ for all $x \in X$.

Ye [26] reduced the NSs to SNSs, defined as below.

Definition 2.2. A NS A in X is defined as [26]

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle \mid x \in X\},$$

and is called as SNS where $T_A(x), I_A(x), F_A(x) \in [0, 1]$. SNS is also denoted by $A = \{\langle T_A(x), I_A(x), F_A(x)\rangle\}$ or $A = \langle a, b, c\rangle$.

Definition 2.3. [29] Let $A_i = \langle a_i, b_i, c_i\rangle$ be n SNSs, then

(i) Neutrosophic weighted average (NWA) operator is defined as

$$NWA(A_1, A_2, \ldots, A_n) = \left\langle 1 - \prod_{i=1}^{n} \left(1 - a_i\right)^{\omega_i}, \prod_{i=1}^{n} b_i^{\omega_i} \prod_{i=1}^{n} c_i^{\omega_i} \right\rangle;$$

(ii) Neutrosophic weighted geometric (NWG) operator is defined as

$$NWG(A_1, A_2, \ldots, A_n) = \left\langle \prod_{i=1}^{n} a_i^{\omega_i}, 1 - \prod_{i=1}^{n} \left(1 - b_i\right)^{\omega_i} 1 - \prod_{i=1}^{n} \left(1 - c_i\right)^{\omega_i} \right\rangle,$$

where ω_i is the weight vector of it such that $\omega_i \in [0, 1]$ and $\sum_{i=1}^{n} \omega_i = 1$.

Definition 2.4. [20] An INS A in X is defined as

$$A = \{\langle [\inf T_A(x), \sup T_A(x)], [\inf I_A(x), \sup I_A(x)], [\inf F_A(x), \sup F_A(x)]\rangle \mid x \in X\},$$

where $T_A(x), I_A(x), F_A(x) \in [0, 1]$ and $0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3, x \in X$. An INS is also denoted by $A = \langle [a^L, a^U], [b^L, b^U], [c^L, c^U]\rangle$.

Definition 2.5. [29] Let $A_i = \langle [a_i^L, a_i^U], [b_i^L, b_i^U], [c_i^L, c_i^U] \rangle$ be n INSs; then

(i) An interval-valued neutrosophic weighted average (INWA) operator is defined as

$$INWA(A_1, A_2, \ldots, A_n) = \left\langle \Big[1 - \prod_{i=1}^{n} \left(1 - a_i^L\right)^{\omega_i}, 1 - \prod_{i=1}^{n} \left(1 - a_i^U\right)^{\omega_i}\Big], \Big[\prod_{i=1}^{n} (b_i^L)^{\omega_i}, \prod_{i=1}^{n} (b_i^U)^{\omega_i}\Big], \Big[\prod_{i=1}^{n} (c_i^L)^{\omega_i}, \prod_{i=1}^{n} (c_i^U)^{\omega_i}\Big] \right\rangle;$$

(ii) An interval-valued neutrosophic weighted geometric (INWG) operator is defined a

$$INWG(A_1, A_2, \ldots, A_n) = \left\langle \Big[\prod_{i=1}^{n} (a_i^L)^{\omega_i}, \prod_{i=1}^{n} (a_i^U)^{\omega_i}\Big], \Big[1 - \prod_{i=1}^{n} \left(1 - b_i^L\right)^{\omega_i}, 1 - \prod_{i=1}^{n} \left(1 - b_i^U\right)^{\omega_i}\Big], \Big[1 - \prod_{i=1}^{n} \left(1 - c_i^L\right)^{\omega_i}, 1 - \prod_{i=1}^{n} \left(1 - c_i^U\right)^{\omega_i}\Big] \right\rangle,$$

where ω_i is the weight vector of it such that $\omega_i \in [0, 1]$ and $\sum_{i=1}^{n} \omega_i = 1$.

Definition 2.6. Consider SNS $A = \langle a, b, c \rangle$; then in order to rank the NS, score functions [29] have been defined as

$$K(A) = \frac{1 + a - 2b - c}{2} \quad ; \quad K(A) \in [0, 1], \tag{1}$$

$$I(A) = a - 2b - c \quad ; \quad I(A) \in [-1, 1]. \tag{2}$$

Example 2.1. Consider two NSs $A_1 = \langle 0.5, 0.2, 0.6 \rangle$ and $A_2 = \langle 0.2, 0.2, 0.3 \rangle$; then by using existing score functions $I(\cdot)$ and $K(\cdot)$, we get $I(A_1) = I(A_2) = -0.5$ and $K(A_1) = K(A_2) = 0.25$. These functions are unable to give the best alternative under some special cases.

Example 2.2. Consider two NSs $A_1 = \langle 0.5, 0, 0.2 \rangle$ and $A_2 = \langle 0.4, 0, 0.1 \rangle$; then $I(A_1) = I(A_2) = 0.3$ and $K(A_1) = K(A_2) = 0.65$. Thus, again they are unable to give the best one under some special cases.

Therefore, there is a need to modify the existing functions for ranking the different alternative(s). Hence, a modified score function has been proposed for NSs as follows.

3. PROPOSED SCORE FUNCTION

In this section, a new score function for ranking NS and INS by overcoming the shortcoming of the above functions has been proposed.

Definition 3.1. Let $A = \langle a, b, c \rangle$ be a SNS, a score function $N(\cdot)$, based on the "truth-membership degree" (a), "indeterminacy-membership degree" (b), and "falsity membership degree" (c) which is defined as

$$N(A) = \frac{1 + (a - 2b - c)(2 - a - c)}{2}. \tag{3}$$

Clearly, if in some cases SNS has $a + c = 1$ then $N(A)$ reduces to $K(A)$. Based on it, a prioritized comparison method for any two SNSs A_1 and A_2 is defined as

(i) if $K(A_1) < K(A_2)$ then $A_1 \prec A_2$,

(ii) if $K(A_1) = K(A_2)$ then

- if $N(A_1) < N(A_2)$ then $A_1 \prec A_2$
- if $N(A_1) > N(A_2)$ then $A_1 \succ A_2$
- if $N(A_1) = N(A_2)$ then $A_1 \sim A_2$

The proposed function can easily solve the above shortcomings of the above functions which can be demonstrated as below.

Example 3.1. If we apply the proposed function on Example 2.1, then we get $N(A_1) = 0.275$ and $N(A_2) = 0.125$ and hence $N(A_1) > N(A_2)$. Therefore, A_1 is a better alternative than A_2.

Example 3.2. If we apply Eq. (3) to Example 2.2 then we get $N(A_1) = 0.695$ and $N(A_2) = 0.725$ and hence A_2 is the best one.

Thus, the proposed function is reasonable and provides an effective algorithm for the decision analysis process.

Property 3.1.
The proposed function $N(A)$ and the existing function $K(A)$ satisfy the relation

$$N(A) = (2 - a - c)K(A) - \frac{1 - a - c}{2}.$$

Proof. By the definitions of $N(A)$ and $K(A)$, we have

$$\begin{aligned} N(A) &= \frac{1 + \{2K((A) - 1)\}(2 - a - c)}{2} \\ &= (2 - a - c)K(A) - \frac{1 - a - c}{2}, \end{aligned}$$

hence the result. □

Property 3.2.
The proposed function $N(A)$ and the existing function $I(A)$ for a SVNS A satisfy the relation

$$N(A) = \frac{1 + (2 - a - c)I(A)}{2}.$$

Proof. Proof follows from Eq. (3). □

Furthermore, we can have the following conclusions:

- When $A = \langle 1, 0, 0\rangle$ then the value of $N(A)$ reaches the maximum value 1.
- When $A = \langle 0, 0, 1\rangle$ then the value of $N(A)$ reaches the minimum value 0.
- For a subset $A = \langle a, b, 1 - a\rangle$, $N(A) = a - b$.

Furthermore, for decision makers it is not always possible to present their preferences in terms of exact numbers and hence corresponding to it they will prefer to give them in the form of interval-valued numbers. Then for an INS $A = \langle [a^L, a^U], [b^L, b^U], [c^L, c^U]\rangle$ where $[a^L, a^U]$ is the "truth-membership degree" interval, $[b^L, b^U]$ is "indeterminacy-membership degree" interval, and $[c^L, c^U]$ is "falsity-membership degree" interval of each element x such that

$$0 \le a^L, a^U \le 1, \quad 0 \le b^L, b^U \le 1, \quad 0 \le c^L, c^U \le 1.$$

Here, we introduce the score function for the INSs:

Definition 3.2. Let $A = \langle [a^L, a^U], [b^L, b^U], [c^L, c^U] \rangle$ be an INS. A score function $M(\cdot)$ of an INS based on the "truth-membership degree" interval, "indeterminacy-membership degree" interval, and "falsity-membership degree" interval is defined as

$$M(A) = \frac{4 + (a^L + a^U - c^L - c^U - 2b^L - 2b^U)(4 - a^L - a^U - c^L - c^U)}{8}, \quad \text{where} \quad M(A) \in [0, 1]. \tag{4}$$

In particular, when $a^L = a^U, b^L = b^U$, and $c^L = c^U$, an INS is degenerated to NS and their corresponding score function reduces to Eq. (3). Clearly, if $A = \langle [1, 1], [0, 0], [0, 0] \rangle$ then $M(A) = 1$ and if $A = \langle [0, 0], [0, 0], [1, 1] \rangle$ then $M(A) = 0$.

Example 3.3. Let $A_1 = \langle [0.4, 0.6], [0.2, 0.3], [0.5, 0.7] \rangle$ and $A_2 = \langle [0.2, 0.7], [0.1, 0.2], [0.1, 0.3] \rangle$ be two alternatives represented in terms of INSs. For getting the best alternative(s), we compute the score function as

$$M(A_1) = \frac{4 + (0.4 + 0.6 - 0.5 - 0.7 - 2 \times 0.2 - 2 \times 0.3)(4 - 0.4 - 0.6 - 0.5 - 0.7)}{8} = 0.2300,$$

$$M(A_2) = \frac{4 + (0.2 + 0.7 - 0.1 - 0.3 - 2 \times 0.1 - 2 \times 0.2)(4 - 0.2 - 0.7 - 0.1 - 0.3)}{8} = 0.2975,$$

so we have $M(A_2) \geq M(A_1)$ which means alternative A_2 is better than A_1.

4. PROPOSED APPROACH

In this section, a MCDM approach has been presented under the NSs and INSs environment based on the proposed functions. For this, consider a problem of m alternatives, $A = \{A_1, A_2, \ldots, A_m\}$, which are evaluated by the decision makers with respect to the n different criteria $C = \{C_1, C_2, \ldots, C_n\}$, whose weight vectors are ω_j, $\omega_j \in [0, 1]$, and $\sum_{j=1}^{n} \omega_j = 1$, in terms of the SNS or INS. Thus the characteristics of the alternative $A_i (i = 1, 2, \ldots, m)$ are expressed by an SNS:

$$A_i = \big\{ \big\langle C_j, a_i(C_j), b_i(C_j), c_i(C_j) \big] \big\rangle \mid C_j \in C \big\},$$

where $0 \leq a_i(C_j), b_i(C_j), c_i(C_j) \leq 1$, and $0 \leq a_i(C_j) + b_i(C_j) + c_i(C_j) \leq 3$. The pairs of these neutrosophic numbers for C_j are denoted by $\alpha_{ij} = \langle a_{ij}, b_{ij}, c_{ij} \rangle$. Therefore, the overall collective neutrosophic matrix is $D = (\alpha_{ij})_{m \times n}$. Since the different criteria may be of different types, namely, benefit or cost, then there is a need to normalize it. For this, the value of the benefit type is converted into the cost type by using the following equation:

$$r_{ij} = \begin{cases} \alpha_{ij}^c ; & \text{for benefit criteria} \\ \alpha_{ij} ; & \text{for cost criteria} \end{cases}, \tag{5}$$

and hence the matrix D is converted into matrix $R = (r_{ij})_{n \times m}$ where $\alpha_{ij}^c = \langle c_{ij}, 1 - b_{ij}, a_{ij} \rangle$ is the complement of SNS α_{ij}. Then, we have the following methods for MCDM based on proposed functions.

Approach I:

Step 1: Obtain the decision matrices R from D, if required, by using Eq. (5).

Step 2: Aggregate the SNSs r_{ij} for each A_i into an overall neutrosophic value r_i by using the NWA or NWG operator.

Step 3: Utilize Eq. (3) to compute the score value of r_i.

Step 4: Rank the r_i according to score values and hence choose the best alternative based on better value.

Step 5: End.

Approach II:
If the decision-makers give their preferences towards each $A_i(i = 1, 2, \ldots, n)$ in the form of interval numbers $\langle [a_{ij}^L, a_{ij}^U], [b_{ij}^L, b_{ij}^U], [c_{ij}^L, c_{ij}^U] \rangle$ rather than the crisp number $\langle a_{ij}, b_{ij}, c_{ij} \rangle$ then we have the following method.

Step 1: Obtain the decision matrices R from D, if required, by using Eq. (5).

Step 2: Aggregate the INSs r_{ij} for each A_i into an overall interval-valued neutrosophic number value r_i by using the $INWA$ or $INWG$ operator.

Step 3: Use Eq. (4) to compute their score value.

Step 4: Find the best one(s) based on its ranking value.

Step 5: End.

5. PRACTICAL EXAMPLE

In this section, two illustrative examples have been given for demonstrating as well as validating the proposed method.

Example 5.1. Consider a multicriteria decision making problem of an investment company where an investor wants to invest some money. To do that, they set a panel for four possible alternatives $A = \{A_1, A_2, A_3, A_4\}$, namely, food, transport, electronic, and tire company, respectively, to invest the money under the three different criteria denoted by C_1, C_2, and C_3 with weight $\omega = (0.35, 0.25, 0.40)^T$. Then by utilizing the proposed Approach I we obtain the most desirable alternative(s) as follows.

Step 1: The decision makers evaluate these alternatives w.r.t. each criterion and give their rating in terms of the SNSs $r_{ij} = \langle a_{ij}, b_{ij}, c_{ij} \rangle, (i = 1, 2, 3, 4; j = 1, 2, 3)$. The normalized information for these is listed in Table 1.

Step 2: Compute the neutrosophic number r_i for each alternative A_i by using the NWA operator as follows.

$$\begin{aligned} r_1 &= \langle 0.3268, 0.2000, 0.3680 \rangle, \qquad r_2 = \langle 0.5626, 0.1319, 0.2000 \rangle, \\ r_3 &= \langle 0.4375, 0.2352, 0.2550 \rangle, \qquad r_4 = \langle 0.5746, 0.0000, 0.1320 \rangle. \end{aligned}$$

Step 3: Use Eq. (3) for finding the score function of r_i and we get

$$N(r_1) = 0.212, \qquad N(r_2) = 0.561, \qquad N(r_3) = 0.312, \qquad N(r_4) = 0.7863.$$

Step 4: Based on the score values, we observe that A_4, i.e., the tire company, is the most desirable alternative.

If we utilize existing score functions to rank these alternatives then we get $I(r_1) = -0.4412$, $I(r_2) = 0.0988$, $I(r_3) = -0.2880$, $I(r_4) = 0.4427$, and $K(r_1) = 0.2794$, $K(r_2) = 0.5494$, $K(r_3) = 0.3560$, $K(r_4) = 0.7213$. Hence, the best alternative remains the same as compared to the proposed approach.

On the other hand, if we utilize the NWG operator for aggregating these SNSs then we get

$$\begin{aligned} r_1 &= \langle 0.3031, 0.2000, 0.3881 \rangle, \qquad r_2 = \langle 0.5578, 0.1414, 0.2000 \rangle, \\ r_3 &= \langle 0.4181, 0.2416, 0.2616 \rangle, \qquad r_4 = \langle 0.5385, 0.1555, 0.1414 \rangle, \end{aligned}$$

TABLE 1: Single-valued neutrosophic decision matrix

	C_1	C_2	C_3
A_1	$\langle 0.4, 0.2, 0.3 \rangle$	$\langle 0.4, 0.2, 0.3 \rangle$	$\langle 0.2, 0.2, 0.5 \rangle$
A_2	$\langle 0.6, 0.1, 0.2 \rangle$	$\langle 0.6, 0.1, 0.2 \rangle$	$\langle 0.5, 0.2, 0.2 \rangle$
A_3	$\langle 0.3, 0.2, 0.3 \rangle$	$\langle 0.5, 0.2, 0.3 \rangle$	$\langle 0.5, 0.3, 0.2 \rangle$
A_4	$\langle 0.7, 0.0, 0.1 \rangle$	$\langle 0.6, 0.1, 0.2 \rangle$	$\langle 0.4, 0.3, 0.2 \rangle$

hence the corresponding score values by using Eq. (3) are

$$N(r_1) = 0.1826, \qquad N(r_2) = 0.5466, \qquad N(r_3) = 0.2844, \qquad N(r_4) = 0.5568.$$

Thus, the best alternative is A_4, i.e., the tire company, for investing the money.

If we utilize the existing score functions $I(r_i)$ and $K(r_i)$, $i = 1, 2, 3, 4$ on these weighted aggregating values then we get

$$I(r_1) = -0.4850, \quad I(r_2) = 0.0750, \quad I(r_3) = -0.3267, \quad I(r_4) = 0.0860,$$

and

$$K(r_1) = 0.2575, \quad K(r_2) = 0.5375, \quad K(r_3) = 0.3367, \quad K(r_4) = 0.5430,$$

so the best alternative is A_4.

Example 5.2. Interval-valued neutrosophic numbers:
Consider the MCDM problem as described in the above example under the INSs environment. Then the following steps of Approach II have been performed to get the best alternative(s).

Step 1: Assume that an expert gives their preferences towards each alternative under each criterion in terms of INSs and their values are summarized in Table 2.

Step 2: The aggregate values of each alternative $A_i (i = 1, 2, 3, 4)$ are computed by using the INWA operator and the results are as follows:

$$\begin{aligned} r_1 &= \langle [0.5452, 0.7516], [0.1681, 0.3000], [0.3041, 0.4373] \rangle, \\ r_2 &= \langle [0.4996, 0.6634], [0.1551, 0.2885], [0.3482, 0.4655] \rangle, \\ r_3 &= \langle [0.3946, 0.5626], [0.2000, 0.3365], [0.4210, 0.5532] \rangle, \\ r_4 &= \langle [0.6383, 0.7396], [0.0000, 0.2070], [0.2297, 0.4039] \rangle. \end{aligned}$$

Step 3: By using Eq. (4), we get $M(r_i)(i = 1, 2, 3, 4)$ corresponding to each alternative as

$$M(r_1) = 0.4066, \quad M(r_2) = 0.3639, \quad M(r_3) = 0.2183, \quad M(r_4) = 0.5821.$$

Step 4: Rank these alternatives and hence we find A_4 is the best company for investment.

If we utilize the INWG operator then we have

Step 2: The aggregated values of each alternative are

$$\begin{aligned} r_1 &= \langle [0.5004, 0.6620], [0.1761, 0.3000], [0.3195, 0.4422] \rangle, \\ r_2 &= \langle [0.4547, 0.6581], [0.1861, 0.3371], [0.5405, 0.6786] \rangle, \\ r_3 &= \langle [0.3824, 0.5578], [0.2000, 0.3419], [0.5012, 0.7070] \rangle, \\ r_4 &= \langle [0.6333, 0.7335], [0.1555, 0.2570], [0.5069, 0.6632] \rangle. \end{aligned}$$

Step 3: We obtain $M(r_i)(i = 1, 2, 3, 4)$ as

$$M(r_1) = 0.3569, \quad M(r_2) = 0.2597, \quad M(r_3) = 0.1872, \quad M(r_4) = 0.3851.$$

Step 4: Rank these alternatives and hence we find A_4 is the most desirable company.

TABLE 2: Interval-valued neutrosophic decision matrix

	C_1	C_2	C_3
A_1	$\langle[0.4,0.5],[0.2,0.3],[0.3,0.4]\rangle$	$\langle[0.4,0.6],[0.1,0.3],[0.2,0.4]\rangle$	$\langle[0.7,0.9],[0.2,0.3],[0.4,0.5]\rangle$
A_2	$\langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle$	$\langle[0.6,0.7],[0.1,0.2],[0.2,0.3]\rangle$	$\langle[0.3,0.6],[0.3,0.5],[0.8,0.9]\rangle$
A_3	$\langle[0.3,0.6],[0.2,0.3],[0.3,0.4]\rangle$	$\langle[0.5,0.6],[0.2,0.3],[0.3,0.4]\rangle$	$\langle[0.4,0.5],[0.2,0.4],[0.7,0.9]\rangle$
A_4	$\langle[0.7,0.8],[0.0,0.1],[0.1,0.2]\rangle$	$\langle[0.6,0.7],[0.1,0.2],[0.1,0.3]\rangle$	$\langle[0.6,0.7],[0.3,0.4],[0.8,0.9]\rangle$

6. CONCLUSION

In the present manuscript, an attempt has been made to present an improved score function for ranking the NSs or INSs. From the existing studies, it has been observed that the existing score functions are unable to give the best alternative(s) under some situations. For overcoming this, the present manuscript considers the hesitation degree between the membership functions and hence proposes a novel score function for ranking NSs. Further, a ranking method for solving MCDM problems based on it has been presented and illustrated with a numerical example in the SNSs and INSs environment. From the results, it has been observed that it can equivalently solve the decision-making problem efficiently. In the future, we will extend this approach to the other domains.

REFERENCES

1. Zadeh, L. A., Fuzzy sets, *Inf. Control*, 8:338–353, 1965.
2. Attanassov, K. T., Intuitionistic fuzzy sets, *Fuzzy Sets Syst.*, 20:87–96, 1986.
3. Atanassov, K. and Gargov, G., Interval-valued intuitionistic fuzzy sets, *Fuzzy Sets Syst.*, 31:343–349, 1989.
4. Smarandache, F., *A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic*, Rehoboth, American Research Press, 1999.
5. Xu, Z. S. and Yager, R. R., Some geometric aggregation operators based on intuitionistic fuzzy sets, *Int. J. Gen. Syst.*, 35:417–433, 2006.
6. Garg, H., A new generalized pythagorean fuzzy information aggregation using Einstein operations and its application to decision making, *Int. J. Intell. Syst.*, 31(9):886–920, 2016.
7. Garg, H., Some series of intuitionistic fuzzy interactive averaging aggregation operators, *SpringerPlus*, 5(1):1– 27, 2016.
8. Garg, H., Generalized intuitionistic fuzzy interactive geometric interaction operators using Einstein t-norm and t-conorm and their application to decision making, *Comput. Ind. Eng.*, 101:53–69, 2016.
9. Xu, Z. S., Intuitionistic fuzzy aggregation operators, *IEEE Trans. Fuzzy Syst.*, 15(6):1179–1187, 2007.
10. Ye, J., Multicriteria fuzzy decision-making method based on a novel accuracy function under interval—valued intuitionistic fuzzy environment, *Expert Syst. Appl.*, 36:6809–6902, 2009.
11. Mitchell, H. B., Ranking intuitionistic fuzzy numbers, *Int. J. Uncertainty, Fuzziness Knowledge Based Syst.*, 12(3):377–386, 2007.
12. Nayagam, V. L. G., Venkateshwari, G., and Sivaraman, G., Ranking of intuitionistic fuzzy numbers, In *Proc. of the IEEE Int. Conf. on Fuzzy Systems (FUZZ-IEEE'08)*, pp. 1971–1974, 2008.
13. Garg, H., A new generalized improved score function of interval-valued intuitionistic fuzzy sets and applications in expert systems, *Appl. Soft Comput.*, 38:988–999, 2016.
14. Xu, Z. S., Method for aggregating interval-valued intuitionistic fuzzy information and their application to decision-making, *Control Decision*, 22(2):1179–1187, 2007.
15. Liu, Y. and Xie, N., Amelioration operators of fuzzy number intuitionistic fuzzy geometric and their application to multi-criteria decision-making, In *Proc. of the 21st Annual Int. Conf. on Chinese Control and Decision Conference*, pp. 6172–6176, 2009.
16. Garg, H., Generalized intuitionistic fuzzy multiplicative interactive geometric operators and their application to multiple criteria decision making, *Int. J. Machine Learn. Cybern.*, 7(6):1075-1092, 2016.

17. Garg, H., A novel accuracy function under interval-valued pythagorean fuzzy environment for solving multicriteria decision making problem, *J. Intell. Fuzzy Syst.*, 31(1):529–540, 2016.
18. Garg, H., A novel correlation coefficients between pythagorean fuzzy sets and its applications to decision-making processes, *Int. J. Intell. Syst.*, 31(12):1234–1253, 2016.
19. Xu, Z. S. and Zhao, N., Information fusion for intuitionistic fuzzy decision making: An overview, *Inf. Fusion*, 28:10–23, 2016.
20. Wang, H., Smarandache, F., Zhang, Y. Q., and Smarandache, R., *Interval Neutrosophic Sets and Logic: Theory and Applications in Computing*, Phoenix, AZ: Hexis, 2005.
21. Wang, H., Smarandache, F., Zhang, Y. Q., and Sunderraman, R., Single valued neutrosophic sets, *Multispace Multistruct.*, 4:410–413, 2010.
22. Majumdar, P. and Samant, S. K., On similarity and entropy of neutrosophic sets, *J. Intell. Fuzzy Syst.*, 26(3):1245–1252, 2014.
23. Ye, J., Similarity measures between interval neutrosophic sets and their applications in multicriteria decision-making, *Int. J. Intell. Fuzzy Syst.*, 26(1):165–172, 2014.
24. Ye, J., Multicriteria decision making method using the correlation coefficient under single-value neutrosophic environment, *Int. J. Gen. Syst.*, 42(4):386–394, 2013.
25. Broumi S. and Smarandache, F., Correlation coefficient of interval neutrosophic set, *Appl. Mech. Mater.*, 436:511–517, 2013.
26. Ye, J., A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets, *Int. J. Gen. Syst.*, 26(5):2459–2466, 2014.
27. Zhang, H. Y., Wang, J. Q., and Chen, X. H., Interval neutrosophic sets and their application in multicriteria decision making problems, *Sci. World J.*, vol. **2014**, doi: 10.1155/2014/645953, 2014.
28. Peng, J. J., Wang, J. Q, Wang, J., Zhang, H. Y., and Chen, Z. H., Simplified neudtrosophic sets and their applications in multi-criteria group decision-making problems, *Int. J. Syst. Sci.*, 47(10):2342–2358, 2016.
29. Şahin, R., Multi-criteria neutrosophic decision making method based on score and accuracy functions under neutrosophic environment, http://arxiv.org/abs/1412.5202, Dec 17, 2014.