Spring 5-4-2015

Using Maya and Mathematica to Create Mathematical Art

Allison Carr
ancarr@bgsu.edu

USING MAYA AND MATHEMATICA TO CREATE MATHEMATICAL ART

ALLISON CARR

HONORS PROJECT

Submitted to the Honors College
At Bowling Green State University in partial
Fulfillment of the requirements for graduation with

UNIVERSITY HONORS

MAY 4, 2015

Bonnie Mitchell,
School of Art, Digital Arts division, Advisor

Dr. Steven M. Seubert,
Department of Mathematics and Statistics, Advisor

Purpose

Whenever I introduce myself and explain that I studied mathematics in college, nine times out of ten the other person will respond with something along the lines of, “ugh, I really hate math”. It frustrates me for my chosen profession to be dismissed so easily. Why can’t everyone see how interesting mathematics is? How beautiful and pure it is? My wish to spread the beauty of math was where this project began, but it evolved into something much larger.

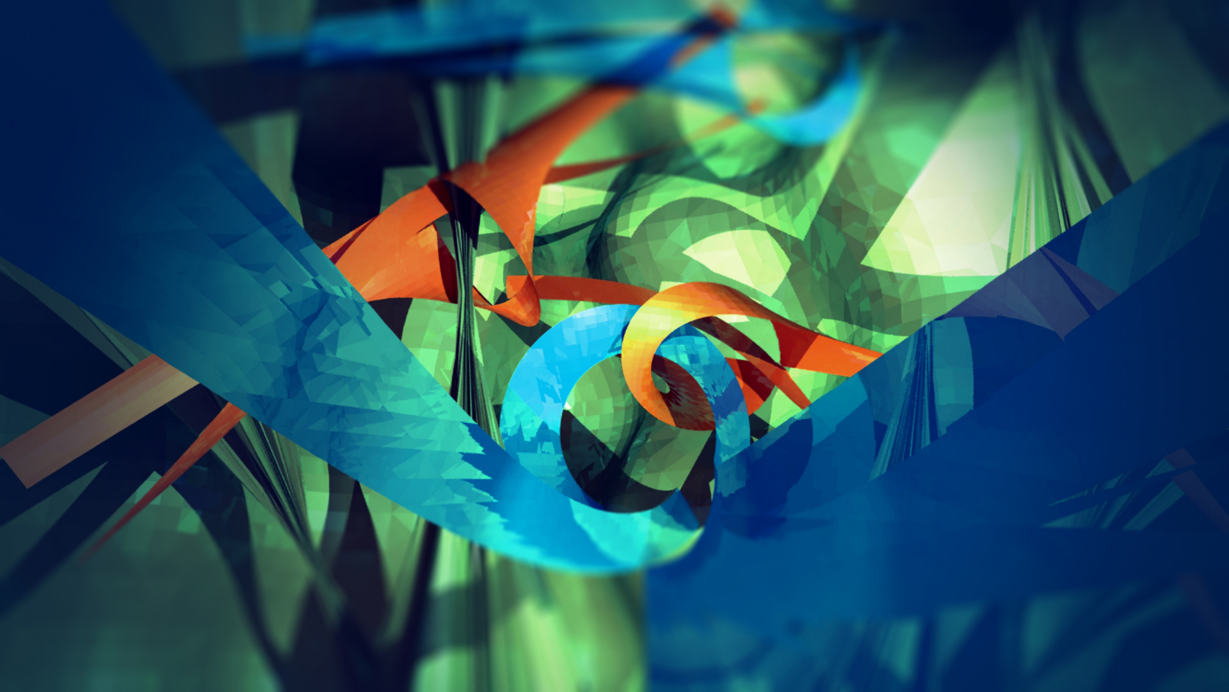
I meant this project to be a learning experience for others to help rid them of their ignorance. In the end, it was I who learned of my ignorance and sought to grow past it.

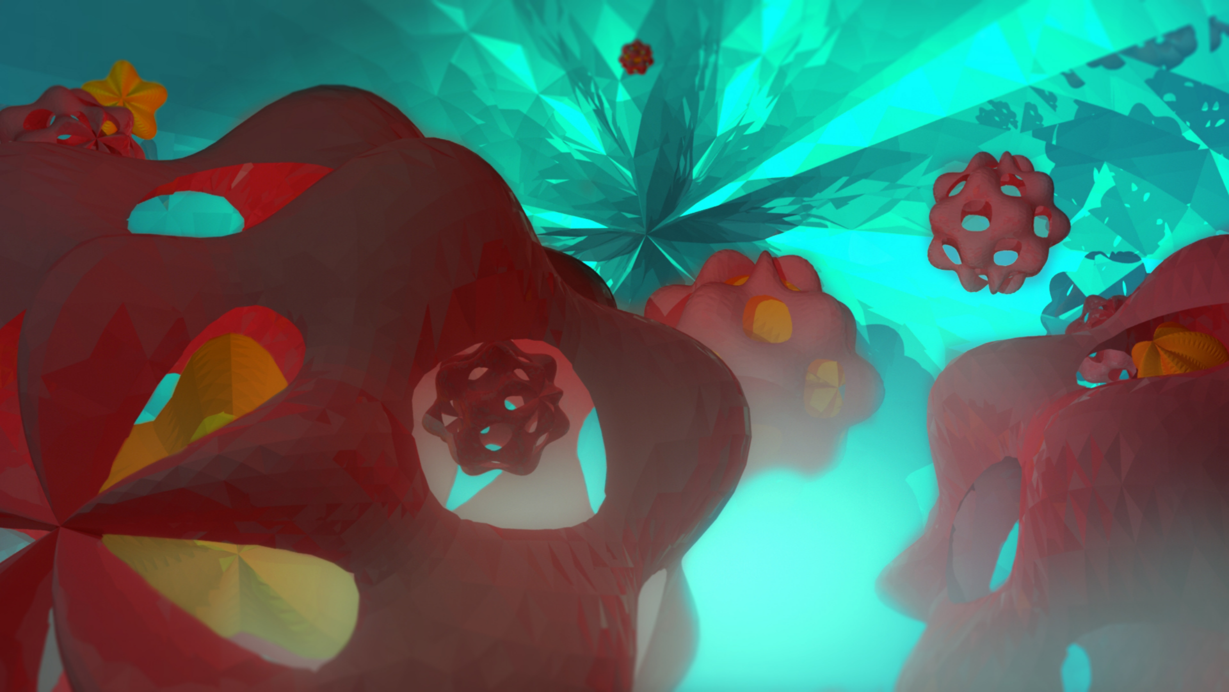
Initially, I thought that creating mathematical art would be simple; after all, how hard could it be to arrange shapes into something pretty or to create three-dimensional objects from formulas? I knew the math, and the art seemed simple, so I assumed this project would go down without any problems. It turns out I didn’t know enough about neither art nor math to complete the art pieces in my mind.

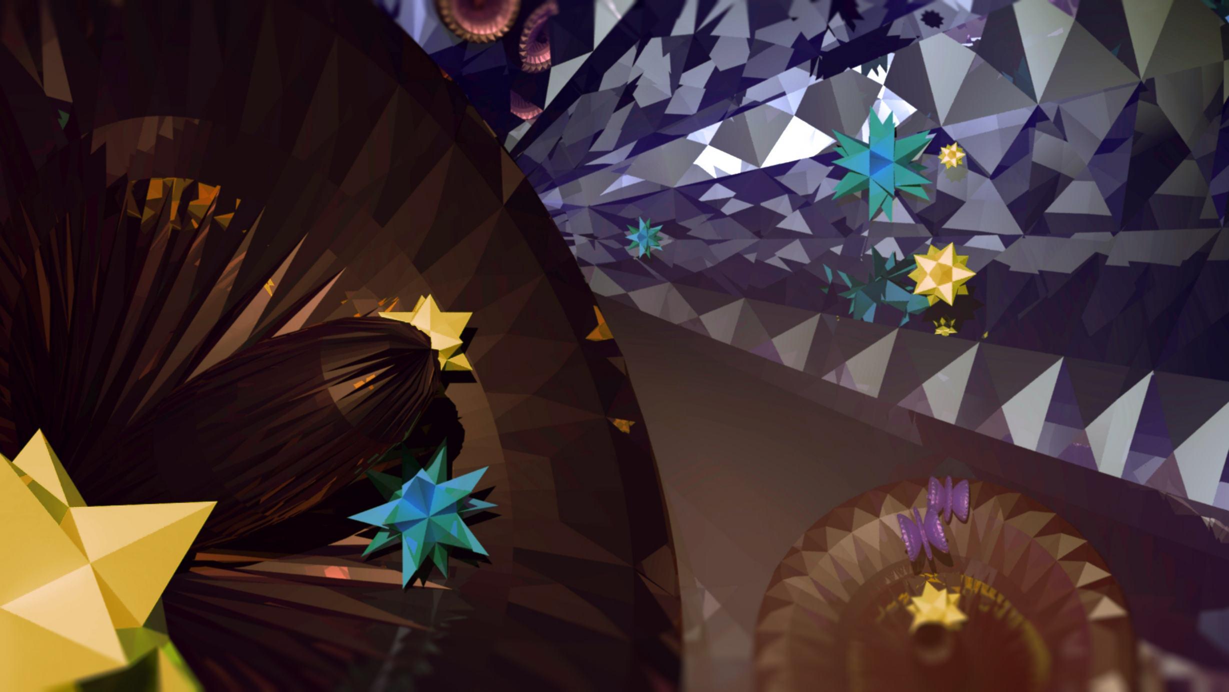
The first thing I learned was that knowing the formulas was very different from knowing how to create a model of the object. After searching for the best program to graph objects, I ended up selecting Mathematica. I knew that it is often used in a wide variety of professions, so I figured it would be an easy program to use. I was wrong. When I opened up Mathematica, the only thing that popped up was a blank screen with a blinking text curser. No tutorials, no user interface, only an empty space for typing. Mathematica works simply by inputting lines of code. Thankfully, there is a wide range of Mathematica tutorials to help newcomers. Finally, and with a lot of assistance from the internet, I was able to create a wide range of mathematical objects and export their meshes to Maya.

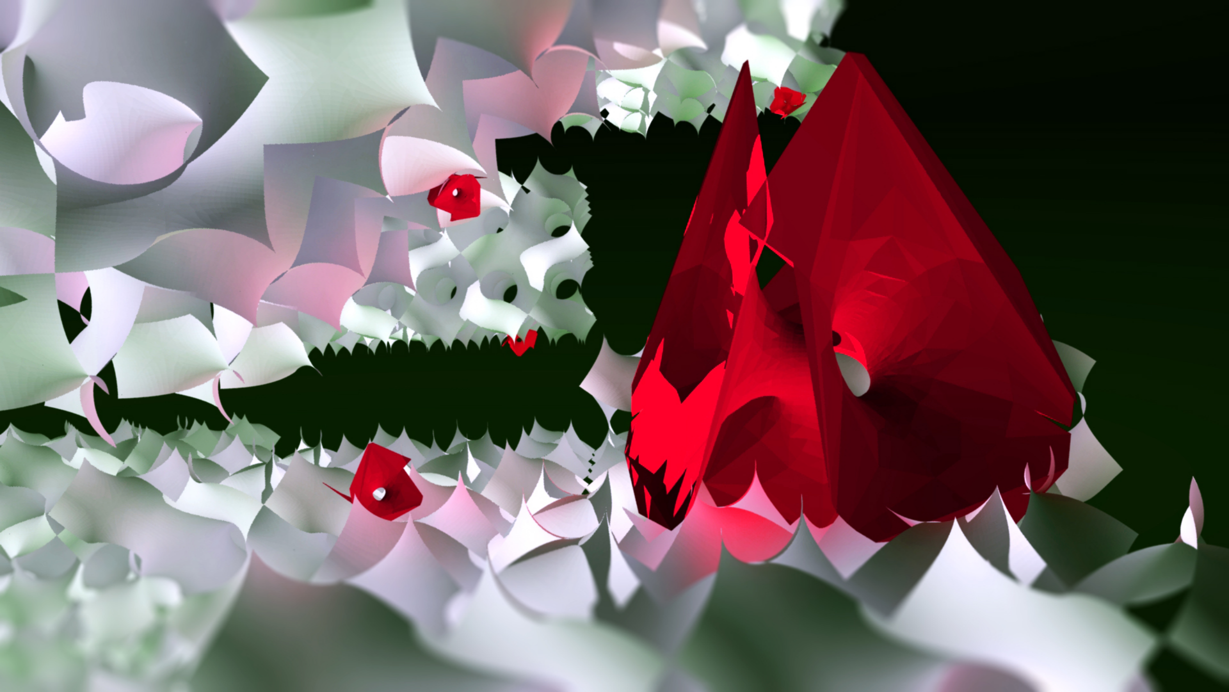
Then I faced the overwhelming challenge that is Maya. I know a lot of artists, and I’ve seen them easily manipulate objects in Maya. Unlike Mathematica, Maya almost exclusively used buttons from the user interface. So taking my freshly created meshes and turning them into art should have been as easy as point and click, right? I must admit, I have never respected digital artists as much as I did in my first few days with Maya; my initiation to Maya involved me frantically watching tutorials and pressing buttons while yelling at my computer to make art.

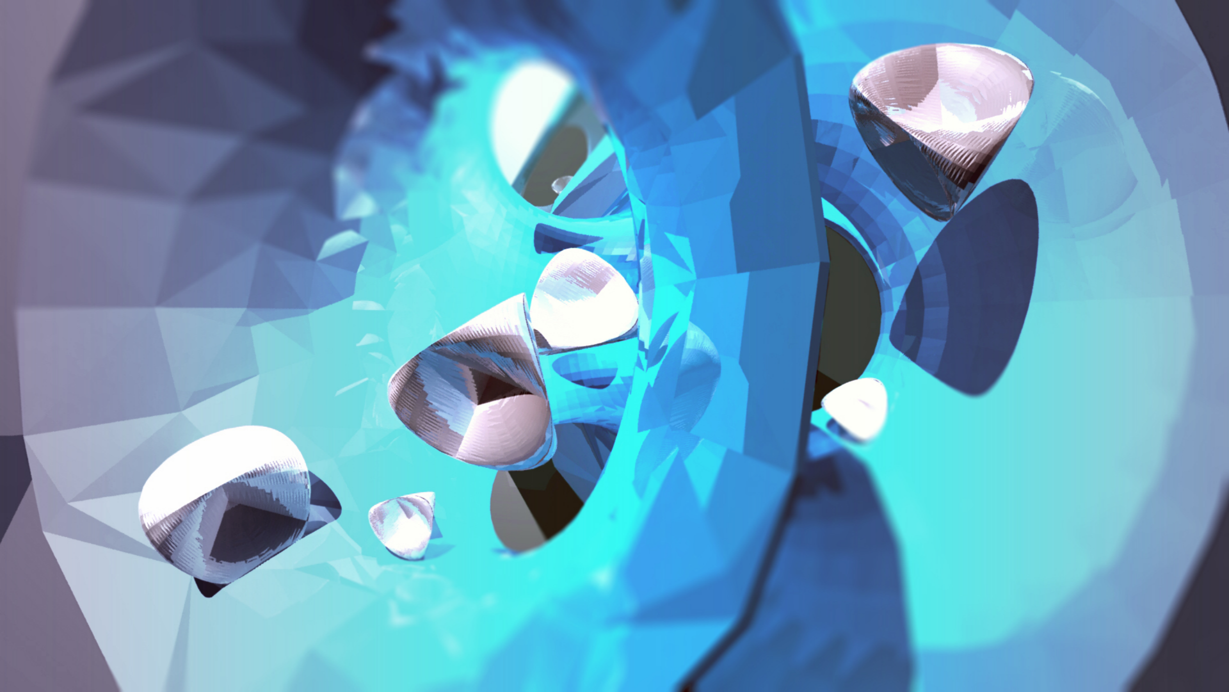
I eventually learned the basics of Mathematica and Maya and created art pieces that I am genuinely proud of. I hope my art as well as my experiences will be a lesson to others. Both math and art are beautiful and complicated disciplines; they both deserve to be respected and praised for the difficultly their creations entail.

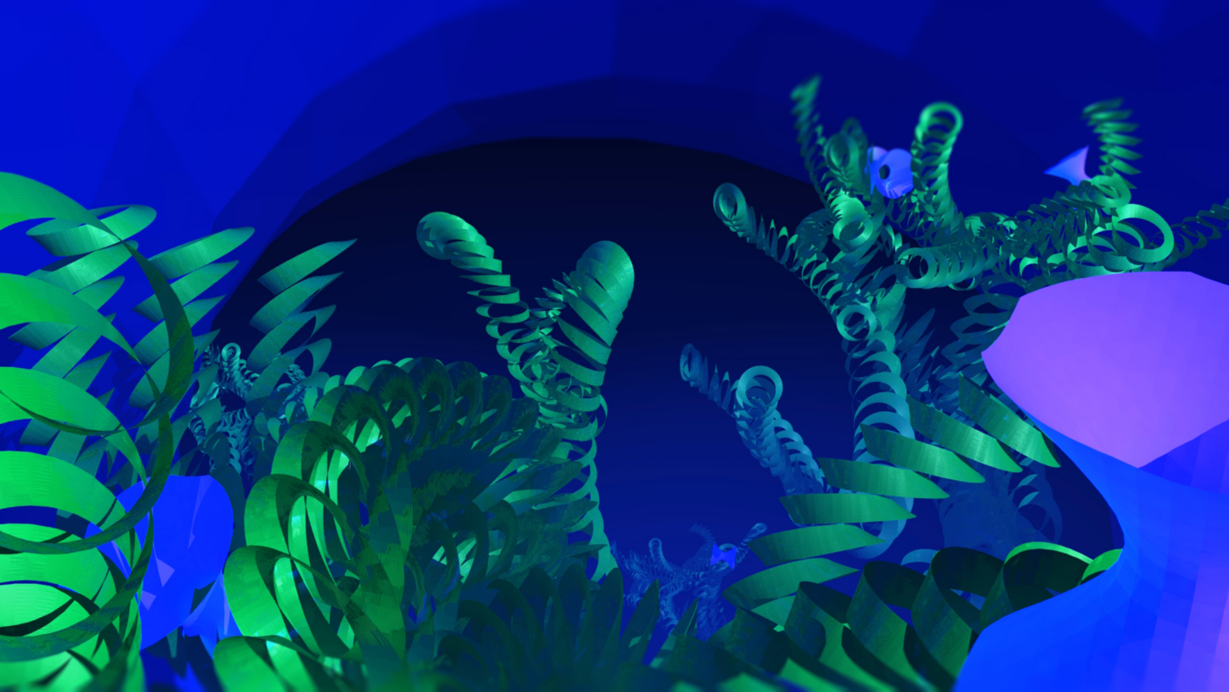












About the Pieces

A Moment in Infinity

This piece contains three different shapes. The most obvious is the Möbius Strip Variation; there are four of them altogether. The background of the piece is a Sine-Cosine Wave. Finally, I use the inner column of Object 1; the most obvious use of this shape is just off-center when one of these columns goes through an orange Möbius strip. The column of Object 1 is used three times in the piece.

I love this variation of the Möbius strip because it feels like a visual representation of infinity. The strips curl together in the middle only to separate and stretch out to the horizon. One strip curls itself around the sharp inner pillar of Object 1. Another weaves around the surface of the Sine-Cosine Wave. Two loop around each other. However, if you see these strips as time lines, this moment of interaction and connection is very brief. Each strip has traveled for an infinite amount of time to reach is single curl in time; and when they pass the loop, they will again travel alone for the rest of infinity.

Summer Morning

This piece uses Objects 2 and 3. The entire scene takes place inside of a large version of Object 2. Object 3 appears eight times in this piece, and Object 2, not including the background, appears four times.

Both of these shapes make me feel slightly nostalgic. To me, Object 2 reminds me of starfruit or oranges and Object 3 resembles a wiffle ball. Together, they remind me of the summer days of my childhood. I played softball almost every day. Team practice was usually in the mornings to avoid the scorching heat of midday. The morning mist still covered the fields as we took turns practicing our batting. As the sun reached its height, we would end practice with a snack of orange slices and juice boxes.

Crystal Flowers

Three shapes where used in this piece: Object 4, Small Stellated Dodecahedron, and Great Stellated Dodecahedron.

In this piece, I experimented with the reflectivity of the shapes. Since Object 4's Maya model has so many faces, increasing reflectivity caused it to take on a crystal-like appearance. I made the stellated dodecahedrons comparatively brighter colors so that their reflections were more apparent. In fact, quite a few of the objects you see in the image are just reflections. This uncertainty of what is real causes this piece to feel sort of dreamlike.

The Trap

Gyroids and Trinoids were used to create this piece.

I was immediately fascinated with the shape of the Gyroid, but I couldn't figure out how to use it. I spend so long trying to find interesting angles from within the Gyroid only to feel lost and tangled within the shape. That journey is how I found inspiration for this piece. The Gyroids appear beautiful and intriguing; their shapes are so unusual and the soft pink and green coloring makes them feel safe. Even more enticing are the Trinoids; they shine like rubies and look as delicate as petals, but they too are only a part of the trickery. For while these are beautiful, the maze of thorns proves to be deadly.

The Impossible Shape

This piece contains the Costa Surface and several Roman surfaces.

Mathematically, these two minimal surfaces are extremely challenging to comprehend. The theories behind these shapes are far more complex than anything I ever covered in my studies. It thus seemed fitting for them to star in a piece together. The Costa surface is confusing to look at. It has many openings, but none of them led to where you think they should, and some seem to disappear when you try to find the other end; it almost seemed impossible for this shape to exist. Even after thoroughly examining it and confirming that it is in fact a completely possible object, my brain still found it visually hard to accept. The Roman surfaces are like little bubbles trying to navigate the seemingly impossible maze.

Möbius Forest

The Möbius Strip and the Hourglass-shaped Surface are contained in this piece.

While manipulating the Möbius Strip, I found that creating a pillar of them like ringlets looked very plant-like. After creating a forest of these, I realized that they looked like they belonged in the ocean. The edges of the Hourglass-shaped Surface are wavy like the tailfin of a fish, so I scattered these shaped about my Möbius forest.

Mathematical Objects: Equations and Creation

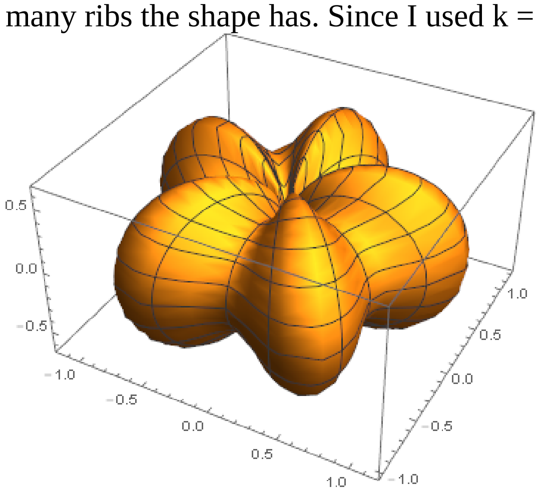
Below I've explained the Mathematica codes used to create the various digital objects, the formulas that describe the shapes and surfaces, and a brief explanation of the more complex objects. I have also included a short statement discussing why I chose to use these objects in my art. There are four sections to organize the digital objects: Spherical Objects, Polyhedrons, Surfaces, and Minimal Surfaces.

Spherical Objects

The spherical coordinate system is fantastic for causally fiddling with equations; almost any combination of sine and cosine will yield interesting results. Even simple changes to an equation can create an extremely different object; in fact, the first three objects below are really only one term apart from each other.

Object 1

Mathematica formula[1]: SphericalPlot3D[Sin[θ] + Sin[5ϕ]/5 , {θ, 0, Pi}, {ϕ, 0,2Pi}]
This is using the spherical coordinate system (r, θ, φ). So the conventional formula in this case is $r = \sin(\theta) + \sin(k\phi)/k, where\ 0 < \theta < \pi\ and\ 0 < \varphi < 2\pi$. The constant, k, dictates how many ribs the shape has. Since I used k = 5, the resulting shape had five ribs.

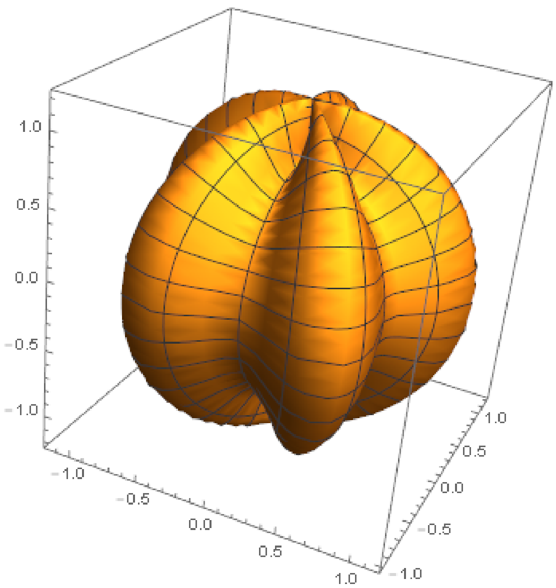


In my opinion, the inside of this shape is the most interesting part. The top and bottom of the shape pinch together sharply creating the image below.

In my mind, this column resembles a stalagmite and stalactite. The texture of this part of the model appears very rough, and the edges are very sharp. Compared to this part of the model, the smooth, flower-like appearance of the shape's exterior is quite dull.

Object 2

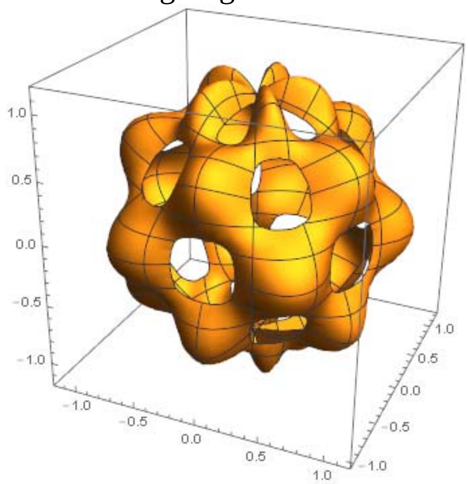
Mathematica formula[6]: SphericalPlot3D$[1+\frac{1}{5}\text{Sin}[5\phi],\{\theta,0,\pi\},\{\phi,0{,}2\pi\}]$
This is using the spherical coordinate system (r, θ, φ). So the conventional formula in this case is $r=1+\sin(k\phi)/k, where\ 0<\theta<\pi\ and\ 0<\varphi<2\pi$. Like with Object 1, k dictates the number of ribs. Again, I used the object created when k=5.

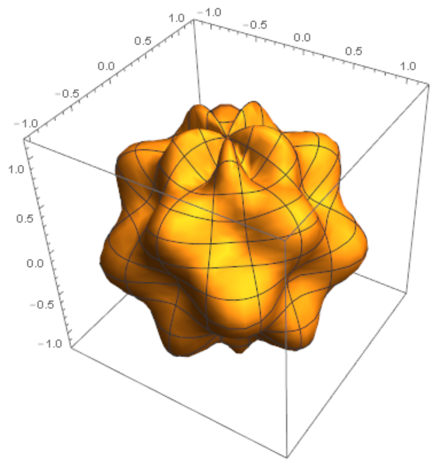


While the only difference between this Object and Object 1 is that the center does not pinch inward, this shape still caught my attention. Probably in part because it resembles starfruit, this shape makes me think of food and nature.

Object 3

Mathematica formula[4]: $\text{SphericalPlot3D}[1 + \text{Sin}[5\theta]\,\text{Sin}[5\phi]/5\,,\ \{\theta, 0, \text{Pi}\},\ \{\phi, 0{,}2\text{Pi}\},$ $\text{RegionFunction} \to (\#6 > 0.95\&)]$
Using the spherical coordinate system, this object’s formula is $r = 1 + \sin(5\theta)\sin(5\phi)/5, \mathit{where}\ 0 < \theta < \pi\ \mathit{and}\ 0 < \varphi < 2\pi$. In the Mathematica formula, “RegionFunction” limits which parts of the shape are graphed. The image below on the left was created using RegionFunction while the image on the right was not.

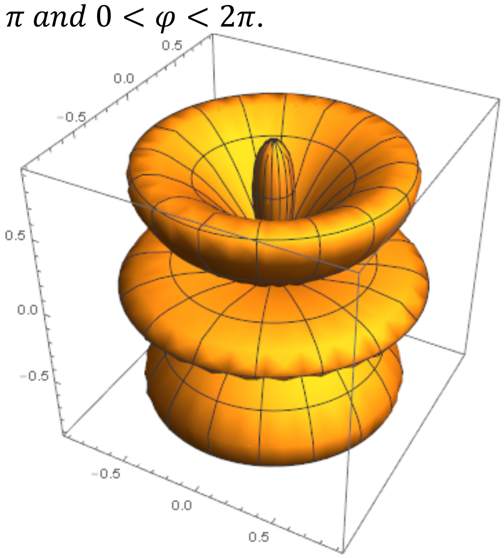




I learned how to create this shape while researching Mathematica’s RegionFunction, and I knew instantly that I wanted to create a piece using it. Every time I look at this object, I think of a wiffle ball. Not only do I find this object nostalgic, I find it artistically versitile; the holes in the object allow for both the inside and the outside of the object to be used simultaneously.

Object 4

Mathematica formula[2]: $\text{SphericalPlot3D}[1 + \text{Cos}[4\theta],\ \{\theta, 0, \text{Pi}\},\ \{\phi, 0{,}2\text{Pi}\}]$
Using the spherical coordinate system, this object’s formula is $r = 1 + \cos(4\theta)\mathit{where}\ 0 < \theta < \pi\ \mathit{and}\ 0 < \varphi < 2\pi$.



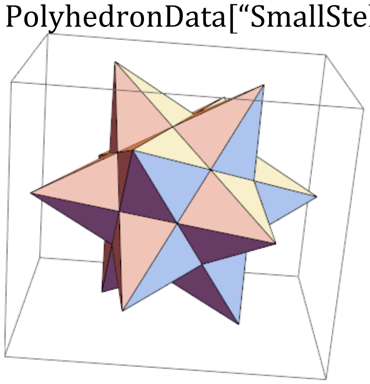
While this is an interesting shape to look at, I initially didn't think it would work well in an art piece. However, after playing around with it in Maya, my opinion changed. This shape looks completely different depending on the angle at which it is viewed. If looked at from the top, it resembles a flower with its pollen tube. On the other hand, when viewed from the side it looks more like wheels or a butterfly. The versitality of this shape allowed me to use it in multiple ways in my art.

Polyhedrons

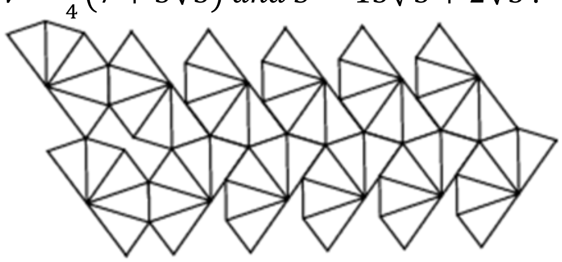
Unlike the other shapes, objects, and planes used in this project, polyhedrons don't really have a mathematical formula. There are equations to find their properties, like volume or suface area, and a set of equations can be assigned to them to create them graphically, but there isn't one formula that completely describes a polyhedron. For both of the polyhedrons I used in this project, the small and great stellated dodecahedrons, I will instead discuss their properties and the formulas that can describe them.

Small Stellated Dodecahedron[12]

In Mathematica, this shape can be created by simply inputing:
PolyhedronData["SmallStellatedDodecahedron"]

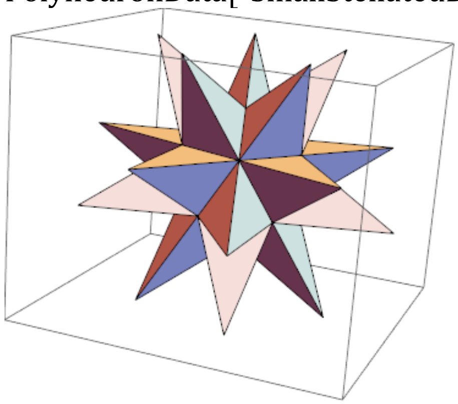


As can be seen below, the small stellated dodecahedron is comprised of twelve pentagonal pyramids. The polyhedron is often described in terms of the twelve pentagrammic faces it contains. A pentagrammic face is a set of five vertices that are coplanar. In the image above, the peach trianges represent one pentagrammic face, and the periwinkle triangles form another. Assuming the pentagrams have unit edge lengths, the circumradius of the small stellated dodecahedron is $\frac{1}{2}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}$. The volume and surface area of this polyhedron are $V = \frac{5}{4}\left(7+3\sqrt{5}\right)$ *and* $S = 15\sqrt{5+2\sqrt{5}}$.



Great Stellated Dodecahedron[11]

In Mathematica, this shape can be created by simply inputing:
PolyhedronData["SmallStellatedDodecahedron"]



Like the small stellated dodecahedron, the great stellated dodecahedron has twelve pentagrammic faces. In the image above, the peach colored faces make up one of the pentagrammic faces. The difference between this polyhedron and the last is that is is comprised of twenty triangular pyramids.
Assuming the pentagrams have unit edge lengths, the circumradius is $\frac{1}{4}\sqrt{3}\left(\sqrt{5}-1\right)$. The volume and surface area are $V = \frac{5}{4}\left(3+\sqrt{5}\right)$ and $S = 15\sqrt{5+2\sqrt{5}}$.

I found both of these polyhedrons very intersting. When I was in elementary school, my math class taught me how to create these shapes using origami, but I never knew their names or the equations that go with them. So learning about the naming convensions of these polyhedrons, the derivations of their equations, and what stellating a polyhedron actually details was a very enjoyable process. After having such a good time learning the mathematics of these shapes, I knew that I had to use them creatively as well.

Surfaces

Traditional Möbius Strip

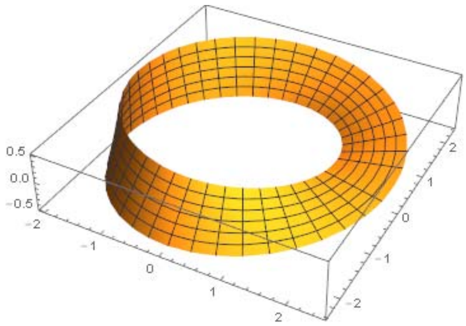
Mathematica formula[7]:
Moebius[R_][s_, t_] := {(R + s Cos[t/2]) Cos[t], (R + s Cos[t/2]) Sin[t], s Sin[t/2]}
ParametricPlot3D[Moebius[2][s, t], {s, –.5, .5}, {t, 0, 2 \[Pi]}, Mesh –> {5, 40}]
The Möbius Strip can be represented parametrically as:

$$x(u,v) = \left(1+\frac{v}{2}\cos\frac{u}{2}\right)\cos u$$

$$y(u,v) = \left(1+\frac{v}{2}\cos\frac{u}{2}\right)\sin u$$

$$z(u,v) = \frac{v}{2}\sin\frac{u}{2}$$

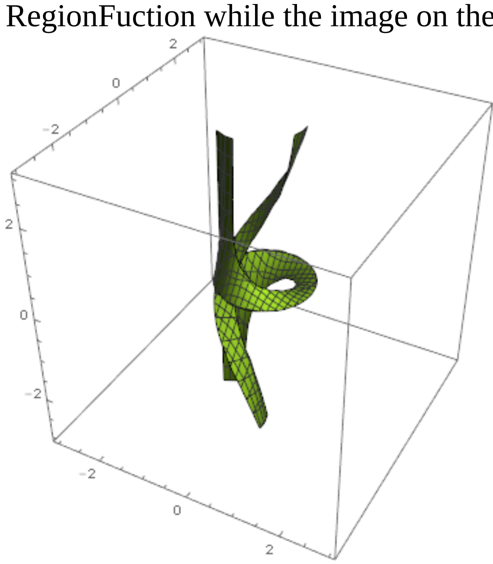
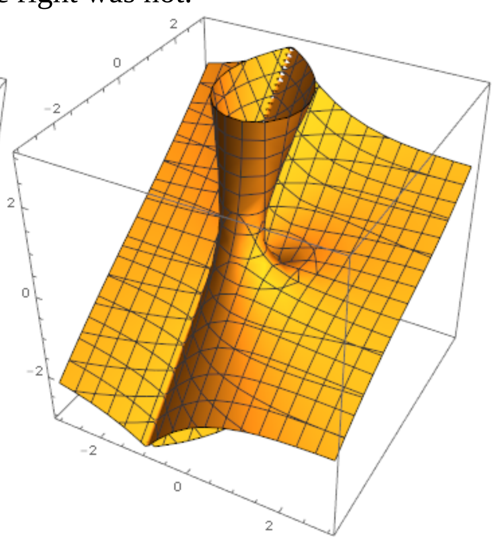
$$where\ 0 \le u \le 2\pi\ and -1 \le v \le 1$$



The Möbius Strip has always fascinated me. I was once told that it could be used to visualize the full number line; the numbers on the outside increase to positive infinity, become imaginary numbers on the inside of the strip, and then go from negative infinity to zero once on the outside again. The strip seems to defy what should be physically possible in our dimension, yet exists nonetheless; in that way, I think it is beautiful.

Möbius Strip Variation

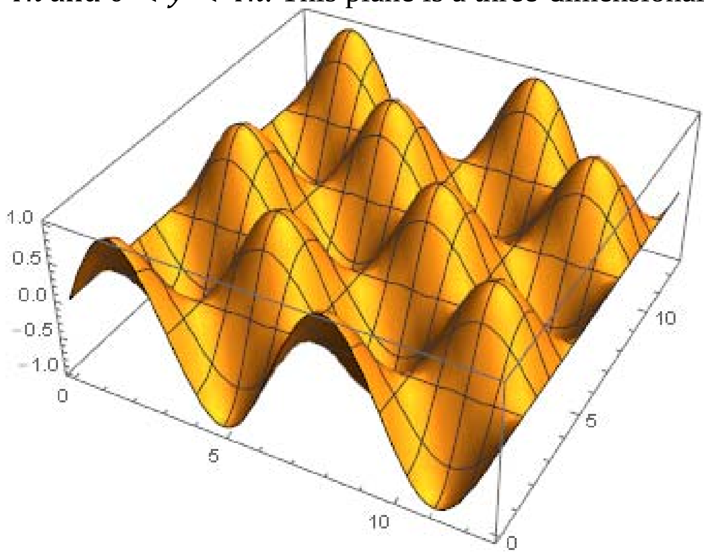
Mathematica formula[7]: ContourPlot3D$[-y + x^2y + y^3 - 2xz - 2x^2z - 2y^2z + yz^2 == 0, \{x, -3,3\}, \{y, -3,3\}, \{z, -3,3\},$ RegionFunction $\rightarrow$ (.6^2 < #1^2 + #2^2 < 1^2&)]
The conventional formula is $0 = -y + x^2y + y^3 - 2xz - 2x^2z - 2y^2z + yz^2, where -3 < x < 3, -3 < y < 3, and -3 < z < 3$. In the mathematica formula, "RegionFunction" limits which parts of the shape are graphed. The image below on the left was created using RegionFuction while the image on the right was not.

This is similar to the traditional Möbius Strip. The obvious difference being that the ends of the strip don't reconnect. As I mentioned earlier, I really love the Möbius Strip; this variation seemed to have even more artistic potential than its traditional form.

Sine-Cosine Wave

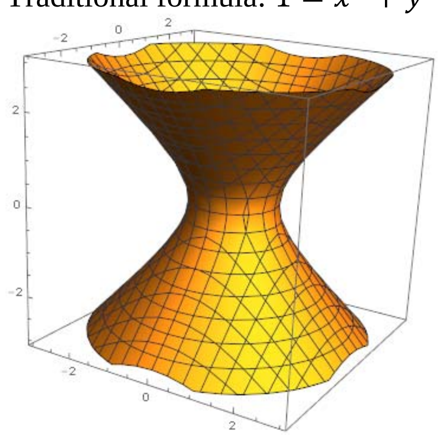
Mathematica formula: Plot3D[Sin[x]Cos[y], {x, 0,4Pi}, {y, 0,4Pi}]
The conventional formula of this object is $f(x, y) = \sin(x) * \cos(y)$, where $0 < x < 4\pi$ and $0 < y < 4\pi$. This plane is a three-dimensional Sine-Cosine wave.



This is really a simple and uninteresting plane, but I love the textured look it creates. When the plane has its reflectivity increased, it creates very interestingly distorted reflections. Due to these two factors, I use this plane as the background in one of my pieces.

Hourglass-shaped Surface

Mathematica formula[2]: ContourPlot3D[x^2 + y^2 − z^2 == 1, {x, −3,3}, {y, −3,3}, {z, −3,3}]
Traditional formula: $1 = x^2 + y^2 - z^2, where - 3 < x < 3, -3 < y < 3, and - 3 < z < 3$.



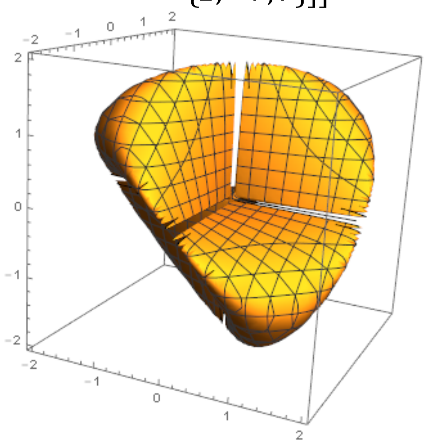
What I found most interesting about this shape is the edges along the top and bottom. At some angles, the slight wave of the edge causes the shape to resemble the tailfin of a beta fish. I use this angle multiple times in the art piece that includes this surface.

Minimal Surfaces

Roman Surface

The traditional equation for this surface is $x^2y^2 + y^2z^2 + z^2x^2 - r^2xyz = 0$, where r is any number that is real and positive. However, using this equation in Mathematica results in a model that is broken into four pieces.

$\text{With}[\{r = 2\}, \text{ContourPlot3D}[x^2y^2 - r^2xyz + x^2z^2 + y^2z^2 == 0, \{x, -r, r\}, \{y, -r, r\}, \{z, -r, r\}]]$



To create an image in Mathematica that looks like a true Roman surface, the following codes are used[8]:

$\text{romeqn} = (x\text{^}2 + y\text{^}2 + z\text{^}2 - k\text{^}2)\text{^}2 == ((z - k)\text{^}2 - 2x\text{^}2)((z + k)\text{^}2 - 2y\text{^}2);$

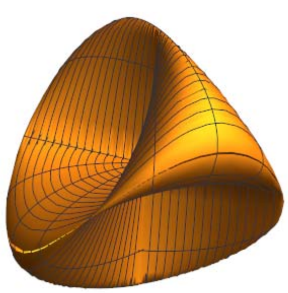
$\text{romsoln} = \text{Solve}[\text{romeqn}, z]$

rom1[x_, y_] := z /. romsoln[[1]] /. k -> 1

rom2[x_, y_] := z /. romsoln[[2]] /. k -> 1

$\text{ParametricPlot3D}[\text{Block}[\{x = r\text{Cos}[t], y = r\text{Sin}[t]\}, \{x, y, \#[x, y]\}], \{r, .0001, .9999\}, \{t, 0, 2\text{Pi}\}, \text{Boxed} -> \text{False}, \text{Axes} -> \text{False}, \text{PlotRange} -> \text{All}]\&/@\{\text{rom1}, \text{rom2}\}$

Show[%]



This method creates two separate surfaces and combines them to create the image of a Roman surface. You can just slightly see the seam between the two planes in the image above. The first time I had ever heard the term “minimal surface” was while learning about the Roman surface; now this shape has a sort of sentimental value to me. Even though I struggled to find an artistic way to incorporate this shape in my art, I was determined to use it.

The Costa Surface

Mathematically, this is a surface quite complicated. According to Wolfram MathWorld[10], the Costa surface can be represented parametrically explicitly by

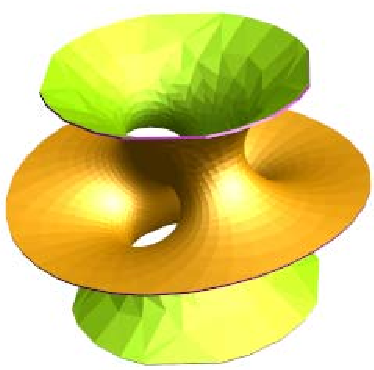
$$x = \frac{1}{2}\,\Re\left\{-\zeta(u+iv)+\pi u+\frac{\pi^2}{4e_1}+\frac{\pi}{2e_1}\left[\zeta\left(u+iv-\tfrac{1}{2}\right)-\zeta\left(u+iv-\tfrac{1}{2}i\right)\right]\right\}$$

$$y = \frac{1}{2}\,\Re\left\{-i\zeta(u+iv)+\pi v+\frac{\pi^2}{4e_1}-\frac{\pi}{2e_1}\left[i\zeta\left(u+iv-\tfrac{1}{2}\right)-i\zeta\left(u+iv-\tfrac{1}{2}i\right)\right]\right\}$$

$$z = \frac{1}{4}\sqrt{2\pi}\,\ln\left|\frac{\wp(u+iv)-e_1}{\wp(u+iv)+e_1}\right|,$$

Where $\zeta(z)$ is the Weierstrass zeta function, $\wp(g_2, g_3; z)$ is the Weierstrass elliptic function with $(g_2, g_3) = (189.072772\ldots, 0)$, and $e_1 = \wp\left(\tfrac{1}{2}; 0, g_3\right) = \wp\left(\tfrac{1}{2} \mid \tfrac{1}{2}, \tfrac{1}{2}i\right) \approx 6.87519$.

In his article "Visualizing Minimal Surfaces"[3], O. Michael Melko describes a method to create a 3D model of the Costa surface in Mathematica. The code for this method can be seen in the Appendix, and the resulting model is shown below.



Mathematically, the Costa surface is the most difficult shape that I incorporated into this project. Differential geometry is far from easy, but this shape takes it to a whole new level. However, I couldn't resist the beauty of this shape. The surface doesn't behave the way my brain thinks it logically should. It makes sense after examining it closer, but I was initially very confused when I turned the model over and didn't see the openings on the other side. I wanted to use this shape in my art to recreate that initial confusion I experienced.

Trinoid

Mathematica code[9]:
Trinoid[r_, ϕ_]:

$$= \{\frac{1}{18}(\frac{6r(r+(1+r^2)\mathrm{Cos}[\phi])}{1+r^2+r^4+2(r+r^3)\mathrm{Cos}[\phi]+2r^2\mathrm{Cos}[2\phi]} - 4\mathrm{Log}[1+r^2 - 2r\mathrm{Cos}[\phi]] + 2\mathrm{Log}[1+r^2+r^4+2(r+r^3)\mathrm{Cos}[\phi] + 2r^2\mathrm{Cos}[2\phi]]), \frac{1}{9}(-2\sqrt{3}\mathrm{ArcTanh}[\frac{\sqrt{3}r\mathrm{Sin}[\phi]}{1+r^2+r\mathrm{Cos}[\phi]}] - \frac{3r(1+r^4+2(r+r^3)\mathrm{Cos}[\phi])\mathrm{Sin}[\phi]}{1+r^6-2r^3\mathrm{Cos}[3\phi]}), \frac{1}{3+\frac{3-3r^6}{-2+2r^3\mathrm{Cos}[3\phi]}}\}$$

ParametricPlot3D[Trinoid[r, t], {t, 0,2Pi}, {r, 0,4}, RegionFunction → (#1^2 + #2^2 + #3^2 < 3^2&)]

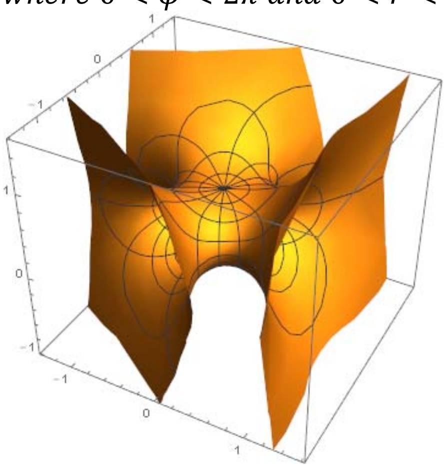
Parametrically, the trinoid can be represented by:

$$x = \frac{1}{18}\left(\frac{6r(r+(1+r^2)\cos\phi)}{1+r^2+r^4+2(r+r^3)\cos\phi+2r^2\cos 2\phi} - 4\log(1+r^2-2r\cos\phi) + 2\log(1+r^2+r^4+2(r+r^3)\cos\phi + 2r^2\cos 2\phi)\right)$$

$$y = \frac{1}{9}\left(-2\sqrt{3}\tanh^{-1}\left(\frac{\sqrt{3}r\sin\phi}{1+r^2+r\cos\phi}\right) - \frac{3r(1+r^4+2(r+r^3)\cos\phi\sin\phi)}{1+r^6-2r^3\cos 3\phi}\right)$$

$$z = \frac{1}{3+\frac{3-3r^6}{-2+2r^3\cos 3\phi}}$$

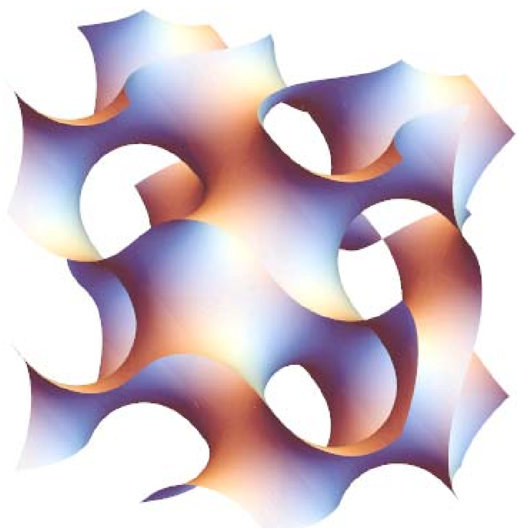
$where\ 0 < \phi < 2\pi\ and\ 0 < r < 4$



The trinoid is interesting surface. At first glance, the three directions of the surface appear to be identical; however, as can be seen in the image above, the rightmost split-off of the surface is larger than the other two. It cannot be seen from this angle, but the bottom side of the surface has a hole in the center.

Gyroid

The gyroid is considered an "infinitely connected triply periodic minimal surface"[13]. This description in layman's terms, it is a structure that has a symmetrical lattice pattern that extends in every direction indefinitely. It was discovered by Alan Schoen in 1970.
The Mathematica code for this surface is quite long and complex; the text is about 34 pages long. Due to this, I will not include the code here. However, Alan Schoen wrote a Mathematica notebook that contains the complete code which can be found at this link: http://mathworld.wolfram.com/notebooks/Surfaces/Gyroid.nb.[5]



This is certainly one of the most interesting objects that I used in this project. The inside of this object feels like a maze; the tunnels twist around creating identical passageways for literally eternity. Even with the code completely laid out in the about Mathematica notebook, it was still a challenge to create. The slightest mistake in the code can cost hours of debugging or redoing the entire code. Artistically, it was worth the patience to create.

Appendix

Costa Surface[3]

Costa surface code as described by Melko in "Visualizing Minimal Surfaces".
The package "MinimalSurfaces" can be found at http://www.mathematica-journal.com/data/uploads/2010/12/MinimalSurfaces.m

```
Get[MinimalSurfaces`]
boundary[δ_,ϵ_,ϕ_]:={
 Line[{{δ,0},{(1/2)-ϵ,0}}],
 Arc[{1/2,0},ϵ,{π,0},Clockwise],
 Line[{{(1/2)+ϵ,0},{1-δ,0}}],
 Arc[{1,0},δ,{π,π/2},Clockwise],

 Line[{{1,δ},{1,(1/2)-ϕ}}],
 Arc[{1,1/2},ϕ,{-π/2,π/2},Clockwise],Line[{{1,(1/2)+ϕ},{1,1-δ}}],
 Arc[{1,1},δ,{-π/2,-π},Clockwise],

 Line[{{1-δ,1},{(1/2)+ϵ,1}}],
 Arc[{1/2,1},ϵ,{0,-π},Clockwise],
 Line[{{(1/2)-ϵ,1},{δ,1}}],
 Arc[{0,1},δ,{0,-π/2},Clockwise],

 Line[{{0,1-δ},{0,(1/2)+ϕ}}],Arc[{0,1/2},ϕ,{π/2,-π/2},Clockwise],Line[{{0,(1/2)-
ϕ},{0,δ}}],
 Arc[{0,0},δ,{π/2,0},Clockwise]
 }
gridlines[q_,s_, n_]:=
 Join[
  Table[
   Line[{{q[[1]] +k s[[1]], q[[2]]+n[[2]][[1]] s[[2]]}, {q[[1]]+k s[[1]],q[[2]]+n[[2]][[2]]
s[[2]]}}],{k,n[[1]][[1]],n[[1]][[2]]}
   ],
  Table[Line[{{q[[1]]+n[[1]] [[1]]s[[1]],q[[2]]+k s[[2]]},{q[[1]]+n[[1]] [[2]]s[[1]],q[[2]]+k
s[[2]]}}],{k,n[[2]][[1]],n[[2]][[2]]}]
  ];
b=boundary[1/6,1/4,1/4];
v=CreateVertexData[b,MeshSize->{10,10}];
Show[Graphics[{{GrayLevel[0.8`],gridlines[{0,0},{0.1`,0.1`},{{0,10},{0,10}}]},{Green,PointSi
ze[0.02`],Point/@v[[1]]},{Red,PointSize[0.02`],Point/@Flatten[v[[2]],1]}}],Frame-
>True,PlotRange->{{-0.2`,1.2`},{-0.2`,1.2`}},AspectRatio->Automatic]
p=Triangulate[v,b];
id[u_,v_]:={u,v,0}
q={Apply[id,p[[1]],{1}],p[[2]],p[[3]]};
gr={RGBColor[3/4,3/4,0],Specularity[GrayLevel[1],5]};
```

```
qw={q[[1]],{{gr,q[[2]]}}};
s=CreatePolyhedron[qw];
ss=Flatten/@s;
Show[Graphics3D[ss],AspectRatio->Automatic,Axes->False,Boxed->False,ViewPoint-
>{0,0,9}]
b=boundary[1/8,1/24,1/24];
v=CreateVertexData[b,MeshSize->{50,50}];
edgeIds = {{1,11},{3,9},{5,15},{7,13}};
vertexIds = {{2,10},{4,16,12,8},{6,14}};
p=Triangulate[v,b,Identifications ->{edgeIds, vertexIds}];
Z[u_,v_,rho_]:=ParallelSurface[CostaSurface[x,y],{x,y},rho]/.{x->u,y->v,d->rho}
X1[u_,v_]:=Re[Z[u,v,0.025]]
X2[u_,v_]:=Re[Z[u,v,-0.025]]
q1={Apply[X1,p[[1]],{1}],p[[2]],p[[3]]};
q2={Apply[X2,p[[1]],{1}],p[[2]],p[[3]]};
zx =GlueComponents[q1,q2];
gr1={EdgeForm[],RGBColor[1/2,3/4,0],Specularity[GrayLevel[0.5`],6]};
gr2={EdgeForm[],RGBColor[3/4,1/2,0],Specularity[GrayLevel[1],9]};
gr3={EdgeForm[],RGBColor[1/2,0,1/2],Specularity[GrayLevel[1],9]};
qw = {zx[[1]],Join[{{gr1,zx[[2]][[1]]},{gr2,zx[[2]][[2]]}},Map[{gr3,#}&,zx[[2]][[3]]]]};
s=CreatePolyhedron[qw];
ss=Map[Flatten,s];
Graphics3D[ss,AspectRatio->Automatic,Axes->False,Boxed->False,Lighting-
>"Neutral",ViewPoint->{-2.417`,-1.984`,1.294`}]
```

Bibliography

[1]Champion, Brett. *Graphics & Visualization: Data Visualization Quick Start*. Champaign, IL: Wolfram Research, Inc., n.d. Wolfram Notebook.

[2]"How to | Create 3D Graphics." *Wolfram Language & System*. Wolfram, 2007. Web. 1 Apr. 2015. <http://reference.wolfram.com/language/howto/Create3DGraphics.html>.

Maya 2015. Computer software. Autodesk Maya. Vers. SP2. Autodesk, 2014. Web. 24 Apr. 2015.

[3]Melko, Michael. "Visualizing Minimal Surfaces." *The Mathematica Journal* 12 (2010): n. pag. Web.

[4]"RegionFunction." *Wolfram Language & System*. Wolfram, 2007. Web. 16 Apr. 2015. <http://reference.wolfram.com/language/ref/RegionFunction.html>.

[5]Schoen, Alan. "Gyroid." *Wolfram MathWorld*. Wolfram Research, Inc., 5 June 2001. Web. 26 Apr. 2015. <http://mathworld.wolfram.com/notebooks/Surfaces/Gyroid.nb>.

[6]"SphericalPlot3D." *Wolfram Language & System*. Wolfram, 2007. Web. 1 Apr. 2015. <http://reference.wolfram.com/language/ref/SphericalPlot3D.html>.

[7]Weisstein, Eric W. "Moebius Strip." *Wolfram MathWorld*. Wolfram Research, Inc., 10 June 2014. Web. 26 Apr. 2015. <http://mathworld.wolfram.com/notebooks/Surfaces/MoebiusStrip.nb>.

[8]Weisstein, Eric W. "Roman Surface." *Wolfram MathWorld*. Wolfram Research, Inc., 10 June 2014. Web. 12 Apr. 2015. <http://mathworld.wolfram.com/notebooks/Surfaces/RomanSurface.nb>.

[9]Weisstein, Eric W. "Trinoid." *Wolfram MathWorld*. Wolfram Research, Inc., 11 June 2014. Web. 22 Apr. 2015. <http://mathworld.wolfram.com/notebooks/Surfaces/Trinoid.nb>.

[10]Weisstein, Eric W. "Costa Minimal Surface." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/CostaMinimalSurface.html

[11]Weisstein, Eric W. "Great Dodecahedron." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/GreatDodecahedron.html

[12]Weisstein, Eric W. "Small Stellated Dodecahedron." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/SmallStellatedDodecahedron.html

[13]Wikipedia Contributors. "Gyroid." *Wikipedia, The Free Encyclopedia*. Wikimedia Foundation, 2 Dec. 2014. Web. 28 Apr. 2015.

Wolfram Mathematica 10. Wolfram. Vers. 10.1. Wolfram Research, Inc., 2015. Web. 24 Apr. 2015.