# An Introduction to *Mathematica*™

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## What Is *Mathematica*™?

*Mathematica* is a general system for doing mathematical computation and can be used in many different ways. This section gives a brief survey of some of them.

One way to use *Mathematica* is as a "calculator". You type in a calculation, and *Mathematica* immediately tries to do it. *Mathematica* does far more than a traditional electronic calculator would: as well as numerical operations, *Mathematica* can do symbolic and algebraic operations, and can generate graphics.

When you use *Mathematica* like a calculator, you are drawing on its built-in mathematical capabilities. But *Mathamatica* is also a language, in which you can make your own definitions. You can write programs in *Mathematica*, working not only with numbers, but also with symbolic expressions and graphical objects.

You can use *Mathematica* as a language for representing mathematical knowledge. You can take mathematical relations from handbooks and textbooks and enter them almost directly into *Mathematica*. The basic approach is to give a sequence of "transformation rules" which specify how *Mathematica* would treat expressions with different forms.

As well as being a language, versions of *Mathematica* on many computers also serve as complete environments for computation. You can, for example, create "notebooks", which consist of ordinary text, mixed with graphics and "live" *Mathematica* input.

*Mathematica* is set up to fit in with other standard programs. You can use *Mathematica* to prepare input, or analyse output, from external programs. What makes this possible is that *Mathematica* supports many standards, such as UNIX pipes and POSTSCRIPT, which are common to many modern programs.

## *Mathematica* Is a System for Doing Calculations

This section gives examples of the three main types of calculations that *Mathematica* can do: numerical, symbolic, and graphical. Each example consists of a short "dialog" with *Mathematica*. The lines labelled IN[$n$] are what you would type in; the ones labelled Out[$n$] are what *Mathematica* would type back.

## Numerical Calculations

At the simplest level, *Mathematica* will do numerical calculations, just like a standard electronic calculator (Figure 1). *Mathematical* can, however, go far beyond a standard calculator. It can, for example, calculate with numbers of arbitrary precision. It can evaluate a wide range of mathematical functions, including all standard special functions of mathematical physics.

*Mathematica* works not only with single numbers, but also with more complicated structures. You can use *Mathematica*, for example, to do operations on matrices. The standard operations of numerical linear algebra are built into *Mathematica*. You can also use *Mathematica* to find Fourier transforms, least-squares fits, and so on. *Mathematica* can do numerical operations on functions, such as numerical integration and numerical minimization.

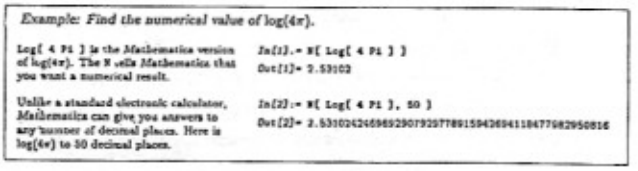


*Figure 1.* Numerical Value of $\log(4\pi)$

## Symbolic Calculations

The ability to deal with symbolic formulae, as well as with numbers, is one of the most powerful features of *Mathematica* (Figure 2). This is what makes it possible to do algebra and calculus with *Mathematica*.

Example: *Find a formula for the integral* $\int x^4/(x^2-1)\,dx$.

Here is the expression $x^4/(x^2-1)$ in Mathematica.

```
In[3]:= x^4 / (x^2 - 1)
                 4
                x
Out[3]= ----------
                2
        -1 + x
```

This tells *Mathematica* to integrate the previous expression. *Mathematica* finds an explicit formula for the integral.

```
In[4]:= Integrate[%, x]
                 3
                x          Log[-1 + x]   Log[1 + x]
Out[4]= x + ---- + ---------- - ----------
            3          2           2
```

*Figure 2.* A formula for $\int x^4/(x^2-1)\,dx$

*Mathematica* does many kinds of algebraic computations. It can expand, factor and simplify polynomials and rational expressions. It can solve polynomial equations, or systems of such equations. It can get algebraic results for many kinds of matrix operations.

*Mathematica* can also do calculus. It can evaluate derivatives and integrals symbolically. It can derive power series approximations.

## Graphics

*Mathematica* does both two and three-dimensional graphics (Figure 3). You can plot functions or lists of data. The three-dimensinal graphics that *Mathematica* produces can be quite realistic: they can, for example, include shading, color, and lighting effects.

You can use *Mathematica* to make two- and three-dimensional pictures. You supply a symbolic representation of the objects, say polygons, in the picture, and *Mathematica* will produce a graphical rendering of them. *Mathematica* also has powerful animation capabilities.

Example: *Plot the function* $\sin(xy)$ *for* $x$ *and* $y$ *between* 0 *and* $\pi$.

This generates a three-dimensional plot of $\sin(xy)$ as a function of $x$ and $y$. There are many options for controlling graphics in *Mathematica*.

```
In[5]:= Plot3D[ Sin[x y], {x, 0, Pi}, {y, 0, Pi} ]
```

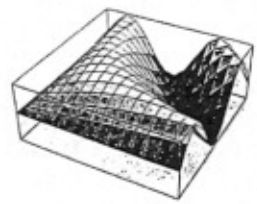

*Figure 3.* The Graph of $\sin(xy)$ for $0 \le x, y \le \pi$

## *Mathematica* is a Programming Language

You can write programs in *Mathematica*, must as you would in a language like C (Figure 4). *Mathematica* is an interpreter: you can run your programs as soon as you have typed them in.

This defines a function f which makes a table of the first n prime numbers.

```
In[6]:= f[n_] := Table[Prime[i], {i, n}]
```

Following the definition of f immediately, here is a table of the first 10 prime numbers.

```
In[7]:= f[10]
Out[7]= {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}
```

*Figure 4.* A Program in *Mathematica*

## *Mathematica* Is a System for Representing Mathematical Knowledge

*Mathematica* gives you a way to represent, and use, the kind of information that appears in tables of mathematical formulae.

Fundamental to much of *Mathematica* is the notion of "transformation rules", which specify

how expressions of one form whould be transformed into expressions of another form. Transformation rules are a very natural way to represent many kinds of mathematical relations.

Some relations that you could use to define your own logarithm function in *Mathematica* are given in Figure 5.

| *Mathematical form* | *Mathematica form* |
|---|---|
| $\log(1) = 0$ | `log[1] = 0` |
| $\log(e) = 1$ | `log[E] = 1` |
| $\log(xy) = \log(x) + \log(y)$ | `log[x_ y_] := log[x] + log[y]` |
| $\log(x^n) = n \log(x)$ | `log[x_^n_] := n log[x]` |

*Figure 5.* Relations to Define the Logarithm Function in *Mathematica*

### *Mathematica* Is a Computing Environment

*Mathematica* gives you an environment in which to set up, run and document your calculations and programs.

There are two pieces to *Mathematica* on most computers: the "kernel", which does computations, and the "front end", which deals with user interaction. (These pieces do not necessarily have to be running on the same computer – you can use *Mathematica* over a network.)

The kernel works in the same way on all computers that run *Mathematica*. The front end, however is set up to take advantage of the different capabilities of different kinds of computers.

On the Macintosh, for example, the *Mathematica* front end lets you use graphical tools to manipulate your input and output, and to insert extra text.

The *Mathematica* front end on the Macintosh takes advantage of the Macintosh's graphical capabilities. Many *Mathematica* front ends support "notebooks", which contain a mixture of text, graphics and *Mathematica* definitions (Figure 6).

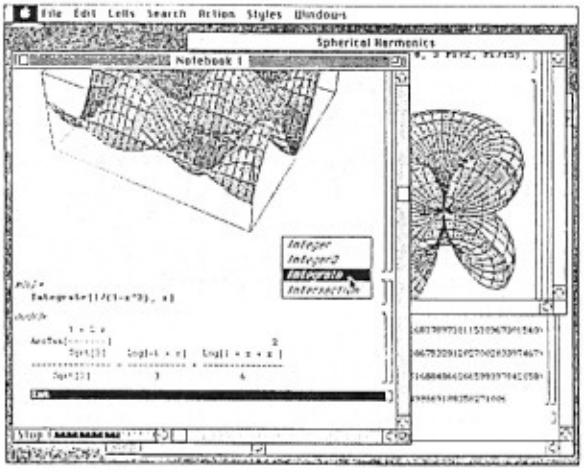


*Figure 6.* A *Mathematica* Front End Example

There is a growing library of *Mathematica* notebooks which serve as "live textbooks" on a variety of different topics. You can read the text in a notebook to learn about a topic, and then use the *Mathematica* definitions in the notebook to do calculations.

A part of a *Mathematica* notebook about orthogonal polynomials is given in Figure 7. The material in the notebook is arranged in a hierarchical fashion, so you do not need to see details unless you want to.

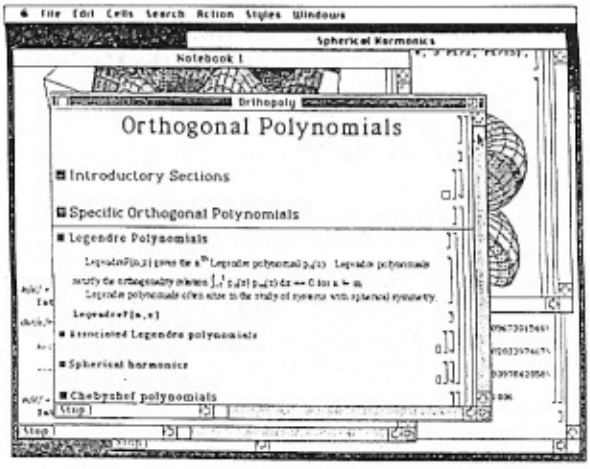


*Figure 7.* Part of a *Mathematica* Notebook about Orthogonal Polynomials

### *Mathematica* Is a Tool in the Standard Computing Environment

*Mathematica* interfaces to many elements of standard computing envioronments. Under UNIX, *Mathematica* can communicate with external programs through pipes.

You can use output from *Mathematica* as input to many kinds of programs. *Mathematica* can write out expressions to use as input for C or FORTRAN programs. It can also produce input for TEX, which you can use to typeset papers and presentations.

*Mathematica* produces graphics using POSTSCRIPT. You can render the POSTSCRIPT on many different kinds of devices. You can also read it into other programs.

### The *Mathematica* Book

The main documentation for *Mathematica* is the book: "*Mathematica*™: A System for Doing Mathematics by Computer", by Stephen Wolfram, published by Addison-Wesley Publishing Company (June 1988). The book is available at most bookstores.

- Softcover version (ISBN 0-=201-19330-2): $29.95
- Hardcover version (ISBN 0-201-19334-5): $44.25

To order the book directly, call Addison-Wesley at 1-800-447-2226.

### Buying *Mathematica*

**Mathematica for the Macintosh.** There are two versions of *Mathematica* for the Macintosh:

- **Standard Version** for Macintosh Plus, SE and II: $495
- **Macintosh II Version** for Macintosh II only: $795

The only difference between the two versions is speed. The Standard version will run on a Macintosh II, and supports color, but does not make use of the Macintosh II 68881 numeric coporcessor, and is consequently several times slower than the Macintosh II version for operations such as graphic rendering.

**Note:** Both versions of *Mathematica* for the Macintosh require 2.5 megabytes of RAM. We recommend 4 megabytes or more.

To buy *Mathematica* for the Macintosh, visit your local software dealer, or call Wolfram Research at 217-398-0700 (for orders only: 1-800-441-MATH).

***Mathematica* Front Ends.** *Mathematica* consists of two pieces: a kernel that does computations, and a front end, which handles interaction with the user. You can run these two pieces on separate computers. Thus, for example, you can have the front end on a Macintosh, with the kernel running on a remote computer server.

The Macintosh from end for *Mathematica* will be made available as a separate product from Wolfram Research. It will run on Macintoshes with only one megabyte of memory. Call Wolfram Research at 217-398-0700 for information.

**University Arrangements.** Special arrangements are available for universities which distribute a copy of *Mathematica* with every Macintosh II (or every Macintosh) that they resell. University administrators interested in this program should contact Wolfram Research at 217-398-0700 for further information.

***Mathematica* for 386-Based MS-DOS Systems.** There are three versions of *Mathematica* for 386-based MS-DOS systems:

- **386 Version** (no numeric coprocessor required): $695
- **386/7 Version** (287 or 387 numeric coprocessor required): $995
- 386/Weitek Version (Weitek numeric coprocessor required): $1295

MS-DOS 386 *Mathematica* requires 640k of RAM, and at least 1 megabyte of extended memory. It also requires 5 megabytes of hard disk space.

MS-DOS 386 *Mathematica* supports CGA, EGA, VGA, Hercules, 8514 and other graphics standards. It supports PostScript, LaserJet, Epson FX and Toshiba P3 compatible printers.

MS-DOS 386 *Mathematica* includes DOS-style editing capabilities, but does not support *Mathematica* Notebooks.

To buy MS-DOS 386 *Mathematica*, visit your local software dealer, or call Wolfram Research at 217-398-0700 (for orders only: 1-800-441-MATH).

**Other Systems.** Versions of *Mathematica* are available for the following systems:

- **Ardent** - for Tital systems
- **IBM** - for RT/AIX systems
- **MIPS** - for M/120 systems
- **NeXT** - bundled as part of standard software on all NeXT Computers
- **Silicon Graphics** - for Iris 4D systems

- **Sony** - for NEWS systems
- **Stellar** - for GS-1000 systems
- **Sun** - for Sun-3, Sun-4 and Sun-386i systems

These versions are available directly from the hardware manufacturers. Contact your local sales representative for information.
For information on possible other versions of *Mathematica*, contact Wolfram Research at 217-398-0700.