

**CLASS NOTES ON**

**ELECTRICAL MEASUREMENTS & INSTRUMENTATION**

**FOR**

**5TH & 6TH SEMESTER OF**

**ELECTRICAL ENGINEERING & EEE (B.TECH PROGRAMME)**



**DEPARTMENT OF ELECTRICAL ENGINEERING**

**VEER SURENDRA SAI UNIVERSITY OF TECHNOLOGY**

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# SYLLABUS

## ELECTRICAL MEASUREMENTS & INSTRUMENTATION (3-1-0)

### MODULE-I (10 HOURS)

Measuring Instruments: Classification, Absolute and secondary instruments, indicating instruments, control, balancing and damping, constructional details, characteristics, errors in measurement, Ammeters, voltmeters: (DC/AC) PMMC, MI, Electro-dynamometer type

Wattmeters: Electro-dynamometer type, induction type, single phase and three phase wattmeter, compensation, Energymeters: AC. Induction type single phase and three phase energy meter, compensation, creep, error, testing, Frequency Meters: Vibrating reed type, electrical resonance type

### MODULE-II (10 HOURS)

Instrument Transformers: Potential and current transformers, ratio and phase angle errors, phasor diagram, methods of minimizing errors; testing and applications.

Galvanometers: General principle and performance equations of D'Arsonval Galvanometers, Vibration Galvanometer and Ballistic Galvanometer.

Potentiometers: DC Potentiometer, Crompton potentiometer, construction, standardization, application. AC Potentiometer, Drysdale polar potentiometer; standardization, application.

### MODULE-III (10 HOURS)

DC/AC Bridges :General equations for bridge balance, measurement of self inductance by Maxwell's bridge (with variable inductance & variable capacitance), Hay's bridge, Owen's bridge, measurement of capacitance by Schering bridge, errors, Wagner's earthing device, Kelvin's double bridge.

Transducer: Strain Gauges, Thermistors, Thermocouples, Linear Variable Differential Transformer (LVDT), Capacitive Transducers, Piezo-Electric transducers, Optical Transducer, Torque meters, inductive torque transducers, electric tachometers, photo-electric tachometers, Hall Effect Transducer

### MODULE-IV (10 HOURS)

CRO: Block diagram, Sweep generation, vertical amplifiers, use of CRG in measurement of frequency, phase, Amplitude and rise time of a pulse.

Digital Multi-meter: Block diagram, principle of operation, Accuracy of measurement, Electronic Voltmeter: Transistor Voltmeter, Block diagram, principle of operation, various types of electronic voltmeter, Digital Frequency meter: Block diagram, principle of operation

### TEXT BOOKS

- [1]. A Course in Elec. & Electronics Measurements & Instrumentation: A K. Sawhney
- [2]. Modern Electronic Instrumentation and Measurement Techniques: Helfrick & Cooper
- [3]. Electrical Measurement and Measuring Instruments - Golding & Waddis

### Model Question Paper: Set-1

Full Marks: 70

Time: 3 hours

Answer any **six** questions including question **No. 1** which is compulsory.

The figures in the right-hand margin indicate marks.

(Answer any six questions including Q.No. 1)

[2X10]

1. Answer the following questions:
  - (a) Differentiate between the spring control and gravity control.
  - (b) Why an ammeter should have a low resistance value.
  - (c) What are the precautions taken while using a DC voltmeter and DC ammeter.
  - (d) What is the major cause of creeping error in an energy meter
  - (e) What are the errors occurs in instrument transformers.
  - (f) Differentiate the principle of dc potentiometer and ac potentiometer.
  - (g) What are the sources of errors in AC bridge measurement.
  - (h) Differentiate between dual trace and dual beam CRO.
  - (i) What are active and passive transducers? Give examples.
  - (j) What is piezoelectric effect.
  
2. (a) With a neat diagram explain in detail the construction of PMMC instrument.  
(b) What are the shunts and multiplier? Derive the expression for both, with reference to meters used in electrical circuits. [5+5]
  
3. (a) Discuss with block diagram, the principle of operation of single phase energy meter.  
(b) An energy meter is designed to make 100 revolutions of the disc for one unit of energy. Calculate the number of revolutions made by it, when connected to a load carrying 40 A at 230V and 0.4 p.f. for 1 hour. If it actually makes 360 revolutions, find the percentage error. [5+5]
  
4. (a) Derive expression for actual transformation ratio, ratio error and phasor angle error of a P.T.  
(b) A current transformer with bar primary has 300 turns in its secondary winding. The resistance and reactance of the secondary circuit are  $1.5\Omega$  and  $1.0\Omega$  respectively, including the transformer winding. With 5A flowing in the secondary winding, the magnetizing mmf is 100AT and the core loss is 1.2 W. Determine the ratio and phase angle errors. [5+5]
  
5. (a) Derive the equation of balance for Anderson bridge and also draw the phasor diagram.  
(b) An AC bridge is balanced at 2KHz with the following components in each arm:  
Arm AB= $10K\Omega$ , Arm BC= $100\mu F$  in series with  $100K\Omega$ , Arm AD= $50K\Omega$   
Find the unknown impedance  $R \pm jX$  in the arm DC, if the detector is between BD.
  
6. (a) What is transducer? Briefly explain the procedure for selecting a transducer.  
(b) Derive an expression for gauge factor in terms of Poission's ration. [5+5]
  
7. (a) With a block diagram, explain the working of CRO  
(b) With a block diagram, explain in detail the digital frequency meter [5+5]
  
8. Write short notes on
  - (a) Kelvin's double bridge
  - (b) D- Arsonaval galvanometer [5+5]

### Model Question Paper: Set-2

Full Marks: 70

Time: 3 hours

Answer any **six** questions including question **No. 1** which is compulsory.

The figures in the right-hand margin indicate marks.

(Answer any six questions including Q.No. 1)

[2X10]

Q.1 Answer the following questions:

- (a) Why is damping required for an electromechanical measuring instrument?
- (b) Why is scale of MI instrument calibrated non-linearly?
- (c) What is the difference between analog and digital frequency meter?
- (d) Which instrument can be used to measure non-sinusoidal voltage?
- (e) What is the major cause of creeping error in an energy meter?
- (f) What do you mean by active and passive transducer? Give suitable examples.
- (g) Draw a suitable AC bridge used for measurement of frequency.
- (h) What are the steps to be taken for minimizing errors in PT?
- (i) Discuss briefly the role of ordinary galvanometer?
- (j) What is the function of time base generator in CRO?

Q.2 (a) Derive the torque equation of a moving iron instrument? [5]

(b) Sketch and explain the working of moving-coil instrument. [5]

Q.3 (a) Discuss a method for measurement of low resistance. [5]

(b) Explain the operation of a Wagner's earthing device. [5]

Q.4 Derive the errors of CT and PT, and discuss its preventives. [10]

Q.5 (a) Discuss about a ac bridge used for measurement of capacitance [5]

(b) Discuss about a galvanometer which is used for measurement of frequency. [5]

Q.6 (a) How is the voltmeter calibrated with DC potentiometer ? [5]

(b) What is the use of LVDT? Discuss its basic principle of operation. [5]

Q.7 (a) How are the frequency and phase measured in CRO. [5]

(b) Draw the block diagram of an electronic voltmeter and explain its operation. [5]

8. Write short notes on any two: [5X2]

- (a) Owen's bridge
- (b) Digital frequency meter
- (c) Hall-effect transducer

### Model Question Paper: Set-3

Full Marks: 70

Time: 3 hours

Answer any **six** questions including question **No. 1** which is compulsory.

The figures in the right-hand margin indicate marks.

(Answer any six questions including Q.No. 1)

[2X10]

1. Answer the following questions:
  - (k) Why scales of the gravity control instruments are not uniform but are crowded?
  - (l) Why eddy current damping cannot be used for moving iron instrument?
  - (m) What is meant by Phantom Load?
  - (n) Why do we use a multiplier with a voltmeter?
  - (o) What is the major cause of creeping error in an energy meter?
  - (p) Why is dynamometer type instrument chiefly used as a wattmeter?
  - (q) Define gauge factor of a strain gauge. How is it related to Poisson's ratio  $\mu$ ?
  - (r) Why the secondary of a CT is never left open circuited?
  - (s) Why there are two conditions of balance in ac bridges, whereas there is only one for dc bridges?
  - (t) How to prevent the loading of a circuit under test when a CRO is used?
  
2. (a) Draw the possible methods of connecting the pressure coil of a wattmeter and compare the errors. Explain the meaning of 'compensating winding' in a wattmeter and show how they help to reduce the error. [5]  
(b) Sketch and explain the working of moving-coil instrument. [5]
  
3. (a) What are the different tests conducted on a single phase energy meter? [5]  
(b) The meter constant of 230V, 10A energy meter is 1700. The meter is tested under half load and rated voltage at unity p.f. The meter is found to make 80 revolutions in 138 sec. Find % error. [5]
  
4. Differentiate between a C.T. and P.T. Discuss the theory of a P.T. with phasor diagrams. Derive expression for actual transformation ratio, ratio error and phasor angle error of a P.T. [10]
  
5. (a) Describe how an AC potentiometer can be used for the calibration of wattmeter? [5]  
(b) Explain how LVDT can be used for measurement of displacement. [5]
  
6. (a) Derive the equation of balance of a Schering Bridge. Draw the phasor diagram under null conditions and explain how loss angle of capacitor can be calculated. [5]  
(b) Describe the general working principle of a D'Arsonval Galvanometer. [5]
  
7. (a) Explain the use of CRG in the measurement of frequency. [5]  
(b) Draw the block diagram of an electronic voltmeter and explain its operation. [5]
  
8. Write short notes on: [5X2]
  - (a) Thermo couple
  - (b) Piezo-Electric Transducers.

### Model Question Paper: Set-4

Full Marks: 70

Time: 3 hours

Answer any **six** questions including question **No. 1** which is compulsory.

The figures in the right-hand margin indicate marks.

(Answer any six questions including Q.No. 1)

1. Answer the following questions: [2 x 10]
- (a) What are the parameters on which the critical damping of galvanometer depends? Why critical damping is important?
  - (b) The current flowing through a resistance of  $10.281 \text{ k}\Omega$  is measured as  $1.217 \text{ mA}$ . Calculate the voltage drop across the resistor to the appropriate number of significant errors.
  - (c) What are the main advantages and disadvantages of PMMC instruments?
  - (d) Why an electro-dynamometer type instrument is called a “Universal Instrument”?
  - (e) Write the working principle of resonance type frequency meter.
  - (f) What are the advantages of electronic voltmeter over electro – mechanical type voltmeter?
  - (g) Why Maxwell Bridge is limited to the measurement of medium – Q coils?
  - (h) What will happen in a current transformer, if the secondary circuit is accidentally opened while the primary winding is energized?
  - (i) Suggest a transducer for the measurement of displacement in the order of one – tenth of millimeter and write the basic principle of measurement.
  - (j) What is the function of delay line in oscilloscope?

Q. 2. [5 + 5]

- a) The coil of a measuring instrument has a resistance of  $1 \Omega$  and the instrument has a full scale deflection of  $250 \text{ V}$  when a resistance of  $4999 \Omega$  is connected with it. Find the current range of the instrument when used as an ammeter with the coil connected across a shunt of  $(1/499) \Omega$  and the value of the shunt resistance for the instrument to give a full scale deflection of  $50 \text{ A}$ .
- b) Distinguish between gross error, systematic error and random error with examples. What are the methods for their elimination/reduction?

Q. 3. [5 + 5]

- a) Draw the circuit diagram of Schering Bridge. Derive the conditions for balancing the bridge and draw the phasor diagram during balanced condition.
- b) Describe the theory and method of measurement of low resistance using Kelvin’s double Bridge. How the effect of thermo – electric emf is taken into account during measurement?

Q. 4.

[10]

A single range student type potentiometer has 20 step dial switch where each step represents 0.1 V. The dial resistors are 20  $\Omega$ . The slide wire of the potentiometer is circular and has 10 turns and a resistance of 10  $\Omega$  each. The slide wire has 200 divisions and interpolation can be done to one fourth of a division. The working battery has a voltage of 10 V and negligible internal resistance. Draw the circuit diagram and calculate

- i) The measuring range of potentiometer
- ii) The resolution
- iii) Working current
- iv) Resistance of series rheostat

Q. 5.

[5 + 5]

- a) What is the requirement of “Screening of bridge components”? Draw the circuit diagram of Wagner’s earthing device and explain its operation.
- b) Define the sensitivity of a strain gauge. Draw the circuit for measurement of strain and derive the expression of output voltage in terms of strain.

Q. 6.

[5 + 5]

- a) Derive the torque equation of moving iron instrument and comment on the shape of the scale.
- b) Prove that for electro-dynamometer type wattmeter  
true power =  $\{\cos \Phi / [\cos \Phi \cos (\Phi - \beta)]\}$  x actual wattmeter reading  
Where  $\cos \Phi$  = power factor of the circuit  
 $\beta = \tan^{-1} (\omega L/R)$  where L and R are the inductance and resistance of the pressure coil of the circuit.

Q. 7.

[5 + 5]

- a) Describe the construction and principle of operation of D’ Arsonval type Galvanometer.
- b) Discuss the major sources of errors in current transformer. What are the means to reduce errors in CT? Explain design and constructional feature to reduce the error.

Q. 8.

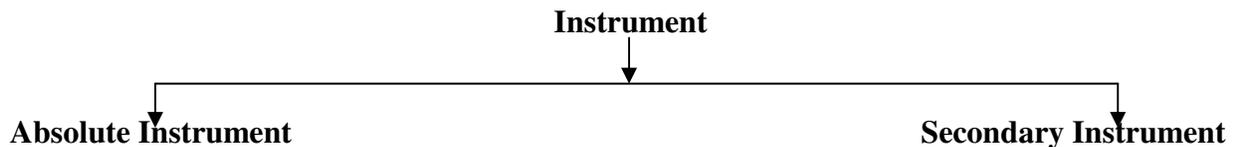
[6 + 4]

- a) Describe the measurement of frequency, phase angle and time delay using oscilloscope with suitable diagrams and mathematical expressions.
- b) With block diagram explain the operation of “Ramp type’ digital voltmeter.

## MEASURING INSTRUMENTS

### 1.1 Definition of instruments

An instrument is a device in which we can determine the magnitude or value of the quantity to be measured. The measuring quantity can be voltage, current, power and energy etc. Generally instruments are classified in to two categories.



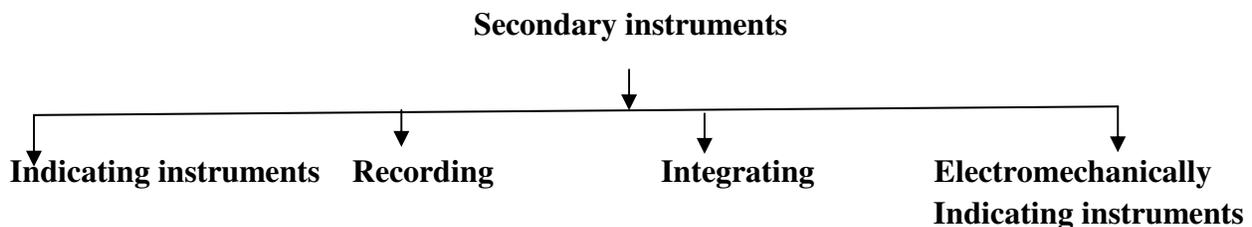
### 1.2 Absolute instrument

An absolute instrument determines the magnitude of the quantity to be measured in terms of the instrument parameter. This instrument is really used, because each time the value of the measuring quantities varies. So we have to calculate the magnitude of the measuring quantity, analytically which is time consuming. These types of instruments are suitable for laboratory use. Example: Tangent galvanometer.

### 1.3 Secondary instrument

This instrument determines the value of the quantity to be measured directly. Generally these instruments are calibrated by comparing with another standard secondary instrument.

Examples of such instruments are voltmeter, ammeter and wattmeter etc. Practically secondary instruments are suitable for measurement.



### 1.3.1 Indicating instrument

This instrument uses a dial and pointer to determine the value of measuring quantity. The pointer indication gives the magnitude of measuring quantity.

### 1.3.2 Recording instrument

This type of instruments records the magnitude of the quantity to be measured continuously over a specified period of time.

### 1.3.3 Integrating instrument

This type of instrument gives the total amount of the quantity to be measured over a specified period of time.

### 1.3.4 Electromechanical indicating instrument

For satisfactory operation electromechanical indicating instrument, three forces are necessary.

They are

- (a) Deflecting force
- (b) Controlling force
- (c) Damping force

### 1.4 Deflecting force

When there is no input signal to the instrument, the pointer will be at its zero position. To deflect the pointer from its zero position, a force is necessary which is known as deflecting force. A system which produces the deflecting force is known as a deflecting system. Generally a deflecting system converts an electrical signal to a mechanical force.

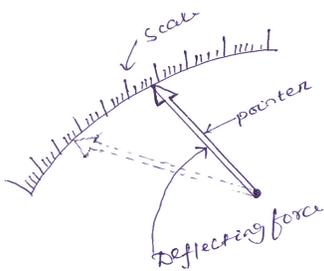


Fig. 1.1 Pointer scale

### 1.4.1 Magnitude effect

When a current passes through the coil (Fig.1.2), it produces a imaginary bar magnet. When a soft-iron piece is brought near this coil it is magnetized. Depending upon the current direction the poles are produced in such a way that there will be a force of attraction between the coil and the soft iron piece. This principle is used in moving iron attraction type instrument.

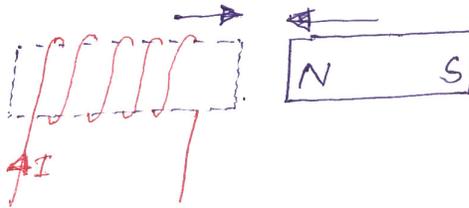


Fig. 1.2

If two soft iron pieces are placed near a current carrying coil there will be a force of repulsion between the two soft iron pieces. This principle is utilized in the moving iron repulsion type instrument.

### 1.4.2 Force between a permanent magnet and a current carrying coil

When a current carrying coil is placed under the influence of magnetic field produced by a permanent magnet and a force is produced between them. This principle is utilized in the moving coil type instrument.

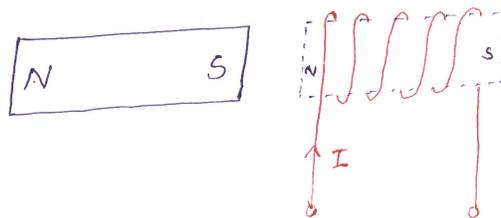


Fig. 1.3

### 1.4.3 Force between two current carrying coil

When two current carrying coils are placed closer to each other there will be a force of repulsion between them. If one coil is movable and other is fixed, the movable coil will move away from the fixed one. This principle is utilized in electro-dynamometer type instrument.

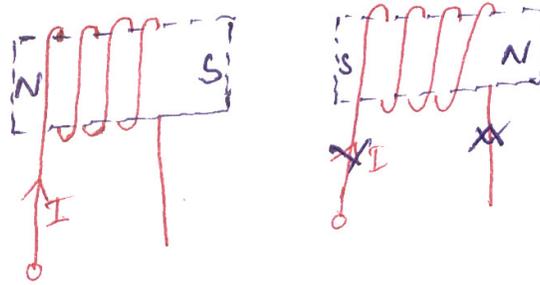


Fig. 1.4

## 1.5 Controlling force

To make the measurement indicated by the pointer definite (constant) a force is necessary which will be acting in the opposite direction to the deflecting force. This force is known as controlling force. A system which produces this force is known as a controlled system. When the external signal to be measured by the instrument is removed, the pointer should return back to the zero position. This is possibly due to the controlling force and the pointer will be indicating a steady value when the deflecting torque is equal to controlling torque.

$$T_d = T_c \quad (1.1)$$

### 1.5.1 Spring control

Two springs are attached on either end of spindle (Fig. 1.5). The spindle is placed in jewelled bearing, so that the frictional force between the pivot and spindle will be minimum. Two springs are provided in opposite direction to compensate the temperature error. The spring is made of phosphorous bronze.

When a current is supply, the pointer deflects due to rotation of the spindle. While spindle is rotate, the spring attached with the spindle will oppose the movements of the pointer. The torque produced by the spring is directly proportional to the pointer deflection  $\theta$ .

$$T_C \propto \theta \quad (1.2)$$

The deflecting torque produced  $T_d$  proportional to 'I'. When  $T_C = T_d$ , the pointer will come to a steady position. Therefore

$$\theta \propto I \quad (1.3)$$

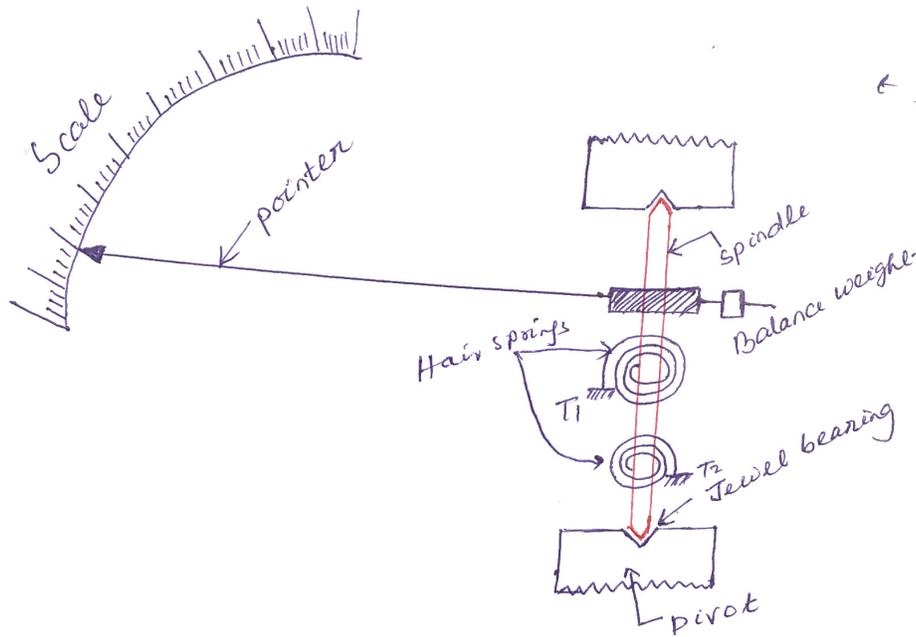


Fig. 1.5

Since,  $\theta$  and  $I$  are directly proportional to the scale of such instrument which uses spring controlled is uniform.

### 1.6 Damping force

The deflection torque and controlling torque produced by systems are electro mechanical. Due to inertia produced by this system, the pointer oscillates about its final steady position before coming to rest. The time required to take the measurement is more. To damp out the oscillation quickly, a damping force is necessary. This force is produced by different systems.

- (a) Air friction damping
- (b) Fluid friction damping
- (c) Eddy current damping

#### 1.6.1 Air friction damping

The piston is mechanically connected to a spindle through the connecting rod (Fig. 1.6). The pointer is fixed to the spindle and moves over a calibrated dial. When the pointer oscillates in clockwise direction, the piston goes inside and the cylinder gets compressed. The air pushes the piston upwards and the pointer tends to move in anticlockwise direction.

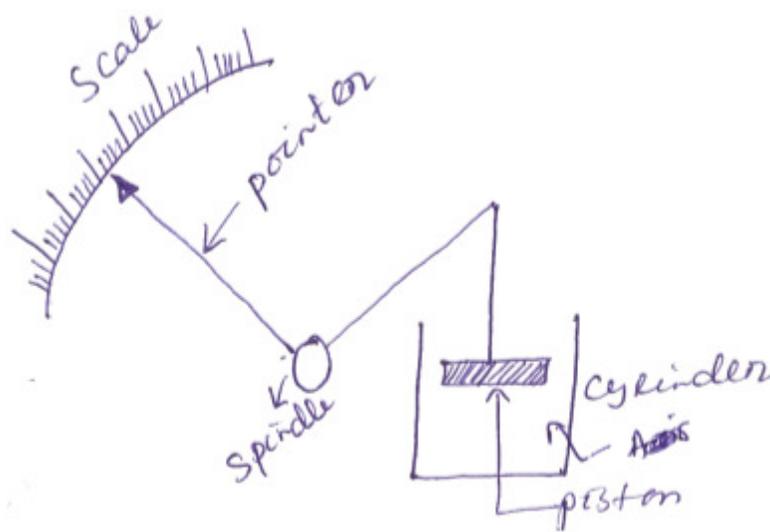


Fig. 1.6

If the pointer oscillates in anticlockwise direction the piston moves away and the pressure of the air inside cylinder gets reduced. The external pressure is more than that of the internal pressure. Therefore the piston moves down wards. The pointer tends to move in clock wise direction.

### 1.6.2 Eddy current damping

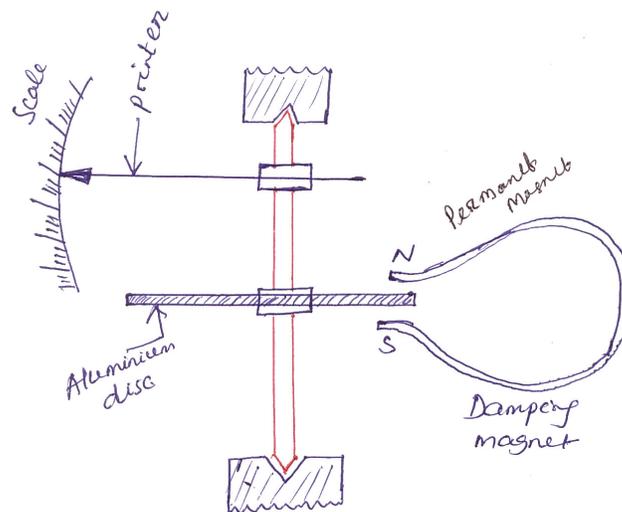


Fig. 1.6 Disc type

An aluminum circular disc is fixed to the spindle (Fig. 1.6). This disc is made to move in the magnetic field produced by a permanent magnet.

When the disc oscillates it cuts the magnetic flux produced by damping magnet. An emf is induced in the circular disc by Faraday's law. Eddy currents are established in the disc since it has several closed paths. By Lenz's law, the current carrying disc produces a force in a direction opposite to oscillating force. The damping force can be varied by varying the projection of the magnet over the circular disc.

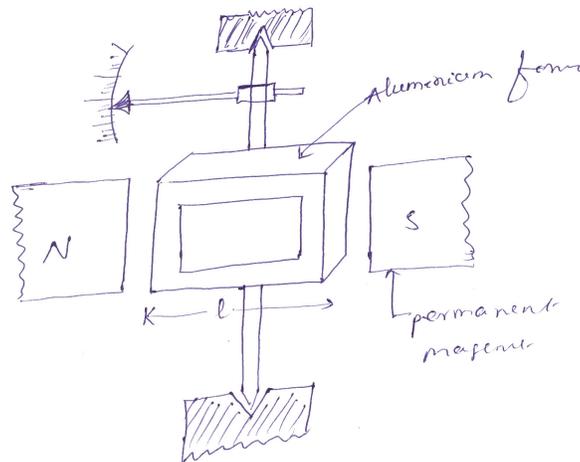


Fig. 1.6 Rectangular type

### 1.7 Permanent Magnet Moving Coil (PMMC) instrument

One of the most accurate type of instrument used for D.C. measurements is PMMC instrument.

**Construction:** A permanent magnet is used in this type instrument. Aluminum former is provided in the cylindrical in between two poles of the permanent magnet (Fig. 1.7). Coils are wound on the aluminum former which is connected with the spindle. This spindle is supported with jeweled bearing. Two springs are attached on either end of the spindle. The terminals of the moving coils are connected to the spring. Therefore the current flows through spring 1, moving coil and spring 2.

**Damping:** Eddy current damping is used. This is produced by aluminum former.

**Control:** Spring control is used.

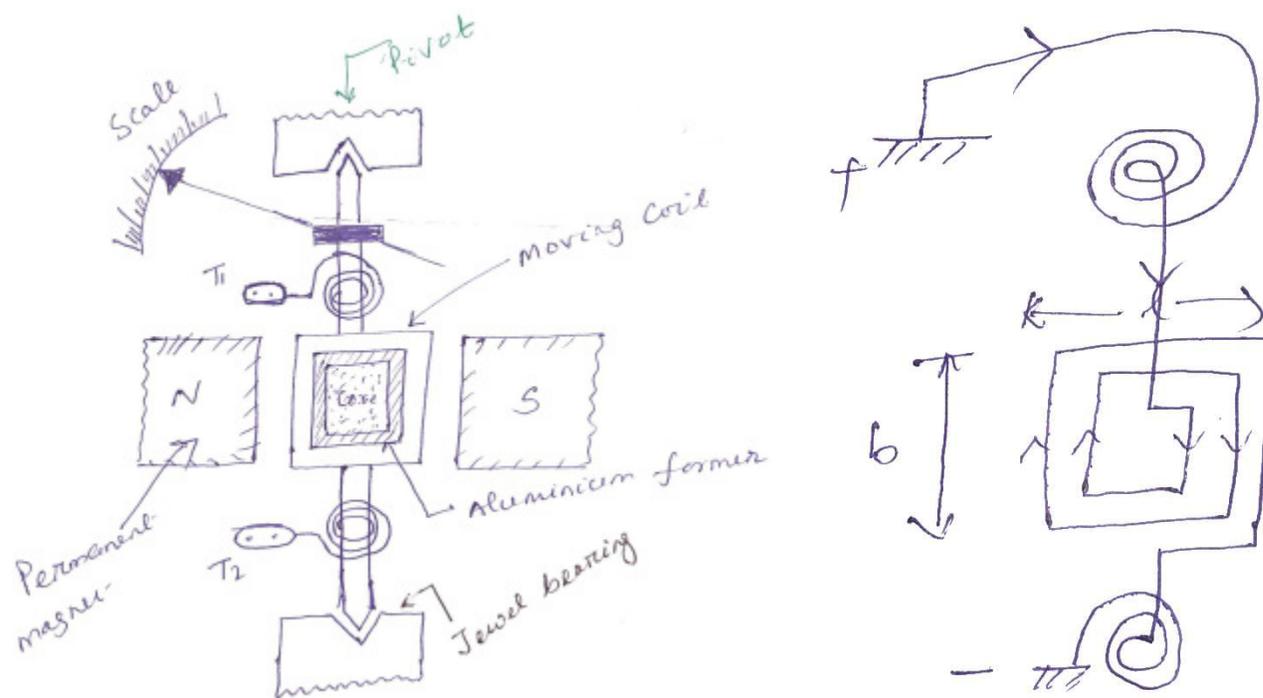


Fig. 1.7

### Principle of operation

When D.C. supply is given to the moving coil, D.C. current flows through it. When the current carrying coil is kept in the magnetic field, it experiences a force. This force produces a torque and the former rotates. The pointer is attached with the spindle. When the former rotates, the pointer moves over the calibrated scale. When the polarity is reversed a torque is produced in the opposite direction. The mechanical stopper does not allow the deflection in the opposite direction. Therefore the polarity should be maintained with PMMC instrument.

If A.C. is supplied, a reversing torque is produced. This cannot produce a continuous deflection. Therefore this instrument cannot be used in A.C.

### Torque developed by PMMC

Let  $T_d$  = deflecting torque  
 $T_C$  = controlling torque  
 $\theta$  = angle of deflection  
 $K$  = spring constant  
 $b$  = width of the coil

$l$ =height of the coil or length of coil

$N$ =No. of turns

$I$ =current

$B$ =Flux density

$A$ =area of the coil

The force produced in the coil is given by

$$F = BIL \sin \theta \quad (1.4)$$

When  $\theta = 90^\circ$

$$\text{For } N \text{ turns, } F = NBIL \quad (1.5)$$

$$\text{Torque produced } T_d = F \times \perp_r \text{ distance} \quad (1.6)$$

$$T_d = NBIL \times b = BINA \quad (1.7)$$

$$T_d = BANl \quad (1.8)$$

$$T_d \propto I \quad (1.9)$$

### **Advantages**

- ✓ Torque/weight is high
- ✓ Power consumption is less
- ✓ Scale is uniform
- ✓ Damping is very effective
- ✓ Since operating field is very strong, the effect of stray field is negligible
- ✓ Range of instrument can be extended

### **Disadvantages**

- ✓ Use only for D.C.
- ✓ Cost is high
- ✓ Error is produced due to ageing effect of PMMC
- ✓ Friction and temperature error are present

### 1.7.1 Extension of range of PMMC instrument

#### Case-I: Shunt

A low shunt resistance connected in parallel with the ammeter to extend the range of current. Large current can be measured using low current rated ammeter by using a shunt.

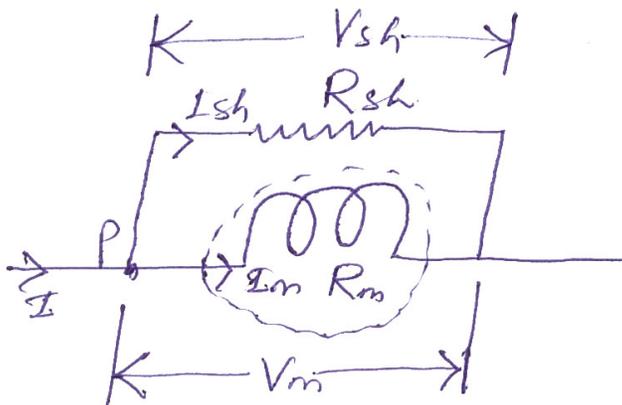


Fig. 1.8

Let  $R_m$  = Resistance of meter

$R_{sh}$  = Resistance of shunt

$I_m$  = Current through meter

$I_{sh}$  = current through shunt

$I$  = current to be measure

$$\therefore V_m = V_{sh} \quad (1.10)$$

$$I_m R_m = I_{sh} R_{sh}$$

$$\frac{I_m}{I_{sh}} = \frac{R_{sh}}{R_m} \quad (1.11)$$

Apply KCL at 'P'  $I = I_m + I_{sh}$  (1.12)

Eq<sup>n</sup> (1.12) ÷ by  $I_m$

$$\frac{I}{I_m} = 1 + \frac{I_{sh}}{I_m} \quad (1.13)$$

$$\frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}} \quad (1.14)$$

$$\therefore I = I_m \left( 1 + \frac{R_m}{R_{sh}} \right) \quad (1.15)$$

$\left( 1 + \frac{R_m}{R_{sh}} \right)$  is called multiplication factor

Shunt resistance is made of manganin. This has least thermoelectric emf. The change in resistance, due to change in temperature is negligible.

### Case (II): Multiplier

A large resistance is connected in series with voltmeter is called multiplier (Fig. 1.9). A large voltage can be measured using a voltmeter of small rating with a multiplier.

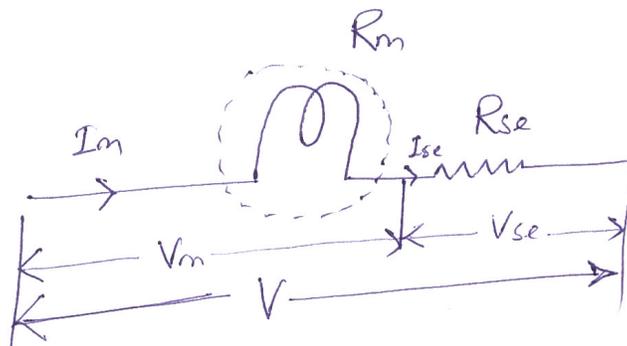


Fig. 1.9

Let  $R_m$  = resistance of meter

$R_{se}$  = resistance of multiplier

$V_m$  = Voltage across meter

$V_{se}$  = Voltage across series resistance

$V$  = voltage to be measured

$$I_m = I_{se} \quad (1.16)$$

$$\frac{V_m}{R_m} = \frac{V_{se}}{R_{se}} \quad (1.17)$$

$$\therefore \frac{V_{se}}{V_m} = \frac{R_{se}}{R_m} \quad (1.18)$$

$$\text{Apply KVL, } V = V_m + V_{se} \quad (1.19)$$

$$\text{Eq}^n (1.19) \div V_m$$

$$\frac{V}{V_m} = 1 + \frac{V_{se}}{V_m} = \left( 1 + \frac{R_{se}}{R_m} \right) \quad (1.20)$$

$$\therefore V = V_m \left( 1 + \frac{R_{se}}{R_m} \right) \quad (1.21)$$

$$\left( 1 + \frac{R_{se}}{R_m} \right) \rightarrow \text{Multiplication factor}$$

## 1.8 Moving Iron (MI) instruments

One of the most accurate instrument used for both AC and DC measurement is moving iron instrument. There are two types of moving iron instrument.

- Attraction type
- Repulsion type

### 1.8.1 Attraction type M.I. instrument

**Construction:** The moving iron fixed to the spindle is kept near the hollow fixed coil (Fig. 1.10). The pointer and balance weight are attached to the spindle, which is supported with jeweled bearing. Here air friction damping is used.

#### Principle of operation

The current to be measured is passed through the fixed coil. As the current is flow through the fixed coil, a magnetic field is produced. By magnetic induction the moving iron gets magnetized. The north pole of moving coil is attracted by the south pole of fixed coil. Thus the deflecting force is produced due to force of attraction. Since the moving iron is attached with the spindle, the spindle rotates and the pointer moves over the calibrated scale. But the force of attraction depends on the current flowing through the coil.

#### Torque developed by M.I

Let ' $\theta$ ' be the deflection corresponding to a current of 'i' amp

Let the current increases by  $di$ , the corresponding deflection is ' $\theta + d\theta$ '

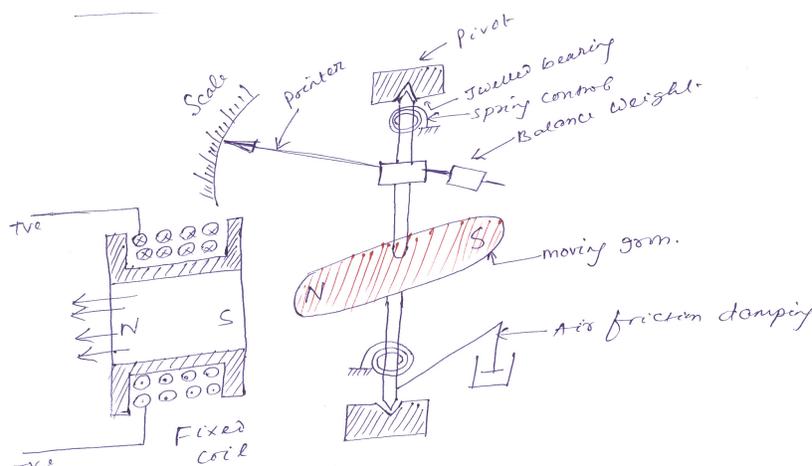


Fig. 1.10

There is change in inductance since the position of moving iron change w.r.t the fixed electromagnets.

Let the new inductance value be ' $L+dL$ '. The current change by ' $di$ ' is  $dt$  seconds.

Let the emf induced in the coil be ' $e$ ' volt.

$$e = \frac{d}{dt}(Li) = L \frac{di}{dt} + i \frac{dL}{dt} \quad (1.22)$$

Multiplying by ' $idt$ ' in equation (1.22)

$$e \times idt = L \frac{di}{dt} \times idt + i \frac{dL}{dt} \times idt \quad (1.23)$$

$$e \times idt = Lidi + i^2 dL \quad (1.24)$$

Eq<sup>n</sup> (1.24) gives the energy is used in to two forms. Part of energy is stored in the inductance. Remaining energy is converted in to mechanical energy which produces deflection.

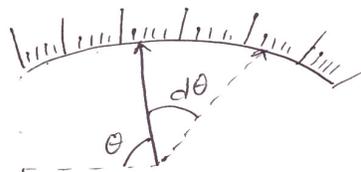
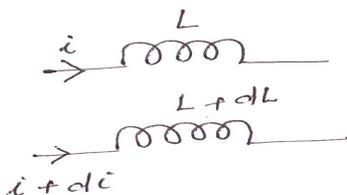


Fig. 1.11

Change in energy stored=Final energy-initial energy stored

$$\begin{aligned}
 &= \frac{1}{2}(L + dL)(i + di)^2 - \frac{1}{2}Li^2 \\
 &= \frac{1}{2}\{(L + dL)(i^2 + di^2 + 2idi) - Li^2\} \\
 &= \frac{1}{2}\{(L + dL)(i^2 + 2idi) - Li^2\} \\
 &= \frac{1}{2}\{Li^2 + 2Lidi + i^2dL + 2ididL - Li^2\} \\
 &= \frac{1}{2}\{2Lidi + i^2dL\} \\
 &= Lidi + \frac{1}{2}i^2dL \tag{1.25}
 \end{aligned}$$

Mechanical work to move the pointer by  $d\theta$

$$= T_d d\theta \tag{1.26}$$

By law of conservation of energy,

Electrical energy supplied=Increase in stored energy+ mechanical work done.

Input energy= Energy stored + Mechanical energy

$$Lidi + i^2dL = Lidi + \frac{1}{2}i^2dL + T_d d\theta \tag{1.27}$$

$$\frac{1}{2}i^2dL = T_d d\theta \tag{1.28}$$

$$T_d = \frac{1}{2}i^2 \frac{dL}{d\theta} \tag{1.29}$$

At steady state condition  $T_d = T_C$

$$\frac{1}{2}i^2 \frac{dL}{d\theta} = K\theta \tag{1.30}$$

$$\theta = \frac{1}{2K}i^2 \frac{dL}{d\theta} \tag{1.31}$$

$$\theta \propto i^2 \tag{1.32}$$

When the instruments measure AC,  $\theta \propto i_{rms}^2$

Scale of the instrument is non uniform.

**Advantages**

- ✓ MI can be used in AC and DC
- ✓ It is cheap
- ✓ Supply is given to a fixed coil, not in moving coil.
- ✓ Simple construction
- ✓ Less friction error.

**Disadvantages**

- ✓ It suffers from eddy current and hysteresis error
- ✓ Scale is not uniform
- ✓ It consumed more power
- ✓ Calibration is different for AC and DC operation

**1.8.2 Repulsion type moving iron instrument**

**Construction:** The repulsion type instrument has a hollow fixed iron attached to it (Fig. 1.12). The moving iron is connected to the spindle. The pointer is also attached to the spindle in supported with jeweled bearing.

**Principle of operation:** When the current flows through the coil, a magnetic field is produced by it. So both fixed iron and moving iron are magnetized with the same polarity, since they are kept in the same magnetic field. Similar poles of fixed and moving iron get repelled. Thus the deflecting torque is produced due to magnetic repulsion. Since moving iron is attached to spindle, the spindle will move. So that pointer moves over the calibrated scale.

Damping: Air friction damping is used to reduce the oscillation.

Control: Spring control is used.

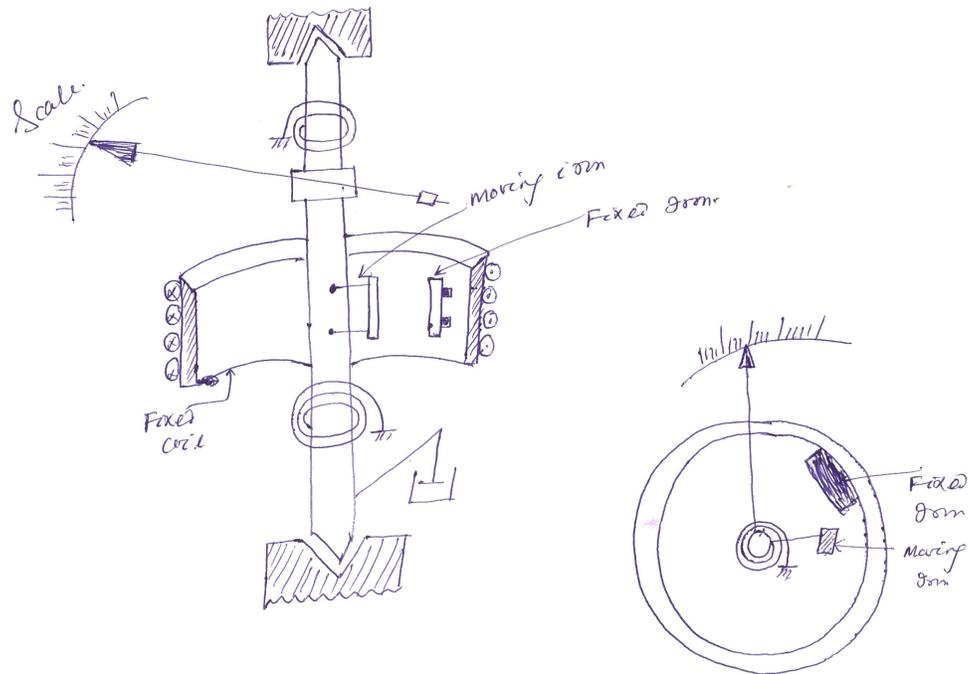


Fig. 1.12

### 1.9 Dynamometer (or) Electromagnetic moving coil instrument (EMMC)

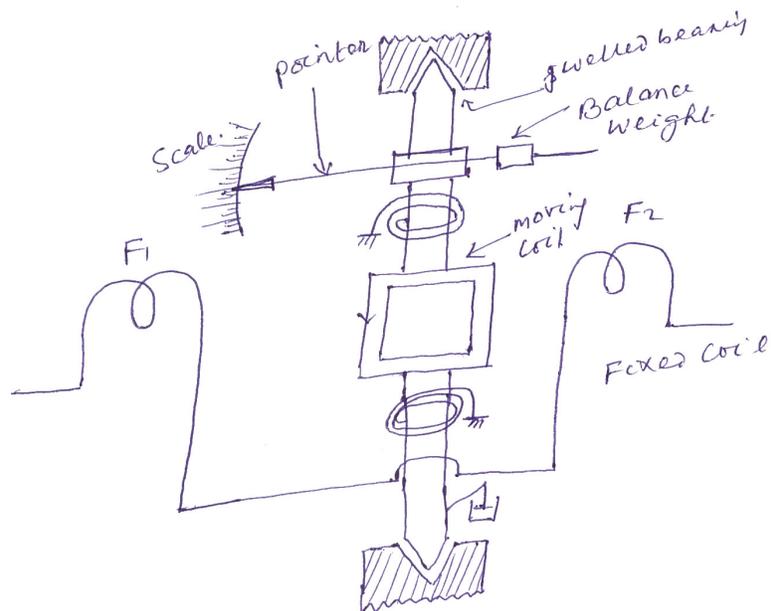


Fig. 1.13

This instrument can be used for the measurement of voltage, current and power. The difference between the PMMC and dynamometer type instrument is that the permanent magnet is replaced by an electromagnet.

**Construction:** A fixed coil is divided in to two equal half. The moving coil is placed between the two half of the fixed coil. Both the fixed and moving coils are air cored. So that the hysteresis effect will be zero. The pointer is attached with the spindle. In a non metallic former the moving coil is wounded.

**Control:** Spring control is used.

**Damping:** Air friction damping is used.

**Principle of operation:**

When the current flows through the fixed coil, it produced a magnetic field, whose flux density is proportional to the current through the fixed coil. The moving coil is kept in between the fixed coil. When the current passes through the moving coil, a magnetic field is produced by this coil.

The magnetic poles are produced in such a way that the torque produced on the moving coil deflects the pointer over the calibrated scale. This instrument works on AC and DC. When AC voltage is applied, alternating current flows through the fixed coil and moving coil. When the current in the fixed coil reverses, the current in the moving coil also reverses. Torque remains in the same direction. Since the current  $i_1$  and  $i_2$  reverse simultaneously. This is because the fixed and moving coils are either connected in series or parallel.

**Torque developed by EMMC**

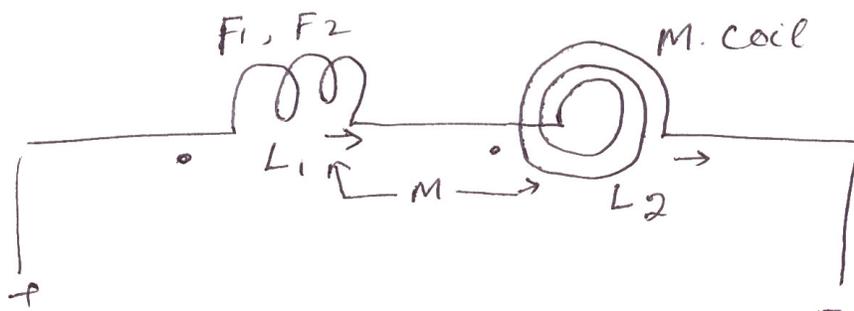


Fig. 1.14

Let

$L_1$ =Self inductance of fixed coil

$L_2$ = Self inductance of moving coil

$M$ =mutual inductance between fixed coil and moving coil

$i_1$ =current through fixed coil

$i_2$ =current through moving coil

Total inductance of system,

$$L_{total} = L_1 + L_2 + 2M \quad (1.33)$$

But we know that in case of M.I

$$T_d = \frac{1}{2} i^2 \frac{d(L)}{d\theta} \quad (1.34)$$

$$T_d = \frac{1}{2} i^2 \frac{d}{d\theta} (L_1 + L_2 + 2M) \quad (1.35)$$

The value of  $L_1$  and  $L_2$  are independent of ' $\theta$ ' but ' $M$ ' varies with  $\theta$

$$T_d = \frac{1}{2} i^2 \times 2 \frac{dM}{d\theta} \quad (1.36)$$

$$T_d = i^2 \frac{dM}{d\theta} \quad (1.37)$$

If the coils are not connected in series  $i_1 \neq i_2$

$$\therefore T_d = i_1 i_2 \frac{dM}{d\theta} \quad (1.38)$$

$$T_C = T_d \quad (1.39)$$

$$\therefore \theta = \frac{i_1 i_2}{K} \frac{dM}{d\theta} \quad (1.40)$$

Hence the deflection of pointer is proportional to the current passing through fixed coil and moving coil.

### 1.9.1 Extension of EMMC instrument

#### Case-I Ammeter connection

Fixed coil and moving coil are connected in parallel for ammeter connection. The coils are designed such that the resistance of each branch is same.

Therefore

$$I_1 = I_2 = I$$

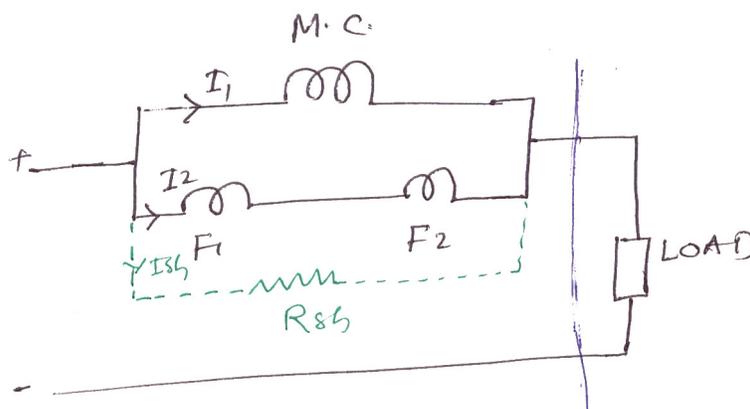


Fig. 1.15

To extend the range of current a shunt may be connected in parallel with the meter. The value  $R_{sh}$  is designed such that equal current flows through moving coil and fixed coil.

$$\therefore T_d = I_1 I_2 \frac{dM}{d\theta} \quad (1.41)$$

$$\text{Or } \therefore T_d = I^2 \frac{dM}{d\theta} \quad (1.42)$$

$$T_C = K\theta \quad (1.43)$$

$$\theta = \frac{I^2}{K} \frac{dM}{d\theta} \quad (1.44)$$

$$\therefore \theta \propto I^2 \text{ (Scale is not uniform)} \quad (1.45)$$

#### Case-II Voltmeter connection

Fixed coil and moving coil are connected in series for voltmeter connection. A multiplier may be connected in series to extent the range of voltmeter.

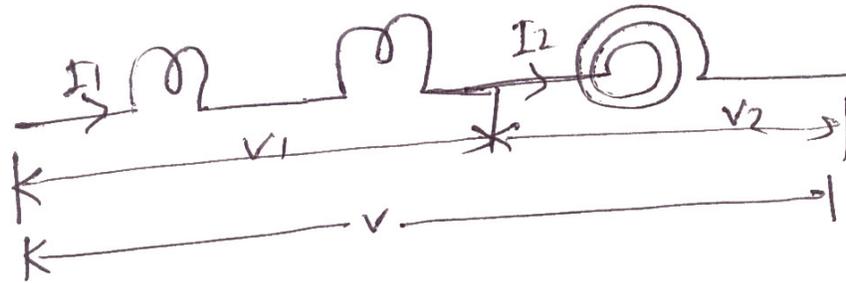


Fig. 1.16

$$I_1 = \frac{V_1}{Z_1}, I_2 = \frac{V_2}{Z_2} \tag{1.46}$$

$$T_d = \frac{V_1}{Z_1} \times \frac{V_2}{Z_2} \times \frac{dM}{d\theta} \tag{1.47}$$

$$T_d = \frac{K_1 V}{Z_1} \times \frac{K_2 V}{Z_2} \times \frac{dM}{d\theta} \tag{1.48}$$

$$T_d = \frac{KV^2}{Z_1 Z_2} \times \frac{dM}{d\theta} \tag{1.49}$$

$$T_d \propto V^2 \tag{1.50}$$

$$\therefore \theta \propto V^2 \text{ (Scale is not uniform)} \tag{1.51}$$

**Case-III As wattmeter**

When the two coils are connected to parallel, the instrument can be used as a wattmeter. Fixed coil is connected in series with the load. Moving coil is connected in parallel with the load. The moving coil is known as voltage coil or pressure coil and fixed coil is known as current coil.

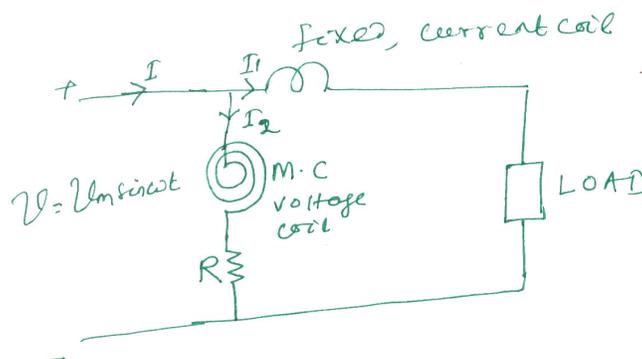


Fig. 1.17

Assume that the supply voltage is sinusoidal. If the impedance of the coil is neglected in comparison with the resistance 'R'. The current,

$$I_2 = \frac{v_m \sin wt}{R} \quad (1.52)$$

Let the phase difference between the currents  $I_1$  and  $I_2$  is  $\phi$

$$I_1 = I_m \sin(wt - \phi) \quad (1.53)$$

$$T_d = I_1 I_2 \frac{dM}{d\theta} \quad (1.54)$$

$$T_d = I_m \sin(wt - \phi) \times \frac{V_m \sin wt}{R} \frac{dM}{d\theta} \quad (1.55)$$

$$T_d = \frac{1}{R} (I_m V_m \sin wt \sin(wt - \phi)) \frac{dM}{d\theta} \quad (1.56)$$

$$T_d = \frac{1}{R} I_m V_m \sin wt \cdot \sin(wt - \phi) \frac{dM}{d\theta} \quad (1.57)$$

The average deflecting torque

$$(T_d)_{avg} = \frac{1}{2\pi} \int_0^{2\pi} T_d \times d(wt) \quad (1.58)$$

$$(T_d)_{avg} = \frac{1}{2\pi} \int_0^{2\pi} \frac{1}{R} \times I_m V_m \sin wt \cdot \sin(wt - \phi) \frac{dM}{d\theta} \times d(wt) \quad (1.59)$$

$$(T_d)_{avg} = \frac{V_m I_m}{2 \times 2\pi} \times \frac{1}{R} \times \frac{dM}{d\theta} \left[ \int \{ \cos \phi - \cos(2wt - \phi) \} dwt \right] \quad (1.60)$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\pi R} \times \frac{dM}{d\theta} \left[ \int_0^{2\pi} \cos \phi \cdot dwt - \int_0^{2\pi} \cos(2wt - \phi) \cdot dwt \right] \quad (1.61)$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\pi R} \times \frac{dM}{d\theta} \left[ \cos \phi [wt]_0^{2\pi} \right] \quad (1.62)$$

$$(T_d)_{avg} = \frac{V_m I_m}{4\pi R} \times \frac{dM}{d\theta} [\cos \phi (2\pi - 0)] \quad (1.63)$$

$$(T_d)_{avg} = \frac{V_m I_m}{2} \times \frac{1}{R} \times \frac{dM}{d\theta} \times \cos \phi \quad (1.64)$$

$$(T_d)_{avg} = V_{rms} \times I_{rms} \times \cos \phi \times \frac{1}{R} \times \frac{dM}{d\theta} \quad (1.65)$$

$$(T_d)_{avg} \propto KVI \cos \phi \quad (1.66)$$

$$T_C \propto \theta \quad (1.67)$$

$$\theta \propto KVI \cos \phi \quad (1.68)$$

$$\theta \propto VI \cos \phi \quad (1.69)$$

### **Advantages**

- ✓ It can be used for voltmeter, ammeter and wattmeter
- ✓ Hysteresis error is nill
- ✓ Eddy current error is nill
- ✓ Damping is effective
- ✓ It can be measure correctively and accurately the rms value of the voltage

### **Disadvantages**

- ✓ Scale is not uniform
- ✓ Power consumption is high(because of high resistance )
- ✓ Cost is more
- ✓ Error is produced due to frequency, temperature and stray field.
- ✓ Torque/weight is low.(Because field strength is very low)

### **Errors in PMMC**

- ✓ The permanent magnet produced error due to ageing effect. By heat treatment, this error can be eliminated.
- ✓ The spring produces error due to ageing effect. By heat treating the spring the error can be eliminated.
- ✓ When the temperature changes, the resistance of the coil vary and the spring also produces error in deflection. This error can be minimized by using a spring whose temperature co-efficient is very low.

### **1.10 Difference between attraction and repulsion type instrument**

An attraction type instrument will usually have a lower inductance, compare to repulsion type instrument. But in other hand, repulsion type instruments are more suitable for economical production in manufacture and nearly uniform scale is more easily obtained. They are therefore much more common than attraction type.

## 1.11 Characteristics of meter

### 1.11.1 Full scale deflection current( $I_{FSD}$ )

The current required to bring the pointer to full-scale or extreme right side of the instrument is called full scale deflection current. It must be as small as possible. Typical value is between  $2 \mu A$  to 30mA.

### 1.11.2 Resistance of the coil( $R_m$ )

This is ohmic resistance of the moving coil. It is due to  $\rho$ , L and A. For an ammeter this should be as small as possible.

### 1.11.3 Sensitivity of the meter(S)

$$S = \frac{1}{I_{FSD}} (\Omega/volt), \uparrow S = \frac{Z \uparrow}{V}$$

It is also called ohms/volt rating of the instrument. Larger the sensitivity of an instrument, more accurate is the instrument. It is measured in  $\Omega/volt$ . When the sensitivity is high, the impedance of meter is high. Hence it draws less current and loading affect is negligible. It is also defend as one over full scale deflection current.

## 1.12 Error in M.I instrument

### 1.12.1 Temperature error

Due to temperature variation, the resistance of the coil varies. This affects the deflection of the instrument. The coil should be made of manganin, so that the resistance is almost constant.

### 1.12.2 Hysteresis error

Due to hysteresis affect the reading of the instrument will not be correct. When the current is decreasing, the flux produced will not decrease suddenly. Due to this the meter reads a higher value of current. Similarly when the current increases the meter reads a lower value of current. This produces error in deflection. This error can be eliminated using small iron parts with narrow hysteresis loop so that the demagnetization takes place very quickly.

### 1.12.3 Eddy current error

The eddy currents induced in the moving iron affect the deflection. This error can be reduced by increasing the resistance of the iron.

### 1.12.4 Stray field error

Since the operating field is weak, the effect of stray field is more. Due to this, error is produced in deflection. This can be eliminated by shielding the parts of the instrument.

### 1.12.5 Frequency error

When the frequency changes the reactance of the coil changes.

$$Z = \sqrt{(R_m + R_S)^2 + X_L^2} \quad (1.70)$$

$$I = \frac{V}{Z} = \frac{V}{\sqrt{(R_m + R_S)^2 + X_L^2}} \quad (1.71)$$

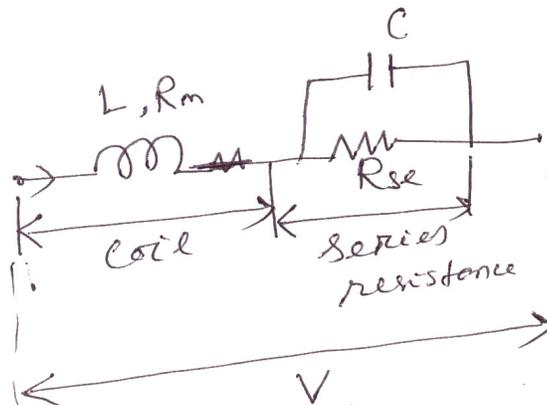


Fig. 1.18

Deflection of moving iron voltmeter depends upon the current through the coil. Therefore, deflection for a given voltage will be less at higher frequency than at low frequency. A capacitor is connected in parallel with multiplier resistance. The net reactance,  $(X_L - X_C)$  is very small, when compared to the series resistance. Thus the circuit impedance is made independent of frequency. This is because of the circuit is almost resistive.

$$C = 0.41 \frac{L}{(R_S)^2} \quad (1.72)$$

### 1.13 Electrostatic instrument

In multi cellular construction several vanes and quadrants are provided. The voltage is to be measured is applied between the vanes and quadrant. The force of attraction between the vanes

and quadrant produces a deflecting torque. Controlling torque is produced by spring control. Air friction damping is used.

The instrument is generally used for measuring medium and high voltage. The voltage is reduced to low value by using capacitor potential divider. The force of attraction is proportional to the square of the voltage.

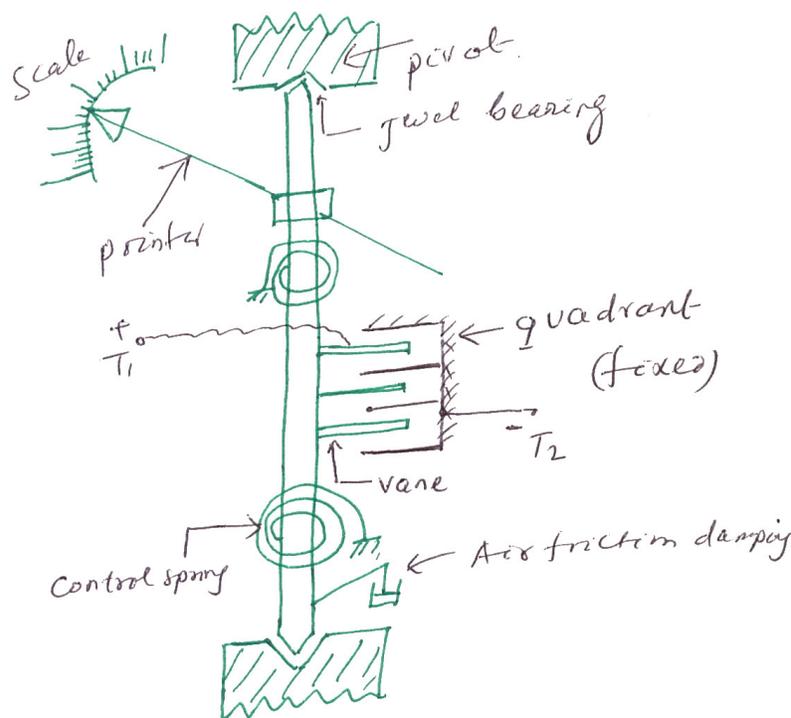


Fig. 1.19

### Torque develop by electrostatic instrument

$V$  = Voltage applied between vane and quadrant

$C$  = capacitance between vane and quadrant

$$\text{Energy stored} = \frac{1}{2} CV^2 \quad (1.73)$$

Let ' $\theta$ ' be the deflection corresponding to a voltage  $V$ .

Let the voltage increases by  $dv$ , the corresponding deflection is ' $\theta + d\theta$ '

When the voltage is being increased, a capacitive current flows

$$i = \frac{dq}{dt} = \frac{d(CV)}{dt} = \frac{dC}{dt} V + C \frac{dV}{dt} \quad (1.74)$$

$V \times dt$  multiply on both side of equation (1.74)

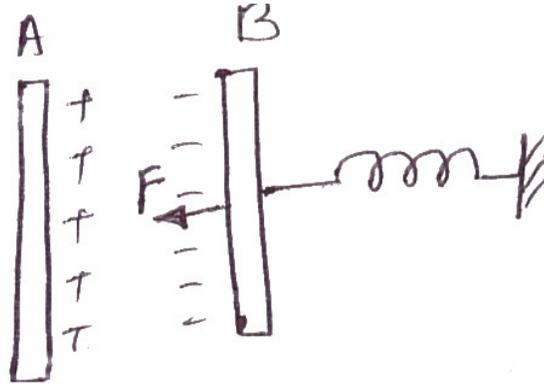


Fig. 1.20

$$Vidt = \frac{dC}{dt} V^2 dt + CV \frac{dV}{dt} dt \quad (1.75)$$

$$Vidt = V^2 dC + CVdV \quad (1.76)$$

$$\text{Change in stored energy} = \frac{1}{2}(C + dC)(V + dV)^2 - \frac{1}{2}CV^2 \quad (1.77)$$

$$= \frac{1}{2}[(C + dC)V^2 + dV^2 + 2VdV] - \frac{1}{2}CV^2$$

$$= \frac{1}{2}[CV^2 + CdV^2 + 2CVdV + V^2dC + dCdV^2 + 2VdVdC] - \frac{1}{2}CV^2$$

$$= \frac{1}{2}V^2dC + CVdV$$

$$V^2dC + CVdV = \frac{1}{2}V^2dC + CVdV + F \times rd\theta \quad (1.78)$$

$$T_d \times d\theta = \frac{1}{2}V^2dC \quad (1.79)$$

$$T_d = \frac{1}{2}V^2 \left( \frac{dC}{d\theta} \right) \quad (1.80)$$

At steady state condition,  $T_d = T_C$

$$K\theta = \frac{1}{2}V^2 \left( \frac{dC}{d\theta} \right) \quad (1.81)$$

$$\theta = \frac{1}{2K}V^2 \left( \frac{dC}{d\theta} \right) \quad (1.82)$$

**Advantages**

- ✓ It is used in both AC and DC.
- ✓ There is no frequency error.
- ✓ There is no hysteresis error.
- ✓ There is no stray magnetic field error. Because the instrument works on electrostatic principle.
- ✓ It is used for high voltage
- ✓ Power consumption is negligible.

**Disadvantages**

- ✓ Scale is not uniform
- ✓ Large in size
- ✓ Cost is more

**1.14 Multi range Ammeter**

When the switch is connected to position (1), the supplied current  $I_1$

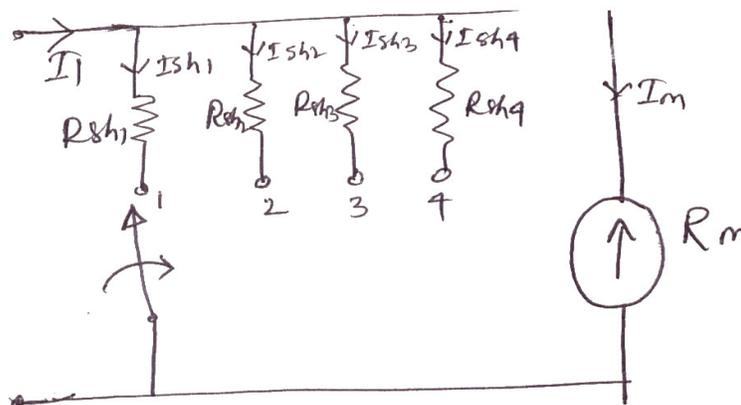


Fig. 1.21

$$I_{sh1}R_{sh1} = I_m R_m \quad (1.83)$$

$$R_{sh1} = \frac{I_m R_m}{I_{sh1}} = \frac{I_m R_m}{I_1 - I_m} \quad (1.84)$$

$$R_{sh1} = \frac{R_m}{\frac{I_1}{I_m} - 1}, R_{sh1} = \frac{R_m}{m_1 - 1}, m_1 = \frac{I_1}{I_m} = \text{Multiplying power of shunt}$$

$$R_{sh2} = \frac{R_m}{m_2 - 1}, m_2 = \frac{I_2}{I_m} \quad (1.85)$$

$$R_{sh3} = \frac{R_m}{m_3 - 1}, m_3 = \frac{I_3}{I_m} \quad (1.86)$$

$$R_{sh4} = \frac{R_m}{m_4 - 1}, m_4 = \frac{I_4}{I_m} \quad (1.87)$$

### 1.15 Ayrton shunt

$$R_1 = R_{sh1} - R_{sh2} \quad (1.88)$$

$$R_2 = R_{sh2} - R_{sh3} \quad (1.89)$$

$$R_3 = R_{sh3} - R_{sh4} \quad (1.90)$$

$$R_4 = R_{sh4} \quad (1.91)$$

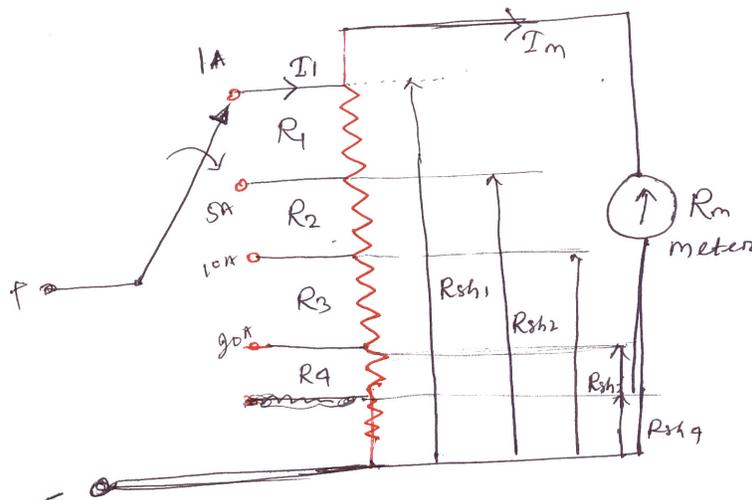


Fig. 1.22

Ayrton shunt is also called universal shunt. Ayrton shunt has more sections of resistance. Taps are brought out from various points of the resistor. The variable points in the o/p can be connected to any position. Various meters require different types of shunts. The Ayrton shunt is used in the lab, so that any value of resistance between minimum and maximum specified can be used. It eliminates the possibility of having the meter in the circuit without a shunt.

### 1.16 Multi range D.C. voltmeter

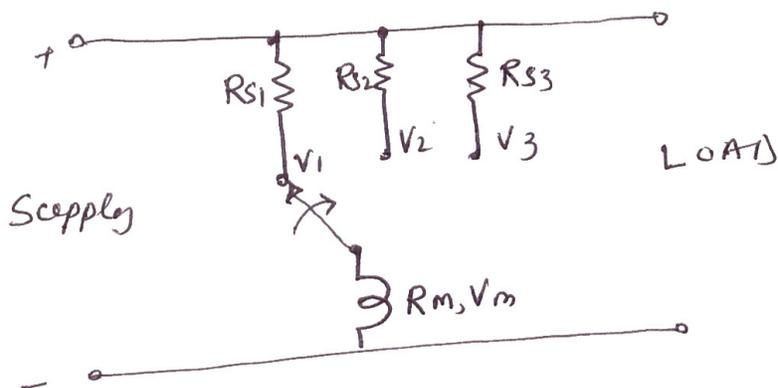


Fig. 1.23

$$R_{s1} = R_m(m_1 - 1)$$

$$R_{s2} = R_m(m_2 - 1) \quad (1.92)$$

$$R_{s3} = R_m(m_3 - 1)$$

$$m_1 = \frac{V_1}{V_m}, m_2 = \frac{V_2}{V_m}, m_3 = \frac{V_3}{V_m} \quad (1.93)$$

We can obtain different Voltage ranges by connecting different value of multiplier resistor in series with the meter. The number of these resistors is equal to the number of ranges required.

### 1.17 Potential divider arrangement

The resistance  $R_1, R_2, R_3$  and  $R_4$  is connected in series to obtained the ranges  $V_1, V_2, V_3$  and  $V_4$

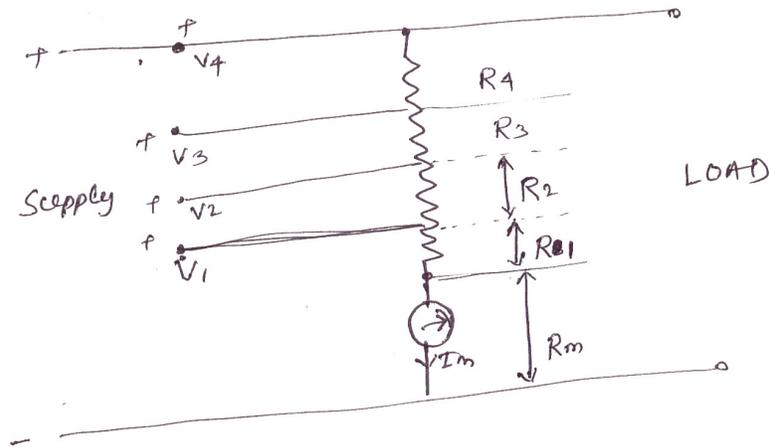


Fig. 1.24

Consider for voltage  $V_1$ ,  $(R_1 + R_m)I_m = V_1$

$$\therefore R_1 = \frac{V_1}{I_m} - R_m = \frac{V_1}{\left(\frac{V_m}{R_m}\right)} - R_m = \left(\frac{V_1}{V_m}\right)R_m - R_m \quad (1.94)$$

$$R_1 = (m_1 - 1)R_m \quad (1.95)$$

For  $V_2$ ,  $(R_2 + R_1 + R_m)I_m = V_2 \Rightarrow R_2 = \frac{V_2}{I_m} - R_1 - R_m \quad (1.96)$

$$R_2 = \frac{V_2}{\left(\frac{V_m}{R_m}\right)} - (m_1 - 1)R_m - R_m \quad (1.97)$$

$$\begin{aligned} R_2 &= m_2 R_m - R_m - (m_1 - 1)R_m \\ &= R_m(m_2 - 1 - m_1 + 1) \end{aligned} \quad (1.98)$$

$$R_2 = (m_2 - m_1)R_m \quad (1.99)$$

For  $V_3$   $(R_3 + R_2 + R_1 + R_m)I_m = V_3$

$$\begin{aligned} R_3 &= \frac{V_3}{I_m} - R_2 - R_1 - R_m \\ &= \frac{V_3}{V_m} R_m - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m \\ &= m_3 R_m - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m \\ R_3 &= (m_3 - m_2)R_m \end{aligned}$$

$$\text{For } V_4 \quad (R_4 + R_3 + R_2 + R_1 + R_m)I_m = V_4$$

$$R_4 = \frac{V_4}{I_m} - R_3 - R_2 - R_1 - R_m$$

$$= \left( \frac{V_4}{V_m} \right) R_m - (m_3 - m_2)R_m - (m_2 - m_1)R_m - (m_1 - 1)R_m - R_m$$

$$R_4 = R_m [m_4 - m_3 + m_2 - m_2 + m_1 - m_1 + 1 - 1]$$

$$R_4 = (m_4 - m_3)R_m$$

### Example: 1.1

A PMMC ammeter has the following specification

Coil dimension are  $1\text{cm} \times 1\text{cm}$ . Spring constant is  $0.15 \times 10^{-6} \text{ N-m/rad}$ , Flux density is  $1.5 \times 10^{-3} \text{ wb/m}^2$ . Determine the no. of turns required to produce a deflection of  $90^\circ$  when a current 2mA flows through the coil.

### Solution:

At steady state **condition**  $T_d = T_C$

$$BANl = K\theta$$

$$\Rightarrow N = \frac{K\theta}{BAI}$$

$$A = 1 \times 10^{-4} \text{ m}^2$$

$$K = 0.15 \times 10^{-6} \frac{\text{N-m}}{\text{rad}}$$

$$B = 1.5 \times 10^{-3} \text{ wb/m}^2$$

$$I = 2 \times 10^{-3} \text{ A}$$

$$\theta = 90^\circ = \frac{\pi}{2} \text{ rad}$$

$$N = 785 \text{ ans.}$$

### Example: 1.2

The pointer of a moving coil instrument gives full scale deflection of 20mA. The potential difference across the meter when carrying 20mA is 400mV. The instrument to be used is 200A for full scale deflection. Find the shunt resistance required to achieve this, if the instrument to be used as a voltmeter for full scale reading with 1000V. Find the series resistance to be connected it?

**Solution:**

**Case-1**

$$V_m = 400 \text{ mV}$$

$$I_m = 20 \text{ mA}$$

$$I = 200 \text{ A}$$

$$R_m = \frac{V_m}{I_m} = \frac{400}{20} = 20 \Omega$$

$$I = I_m \left( 1 + \frac{R_m}{R_{sh}} \right)$$

$$200 = 20 \times 10^{-3} \left[ 1 + \frac{20}{R_{sh}} \right]$$

$$R_{sh} = 2 \times 10^{-3} \Omega$$

**Case-II**

$$V = 1000 \text{ V}$$

$$V = V_m \left( 1 + \frac{R_{se}}{R_m} \right)$$

$$4000 = 400 \times 10^{-3} \left( 1 + \frac{R_{se}}{20} \right)$$

$$R_{se} = 49.98 \text{ k}\Omega$$

**Example: 1.3**

A 150 v moving iron voltmeter is intended for 50HZ, has a resistance of 3kΩ. Find the series resistance required to extent the range of instrument to 300v. If the 300V instrument is used to measure a d.c. voltage of 200V. Find the voltage across the meter?

**Solution:**

$$R_m = 3 \text{ k}\Omega, V_m = 150 \text{ V}, V = 300 \text{ V}$$

$$V = V_m \left( 1 + \frac{R_{se}}{R_m} \right)$$

$$300 = 150 \left( 1 + \frac{R_{se}}{3} \right) \Rightarrow R_{se} = 3k\Omega$$

Case-II  $V = V_m \left( 1 + \frac{R_{se}}{R_m} \right)$

$$200 = V_m \left( 1 + \frac{3}{3} \right)$$

$$\therefore V_m = 100V \quad \text{Ans}$$

#### Example: 1.4

What is the value of series resistance to be used to extent '0' to 200V range of 20,000Ω/volt voltmeter to 0 to 2000 volt?

#### Solution:

$$V_{se} = V - V = 1800$$

$$I_{FSD} = \frac{1}{20000} = \frac{1}{\text{Sensitivity}}$$

$$V_{se} = R_{se} \times i_{FSD} \Rightarrow R_{se} = 36M\Omega \quad \text{ans.}$$

#### Example: 1.5

A moving coil instrument whose resistance is 25Ω gives a full scale deflection with a current of 1mA. This instrument is to be used with a manganin shunt, to extent its range to 100mA. Calculate the error caused by a 10<sup>0</sup>C rise in temperature when:

- Copper moving coil is connected directly across the manganin shunt.
- A 75 ohm manganin resistance is used in series with the instrument moving coil.

The temperature co-efficient of copper is 0.004/<sup>0</sup>C and that of manganin is 0.00015/<sup>0</sup>C.

#### Solution:

##### Case-1

$$I_m = 1mA$$

$$R_m = 25\Omega$$

$I=100\text{mA}$

$$I = I_m \left( 1 + \frac{R_m}{R_{sh}} \right)$$

$$100 = 1 \left( 1 + \frac{25}{R_{sh}} \right) \Rightarrow \frac{25}{R_{sh}} = 99$$

$$\Rightarrow R_{sh} = \frac{25}{99} = 0.2525\Omega$$

Instrument resistance for  $10^{\circ}\text{C}$  rise in temperature,  $R_{mt} = 25(1 + 0.004 \times 10)$

$$R_t = R_o(1 + \rho_t \times t)$$

$$R_{m/t=10^{\circ}} = 26\Omega$$

Shunt resistance for  $10^{\circ}\text{C}$ , rise in temperature

$$R_{sh/t=10^{\circ}} = 0.2525(1 + 0.00015 \times 10) = 0.2529\Omega$$

Current through the meter for 100mA in the main circuit for  $10^{\circ}\text{C}$  rise in temperature

$$I = I_m \left( 1 + \frac{R_m}{R_{sh}} \right) \Big|_{t=10^{\circ}\text{C}}$$

$$100 = I_{mt} \left( 1 + \frac{26}{0.2529} \right)$$

$$I_{m|_{t=10}} = 0.963\text{mA}$$

But normal meter current=1mA

Error due to rise in temperature= $(0.963-1) \times 100 = -3.7\%$

**Case-b** As voltmeter

Total resistance in the meter circuit= $R_m + R_{sh} = 25 + 75 = 100\Omega$

$$I = I_m \left( 1 + \frac{R_m}{R_{sh}} \right)$$

$$100 = 1 \left( 1 + \frac{100}{R_{sh}} \right)$$

$$R_{sh} = \frac{100}{100-1} = 1.01\Omega$$

Resistance of the instrument circuit for  $10^{\circ}\text{C}$  rise in temperature

$$R_m|_{t=10} = 25(1 + 0.004 \times 10) + 75(1 + 0.00015 \times 10) = 101.11\Omega$$

Shunt resistance for  $10^{\circ}\text{C}$  rise in temperature

$$R_{sh}|_{t=10} = 1.01(1 + 0.00015 \times 10) = 1.0115\Omega$$

$$I = I_m \left( 1 + \frac{R_m}{R_{sh}} \right)$$

$$100 = I_m \left( 1 + \frac{101.11}{1.0115} \right)$$

$$I_m|_{t=10^{\circ}} = 0.9905\text{mA}$$

$$\text{Error} = (0.9905 - 1) \times 100 = -0.95\%$$

### Example: 1.6

The coil of a 600V M.I meter has an inductance of 1 henry. It gives correct reading at 50HZ and requires 100mA. For its full scale deflection, what is % error in the meter when connected to 200V D.C. by comparing with 200V A.C?

### Solution:

$$V_m = 600\text{V}, I_m = 100\text{mA}$$

Case-I A.C.

$$Z_m = \frac{V_m}{I_m} = \frac{600}{0.1} = 6000\Omega$$

$$X_L = 2\pi fL = 314\Omega$$

$$R_m = \sqrt{Z_m^2 - X_L^2} = \sqrt{(6000)^2 - (314)^2} = 5990\Omega$$

$$I_{AC} = \frac{V_{AC}}{Z} = \frac{200}{6000} = 33.33\text{mA}$$

Case-II D.C

$$I_{DC} = \frac{V_{DC}}{R_m} = \frac{200}{5990} = 33.39\text{mA}$$

$$\text{Error} = \frac{I_{DC} - I_{AC}}{I_{AC}} \times 100 = \frac{33.39 - 33.33}{33.33} \times 100 = 0.18\%$$

**Example: 1.7**

A 250V M.I. voltmeter has coil resistance of  $500\Omega$ , coil inductance of  $1.04\text{ H}$  and series resistance of  $2\text{k}\Omega$ . The meter reads correctly at  $250\text{V D.C.}$  What will be the value of capacitance to be used for shunting the series resistance to make the meter read correctly at  $50\text{HZ}$ ? What is the reading of voltmeter on A.C. without capacitance?

**Solution:**

$$C = 0.41 \frac{L}{(R_S)^2}$$

$$= 0.41 \times \frac{1.04}{(2 \times 10^3)^2} = 0.1 \mu\text{F}$$

For A.C  $Z = \sqrt{(R_m + R_{Se})^2 + X_L^2}$

$$Z = \sqrt{(500 + 2000)^2 + (314)^2} = 2520\Omega$$

With D.C

$$R_{total} = 2500\Omega$$

For  $2500\Omega \rightarrow 250\text{V}$

$$1\Omega \rightarrow \frac{250}{2500}$$

$$2520\Omega \rightarrow \frac{250}{2500} \times 2520 = 248\text{V}$$

**Example: 1.8**

The relationship between inductance of moving iron ammeter, the current and the position of pointer is as follows:

Reading (A)	1.2	1.4	1.6	1.8
Deflection (degree)	36.5	49.5	61.5	74.5
Inductance ( $\mu\text{H}$ )	575.2	576.5	577.8	578.8

Calculate the deflecting torque and the spring constant when the current is  $1.5\text{A}$ ?

**Solution:**

For current  $I=1.5\text{A}$ ,  $\theta=55.5$  degree  $=0.96865$  rad

$$\frac{dL}{d\theta} = \frac{577.65 - 576.5}{60 - 49.5} = 0.11 \mu\text{H} / \text{deg} = 6.3 \mu\text{H} / \text{rad}$$

$$\text{Deflecting torque, } T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} = \frac{1}{2} (1.5)^2 \times 6.3 \times 10^{-6} = 7.09 \times 10^{-6} \text{ N-m}$$

$$\text{Spring constant, } K = \frac{T_d}{\theta} = \frac{7.09 \times 10^{-6}}{0.968} = 7.319 \times 10^{-6} \frac{\text{N-m}}{\text{rad}}$$

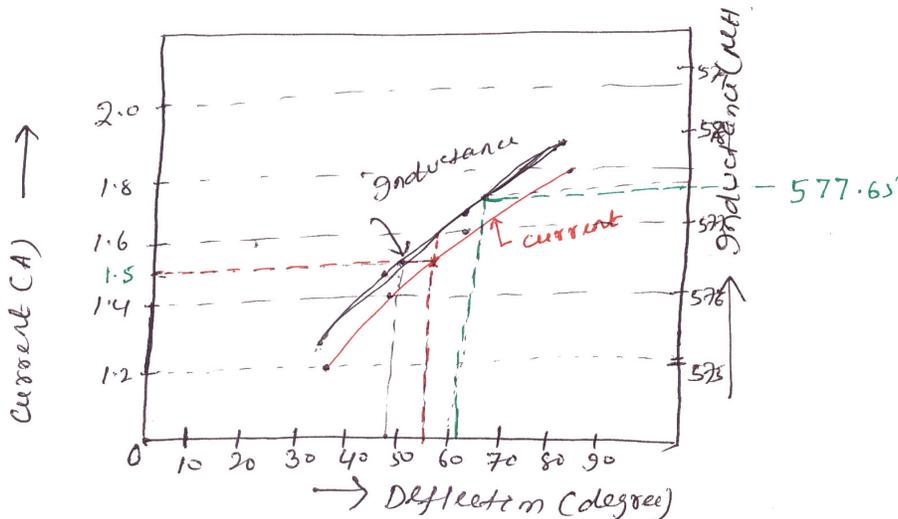


Fig. 1.25

**Example: 1.9**

For a certain dynamometer ammeter the mutual inductance 'M' varies with deflection  $\theta$  as  $M = -6 \cos(\theta + 30^\circ) \text{ mH}$ . Find the deflecting torque produced by a direct current of 50mA corresponding to a deflection of  $60^\circ$ .

**Solution:**

$$T_d = I_1 I_2 \frac{dM}{d\theta} = I^2 \frac{dM}{d\theta}$$

$$M = -6 \cos(\theta + 30^\circ)$$

$$\frac{dM}{d\theta} = 6 \sin(\theta + 30^\circ) \text{ mH}$$

$$\left. \frac{dM}{d\theta} \right|_{\theta=60} = 6 \sin 90 = 6 \text{ mH} / \text{deg}$$

$$T_d = I^2 \frac{dM}{d\theta} = (50 \times 10^{-3})^2 \times 6 \times 10^{-3} = 15 \times 10^{-6} \text{ N-m}$$

**Example: 1.10**

The inductance of a moving iron ammeter with a full scale deflection of  $90^\circ$  at 1.5A, is given by the expression  $L = 200 + 40\theta - 4\theta^2 - \theta^3 \mu H$ , where  $\theta$  is deflection in radian from the zero position. Estimate the angular deflection of the pointer for a current of 1.0A.

**Solution:**

$$L = 200 + 40\theta - 4\theta^2 - \theta^3 \mu H$$

$$\frac{dL}{d\theta} \Big|_{\theta=90^\circ} = 40 - 8\theta - 3\theta^2 \mu H / rad$$

$$\frac{dL}{d\theta} \Big|_{\theta=90^\circ} = 40 - 8 \times \frac{\Pi}{2} - 3 \left( \frac{\Pi}{2} \right)^2 \mu H / rad = 20 \mu H / rad$$

$$\therefore \theta = \frac{1}{2K} I^2 \left( \frac{dL}{d\theta} \right)$$

$$\frac{\Pi}{2} = \frac{1}{2} \frac{(1.5)^2}{K} \times 20 \times 10^{-6}$$

$$K = \text{Spring constant} = 14.32 \times 10^{-6} N - m / rad$$

$$\text{For } I=1A, \therefore \theta = \frac{1}{2K} I^2 \left( \frac{dL}{d\theta} \right)$$

$$\therefore \theta = \frac{1}{2} \times \frac{(1)^2}{14.32 \times 10^{-6}} (40 - 8\theta - 3\theta^2)$$

$$3\theta + 36.64\theta^2 - 40 = 0$$

$$\theta = 1.008 rad, 57.8^\circ$$

**Example: 1.11**

The inductance of a moving iron instrument is given by  $L = 10 + 5\theta - \theta^2 - \theta^3 \mu H$ , where  $\theta$  is the deflection in radian from zero position. The spring constant is  $12 \times 10^{-6} N - m / rad$ . Estimate the deflection for a current of 5A.

**Solution:**

$$\frac{dL}{d\theta} = (5 - 2\theta) \frac{\mu H}{rad}$$

$$\therefore \theta = \frac{1}{2K} I^2 \left( \frac{dL}{d\theta} \right)$$

$$\therefore \theta = \frac{1}{2} \times \frac{(5)^2}{12 \times 10^{-6}} (5 - 2\theta) \times 10^{-6}$$

$$\therefore \theta = 1.69 rad, 96.8^\circ$$

### Example: 1.12

The following figure gives the relation between deflection and inductance of a moving iron instrument.

Deflection (degree)	20	30	40	50	60	70	80	90
Inductance ( $\mu H$ )	335	345	355.5	366.5	376.5	385	391.2	396.5

Find the current and the torque to give a deflection of (a)  $30^\circ$  (b)  $80^\circ$ . Given that control spring constant is  $0.4 \times 10^{-6} N - m / degree$

### Solution:

$$\theta = \frac{1}{2K} I^2 \left( \frac{dL}{d\theta} \right)$$

(a) For  $\theta = 30^\circ$

The curve is linear

$$\therefore \left( \frac{dL}{d\theta} \right)_{\theta=30} = \frac{355.5 - 335}{40 - 20} = 1.075 \mu H / degree = 58.7 \mu H / rad$$

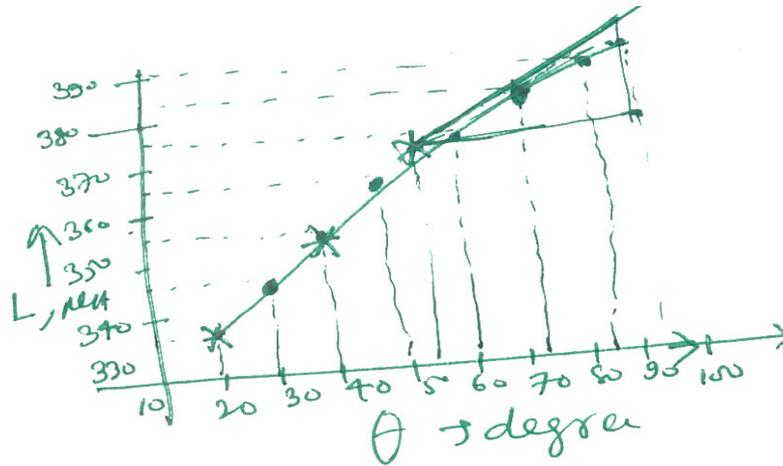


Fig. 1.26

**Example: 1.13**

In an electrostatic voltmeter the full scale deflection is obtained when the moving plate turns through  $90^\circ$ . The torsional constant is  $10 \times 10^{-6} \text{ N-m/rad}$ . The relation between the angle of deflection and capacitance between the fixed and moving plates is given by

Deflection (degree)	0	10	20	30	40	50	60	70	80	90
Capacitance (PF)	81.4	121	156	189.2	220	246	272	294	316	334

Find the voltage applied to the instrument when the deflection is  $90^\circ$ ?

**Solution:**

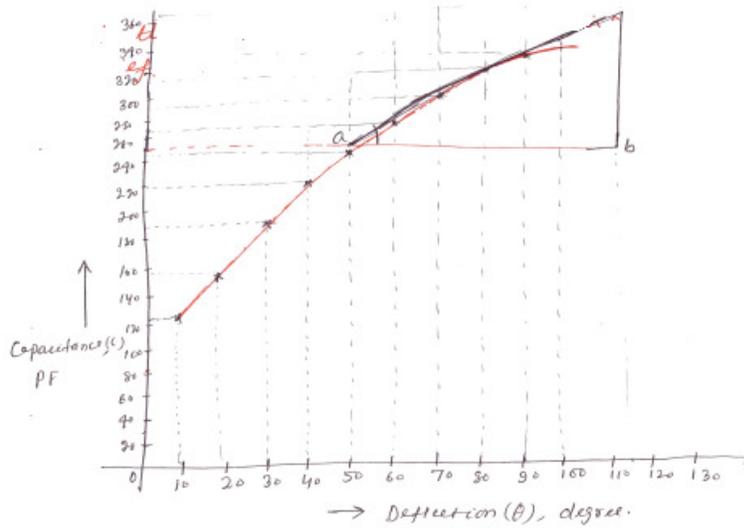


Fig. 1.27

$$\frac{dC}{d\theta} = \tan \theta = \frac{bc}{ab} = \frac{370 - 250}{110 - 44} = 1.82 PF / \text{deg } ree = 104.2 PF / \text{rad}$$

$$\text{Spring constant } K = 10 \times 10^{-6} \frac{N - m}{\text{rad}} = 0.1745 \times 10^{-6} N - m / \text{deg } ree$$

$$\theta = \frac{1}{2K} V^2 \left( \frac{dC}{d\theta} \right) \Rightarrow V = \sqrt{\frac{2K\theta}{\frac{dC}{d\theta}}}$$

$$V = \sqrt{\frac{2 \times 0.1745 \times 10^{-6} \times 90}{104.2 \times 10^{-12}}} = 549 \text{ volt}$$

### Example: 1.14

Design a multi range d.c. mille ammeter using a basic movement with an internal resistance  $R_m = 50\Omega$  and a full scale deflection current  $I_m = 1\text{mA}$ . The ranges required are 0-10mA; 0-50mA; 0-100mA and 0-500mA.

#### Solution:

**Case-I** 0-10mA

$$\text{Multiplying power } m = \frac{I}{I_m} = \frac{10}{1} = 10$$

$$\therefore \text{Shunt resistance } R_{sh1} = \frac{R_m}{m-1} = \frac{50}{10-1} = 5.55\Omega$$

**Case-II** 0-50mA

$$m = \frac{50}{1} = 50$$

$$R_{sh2} = \frac{R_m}{m-1} = \frac{50}{50-1} = 1.03\Omega$$

**Case-III** 0-100mA,  $m = \frac{100}{1} = 100\Omega$

$$R_{sh3} = \frac{R_m}{m-1} = \frac{50}{100-1} = 0.506\Omega$$

**Case-IV** 0-500mA,  $m = \frac{500}{1} = 500\Omega$

$$R_{sh4} = \frac{R_m}{m-1} = \frac{50}{500-1} = 0.1\Omega$$

### Example: 1.15

A moving coil voltmeter with a resistance of  $20\Omega$  gives a full scale deflection of  $120^\circ$ , when a potential difference of  $100\text{mV}$  is applied across it. The moving coil has dimension of  $30\text{mm} \times 25\text{mm}$  and is wound with 100 turns. The control spring constant is  $0.375 \times 10^{-6} \text{ N - m / deg ree}$ . Find the flux density, in the air gap. Find also the diameter of copper wire of coil winding if 30% of instrument resistance is due to coil winding. The specific resistance for copper =  $1.7 \times 10^{-8} \Omega \text{ m}$ .

**Solution:**

Data given

$$V_m = 100\text{mV}$$

$$R_m = 20\Omega$$

$$\theta = 120^\circ$$

$$N = 100$$

$$K = 0.375 \times 10^{-6} \text{ N - m / deg ree}$$

$$R_C = 30\% \text{ of } R_m$$

$$\rho = 1.7 \times 10^{-8} \Omega \text{ m}$$

$$I_m = \frac{V_m}{R_m} = 5 \times 10^{-3} \text{ A}$$

$$T_d = BAN I, T_C = K\theta = 0.375 \times 10^{-6} \times 120 = 45 \times 10^{-6} \text{ N - m}$$

$$B = \frac{T_d}{AN I} = \frac{45 \times 10^{-6}}{30 \times 25 \times 10^{-6} \times 100 \times 5 \times 10^{-3}} = 0.12 \text{ wb / m}^2$$

$$R_C = 0.3 \times 20 = 6\Omega$$

Length of mean turn path =  $2(a+b) = 2(55) = 110\text{mm}$

$$R_C = N \left( \frac{\rho l}{A} \right)$$

$$A = \frac{N \times \rho \times (l_t)}{R_C} = \frac{100 \times 1.7 \times 10^{-8} \times 110 \times 10^{-3}}{6}$$

$$= 3.116 \times 10^{-8} m^2$$

$$= 31.16 \times 10^{-3} mm^2$$

$$A = \frac{\Pi}{4} d^2 \Rightarrow d = 0.2 mm$$

### Example: 1.16

A moving coil instrument gives a full scale deflection of 10mA, when the potential difference across its terminal is 100mV. Calculate

- (1) The shunt resistance for a full scale deflection corresponding to 100A
- (2) The resistance for full scale reading with 1000V.

Calculate the power dissipation in each case?

### Solution:

Data given

$$I_m = 10 mA$$

$$V_m = 100 mV$$

$$I = 100 A$$

$$I = I_m \left( 1 + \frac{R_m}{R_{sh}} \right)$$

$$100 = 10 \times 10^{-3} \left( 1 + \frac{10}{R_{sh}} \right)$$

$$R_{sh} = 1.001 \times 10^{-3} \Omega$$

$$R_{se} = ??, V = 1000 V$$

$$R_m = \frac{V_m}{I_m} = \frac{100}{10} = 10 \Omega$$

$$V = V_m \left( 1 + \frac{R_{se}}{R_m} \right)$$

$$1000 = 100 \times 10^{-3} \left( 1 + \frac{R_{se}}{10} \right)$$

$$\therefore R_{se} = 99.99 K\Omega$$

### Example: 1.17

Design an Aryton shunt to provide an ammeter with current ranges of 1A,5A,10A and 20A. A basic meter with an internal resistance of  $50\Omega$  and a full scale deflection current of  $1\text{mA}$  is to be used.

**Solution:** Data given

$$I_m = 1 \times 10^{-3} \text{ A} \quad \left| \begin{array}{l} I_1 = 1\text{A} \\ I_2 = 5\text{A} \\ I_3 = 10\text{A} \\ I_4 = 20\text{A} \end{array} \right. \quad \left| \begin{array}{l} m_1 = \frac{I_1}{I_m} = 1000\text{A} \\ m_2 = \frac{I_2}{I_m} = 5000\text{A} \\ m_3 = \frac{I_3}{I_m} = 10000\text{A} \\ m_4 = \frac{I_4}{I_m} = 20000\text{A} \end{array} \right.$$

$$R_{sh1} = \frac{R_m}{m_1 - 1} = \frac{50}{1000 - 1} = 0.05\Omega$$

$$R_{sh2} = \frac{R_m}{m_2 - 1} = \frac{50}{5000 - 1} = 0.01\Omega$$

$$R_{sh3} = \frac{R_m}{m_3 - 1} = \frac{50}{10000 - 1} = 0.005\Omega$$

$$R_{sh4} = \frac{R_m}{m_4 - 1} = \frac{50}{20000 - 1} = 0.0025\Omega$$

∴ The resistances of the various section of the universal shunt are

$$R_1 = R_{sh1} - R_{sh2} = 0.05 - 0.01 = 0.04\Omega$$

$$R_2 = R_{sh2} - R_{sh3} = 0.01 - 0.005 = 0.005\Omega$$

$$R_3 = R_{sh3} - R_{sh4} = 0.005 - 0.0025 = 0.0025\Omega$$

$$R_4 = R_{sh4} = 0.0025\Omega$$

### Example: 1.18

A basic d' Arsonval meter movement with an internal resistance  $R_m = 100\Omega$  and a full scale current of  $I_m = 1\text{mA}$  is to be converted in to a multi range d.c. voltmeter with ranges of 0-10V, 0-50V, 0-250V, 0-500V. Find the values of various resistances using the potential divider arrangement.

**Solution:**

**Data given**

$$R_m = 100\Omega$$

$$I_m = 1mA$$

$$V_m = I_m \times R_m$$

$$V_m = 100 \times 1 \times 10^{-3}$$

$$V_m = 100mV$$

$$m_1 = \frac{V_1}{V_m} = \frac{10}{100 \times 10^{-3}} = 100$$

$$m_2 = \frac{V_2}{V_m} = \frac{50}{100 \times 10^{-3}} = 500$$

$$m_3 = \frac{V_3}{V_m} = \frac{250}{100 \times 10^{-3}} = 2500$$

$$m_4 = \frac{V_4}{V_m} = \frac{500}{100 \times 10^{-3}} = 5000$$

$$R_1 = (m_1 - 1)R_m = (100 - 1) \times 100 = 9900\Omega$$

$$R_2 = (m_2 - m_1)R_m = (500 - 100) \times 100 = 40K\Omega$$

$$R_3 = (m_3 - m_2)R_m = (2500 - 500) \times 100 = 200K\Omega$$

$$R_4 = (m_4 - m_3)R_m = (5000 - 2500) \times 100 = 250K\Omega$$

## AC BRIDGES

### 2.1 General form of A.C. bridge

AC bridge are similar to D.C. bridge in topology (way of connecting). It consists of four arm AB, BC, CD and DA. Generally the impedance to be measured is connected between 'A' and 'B'. A detector is connected between 'B' and 'D'. The detector is used as null deflection instrument. Some of the arms are variable element. By varying these elements, the potential values at 'B' and 'D' can be made equal. This is called balancing of the bridge.

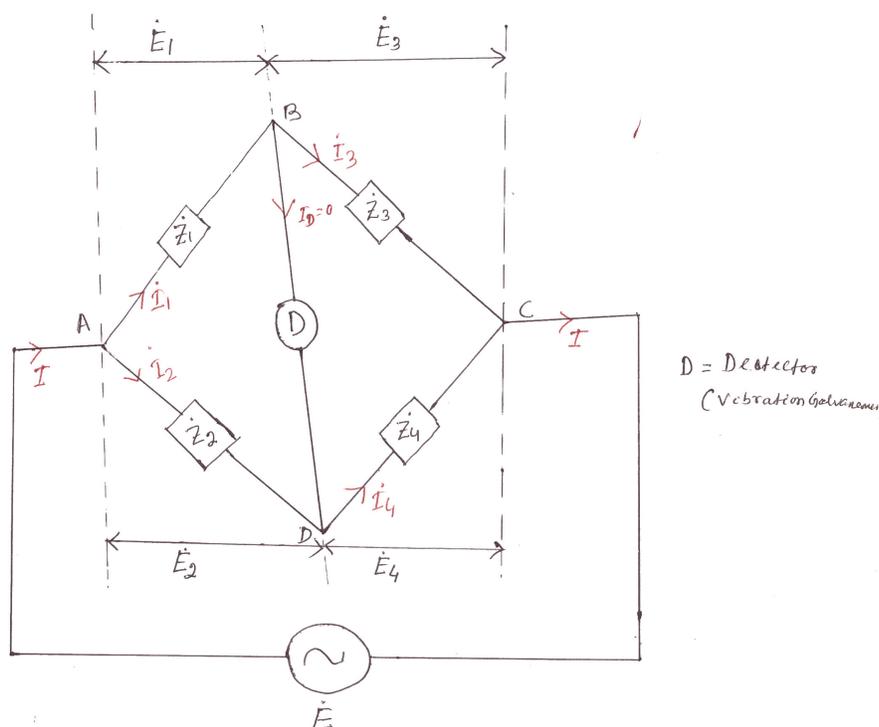


Fig. 2.1 General form of A.C. bridge

At the balance condition, the current through detector is zero.

$$\therefore \dot{I}_1 = \dot{I}_3$$

$$\dot{I}_2 = \dot{I}_4$$

$$\therefore \frac{\dot{I}_1}{\dot{I}_2} = \frac{\dot{I}_3}{\dot{I}_4} \quad (2.1)$$

At balance condition,

Voltage drop across 'AB'=voltage drop across 'AD'.

$$\dot{E}_1 = \dot{E}_2$$

$$\therefore \dot{I}_1 \dot{Z}_1 = \dot{I}_2 \dot{Z}_2 \quad (2.2)$$

Similarly, Voltage drop across 'BC'=voltage drop across 'DC'

$$\dot{E}_3 = \dot{E}_4$$

$$\therefore \dot{I}_3 \dot{Z}_3 = \dot{I}_4 \dot{Z}_4 \quad (2.3)$$

From Eqn. (2.2), we have  $\therefore \frac{\dot{I}_1}{\dot{I}_2} = \frac{\dot{Z}_2}{\dot{Z}_1}$  (2.4)

From Eqn. (2.3), we have  $\therefore \frac{\dot{I}_3}{\dot{I}_4} = \frac{\dot{Z}_4}{\dot{Z}_3}$  (2.5)

From equation -2.1, it can be seen that, equation -2.4 and equation-2.5 are equal.

$$\therefore \frac{\dot{Z}_2}{\dot{Z}_1} = \frac{\dot{Z}_4}{\dot{Z}_3}$$

$$\therefore \dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$$

Products of impedances of opposite arms are equal.

$$\therefore |Z_1| \angle \theta_1 |Z_4| \angle \theta_4 = |Z_2| \angle \theta_2 |Z_3| \angle \theta_3$$

$$\Rightarrow |Z_1| |Z_4| \angle \theta_1 + \theta_4 = |Z_2| |Z_3| \angle \theta_2 + \theta_3$$

$$|Z_1| |Z_4| = |Z_2| |Z_3|$$

$$\theta_1 + \theta_4 = \theta_2 + \theta_3$$

- \* For balance condition, magnitude on either side must be equal.
- \* Angle on either side must be equal.

### **Summary**

For balance condition,

- $I_1 = I_3, I_2 = I_4$
- $|Z_1||Z_4| = |Z_2||Z_3|$
- $\theta_1 + \theta_4 = \theta_2 + \theta_3$
- $E_1 = E_2 \quad \& \quad E_3 = E_4$

### **2.2 Types of detector**

The following types of instruments are used as detector in A.C. bridge.

- Vibration galvanometer
- Head phones (speaker)
- Tuned amplifier

#### **2.2.1 Vibration galvanometer**

Between the point 'B' and 'D' a vibration galvanometer is connected to indicate the bridge balance condition. This A.C. galvanometer which works on the principle of resonance. The A.C. galvanometer shows a dot, if the bridge is unbalanced.

#### **2.2.2 Head phones**

Two speakers are connected in parallel in this system. If the bridge is unbalanced, the speaker produced more sound energy. If the bridge is balanced, the speaker do not produced any sound energy.

#### **2.2.3 Tuned amplifier**

If the bridge is unbalanced the output of tuned amplifier is high. If the bridge is balanced, output of amplifier is zero.

## 2.3 Measurements of inductance

### 2.3.1 Maxwell's inductance bridge

The choke for which  $R_1$  and  $L_1$  have to measure connected between the points 'A' and 'B'. In this method the unknown inductance is measured by comparing it with the standard inductance.

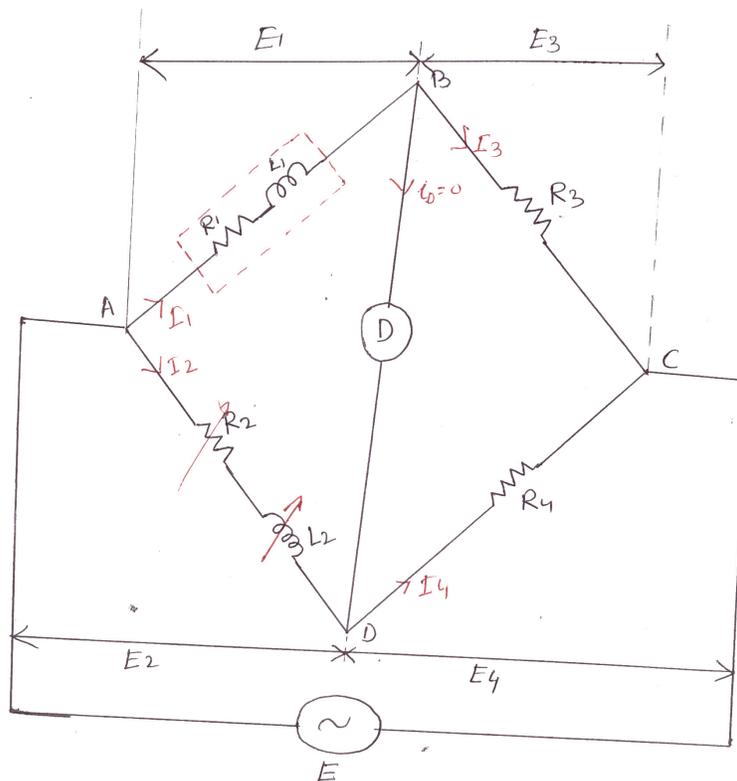


Fig. 2.2 Maxwell's inductance bridge

$L_2$  is adjusted, until the detector indicates zero current.

Let  $R_1$  = unknown resistance

$L_1$  = unknown inductance of the choke.

$L_2$  = known standard inductance

$R_1, R_2, R_4$  = known resistances.

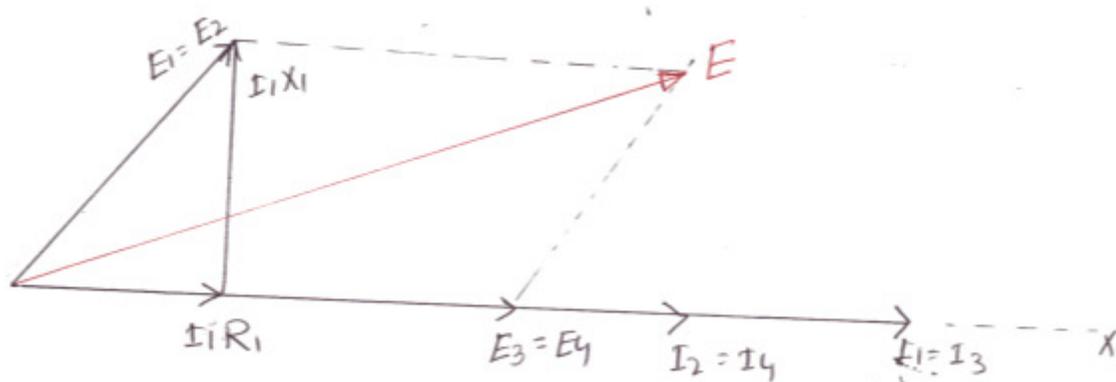


Fig 2.3 Phasor diagram of Maxwell's inductance bridge

At balance condition,  $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$(R_1 + jXL_1)R_4 = (R_2 + jXL_2)R_3$$

$$(R_1 + j\omega L_1)R_4 = (R_2 + j\omega L_2)R_3$$

$$R_1R_4 + j\omega L_1R_4 = R_2R_3 + j\omega L_2R_3$$

Comparing real part,

$$R_1R_4 = R_2R_3$$

$$\therefore R_1 = \frac{R_2R_3}{R_4} \quad (2.6)$$

Comparing the imaginary parts,

$$\omega L_1R_4 = \omega L_2R_3$$

$$L_1 = \frac{L_2R_3}{R_4} \quad (2.7)$$

$$\text{Q-factor of choke, } Q = \frac{\omega L_1}{R_1} = \frac{\omega L_2R_3R_4}{R_4R_2R_3}$$

$$Q = \frac{\omega L_2}{R_2} \quad (2.8)$$

### Advantages

- ✓ Expression for  $R_1$  and  $L_1$  are simple.
- ✓ Equations are simple
- ✓ They do not depend on the frequency (as  $\omega$  is cancelled)
- ✓  $R_1$  and  $L_1$  are independent of each other.

### Disadvantages

- ✓ Variable inductor is costly.
- ✓ Variable inductor is bulky.

### 2.3.2 Maxwell's inductance capacitance bridge

Unknown inductance is measured by comparing it with standard capacitance. In this bridge, balance condition is achieved by varying ' $C_4$ '.

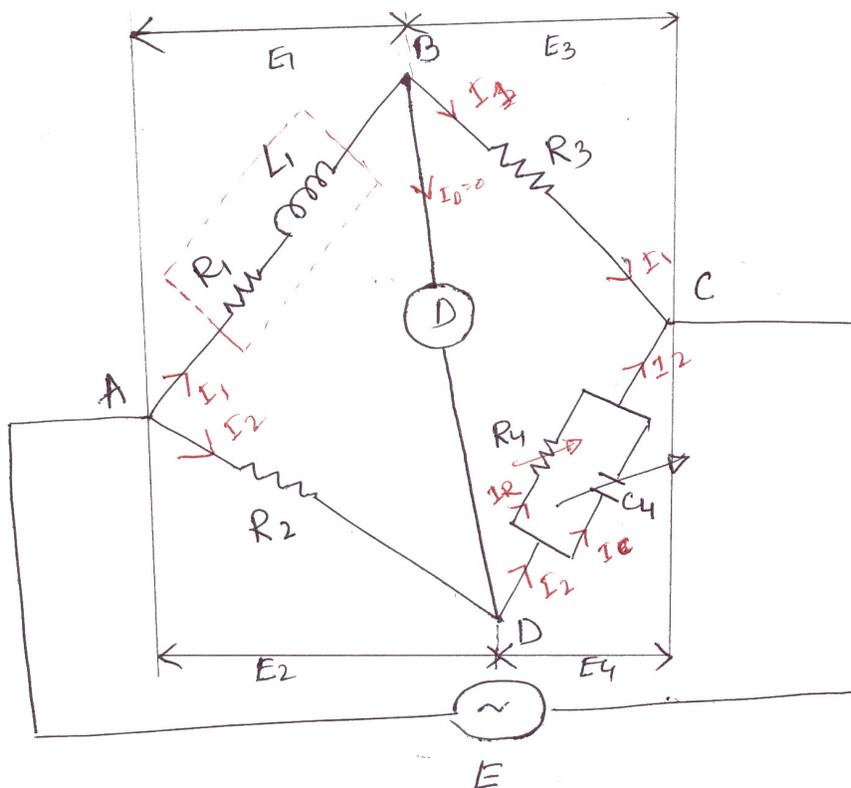


Fig 2.4 Maxwell's inductance capacitance bridge

At balance condition,  $Z_1 Z_4 = Z_3 Z_2$  (2.9)

$$Z_4 = R_4 \parallel \frac{1}{j\omega C_4} = \frac{R_4 \times \frac{1}{j\omega C_4}}{R_4 + \frac{1}{j\omega C_4}}$$

$$Z_4 = \frac{R_4}{j\omega R_4 C_4 + 1} = \frac{R_4}{1 + j\omega R_4 C_4} \quad (2.10)$$

∴ Substituting the value of  $Z_4$  from eqn. (2.10) in eqn. (2.9) we get

$$(R_1 + j\omega L_1) \times \frac{R_4}{1 + j\omega R_4 C_4} = R_2 R_3$$

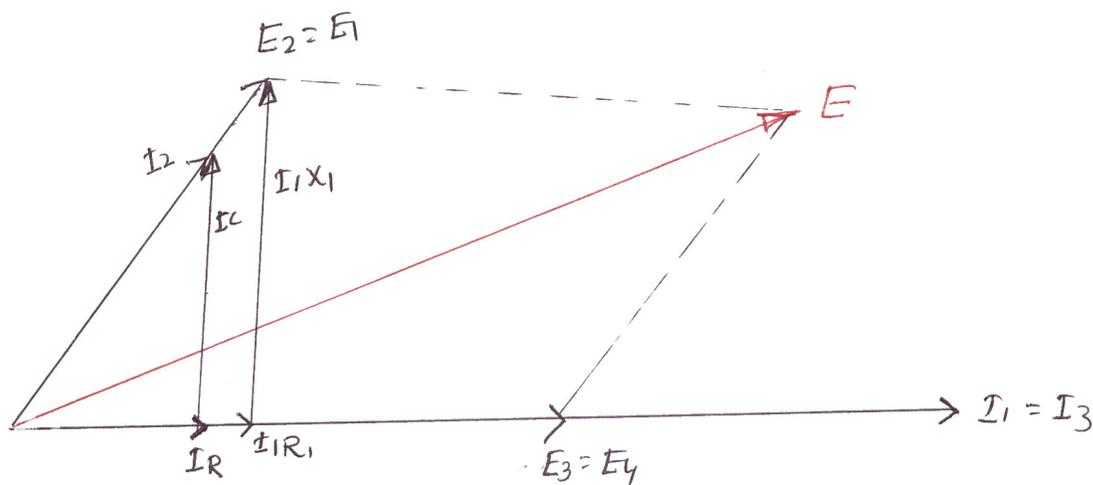


Fig 2.5 Phasor diagram of Maxwell's inductance capacitance bridge

$$(R_1 + j\omega L_1)R_4 = R_2 R_3 (1 + j\omega R_4 C_4)$$

$$R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega C_4 R_4 R_2 R_3$$

Comparing real parts,

$$R_1 R_4 = R_2 R_3$$

$$\Rightarrow R_1 = \frac{R_2 R_3}{R_4} \quad (2.11)$$

Comparing imaginary part,

$$\omega L_1 R_4 = \omega C_4 R_4 R_2 R_3$$

$$L_1 = C_4 R_2 R_3 \quad (2.12)$$

Q-factor of choke,

$$Q = \frac{\omega L_1}{R_1} = \omega \times C_4 R_2 R_3 \times \frac{R_4}{R_2 R_3}$$

$$Q = \omega C_4 R_4 \quad (2.13)$$

### Advantages

- ✓ Equation of  $L_1$  and  $R_1$  are simple.
- ✓ They are independent of frequency.
- ✓ They are independent of each other.
- ✓ Standard capacitor is much smaller in size than standard inductor.

### Disadvantages

- ✓ Standard variable capacitance is costly.
- ✓ It can be used for measurements of Q-factor in the ranges of 1 to 10.
- ✓ It cannot be used for measurements of choke with Q-factors more than 10.

We know that  $Q = \omega C_4 R_4$

For measuring chokes with higher value of Q-factor, the value of  $C_4$  and  $R_4$  should be higher. Higher values of standard resistance are very expensive. Therefore this bridge cannot be used for higher value of Q-factor measurements.

### 2.3.3 Hay's bridge

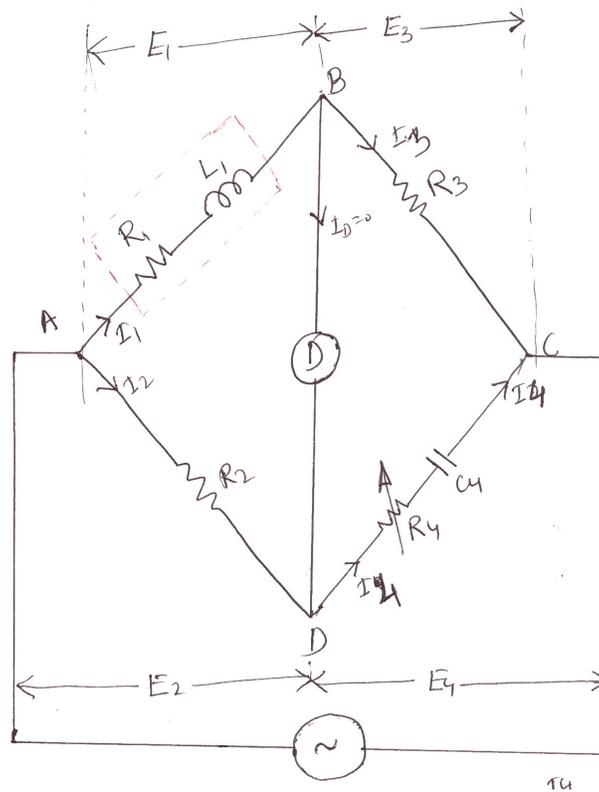


Fig 2.6 Hay's bridge

$$\text{➤ } \dot{E}_1 = I_1 R_1 + j I_1 X_1$$

$$\text{➤ } \dot{E} = \dot{E}_1 + \dot{E}_3$$

$$\text{➤ } \dot{E}_4 = I_4 R_4 + \frac{I_4}{j \omega C_4}$$

$$\text{➤ } \dot{E}_3 = I_3 R_3$$

$$Z_4 = R_4 + \frac{1}{j \omega C_4} = \frac{1 + j \omega R_4 C_4}{j \omega C_4}$$

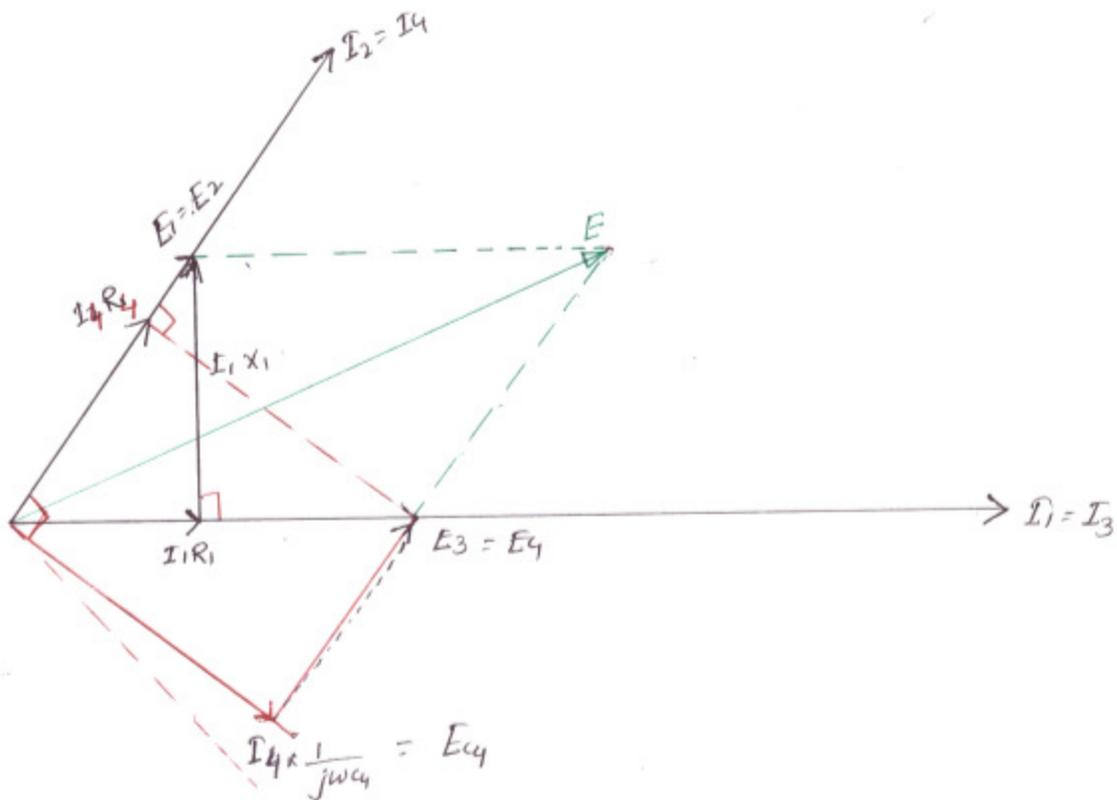


Fig 2.7 Phasor diagram of Hay's bridge

At balance condition,  $Z_1 Z_4 = Z_3 Z_2$

$$(R_1 + j\omega L_1) \left( \frac{1 + j\omega R_4 C_4}{j\omega C_4} \right) = R_2 R_3$$

$$(R_1 + j\omega L_1)(1 + j\omega R_4 C_4) = j\omega R_2 C_4 R_3$$

$$R_1 + j\omega C_4 R_4 R_1 + j\omega L_1 + j^2 \omega^2 L_1 C_4 R_4 = j\omega C_4 R_2 R_3$$

$$(R_1 - \omega^2 L_1 C_4 R_4) + j(\omega C_4 R_4 R_1 + \omega L_1) = j\omega C_4 R_2 R_3$$

Comparing the real term,

$$R_1 - \omega^2 L_1 C_4 R_4 = 0$$

$$R_1 = \omega^2 L_1 C_4 R_4 \quad (2.14)$$

Comparing the imaginary terms,

$$wC_4R_4R_1 + wL_1 = wC_4R_2R_3$$

$$C_4R_4R_1 + L_1 = C_4R_2R_3$$

$$L_1 = C_4R_2R_3 - C_4R_4R_1 \quad (2.15)$$

Substituting the value of  $R_1$  from eqn. 2.14 into eqn. 2.15, we have,

$$L_1 = C_4R_2R_3 - C_4R_4 \times w^2L_1C_4R_4$$

$$L_1 = C_4R_2R_3 - w^2L_1C_4^2R_4^2$$

$$L_1(1 + w^2L_1C_4^2R_4^2) = C_4R_2R_3$$

$$L_1 = \frac{C_4R_2R_3}{1 + w^2L_1C_4^2R_4^2} \quad (2.16)$$

Substituting the value of  $L_1$  in eqn. 2.14, we have

$$R_1 = \frac{w^2C_4^2R_2R_3R_4}{1 + w^2C_4^2R_4^2} \quad (2.17)$$

$$Q = \frac{wL_1}{R_1} = \frac{w \times C_4R_2R_3}{1 + w^2C_4^2R_4^2} \times \frac{1 + w^2C_4^2R_4^2}{w^2C_4^2R_4R_2R_3}$$

$$Q = \frac{1}{wC_4R_4} \quad (2.18)$$

### Advantages

- ✓ Fixed capacitor is cheaper than variable capacitor.
- ✓ This bridge is best suitable for measuring high value of Q-factor.

### Disadvantages

- ✓ Equations of  $L_1$  and  $R_1$  are complicated.
- ✓ Measurements of  $R_1$  and  $L_1$  require the value of frequency.
- ✓ This bridge cannot be used for measuring low Q- factor.

### 2.3.4 Owen's bridge

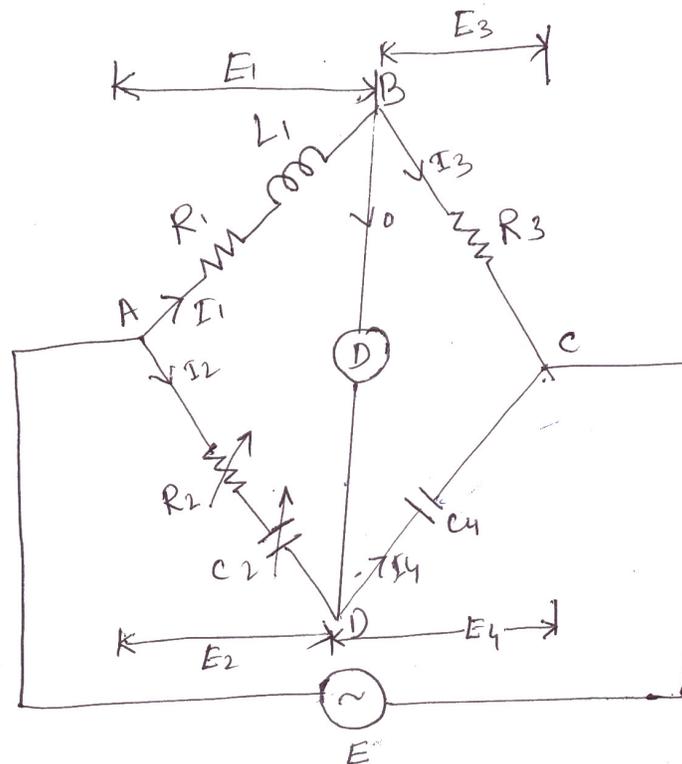


Fig 2.8 Owen's bridge

- $E_1 = I_1 R_1 + j I_1 X_1$
- $I_4$  leads  $E_4$  by  $90^\circ$

$$\triangleright \dot{E} = \dot{E}_1 + \dot{E}_3$$

$$\triangleright \dot{E}_2 = I_2 R_2 + \frac{I_2}{j\omega C_2}$$

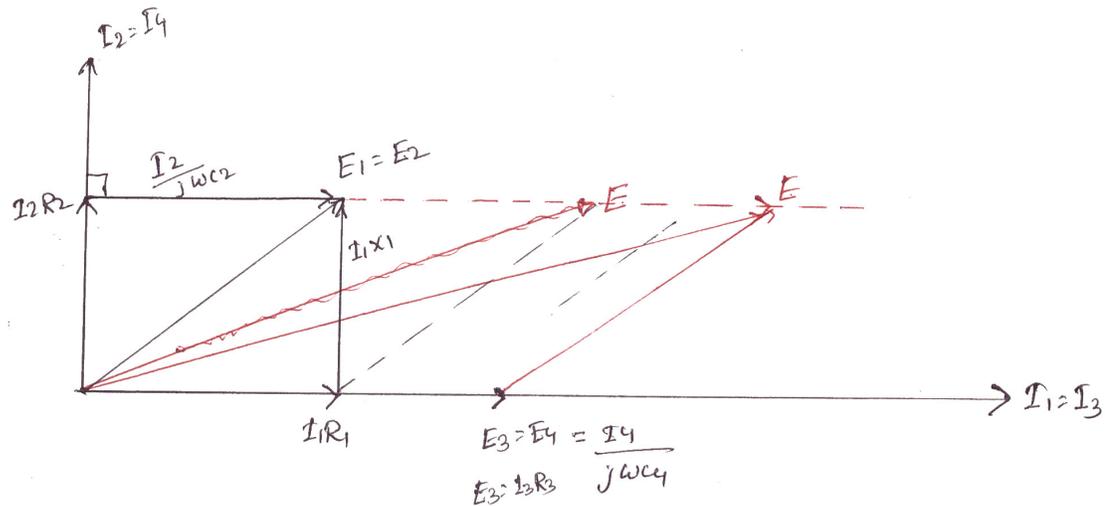


Fig 2.9 Phasor diagram of Owen's bridge

**Balance condition,**  $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$Z_2 = R_2 + \frac{1}{j\omega C_2} = \frac{j\omega C_2 R_2 + 1}{j\omega C_2}$$

$$\therefore (R_1 + j\omega L_1) \times \frac{1}{j\omega C_4} = \frac{(1 + j\omega R_2 C_2) \times R_3}{j\omega C_2}$$

$$C_2 (R_1 + j\omega L_1) = R_3 C_4 (1 + j\omega R_2 C_2)$$

$$R_1 C_2 + j\omega L_1 C_2 = R_3 C_4 + j\omega R_2 C_2 R_3 C_4$$

Comparing real terms,

$$R_1 C_2 = R_3 C_4$$

$$R_1 = \frac{R_3 C_4}{C_2}$$

Comparing imaginary terms,

$$\omega L_1 C_2 = \omega R_2 C_2 R_3 C_4$$

$$L_1 = R_2 R_3 C_4$$

$$Q\text{-factor} = \frac{\omega L_1}{R_1} = \frac{\omega R_2 R_3 C_4 C_2}{R_3 C_4}$$

$$Q = \omega R_2 C_2$$

### Advantages

- ✓ Expression for  $R_1$  and  $L_1$  are simple.
- ✓  $R_1$  and  $L_1$  are independent of Frequency.

### Disadvantages

- ✓ The Circuits used two capacitors.
- ✓ Variable capacitor is costly.
- ✓ Q-factor range is restricted.

### 2.3.5 Anderson's bridge

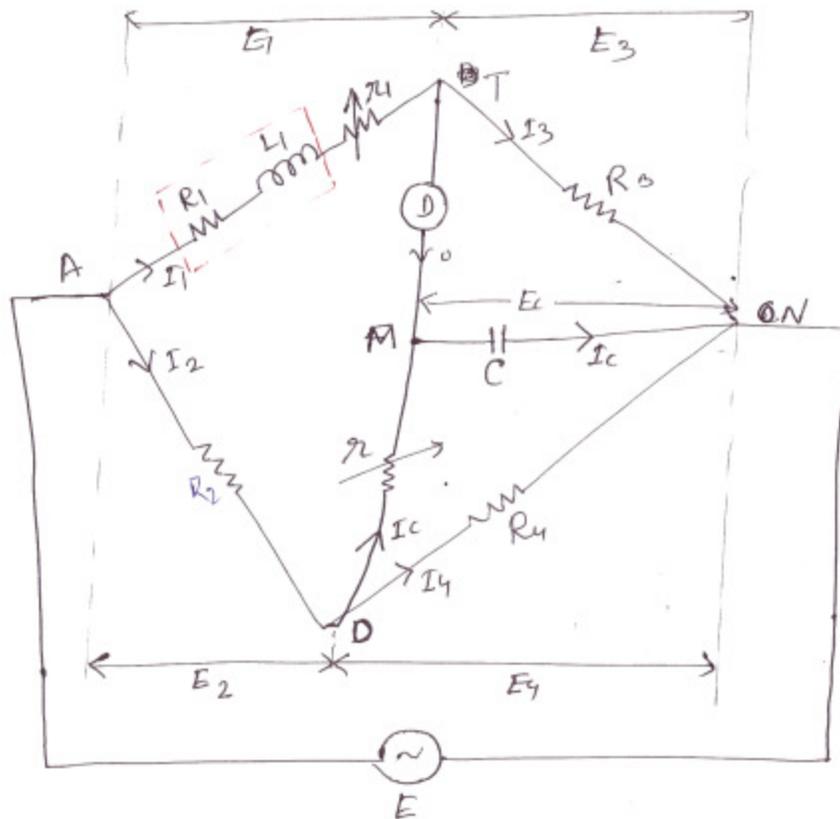


Fig 2.10 Anderson's bridge

- $\dot{E}_1 = I_1(R_1 + r_1) + jI_1X_1$
- $E_3 = E_C$
- $\dot{E}_4 = I_C r + E_C$
- $I_2 = I_4 + I_C$
- $\bar{E}_2 + \bar{E}_4 = \bar{E}$
- $\bar{E}_1 + \bar{E}_3 = \bar{E}$

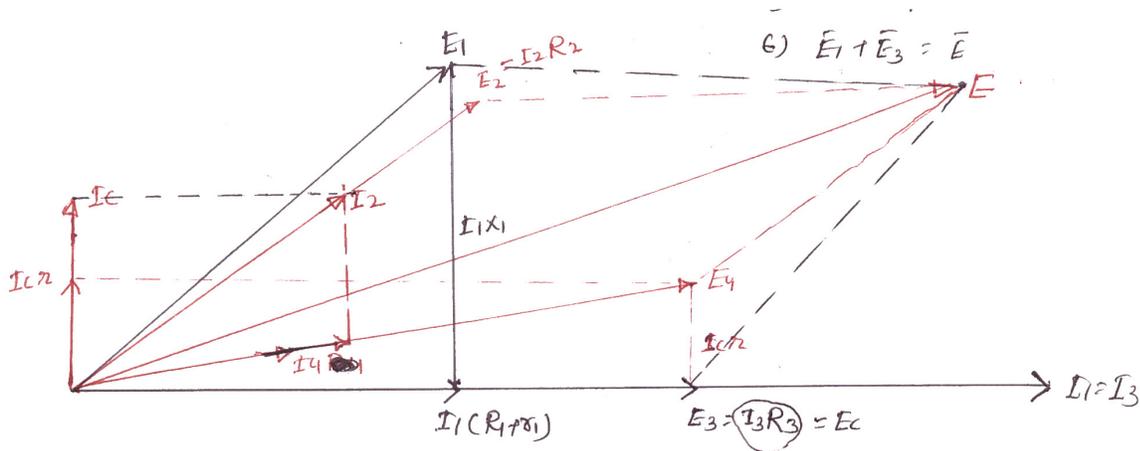


Fig 2.11 Phasor diagram of Anderson's bridge

**Step-1** Take  $I_1$  as reference vector. Draw  $I_1 R_1^1$  in phase with  $I_1$

$$R_1^1 = (R_1 + r_1) , I_1 X_1 \text{ is } \perp_r \text{ to } I_1 R_1^1$$

$$E_1 = I_1 R_1^1 + j I_1 X_1$$

**Step-2**  $I_1 = I_3$  ,  $E_3$  is in phase with  $I_3$  , From the circuit ,

$$E_3 = E_C , I_C \text{ leads } E_C \text{ by } 90^\circ$$

**Step-3**  $E_4 = I_C r + E_C$

**Step-4** Draw  $I_4$  in phase with  $E_4$  , By KCL ,  $I_2 = I_4 + I_C$

**Step-5** Draw  $E_2$  in phase with  $I_2$

**Step-6** By KVL ,  $\bar{E}_1 + \bar{E}_3 = \bar{E}$  or  $\bar{E}_2 + \bar{E}_4 = \bar{E}$

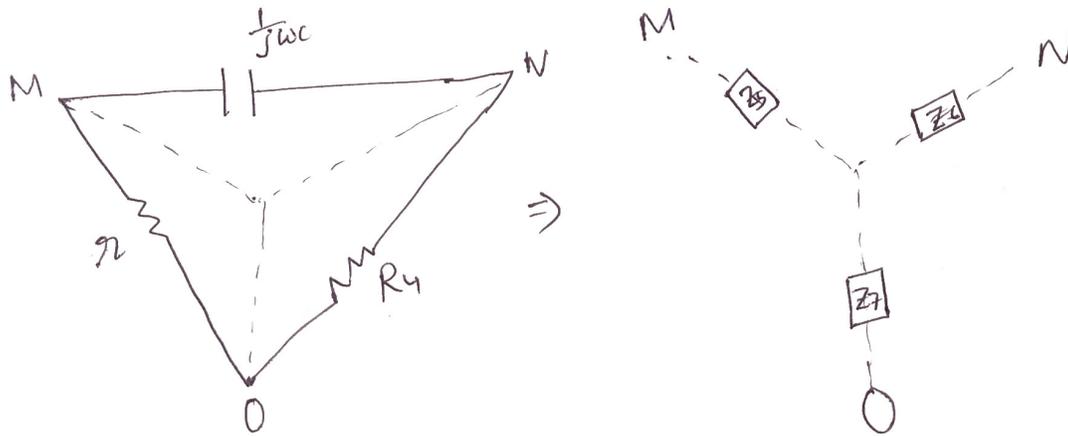


Fig 2.12 Equivalent delta to star conversion for the loop MON

$$Z_7 = \frac{R_4 \times r}{R_4 + r + \frac{1}{j\omega C}} = \frac{j\omega C R_4 r}{1 + j\omega C (R_4 + r)}$$

$$Z_6 = \frac{R_4 \times \frac{1}{j\omega C}}{R_4 + r + \frac{1}{j\omega C}} = \frac{R_4}{1 + j\omega C (R_4 + r)}$$

$$(R_1^1 + j\omega L_1) \times \frac{R_4}{1 + j\omega C (R_4 + r)} = R_3 \left( R_2 + \frac{j\omega C R_4 r}{1 + j\omega C (R_4 + r)} \right)$$

$$\Rightarrow \frac{(R_1^1 + j\omega L_1) R_4}{1 + j\omega C (R_4 + r)} = R_3 \left[ \frac{R_2 (1 + j\omega C (R_4 + r)) + j\omega C r R_4}{1 + j\omega C (R_4 + r)} \right]$$

$$\Rightarrow R_1^1 R_4 + j\omega L_1 R_4 = R_2 R_3 + j\omega C R_2 R_3 (r + R_4) + j\omega C r R_4 R_3$$

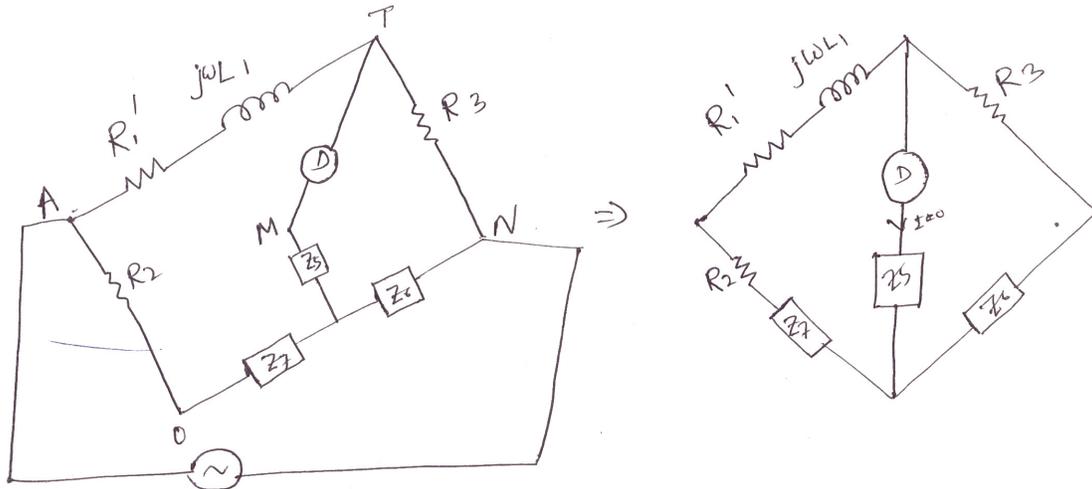


Fig 2.13 Simplified diagram of Anderson's bridge

Comparing real term,

$$R_1^1 R_4 = R_2 R_3$$

$$(R_1 + r_1) R_4 = R_2 R_3$$

$$R_1 = \frac{R_2 R_3}{R_4} - r_1$$

Comparing the imaginary term,

$$\omega L_1 R_4 = \omega C R_2 R_3 (r + R_4) + \omega C R_3 R_4$$

$$L_1 = \frac{R_2 R_3 C}{R_4} (r + R_4) + R_3 r C$$

$$L_1 = R_3 C \left[ \frac{R_2}{R_4} (r + R_4) + r \right]$$

### Advantages

- ✓ Variable capacitor is not required.
- ✓ Inductance can be measured accurately.
- ✓  $R_1$  and  $L_1$  are independent of frequency.
- ✓ Accuracy is better than other bridges.

## Disadvantages

- ✓ Expression for  $R_1$  and  $L_1$  are complicated.
- ✓ This is not in the standard form A.C. bridge.

## 2.4 Measurement of capacitance and loss angle. (Dissipation factor)

### 2.4.1 Dissipation factors (D)

A practical capacitor is represented as the series combination of small resistance and ideal capacitance.

From the vector diagram, it can be seen that the angle between voltage and current is slightly less than  $90^\circ$ . The angle ' $\delta$ ' is called loss angle.

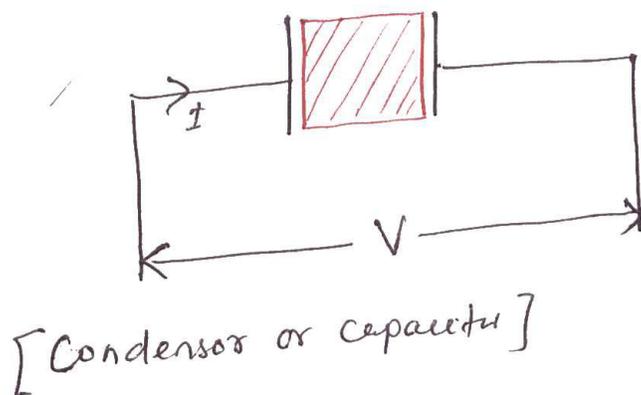


Fig 2.14 Condensor or capacitor

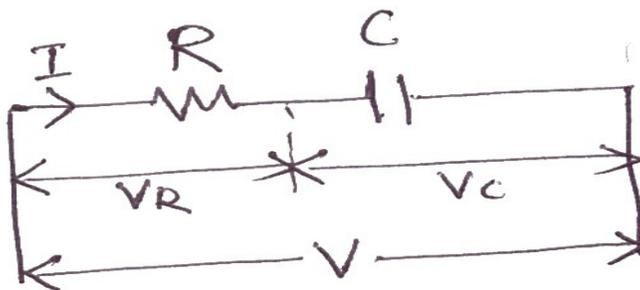


Fig 2.15 Representation of a practical capacitor

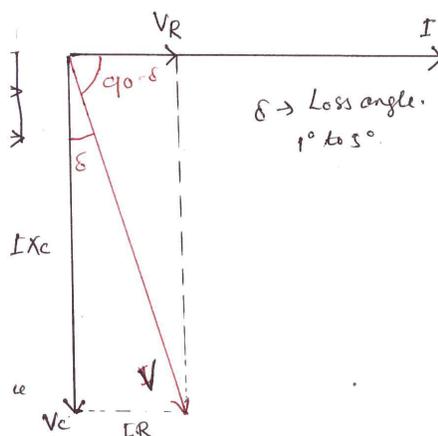


Fig 2.16 Vector diagram for a practical capacitor

A dissipation factor is defined as 'tan  $\delta$ '.

$$\therefore \tan \delta = \frac{IR}{IX_C} = \frac{R}{X_C} = \omega CR$$

$$D = \omega CR$$

$$D = \frac{1}{Q}$$

$$D = \tan \delta = \frac{\sin \delta}{\cos \delta} \cong \frac{\delta}{1} \quad \text{For small value of ' } \delta \text{ ' in radians}$$

$$D \cong \delta \cong \text{Loss Angle} \quad (\delta \text{ must be in radian)}$$

### 2.4.2 Desauty's Bridge

$C_1 =$  Unknown capacitance

At balance condition,

$$\frac{1}{j\omega C_1} \times R_4 = \frac{1}{j\omega C_2} \times R_3$$

$$\frac{R_4}{C_1} = \frac{R_3}{C_2}$$

$$\Rightarrow C_1 = \frac{R_4 C_2}{R_3}$$

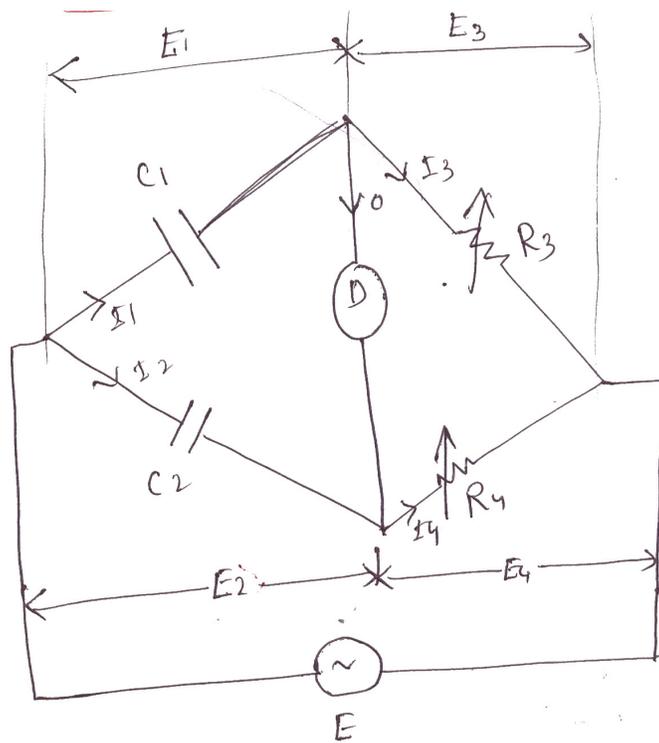


Fig 2.17 Desauty's bridge

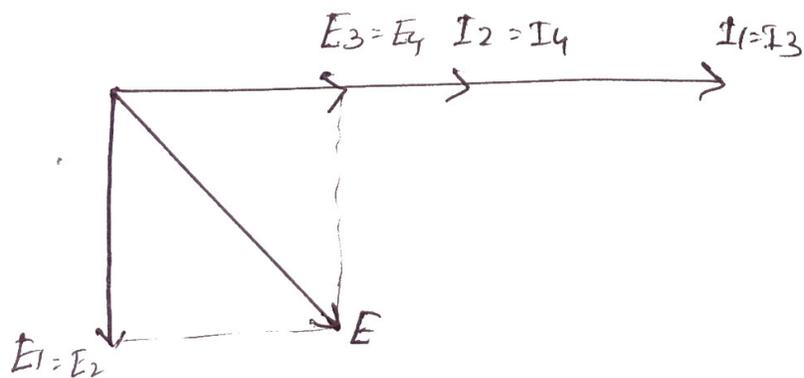


Fig 2.18 Phasor diagram of Desauty's bridge

2.4.3 Modified desauty's bridge

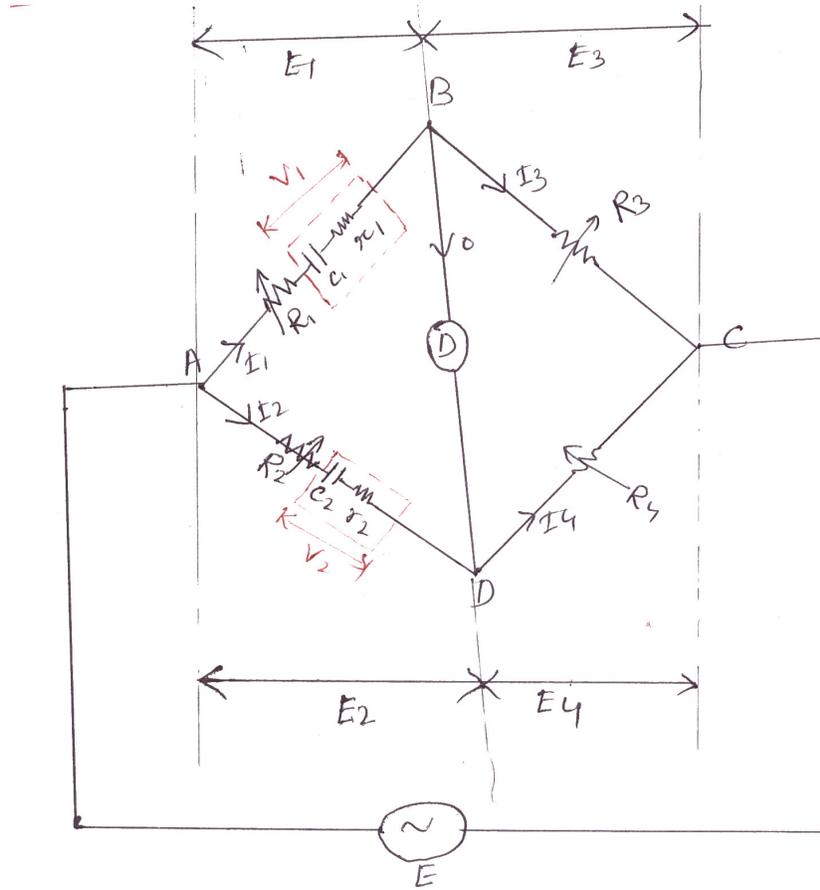


Fig 2.19 Modified Desauty's bridge

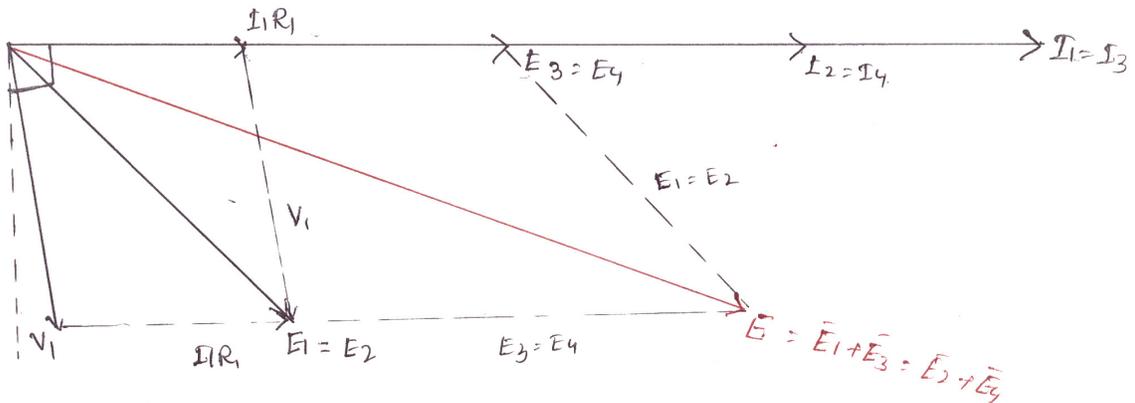


Fig 2.20 Phasor diagram of Modified Desauty's bridge

$$R_1^1 = (R_1 + r_1)$$

$$R_2^1 = (R_2 + r_2)$$

$$\text{At balance condition, } (R_1^1 + \frac{1}{j\omega C_1})R_4 = R_3(R_2^1 + \frac{1}{j\omega C_2})$$

$$R_1^1 R_4 + \frac{R_4}{j\omega C_1} = R_3 R_2^1 + \frac{R_3}{j\omega C_2}$$

$$\text{Comparing the real term, } R_1^1 R_4 = R_3 R_2^1$$

$$R_1^1 = \frac{R_3 R_2^1}{R_4}$$

$$R + r_1 = \frac{(R_2 + r_2)R_3}{R_4}$$

Comparing imaginary term,

$$\frac{R_4}{\omega C_1} = \frac{R_3}{\omega C_2}$$

$$C_1 = \frac{R_4 C_2}{R_3}$$

Dissipation factor  $D = \omega C_1 r_1$

### Advantages

- ✓  $r_1$  and  $c_1$  are independent of frequency.
- ✓ They are independent of each other.
- ✓ Source need not be pure sine wave.

### 2.4.4 Schering bridge

$$E_1 = I_1 r_1 - jI_1 X_4$$

$C_2 = C_4 =$  Standard capacitor (Internal resistance=0)

$C_4 =$  Variable capacitance.

$C_1 =$  Unknown capacitance.

$r_1 =$  Unknown series equivalent resistance of the capacitor.

$R_3=R_4=$  Known resistor.

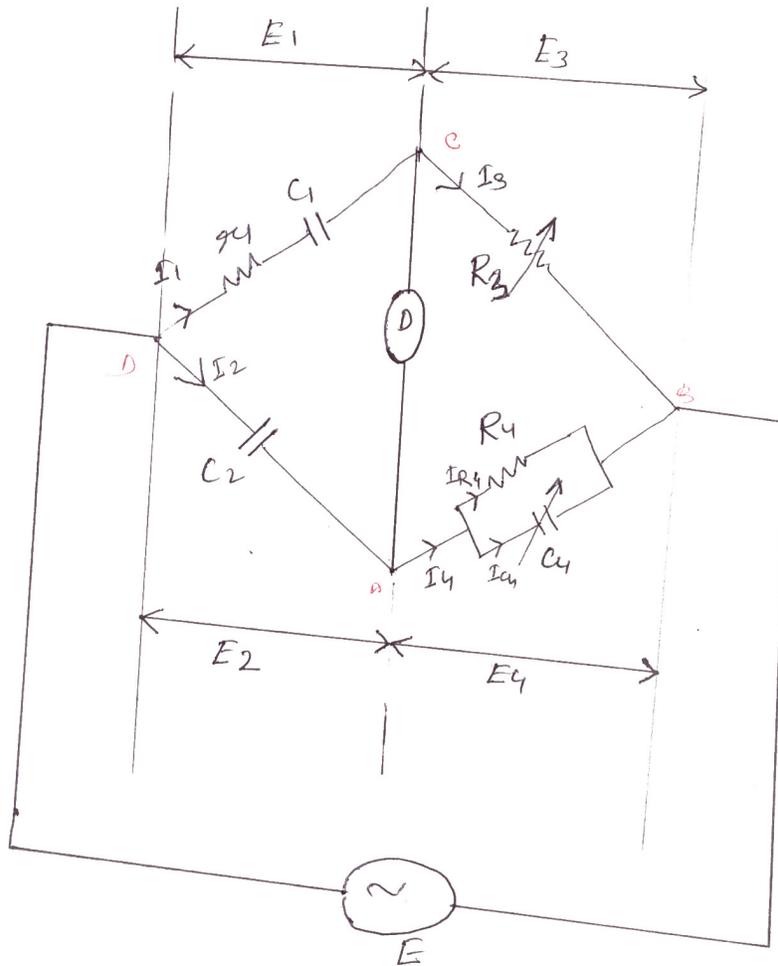


Fig 2.21 Schering bridge

$$Z_1 = r_1 + \frac{1}{j\omega C_1} = \frac{j\omega C_1 r_1 + 1}{j\omega C_1}$$

$$Z_4 = \frac{R_4 \times \frac{1}{j\omega C_4}}{R_4 + \frac{1}{j\omega C_4}} = \frac{R_4}{1 + j\omega C_4 R_4}$$



$$D = wC_1r_1 = w \times \frac{R_4C_2}{R_3} \times \frac{C_4R_3}{C_2}$$

$$\therefore D = wC_4R_4$$

### Advantages

- ✓ In this type of bridge, the value of capacitance can be measured accurately.
- ✓ It can measure capacitance value over a wide range.
- ✓ It can measure dissipation factor accurately.

### Disadvantages

- ✓ It requires two capacitors.
- ✓ Variable standard capacitor is costly.

## 2.5 Measurements of frequency

### 2.5.1 Wein's bridge

Wein's bridge is popularly used for measurements of frequency of frequency. In this bridge, the value of all parameters are known. The source whose frequency has to measure is connected as shown in the figure.

$$Z_1 = r_1 + \frac{1}{j\omega C_1} = \frac{j\omega C_1 r_1 + 1}{j\omega C_1}$$

$$Z_2 = \frac{R_2}{1 + j\omega C_2 R_2}$$

At balance condition,  $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$\frac{j\omega C_1 r_1 + 1}{j\omega C_1} \times R_4 = \frac{R_2}{1 + j\omega C_2 R_2} \times R_3$$

$$(1 + j\omega C_1 r_1)(1 + j\omega C_2 R_2)R_4 = R_2 R_3 \times j\omega C_1$$

$$\left[ 1 + j\omega C_2 R_2 + j\omega C_1 r_1 - \omega^2 C_1 C_2 r_1 R_2 \right] = j\omega C_1 \frac{R_2 R_3}{R_4}$$

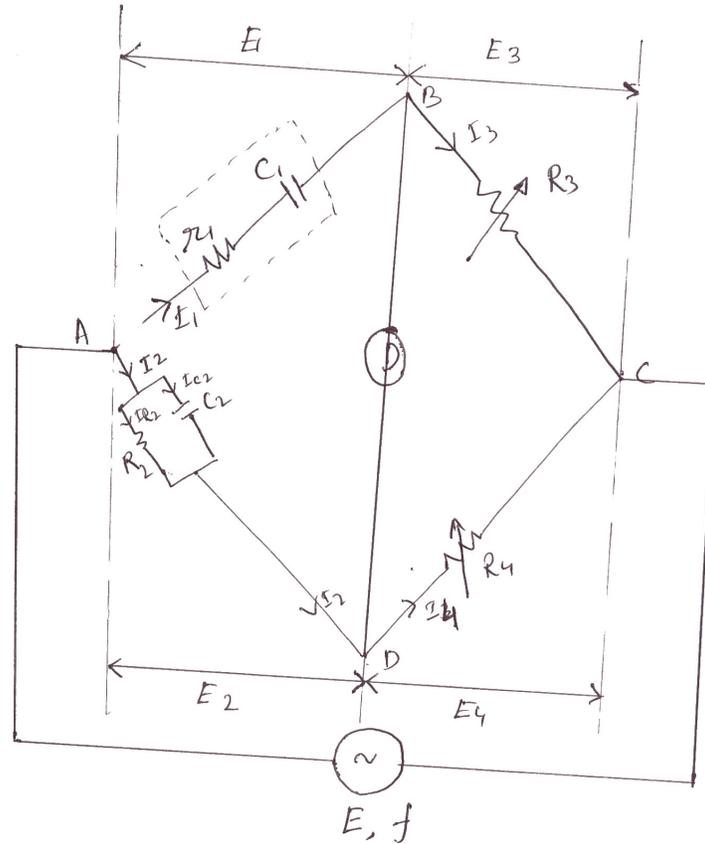


Fig 2.23 Wein's bridge

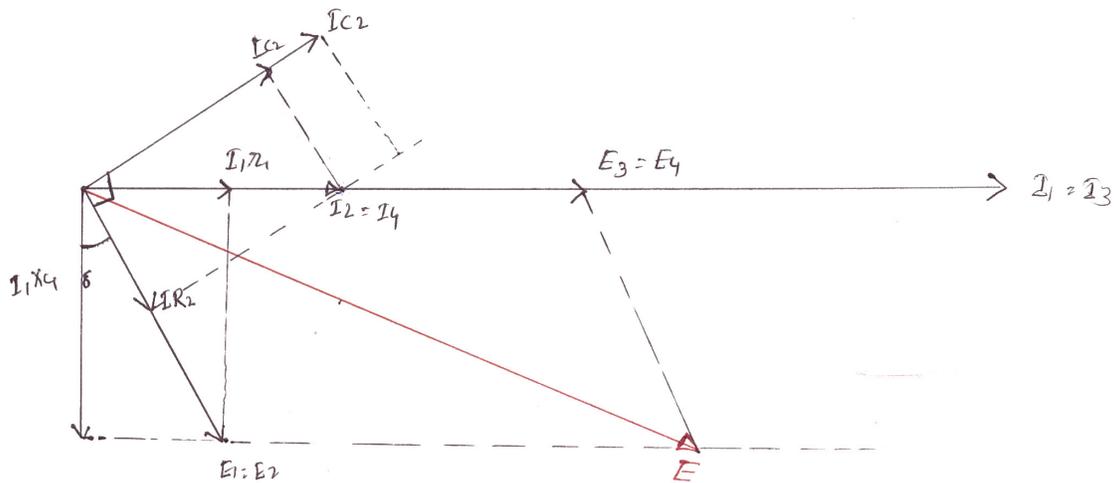


Fig 2.24 Phasor diagram of Wein's bridge

Comparing real term,

$$1 - \omega^2 C_1 C_2 r_1 R_2 = 0$$

$$\omega^2 C_1 C_2 r_1 R_2 = 1$$

$$\omega^2 = \frac{1}{C_1 C_2 r_1 R_2}$$

$$\omega = \frac{1}{\sqrt{C_1 C_2 r_1 R_2}}, \quad f = \frac{1}{2\pi \sqrt{C_1 C_2 r_1 R_2}}$$

### **NOTE**

The above bridge can be used for measurements of capacitance. In such case,  $r_1$  and  $C_1$  are unknown and frequency is known. By equating real terms, we will get  $R_1$  and  $C_1$ . Similarly by equating imaginary term, we will get another equation in terms of  $r_1$  and  $C_1$ . It is only used for measurements of Audio frequency.

A.F=20 HZ to 20 KHZ

R.F=>> 20 KHZ

Comparing imaginary term,

$$\omega C_2 R_2 + \omega C_1 r_1 = \omega C_1 \frac{R_2 R_3}{R_4}$$

$$C_2 R_2 + C_1 r_1 = \frac{C_1 R_2 R_3}{R_4} \dots\dots\dots(2.19)$$

$$C_1 = \frac{1}{\omega^2 C_2 r_1 R_2}$$

Substituting in eqn. (2.19), we have

$$C_2 R_2 + \frac{r_1}{\omega^2 C_2 r_1 R_2} = \frac{R_2 R_3}{R_4} C_1$$

Multiplying  $\frac{R_4}{R_2 R_3}$  in both sides, we have

$$C_2 R_2 \times \frac{R_4}{R_2 R_3} + \frac{1}{\omega^2 C_2 R_2} \times \frac{R_4}{R_2 R_3} = C_1$$

$$C_1 = \frac{C_2 R_4}{R_3} + \frac{R_4}{w^2 C_2 R_2^2 R_3}$$

$$w^2 C_1 \eta_1 C_2 R_2 = 1$$

$$\eta_1 = \frac{1}{w^2 C_2 R_2 C_1} = \frac{1}{w^2 C_2 R_2 \left[ \frac{C_2 R_4}{R_3} + \frac{R_4}{w^2 C_2 R_2^2 R_3} \right]}$$

$$= \frac{1}{\left[ \frac{w^2 C_2^2 R_2 R_4}{R_3} + \frac{R_4}{R_2 R_3} \right]}$$

$$\therefore \eta_1 = \frac{1}{\frac{R_3}{R_4} \left[ w^2 C_2^2 R_2 + \frac{1}{R_2} \right]}$$

$$\therefore \eta_1 = \frac{R_3}{R_4} \left[ \frac{1}{w^2 C_2^2 R_2 + \frac{1}{R_2}} \right]$$

### 2.5.2 High Voltage Schering Bridge

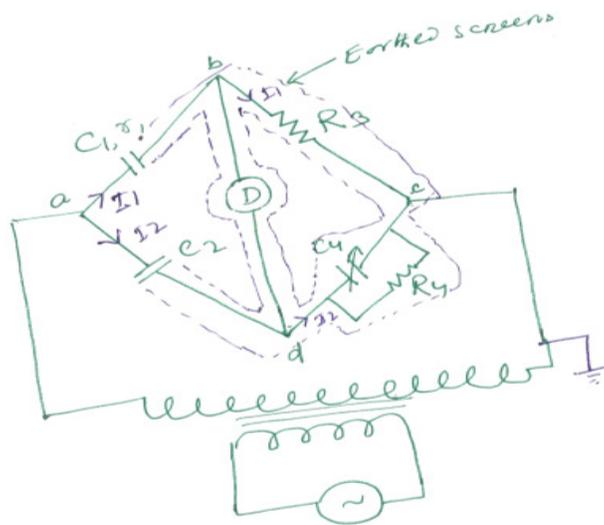


Fig 2.25 High Voltage Schering bridge



- ✓ Waveform error (due to harmonics)
- ✓ Residual error: small inductance and small capacitance of the resistor produce this error.

### **Precaution**

- ✓ The load inductance is eliminated by twisting the connecting the connecting lead.
- ✓ In the case of capacitive bridge, the connecting lead are kept apart. ( $\because C = \frac{A\epsilon_0\epsilon_r}{d}$ )
- ✓ In the case of inductive bridge, the various arm are magnetically screen.
- ✓ In the case of capacitive bridge, the various arm are electro statically screen to reduced the stray capacitance between various arm.
- ✓ To avoid the problem of spike, an inter bridge transformer is used in between the source and bridge.
- ✓ The stray capacitance between the ends of detector to the ground, cause difficulty in balancing as well as error in measurements. To avoid this problem, we use wagner earthing device.

### **2.7 Ballistic galvanometer**

This is a sophisticated instrument. This works on the principle of PMMC meter. The only difference is the type of suspension is used for this meter. Lamp and glass scale method is used to obtain the deflection. A small mirror is attached to the moving system. Phosphorous bronze wire is used for suspension.

When the D.C. voltage is applied to the terminals of moving coil, current flows through it. When a current carrying coil kept in the magnetic field, produced by permanent magnet, it experiences a force. The coil deflects and mirror deflects. The light spot on the glass scale also move. This deflection is proportional to the current through the coil.

$$i = \frac{Q}{t}, Q = it = \int idt$$

$$\theta \propto Q, \text{ deflection} \propto \text{Charge}$$

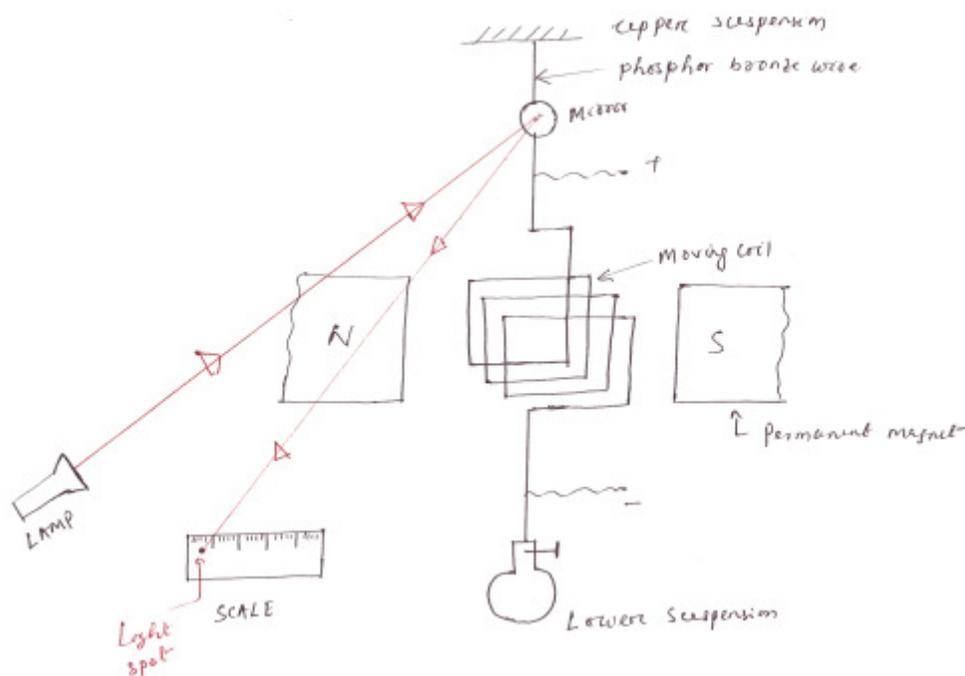


Fig 2.27 Ballistic galvanometer

### 2.8 Measurements of flux and flux density (Method of reversal)

D.C. voltage is applied to the electromagnet through a variable resistance  $R_1$  and a reversing switch. The voltage applied to the toroid can be reversed by changing the switch from position 2 to position '1'. Let the switch be in position '2' initially. A constant current flows through the toroid and a constant flux is established in the core of the magnet.

A search coil of few turns is provided on the toroid. The B.G. is connected to the search coil through a current limiting resistance. When it is required to measure the flux, the switch is changed from position '2' to position '1'. Hence the flux reduced to zero and it starts increasing in the reverse direction. The flux goes from  $+\phi$  to  $-\phi$ , in time 't' second. An emf is induced in the search coil, since the flux changes with time. This emf circulates a current through  $R_2$  and B.G. The meter deflects. The switch is normally closed. It is opened when it is required to take the reading.

### 2.8.1 Plotting the BH curve

The curve drawn with the current on the X-axis and the flux on the Y-axis, is called magnetization characteristics. The shape of B-H curve is similar to shape of magnetization characteristics. The residual magnetism present in the specimen can be removed as follows.

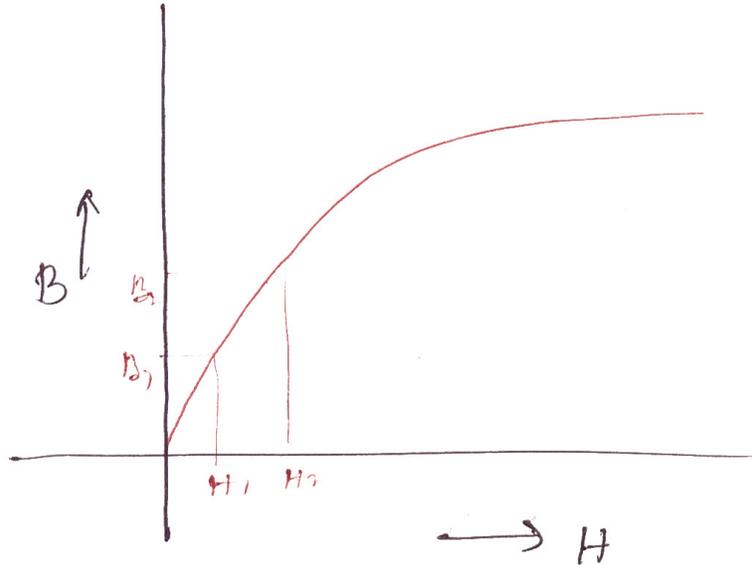


Fig 2.28 BH curve

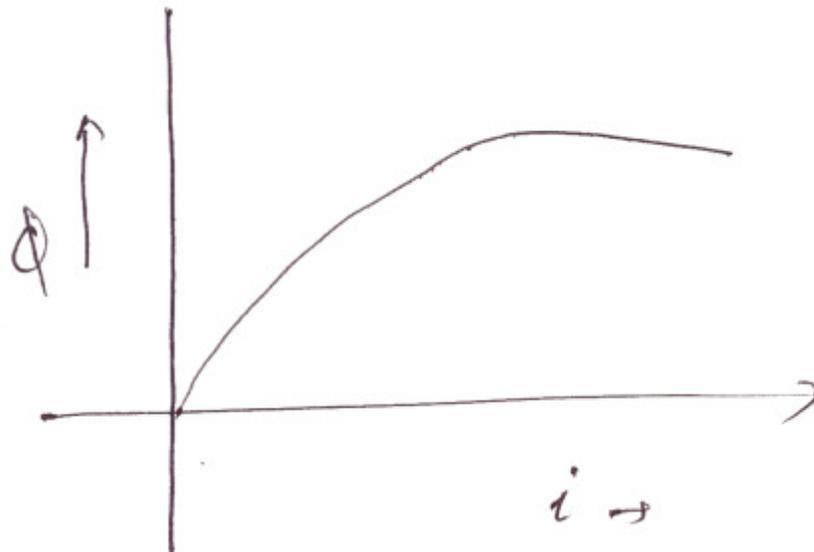


Fig 2.29 Magnetization characteristics

Close the switch 'S<sub>2</sub>' to protect the galvanometer, from high current. Change the switch S1 from position '1' to '2' and vice versa for several times.

To start with the resistance 'R<sub>1</sub>' is kept at maximum resistance position. For a particular value of current, the deflection of B.G. is noted. This process is repeated for various value of current. For each deflection flux can be calculated. ( $B = \frac{\phi}{A}$ )

Magnetic field intensity value for various current can be calculated.().The B-H curve can be plotted by using the value of 'B' and 'H'.

### 2.8.2 Measurements of iron loss:

Let R<sub>p</sub>= pressure coil resistance

R<sub>s</sub> = resistance of coil S1

E= voltage reading= Voltage induced in S<sub>2</sub>

I= current in the pressure coil

V<sub>p</sub>= Voltage applied to wattmeter pressure coil.

W= reading of wattmeter corresponding voltage V

W<sub>1</sub>= reading of wattmeter corresponding voltage E

$$\begin{array}{l} W \rightarrow V \\ W_1 \rightarrow E_p \end{array} \quad \frac{W_1}{W} = \frac{E}{V} \Rightarrow W_1 = \frac{E \times W}{V}$$

W<sub>1</sub>=Total loss=Iron loss+ Copper loss.

The above circuit is similar to no load test of transformer.

In the case of no load test the reading of wattmeter is approximately equal to iron loss. Iron loss depends on the emf induced in the winding. Science emf is directly proportional to flux. The voltage applied to the pressure coil is V. The corresponding of wattmeter is 'W'. The iron loss corresponding E is  $E = \frac{WE}{V}$ . The reading of the wattmeter includes the losses in the pressure coil and copper loss of the winding S1. These loses have to be subtracted to get the actual iron loss.

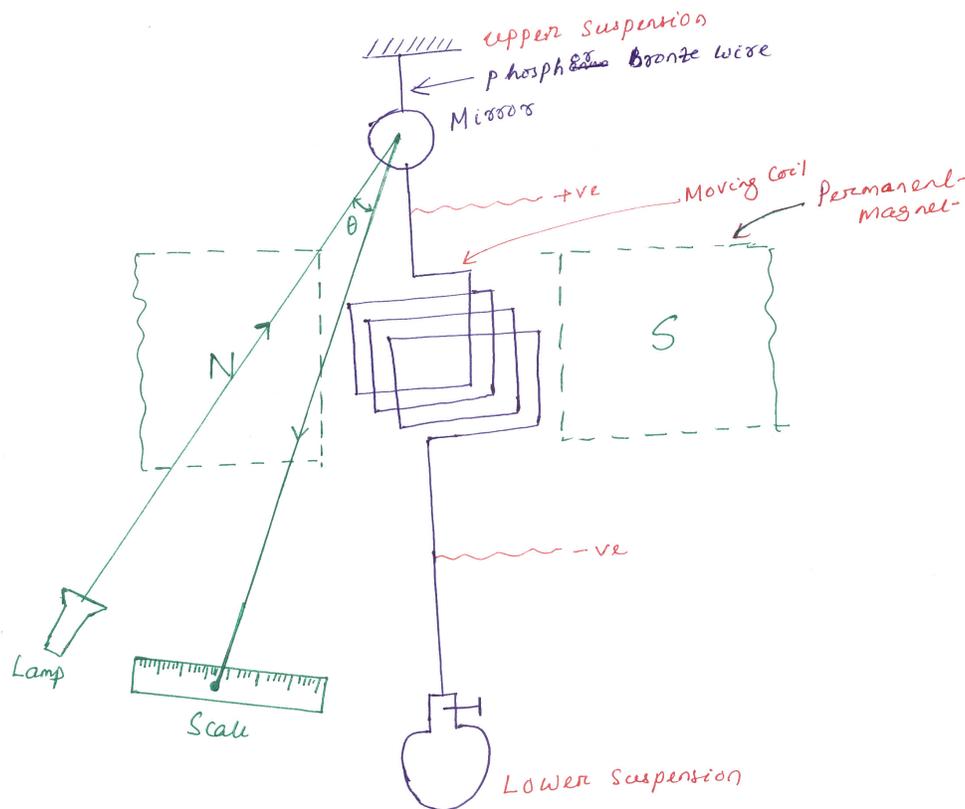
## 2.9 Galvanometers

D-Arsonval Galvanometer

Vibration Galvanometer

Ballistic C

### 2.9.1 D-aronval galvanometer (d.c. galvanometer)



**Fig 2.30** D-Arsonval Galvanometer

Galvanometer is a special type of ammeter used for measuring  $\mu A$  or mA. This is a sophisticated instrument. This works on the principle of PMMC meter. The only difference is the type of suspension used for this meter. It uses a sophisticated suspension called taut suspension, so that moving system has negligible weight.

Lamp and glass scale method is used to obtain the deflection. A small mirror is attached to the moving system. Phosphor bronze is used for suspension.

When D.C. voltage is applied to the terminal of moving coil, current flows through it. When current carrying coil is kept in the magnetic field produced by P.M. , it experiences a force. The light spot on the glass scale also move. This deflection is proportional to the current through the coil. This instrument can be used only with D.C. like PMMC meter.

The deflecting Torque,

$$T_D = BINA$$

$$T_D = GI, \quad \text{Where } G = BAN$$

$$T_C = K_S \theta = S \theta$$

$$\text{At balance, } T_C = T_D \Rightarrow S \theta = GI$$

$$\therefore \theta = \frac{GI}{S}$$

Where G= Displacements constant of Galvanometer

S=Spring constant

### **2.9.2 Vibration Galvanometer (A.C. Galvanometer)**

The construction of this galvanometer is similar to the PMMC instrument except for the moving system. The moving coil is suspended using two ivory bridge pieces. The tension of the system can be varied by rotating the screw provided at the top suspension. The natural frequency can be varied by varying the tension wire of the screw or varying the distance between ivory bridge piece.

When A.C. current is passed through coil an alternating torque or vibration is produced. This vibration is maximum if the natural frequency of moving system coincide with supply frequency. Vibration is maximum, since resonance takes place. When the coil is vibrating , the mirror oscillates and the dot moves back and front. This appears as a line on the glass scale. Vibration galvanometer is used for null deflection of a dot appears on the scale. If the bridge is unbalanced, a line appears on the scale

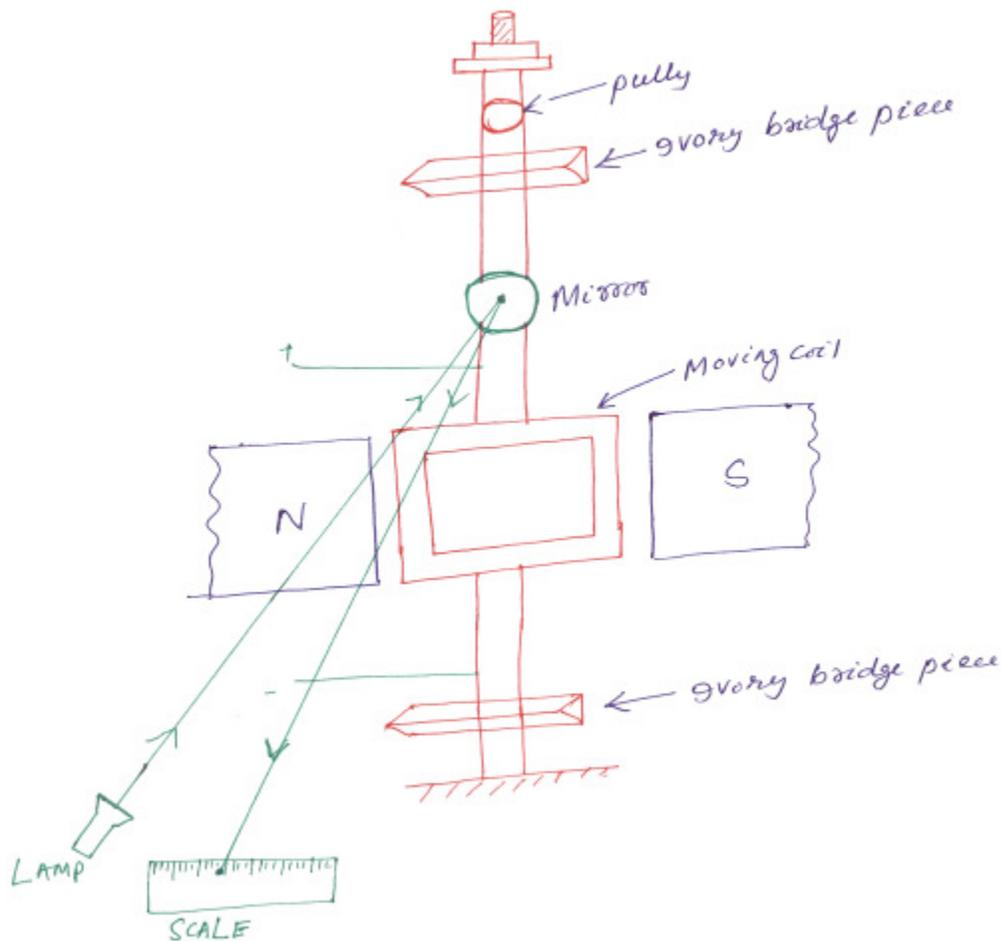
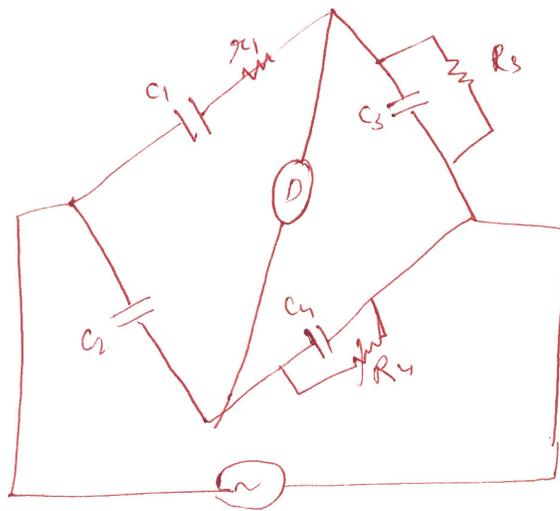


Fig 2.31 Vibration Galvanometer

**Example 2.2-**In a low- Voltage Schering bridge designed for the measurement of permittivity, the branch 'ab' consists of two electrodes between which the specimen under test may be inserted, arm 'bc' is a non-reactive resistor  $R_3$  in parallel with a standard capacitor  $C_3$ , arm CD is a non-reactive resistor  $R_4$  in parallel with a standard capacitor  $C_4$ , arm 'da' is a standard air capacitor of capacitance  $C_2$ . Without the specimen between the electrode, balance is obtained with following values ,  $C_3=C_4=120$  pF,  $C_2=150$  pF,  $R_3=R_4=5000\Omega$ . With the specimen inserted, these values become  $C_3=200$  pF,  $C_4=1000$  pF,  $C_2=900$  pF and  $R_3=R_4=5000\Omega$ . In such test  $\omega=5000$  rad/sec. Find the relative permittivity of the specimen?

**Sol:** Relative permittivity( $\epsilon_r$ ) =  $\frac{\text{capacitance measured with given medium}}{\text{capacitance measured with air medium}}$



**Fig 2.32** Schering bridge

$$C_1 = C_2 \left( \frac{R_4}{R_3} \right)$$

Let capacitance value  $C_0$ , when without specimen dielectric.

Let the capacitance value  $C_S$  when with the specimen dielectric.

$$C_0 = C_2 \left( \frac{R_4}{R_3} \right) = 150 \times \frac{5000}{5000} = 150 \text{ pF}$$

$$C_S = C_2 \left( \frac{R_4}{R_3} \right) = 900 \times \frac{5000}{5000} = 900 \text{ pF}$$

$$\epsilon_r = \frac{C_S}{C_0} = \frac{900}{150} = 6$$

**Example 2.3-** A specimen of iron stamping weighting 10 kg and having a area of  $16.8 \text{ cm}^2$  is tested by an Epstein square. Each of the two winding  $S_1$  and  $S_2$  have 515 turns. A.C. voltage of 50 HZ frequency is given to the primary. The current in the primary is 0.35 A. A voltmeter connected to  $S_2$  indicates 250 V. Resistance of  $S_1$  and  $S_2$  each equal to  $40 \Omega$ . Resistance of pressure coil is  $80 \text{ k}\Omega$ . Calculate maximum flux density in the specimen and iron loss/kg if the wattmeter indicates 80 watt?

$$\text{Sol}^n \quad E = 4.44 f \phi_m N$$

$$B_m = \frac{E}{4.44 f AN} = 1.3 \text{wb/m}^2$$

$$\text{Iron loss} = W \left(1 + \frac{R_S}{R_P}\right) - \frac{E^2}{(R_S + R_P)}$$

$$= 80 \left(1 + \frac{40}{80 \times 10^3}\right) - \frac{250^2}{(40 + 80 \times 10^3)} = 79.26 \text{watt}$$

$$\text{Iron loss/ kg} = 79.26/10 = 7.926 \text{ w/kg.}$$