Comparing Massive MIMO at Sub-6 GHz and Millimeter Wave Using Stochastic Geometry

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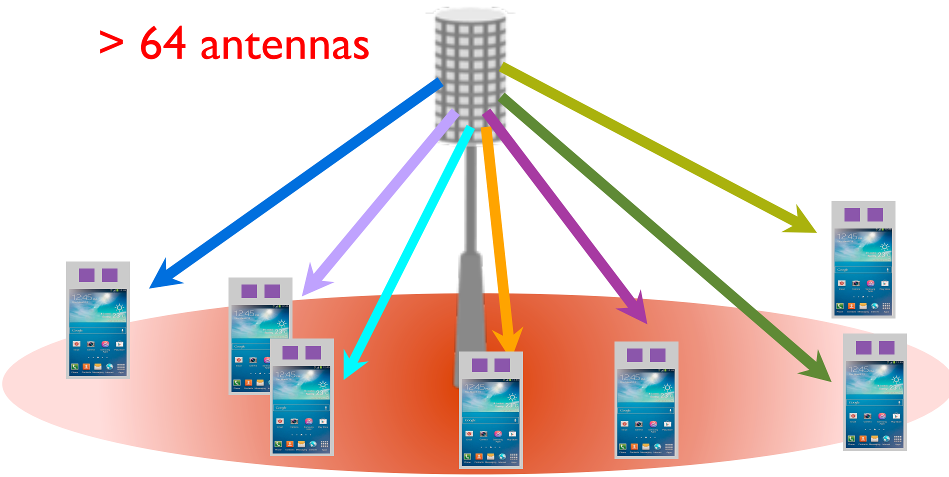
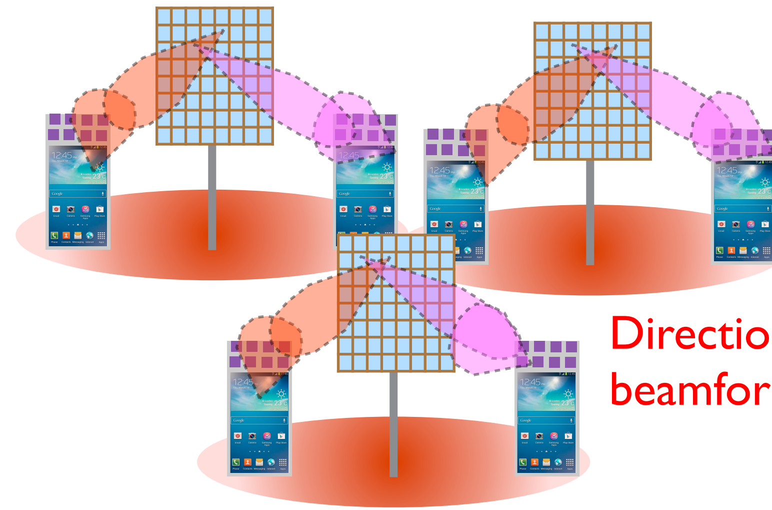
Going massive in 5G

Massive MIMO at sub 6 GHz



10 to 30 users sharing same resources

Massive MIMO at mmWave with small cells



Directional beamforming

Fewer than 4 users sharing same resources

Massive MIMO and small cells are a competing or complimentary technology depending on the carrier frequency

* T. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," IEEE Trans. Wireless Commun., Nov. 2011
** T. Rappaport et al., "Millimeter wave mobile communications for 5G cellular: It will work!" IEEE Access, vol. 1, pp. 335–349, 2013.
*** W. Roh et al., "Millimeter-wave beamforming as an enabling technology for 5G cellular communications: theoretical feasibility and prototype results," IEEE Commun. Magazine, vol. 52, no. 2, pp. 106–113, February 2014.

Outline

- Features of massive MIMO at sub-6 GHz and mmWave
- Framework for comparison
- Analytical results with infinite & finite #s of antennas
- Visualizing the gains of going massive

Some results are described here:

Tianyang Bai and R.W. Heath, Jr., `` Asymptotic Coverage and Rate in Massive MIMO Networks," Proc. of the IEEE Global Signal and Information Processing Conference, Atlanta, GA, Dec. 3-5, 2014

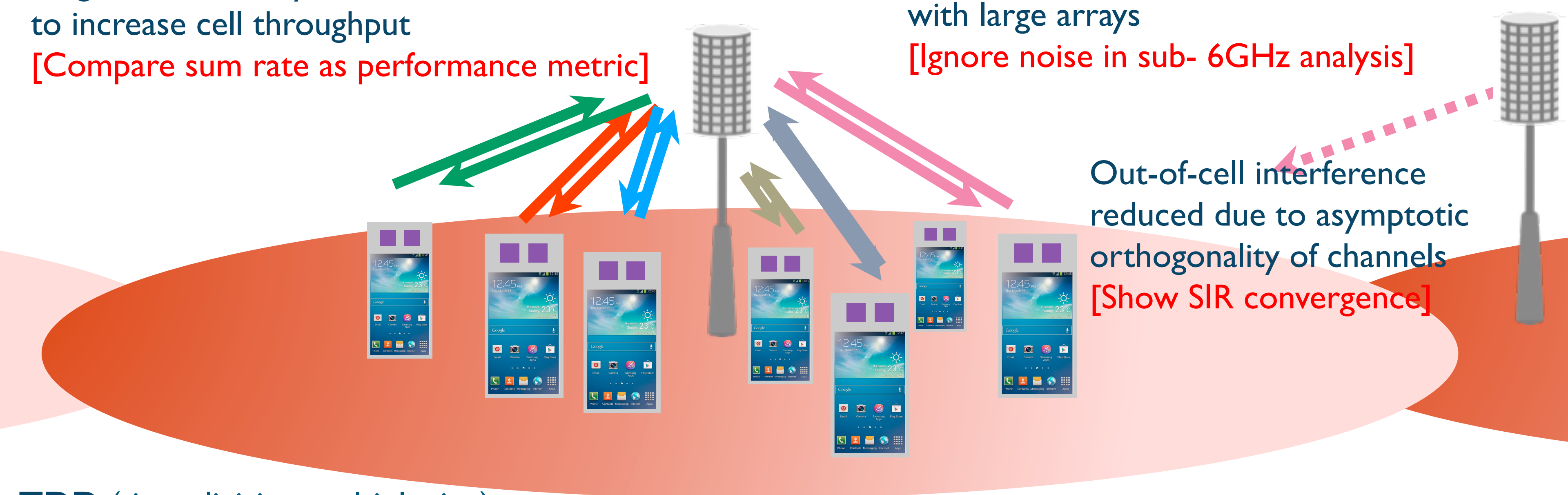
Other results are in various submitted papers

Massive MIMO at sub-6 GHz

Features of massive MIMO & implications

Large antenna arrays serve more users to increase cell throughput
[Compare sum rate as performance metric]

Fading and noise become minor with large arrays
[Ignore noise in sub- 6GHz analysis]



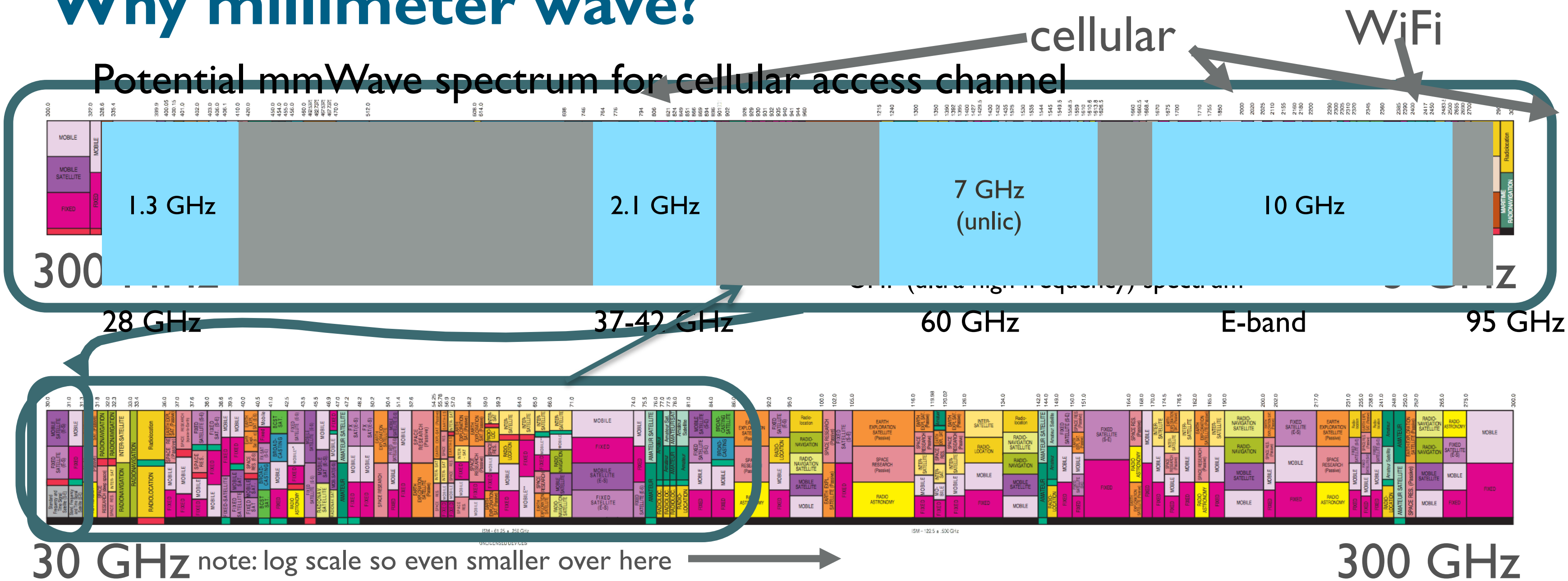
Out-of-cell interference reduced due to asymptotic orthogonality of channels
[Show SIR convergence]

TDD (time-division multiplexing) avoids **downlink** training overhead
[Include pilot contamination]

Simple signal processing becomes near-optimal, with large arrays
[Assume matched filter beamforming]

Massive MIMO at millimeter wave

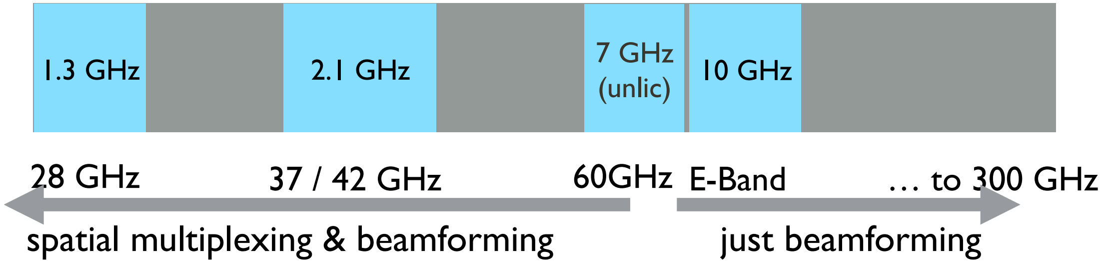
Why millimeter wave?



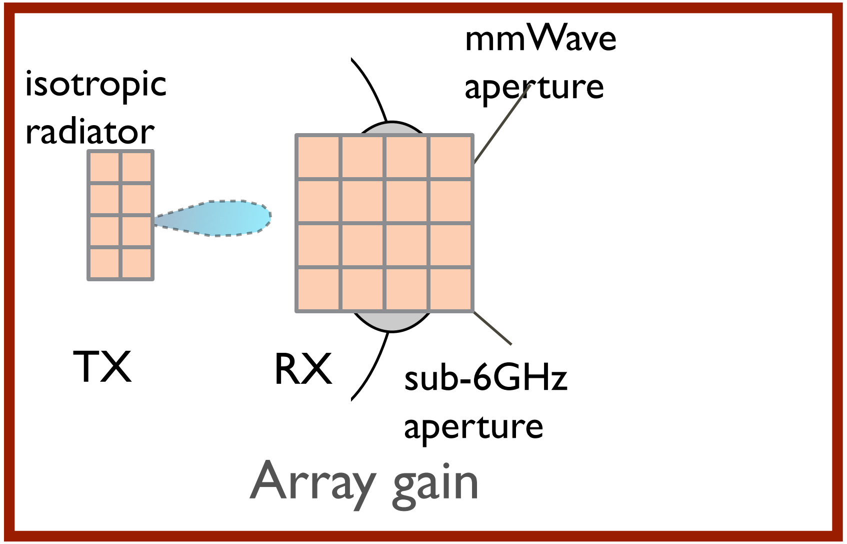
- Huge amount of spectrum possibly available in mmWave bands
- Technology advances make mmWave possible for low cost consumer devices
- MmWave research is as old as wireless itself, e.g. Bose 1895 and Lebedow 1895

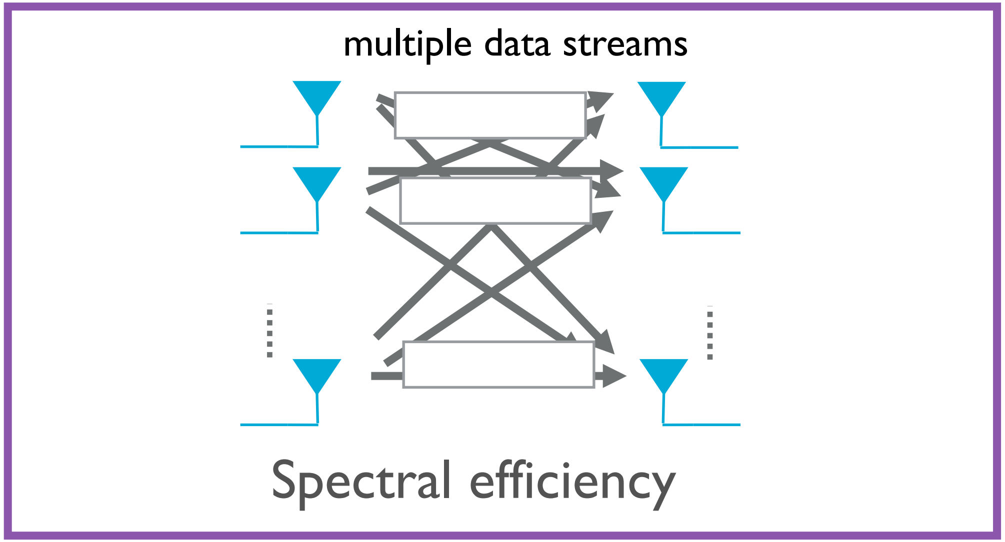
Why large arrays at mmWave?

millimeter wave band possible bands used for cellular



MIMO is a key feature of 5G mmWave systems





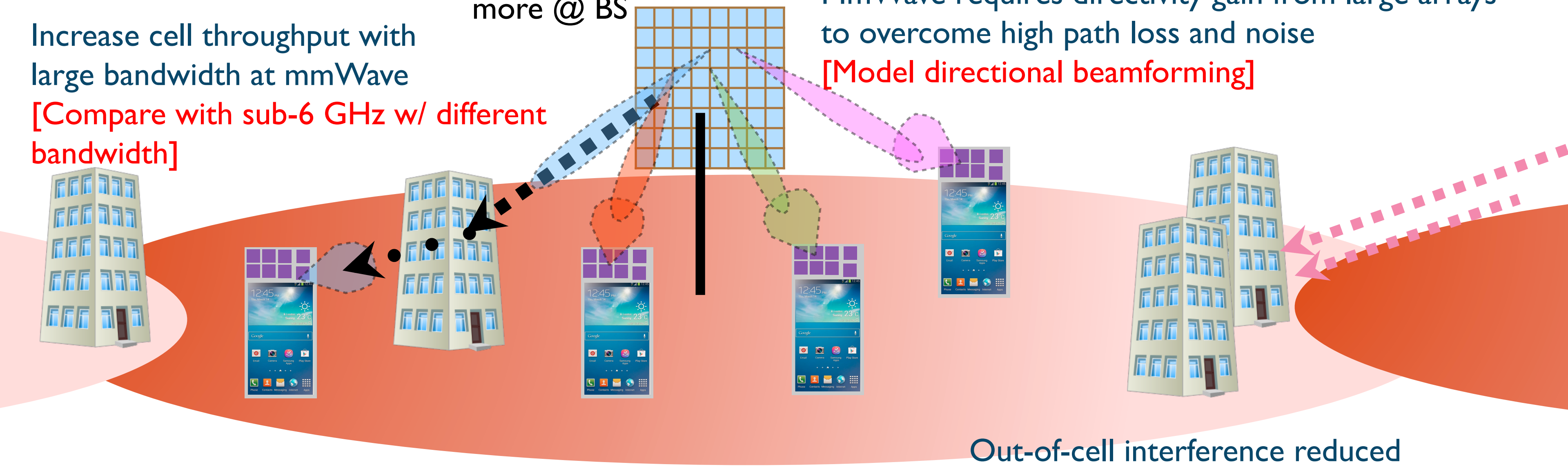
* Shu Sun, T. Rappapport, R.W. Heath, Jr., A. Nix, and S. Rangan, `` MIMO for Millimeter Wave Wireless Communications: Beamforming, Spatial Multiplexing, or Both?," IEEE Communications Magazine, December 2014.

Features of mmWave massive MIMO & implications

256 antennas or more @ BS

Increase cell throughput with large bandwidth at mmWave
[Compare with sub-6 GHz w/ different bandwidth]

MmWave requires directivity gain from large arrays to overcome high path loss and noise
[Model directional beamforming]



Exploit channel sparsity to reduce training overhead
[Apply compressed sensing channel estimation (future work)]

Out-of-cell interference reduced due to directional transmission and blockage
[Incorporate blockages]

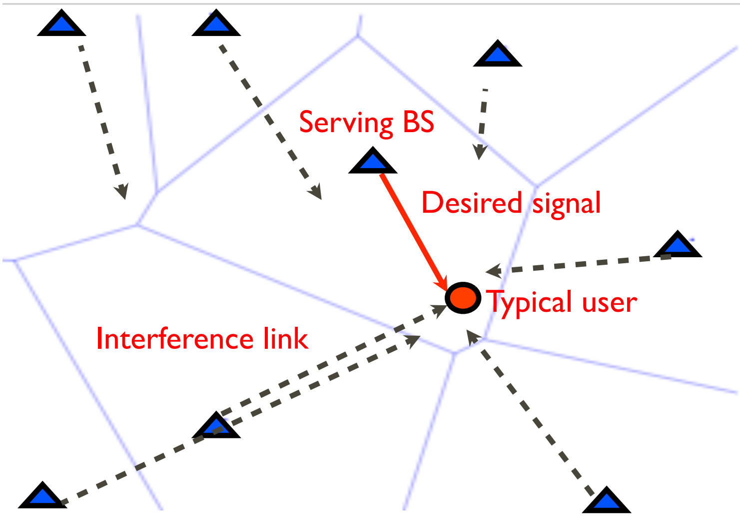
Need common framework to make a fair comparison

Differentiating features between sub-6 GHz & mmWave included in the analysis

	sub-6 GHz	mmWave
bandwidth	~100 MHz	500 GHz @28 GHz 2 GHz @E-Band
small-scale fading	correlated with high rank	correlated with low rank, varies with LOS or NLOS
large-scale fading	distant dependent pathloss	distant dependent with random blockage model and total outage
network deployment	low BS density	high BS density
UE array configuration	single antenna	directional antenna with gain
# users served simultaneously	higher (10 or more)	1 to 4 users (limited by hardware)

Comparisons built around a stochastic geometry framework

Stochastic geometry in cellular systems



Modeling base stations locations as Poisson point process

Shows reasonable fits with real BS distributions

Analyzes the system performance in large networks (in closed form for certain cases)

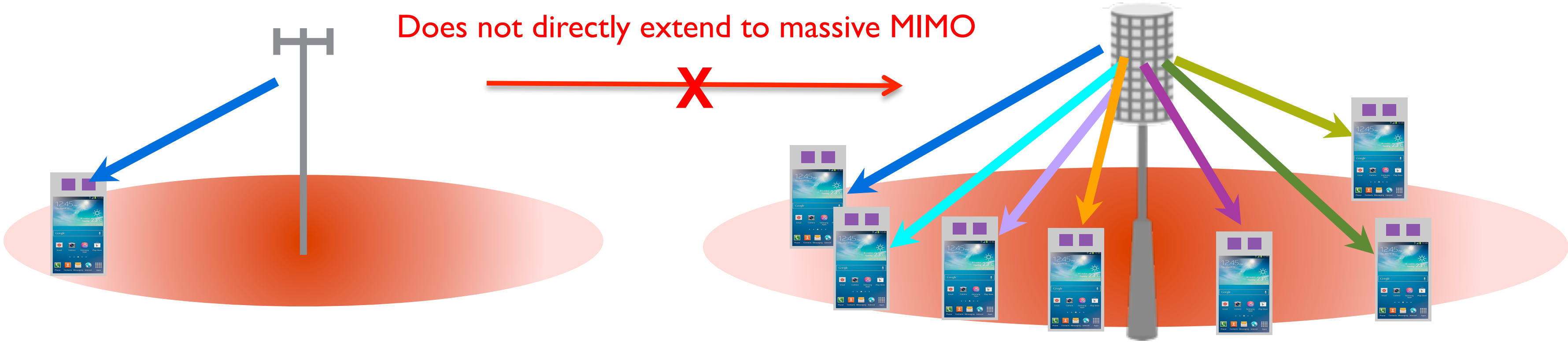
Extends to many applications: Heterogeneous, offloading, mmWave …

Apply stochastic geometry to compare massive MIMO @ sub-6 GHz and mmWave

[1] T. X Brown, ``Practical Cellular Performance Bounds via Shotgun Cellular System," IEEE JSAC, Nov. 2000.
[2] M. Haenggi, J. G. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, " Stochastic geometry and random graph for the analysis and design of wireless networks", IEEEJSAC 09
[3] J. G. Andrews, F. Baccelli, and R. K. Ganti, " A tractable approach to coverage and rate in cellular networks", IEEE TCOM 2011.
[4] H. S. Dhillon, R. K. Ganti, F. Baccelli, and J. G. Andrews, " Modeling and analysis of K-tier downlink heterogeneous cellular networks", IEEE JSAC, 2012

& many more…

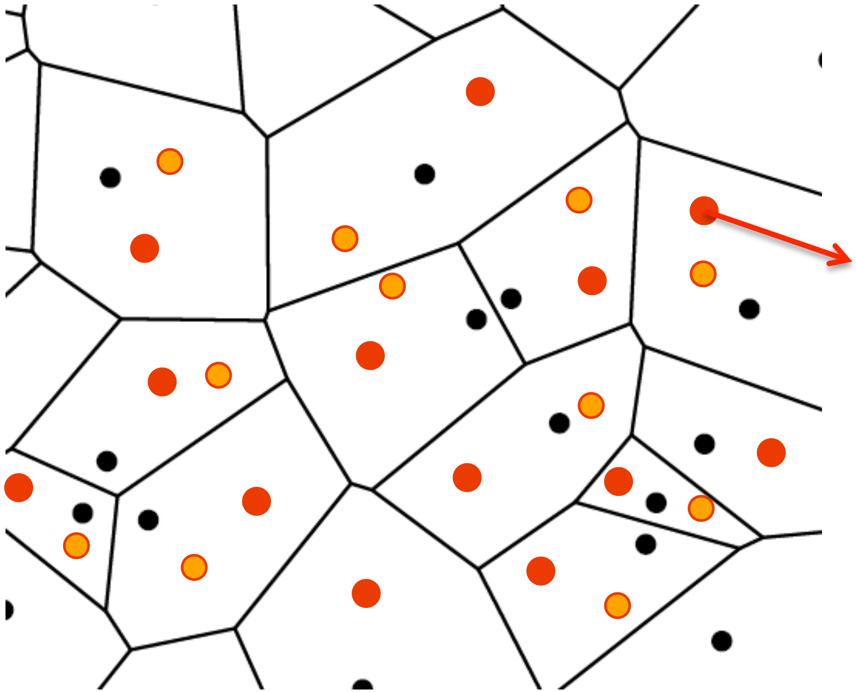
Challenges of analyzing massive MIMO using SG



Most prior SG cellular models	Massive MIMO model
Single user per cell	Multiple user per cell
Single base station antenna	Massive base station antennas
Rayleigh fading	Correlated fading MIMO channel
No channel estimation	Pilot contamination
Mainly focus on downlink	Analyze both uplink and downlink

Sub-6 GHz massive MIMO: system model

System model



- Base station w/ M antennas
- 1st scheduled user
- 2nd scheduled user

Presence of a "red" user in one cell prevents those of the other red

Base stations distributed as a PPP

Users PPP w/ high density
BS randomly schedules K users

Scheduled users do not form a PPP (# of scheduled users fixed)
→ Use certain hardcore Matérn process

[1] H. El Sawy and E. Hossain, "On stochastic geometry modeling of cellular uplink transmission with truncated channel inversion power control" IEEE TCOM, 2014
[2] S. Singh, X. Zhang, and J. Andrews, " Joint rate and SINR coverage analysis for decoupled uplink-downlink biased cell association in HetNet," Arxiv, 2014

Channel model

Channel vector from BS l to user k in cell n

IID Gaussian vector for fading

$$\mathbf{h}_{\ell n}^{(k)} = \left(\beta_{\ell n}^{(k)}\right)^{1/2} \mathbf{\Phi}_{\ell n}^{(k)1/2} \mathbf{w}_{\ell n}^{(k)}$$

Bounded path loss model

Covariance matrix for correlated fading

Path loss of a link with length R

$$C \max(\delta, R)^{-\alpha}$$

Mean square of eigenvalues uniformly bounded

$$\lim_{M\to\infty} \sup \frac{1}{M} \sum_{m=1}^{M} \lambda_{\ell n}^{(k)}[m] \le \gamma$$

Address near-field effects in path loss

Reasonable for rich scattering channel

Uplink channel estimation

Assume perfect time synchronization
& full pilot reuse in the network

Channel estimate of ℓ-th BS to its k-th user

$$\bar{\mathbf{h}}_{\ell\ell}^{(k)} = \mathbf{h}_{\ell\ell}^{(k)} + \sum_{\ell' \neq \ell} \mathbf{h}_{\ell\ell'}^{(k)}$$

Error from pilot contamination

Need to incorporate pilot contamination in system analysis

Uplink data transmission

BSs perform maximum ratio combining based on channel estimates

$$\mathrm{SIR_U} = \frac{|\bar{\mathbf{h}}_{00}^{(1)*}\mathbf{h}_{00}^{(1)}|^2}{\sum_{\ell>0} |\bar{\mathbf{h}}_{00}^{(1)*}\mathbf{h}_{0\ell}^{(1)}|^2 + \sum_{k>1}^{K}\sum_{\ell\geq 0} |\bar{\mathbf{h}}_{00}^{(1)*}\mathbf{h}_{0\ell}^{(k)}|^2}$$

As M grows large

Out-of-cell interference with different pilots disappears from expression

Downlink data transmission

BSs perform match-filtering beamforming based on channel estimates

$$\mathrm{SIR_D} = \frac{|\mathbf{h}_{00}^{(1)*}\mathbf{f}_0^{(1)}|^2}{\sum_{\ell \neq 1} |\mathbf{h}_{\ell 0}^{(1)*}\mathbf{f}_\ell^{(1)}|^2 + \sum_{k=2}^{K}\sum_{\ell>0} |\mathbf{h}_{\ell 0}^{(k)*}\mathbf{f}_\ell^{(k)}|^2}$$

Match-filtering precoder:

$$\mathbf{f}_\ell^{(k)} = \frac{\bar{\mathbf{h}}_{\ell\ell}^{(k)}}{||\bar{\mathbf{h}}_{\ell\ell}^{(k)}||}$$

As M grows large

Out-of-cell interference with different pilots disappears from expression

Sub-6 GHz massive MIMO: asymptotic performance analysis when # of BS antennas goes to infinity

Prior results assuming IID fading & finite # BSs

UL received signal — desired signal — pilot contamination — interference

$$\hat{s}_0^{(1)} = \frac{\|\mathbf{h}_{00}^{(1)}\|^2 s_0^{(1)}}{M} + \frac{\sum_{\ell>1} \|\mathbf{h}_{0\ell}^{(1)}\|^2 s_\ell^{(1)}}{M} + \frac{\sum_{k=2}^{K} \mathbf{h}_{0\ell}^{(1)*} \mathbf{h}_{0\ell'}^{(k)} s_\ell^{(k)}}{M}$$

By LLN for IID variables

$$\lim_{M\to\infty} \mathbf{h}_{st}^{(k)*} \mathbf{h}_{st}^{(k)} / M \overset{p.}{=} \beta_{st}^{(k)}$$

swap limit and sum in finite sum

$$\lim_{M\to\infty} \mathbf{h}_{st}^{(k)*} \mathbf{h}_{s't'}^{(k')} / M \overset{p.}{=} 0$$

$$\lim_{M\to\infty} \hat{s}_0^{(1)} \overset{p.}{=} \beta_{00}^{(1)} s_0^{(1)} + \sum_{\ell>0} \beta_{0\ell}^{(1)} s_\ell^{(1)} + \mathbf{0}$$

$$\mathrm{SIR}_{\mathrm{UL}} \overset{p.}{\to} \frac{\left(\beta_{00}^{(1)}\right)^2}{\sum_{\ell\neq 0} \left(\beta_{0\ell}^{(1)}\right)^2}.$$

What about spatial correlation and infinite number of BSs??

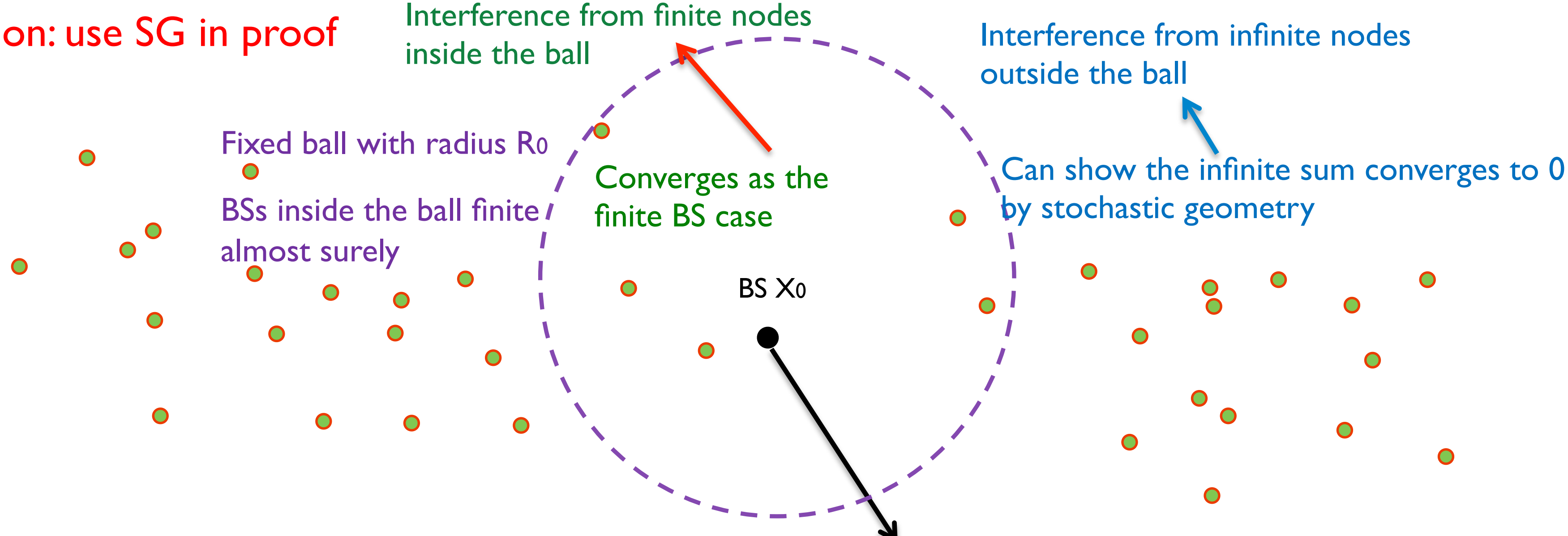
* T. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," IEEE Trans. Wireless Commun., Nov. 2011

Dealing with infinite interferers

Difficulty: cannot swap limit and infinite sum directly, with infinite BSs

$$\sum_{\ell>0}\left(\frac{||\mathbf{h}_{0\ell}^{(1)}||^2}{M}-\beta_{0\ell}^{(1)}\right)=\sum_{\ell:\|Y_\ell^{(1)}-X_0\|\le R_0}\left(\frac{||\mathbf{h}_{0\ell}^{(1)}||^2}{M}-\beta_{0\ell}^{(1)}\right)+\sum_{\ell:\|Y_\ell^{(1)}-X_0\|>R_0}\left(\frac{||\mathbf{h}_{0\ell}^{(1)}||^2}{M}-\beta_{0\ell}^{(1)}\right)$$

Solution: use SG in proof



Use stochastic geometry to prove convergence of infinite sum

Asymptotic SINR results

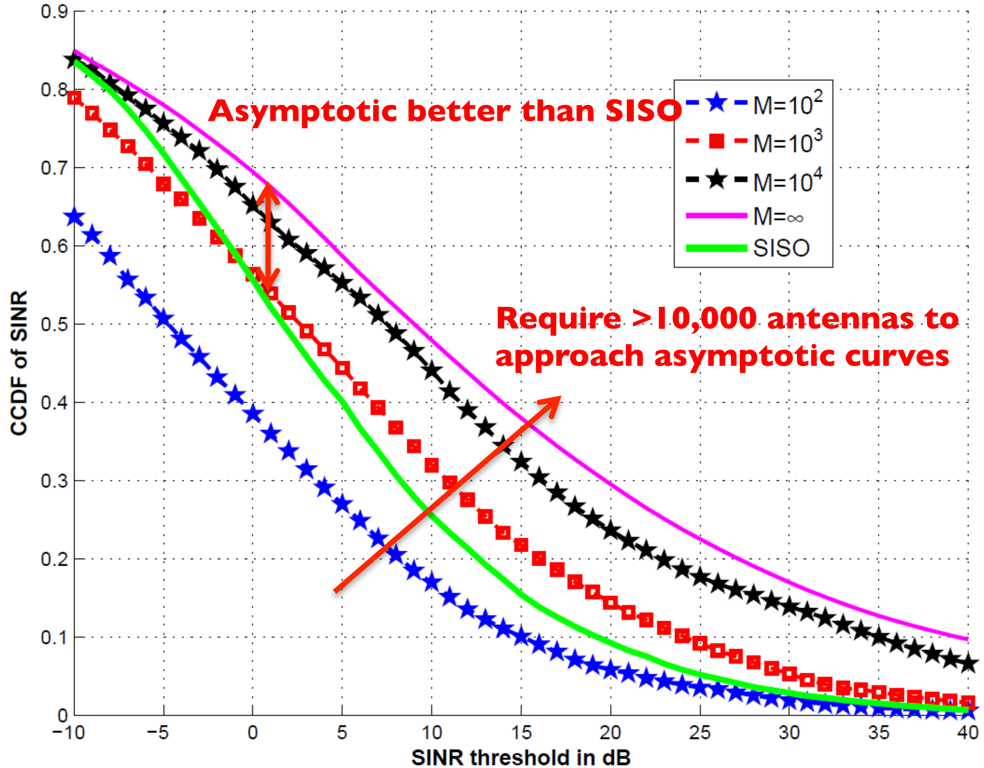
under the bounded spatial correlation model the following hold

Compared with SISO: Path loss exponent **doubles** Fading vanishes	Asymptotic SINR expression	CCDF of SINR
Uplink	$\frac{\left(\beta_{00}^{(1)}\right)^2}{\sum_{\ell \neq 0}\left(\beta_{0\ell}^{(1)}\right)^2}$	$1-\mathrm{e}^{-\left(\frac{\alpha-1}{T}\right)^{1/\alpha}}$ DL and UL SIR distribution are different
Downlink	$\frac{\beta_{00}^{(1)2}/a_0^{(1)}}{\sum_{\ell \neq 0}\beta_{\ell 0}^{(1)2}/a_\ell^{(1)}}$	$\min\left(1, \frac{\alpha \sin(\pi/\alpha)}{\pi T^{1/\alpha}}\right)$

$a_\ell^{(k)} = \sum_{\ell'} \beta_{\ell\ell'}^{(k)}$ due to power normalization in DL

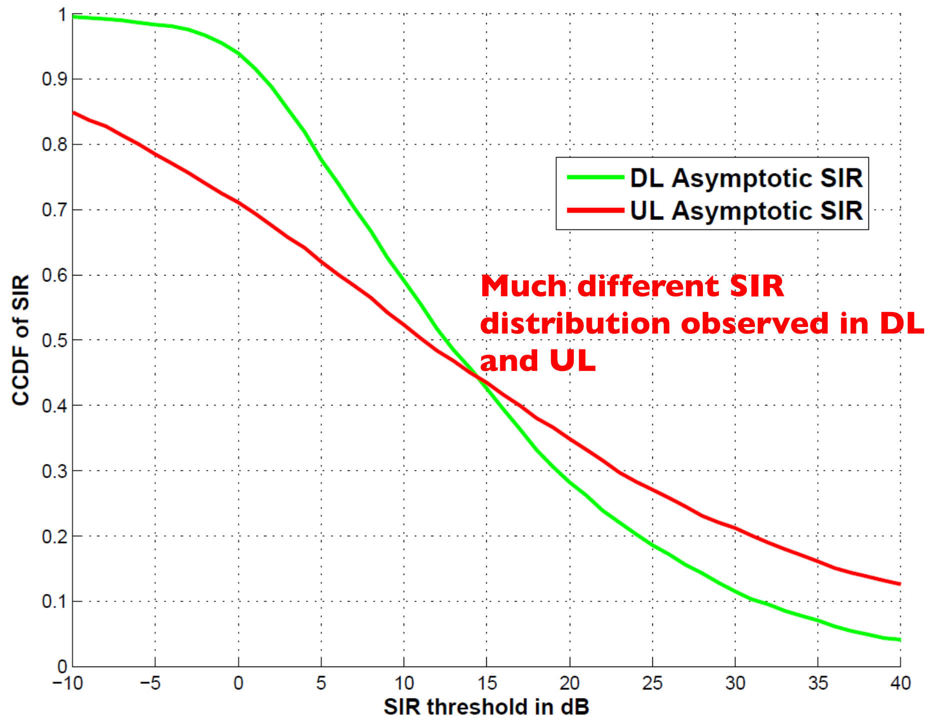
Stochastic geometry allows simple expressions for coverage

Asymptotic uplink SIR plots



Convergence to asymptotic SIR
(IID fading, K=10, α=4)

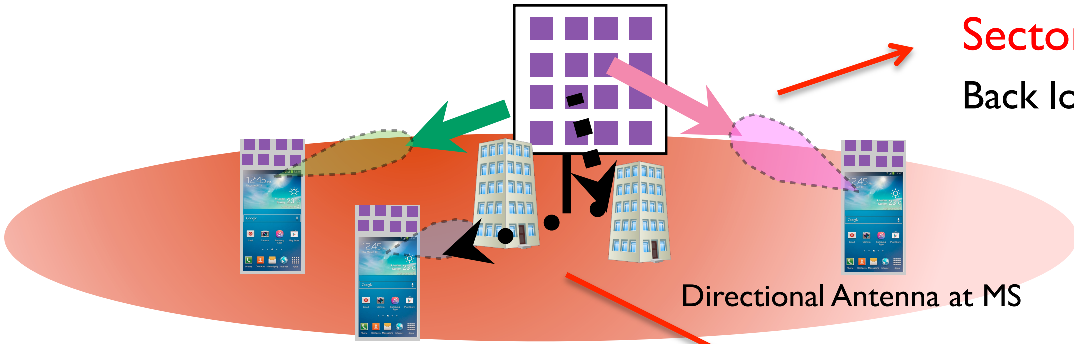
Comparing UL and DL distribution



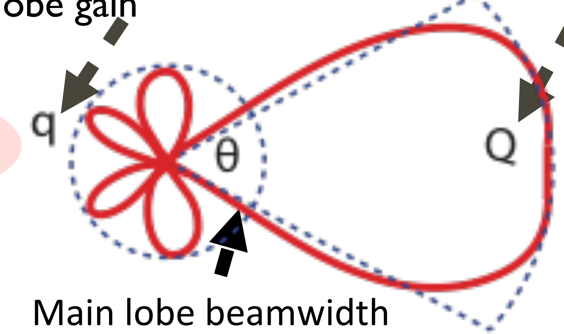
Indicate decoupled system design for DL and UL

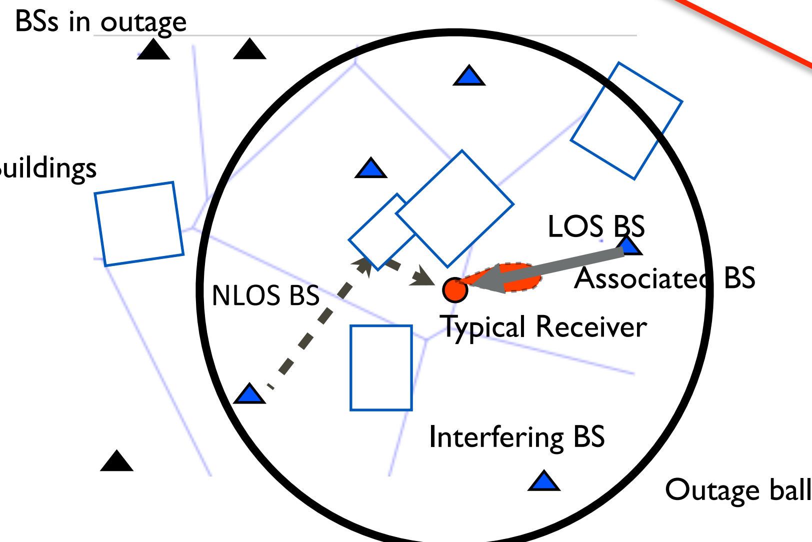
MmWave massive MIMO

MmWave massive MIMO network model



Sectored beamforming pattern model @ UE





Modeling blockage effects of buildings

Use LOS probability function of the link length to determine LOS/ NLOS/ outage

	LOS	NLOS	Total outage
Path loss	LOS path loss	NLOS path loss	No signal
Fading	Deterministic (no fading)	Sub-6 GHz fading (w/ more correlations)	NA

M. R. Akdeniz, Y. Liu, M. K. Samimi, S. Sun, S. Rangan, T. S. Rappaport, and E. Erkip, " Millimeter Wave Channel Modeling and Cellular Capacity Evaluation," IEEE JSAC, 2014.
T. Bai, R. Vaze, and R. W Heath, Jr., " Analysis of Blockage Effects on Urban Cellular Networks", IEEE Trans. Wireless, 2014.

MmWave asymptotic SINR results

	Asymptotic SINR expression	CCDF of SINR
Asymptotic mmWave uplink	$\frac{Q^2 \beta_{00}^{(1)2}}{\sum_{\ell \neq 0} D_{0\ell}^{(1)2} \beta_{0\ell}^{(1)2}}$	$A \sum_{n=1}^{N} \binom{N}{n} (-1)^n \times \int_0^\infty \mathrm{e}^{-W_n(T,t) - V_n(T,t) - \Xi(t)} \Xi(\mathrm{d}t)$
Asymptotic mmWave downlink	$\frac{Q^2 \beta_{00}^{(1)2} / a_0^{(1)}}{\sum_{\ell \neq 0} D_{\ell 0}^{(1)2} \beta_{\ell 0}^{(1)2} / a_\ell^{(1)}}$	$A \sum_{n=1}^{N} \binom{N}{n} (-1)^n \int_0^\infty \mathrm{e}^{-Z_n(T,t) - \Xi(t)} \Xi(\mathrm{d}t)$

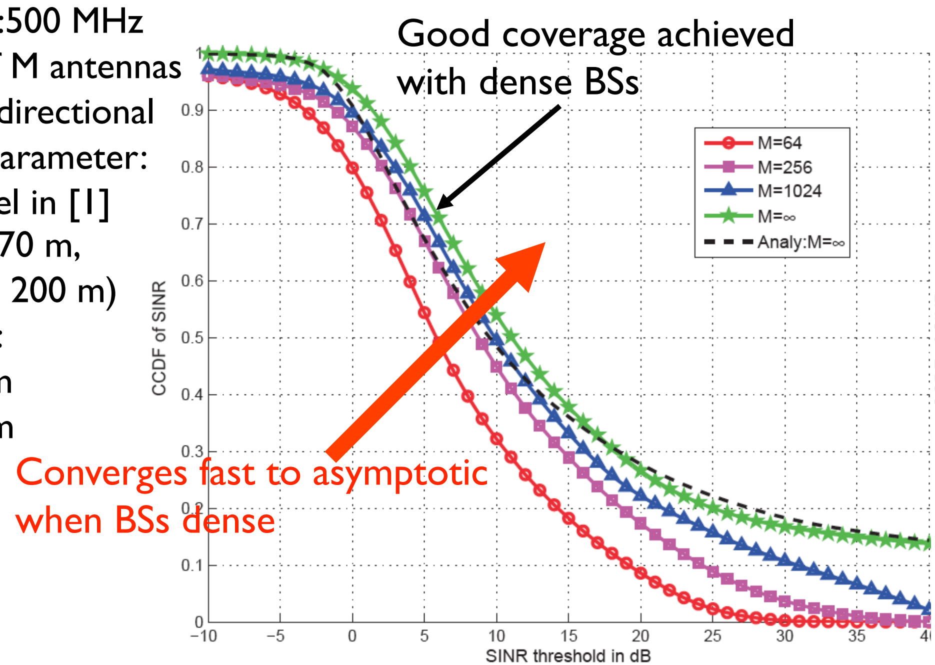
Directivity gain from UE beamforming

Can be computed through numerical integration

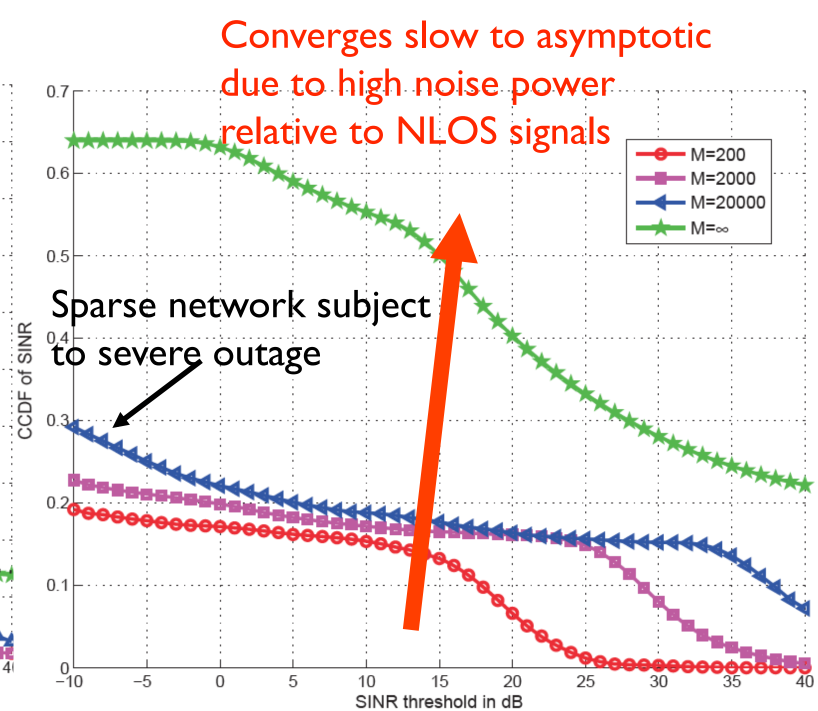
LOS/ NLOS effects make expressions complicated

MmWave SINR sensitive to BS densities

Carrier frequency: 28 GHz
Bandwidth:500 MHz
BS: ULA of M antennas
UE: Omni-directional
Blockage parameter:
NYU model in [1]
(Avg. LOS 70 m,
no signal > 200 m)
TX power:
UL: 20 dBm
DL: 30 dBm



Dense DL mmWave (ISD=100 m)



Sparse DL mmWave (ISD=400 m)

MmWave massive MIMO needs dense BS deployment

Rate comparison

Comparing sub-6 GHz and mmWave massive MIMO

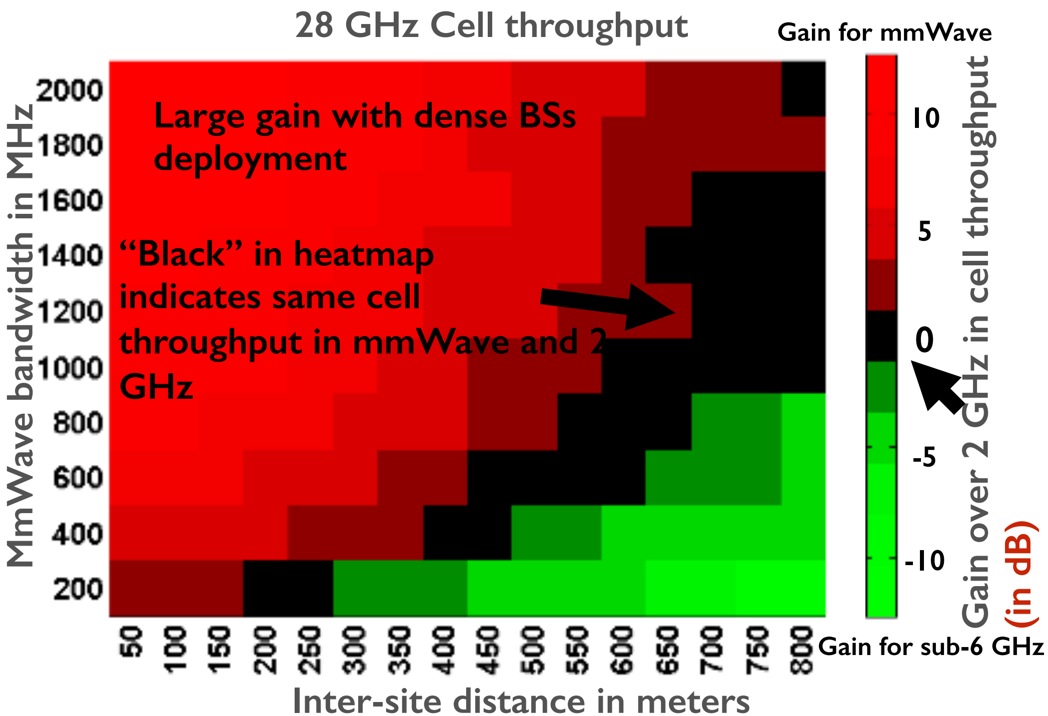
Carrier freq.	2 GHz	28 GHz	73 GHz
bandwidth	100 MHz	Varies	Varies
# of scheduled user per cell	10	4	1
# of base station antennas	8X8	16X16	40X40
# of UE antennas	1	2X2	5X5
TX power (DL/ UL)	46/ 20 dBm	30/ 20 dBm	30/ 20 dBm

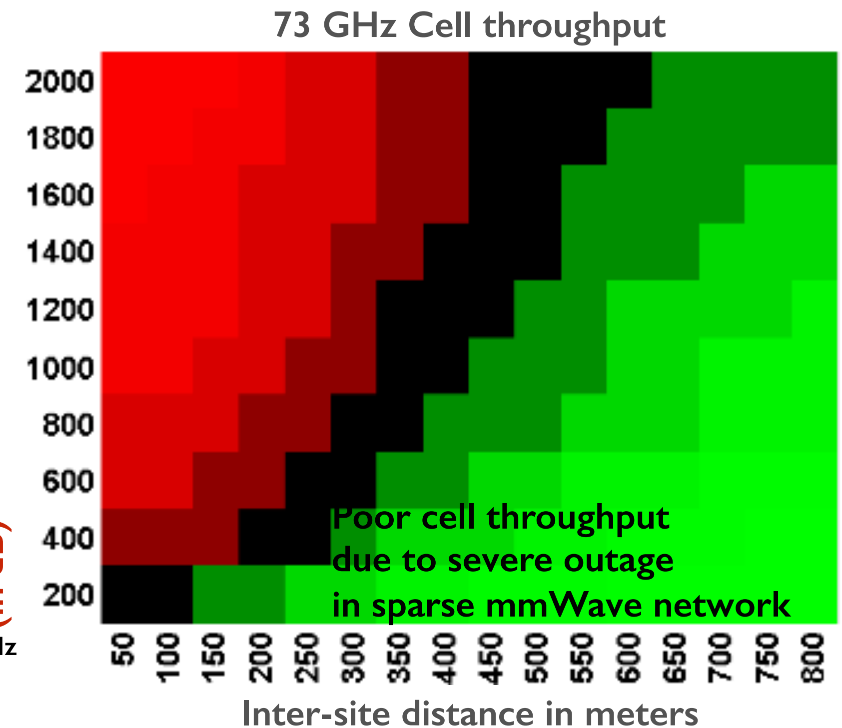
Keep the same aperture in 28 and 73 GHz

1. We vary the bandwidth of mmWave systems in the simulations
2. We assume the same amount of overhead for all systems
3. Use the parameters in the blockage model from [1] based on NYU measurements

[1] M. R. Akdeniz, Y. Liu, M. K. Samimi, S. Sun, S. Rangan, T. S. Rappaport, and E. Erkip, " Millimeter Wave Channel Modeling and Cellular Capacity Evaluation," IEEE JSAC, 2014.

Comparison of average cell throughput





2GHz setup: bandwidth fixed as 100 MHz, while ISD varies

100 m in ISD = 128 BS/ km^2

200 m in ISD = 32 BS/ km^2

MmWave benefits more from network densifications

Comparing massive MIMO w/ small cells

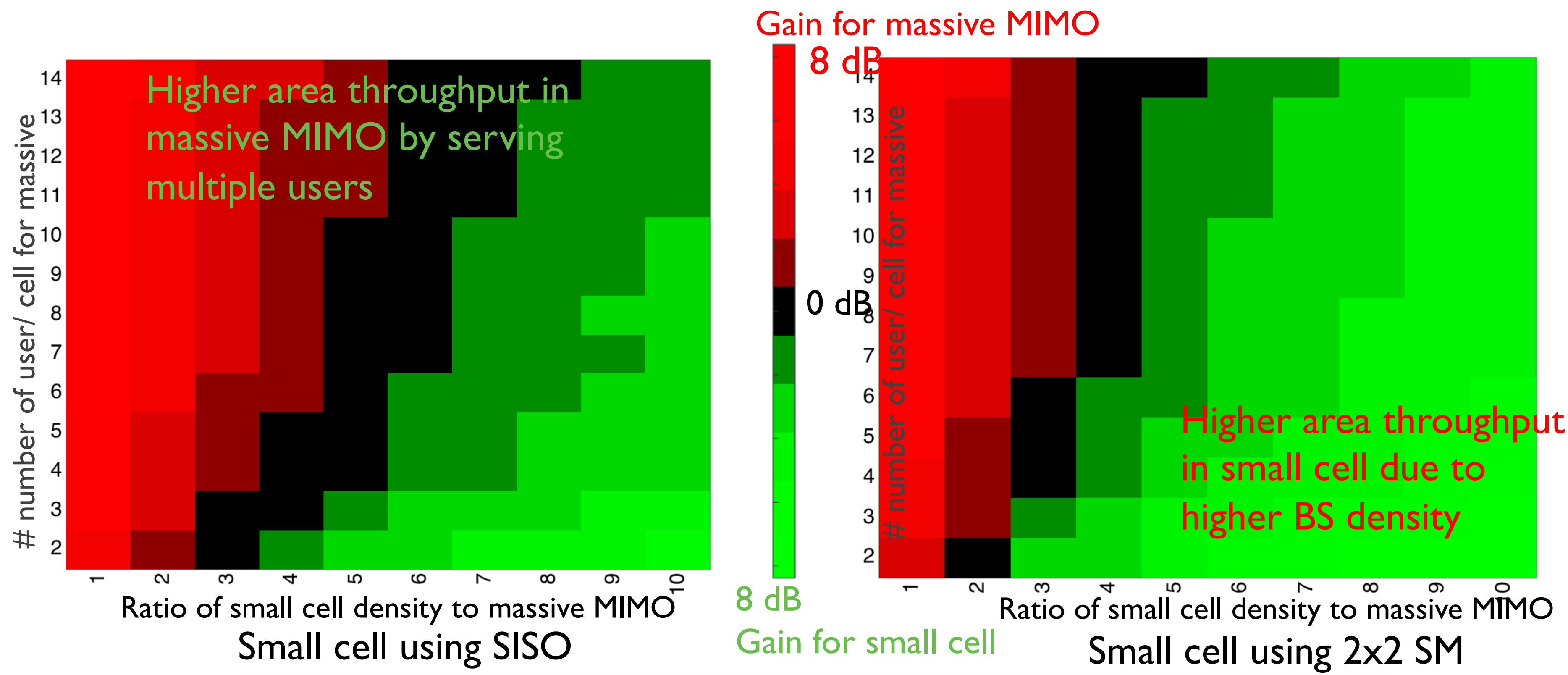
	Sub-6 GHz massive MIMO	28 GHz massive MIMO	73 GHz massive MIMO	Sub-6 GHz Small cell MIMO
# user/ cell	Varies	4	1	1
# BS antenna	8x8	16 x 16	40 x 40	2
# User antenna	1	2x2	5x5	2
Bandwidth	100 MHz	varies	varies	100 MHz

1. Small cell serves its user by 2x2 spatial multiplexing or SISO
2. Assume perfect channel knowledge for small cell case
3. Assume user density 40x macro massive MIMO BS density

Compare throughput per unit area b/w massive MIMO and small cell

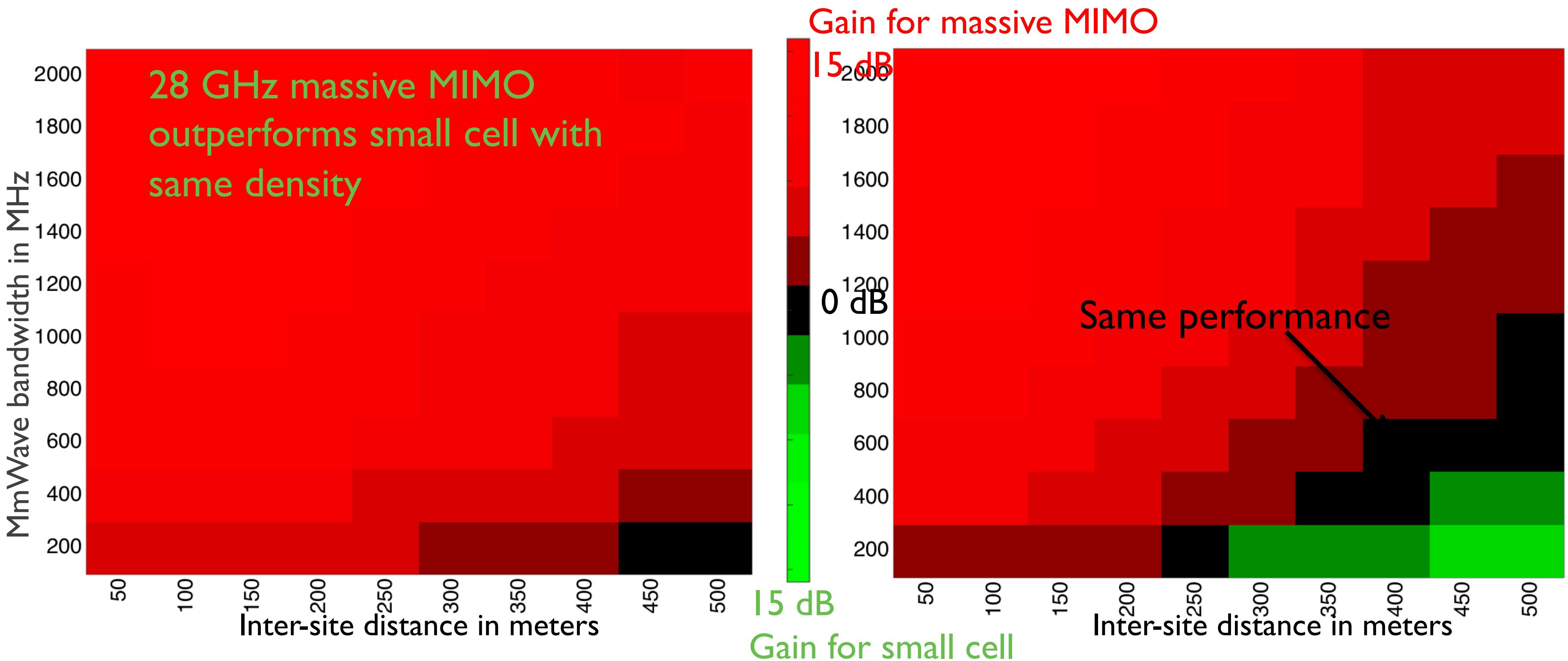
[1] T. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas," IEEE Trans. Wireless Commun., Nov. 2011

Sub-6 GHz massive MIMO vs. Small cell



Sub 6-GHz massive MIMO achieves comparable area throughput using sparser BS deployment

MmWave massive MIMO vs. Small cell



28 GHz vs 2x2 SM small cell

73 GHz vs 2x2 SM small cell

MmWave provides large gain in area throughput in small-cell regime

Conclusions

go massive @ mmWave w/ small cells
go massive @ sub-6 GHz w/ large cells