The Area of a Triangle Using Its Semi-perimeter and the Radius of the In-circle: An Algebraic and Geometric Approach

Lesson Summary:

This lesson is for more advanced geometry students. In this lesson, the students will algebraically prove that the area of a triangle is equal to its semi-perimeter times the radius of its in-circle (A=sr). Then, using Cabri Geometry II, the students will use rotations and translations to transform the triangle into a rectangle. They will then show that the area of the resulting rectangle is equal to the area of the original triangle.

Keywords:
In-circle, semi-perimeter, area of triangle

Existing Knowledge:

Students should have previous knowledge in constructing the in-center and in-circle of a triangle using Cabri Geometry II.

NCTM Standards:

Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Learning Objectives:

Students will prove algebraically that the area of a triangle is equal to its semi-perimeter times the radius of its in-circle (A = sr)
Students will use Cabri Geometry II to transform a triangle into a rectangle that has an area equal to the area of the triangle.

Materials:

Cabri Geometry II or Geometer's Sketchpad

Procedure:

Students should complete Lab 1. The teacher could then have a classroom discussion about the results. Then the students should use Cabri Geometry II to complete Lab 2.

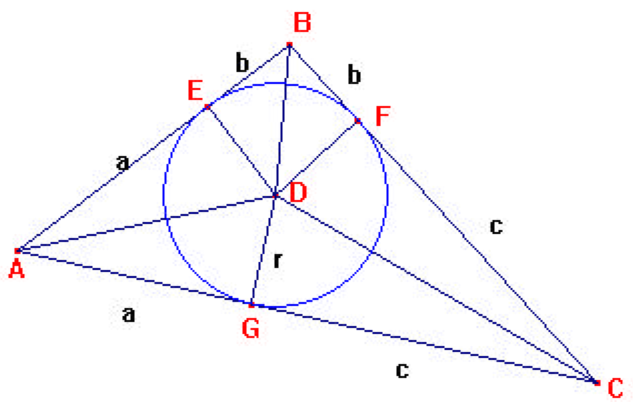
Team Members: ______________________
____

File Name: ______________________

Activity Goals:
In this activity you will algebraically prove that the area of a triangle is equal to its semi-perimeter times the radius of the inscribed circle (A=sr) Then in Laboratory Two, you will use Cabri Geometry II to transform the triangle, using rotations and translations, into a rectangle.

Laboratory One

Notes:
1. This lab involves using in-centers and in-circles of triangles. If you need a review on these topics, refer to the "In-centers and In-circles" lab.
2. The semi-perimeter of a triangle, s, is the same as half the perimeter, p, of the triangle.



Let *D* be the center of the in-circle of $\triangle ABC$. Let p be the perimeter of $\triangle ABC$. Let *s* be the semi-perimeter of $\triangle ABC$. Let *r* be the radius of circle *D*. Prove that the area of $\triangle ABC$ is equal to the length of its semi-perimeter times the radius of circle *D*. (A=*sr*)

1. What is the relationship between the radii of circle *D* and the tangent segments $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$?

   ____

2. What can you conclude about the six triangles formed? ______________________

3. Are $\triangle AGD$ and $\triangle AED$ congruent? Why or why not? __________________

   ____

4. Would this also be true for the other pairs of triangles shown? Why or why not?

____

5. Why would the area of $\triangle ABC$ be equal to the sum of the areas of the six triangles?
____

6. Write an equation for the area of $\triangle ABC$ using the sum of the areas of the six right triangles formed (in terms of a, b, c, and r).
____

7. Using the space provided, simplify and factor your above equation and write the resulting equation here. ____

8. What is the relationship of $(a + b + c)$ to the perimeter of $\triangle ABC$?

____

9. In your own words, what is the area of $\triangle ABC$? _________________________

____

Laboratory Two

1. Construct $\triangle ABC$. ***(Triangle tool)***

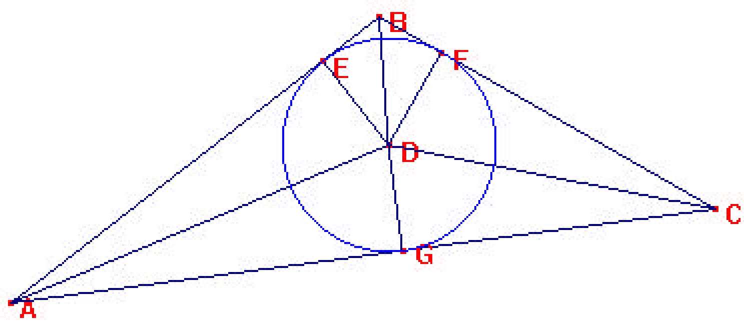
2. Construct the angle bisector of $\angle A$, $\angle B$ and $\angle C$, and label their intersection point *D*. ***(Angle bisector tool and point tool)***

3. Construct $\overline{AD}$, $\overline{BD}$ and $\overline{CD}$ and then hide the angle bisectors. ***(Segment tool and hide/show tool)***

4. Construct perpendicular lines from point *D* to sides $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ and label the points of intersection as *E*, *F*, and *G*, respectively. ***(Perpendicular tool and point tool)***

5. Construct $\overline{DE}$, $\overline{DF}$ and $\overline{DG}$, then hide the perpendicular lines. ***(Segment tool and hide/show tool)***

6. Construct a circle with center *D* and radius $\overline{DE}$. ***(circle tool)***



7. Construct the midpoint of segments $\overline{AD}$, $\overline{CD}$ and $\overline{BD}$. ***(Midpoint tool)***

8. Construct $\triangle AGD$, $\triangle CGD$ and $\triangle BFD$. ***(Triangle tool)***

9. Make a numerical edit of 180 degrees. ***(Numerical edit tool)***

10. Rotate $\triangle AGD$ using the 180 degrees about the midpoint of $\overline{AD}$. Name the rotated triangle as $\triangle ADH$. ***(Rotation tool and label tool)***

11. Rotate $\triangle CGD$ using the 180 degrees about the midpoint of $\overline{CD}$. Name the rotated triangle as $\triangle CDI$. ***(Rotation tool and label tool)***

12. Rotate $\triangle BFD$ using the 180 degrees about the midpoint of $\overline{BD}$.

Name the rotated triangle as $\triangle BDJ$. ***(Rotation tool and label tool)***

13. Construct polygon *BFDJ* and then hide $\triangle BDJ$. ***(polygon tool and hide/show tool)***

14. Construct vector *DI* and translate polygon *BFDJ* using vector *DI*. Name the translated polygon clockwise *IKLM*. ***(vector tool, translation tool, label tool)***

15. Hide polygon *BFDJ* point *J*, and vector *DI*. ***(hide/show tool)***

16. Measure $\angle KID$ and rotate polygon *IKLM* using the measure of $\angle KID$ about point *me*. Name the new polygon clockwise *INOP*. ***(angle tool, rotation tool, label tool)***

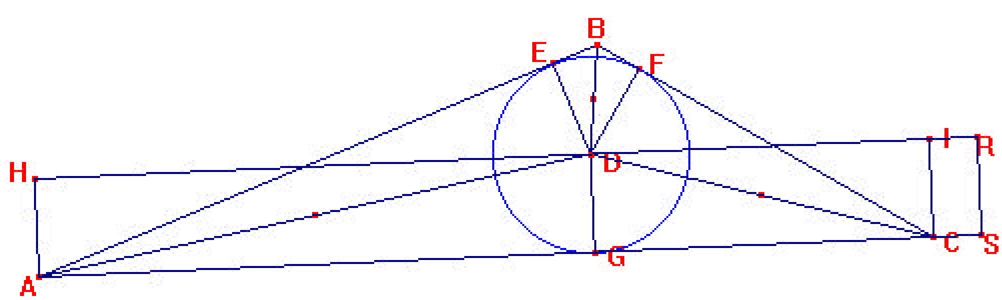
17. Hide polygon *IKLM* and points *K*, *L*, and *M*. Also hide the angle measure. ***(Hide/show tool)***

18. Construct vector *NI* and translate polygon *INOP* using vector *NI*. Name the translated polygon clockwise *IPQR*. ***(Vector tool, translation tool, label tool)***

19. Hide polygon *INOP*, vector *NI*, and points *N* and *O*. ***(hide/show tool)***

20. Construct vector *IC* and translate polygon *IPQR* using vector *IC*. Name the new polygon *CIRS*. ***(Vector tool, translation tool, label tool)***

21. Hide polygon *IPQR*, points *P* and *Q*, and vector *IC*. ***(hide/show tool)***



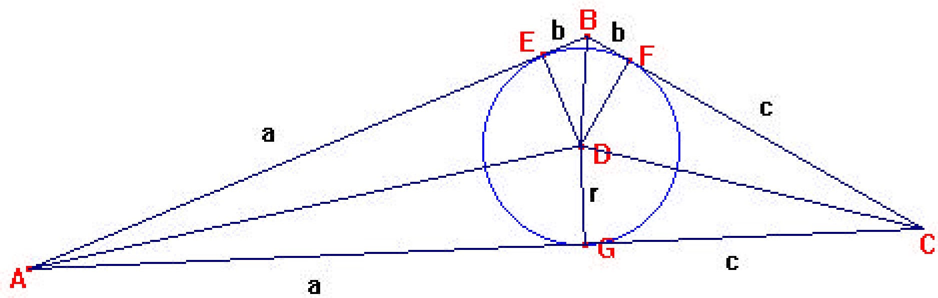
22. Construct polygon *AHRS* and find its area. Also find the area of $\triangle ABC$. [polygon tool and area tool]

23. What is the relationship of the area of polygon *AHRS* and the area of $\triangle ABC$?

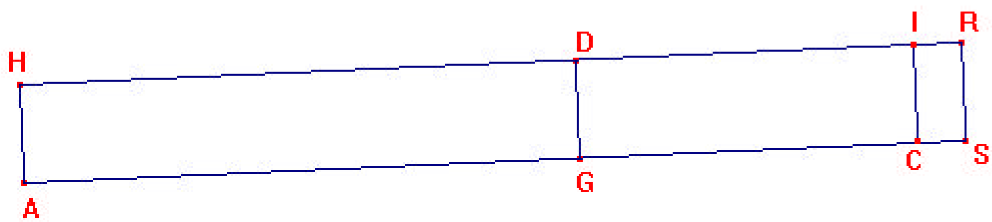
____

24. Grab and move point *A*. Do the areas always stay equal? If not, when are they different?

____



25. The rectangle below is the transformation of the given $\triangle ABC$ above. Mark all the segments of the rectangle with the appropriate lengths.



26. What is the equation for the area of rectangle *AHRS*?

____

27. The equation for the area of $\triangle ABC$ was completed in Lab 1. What do you notice about the equation for the area of rectangle AHRS and the equation for the area of $\triangle ABC$?

____