The Mathematics of Aircraft Navigation
Thales
Aeronautical Engineering

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Aircraft Navigation is the art and science of getting from a departure point to a destination in the least possible time without losing your way. If you are a pilot of a rescue helicopter, you need to know the following:

- Start point (point of departure)
- End point (final destination)
- Direction of travel
- Distance to travel
- Characteristics of type of aircraft being flown
- Aircraft cruising speed
- Aircraft fuel capacity
- Aircraft weight and balance information
- Capability of on-board navigation equipment

Some of this information can be obtained from the aircraft operation handbook. Also, if taken into consideration at the start of the aircraft design they help an aeronautical engineer to develop a better aircraft.

Vectors: A Reminder

A vector quantity has both magnitude and direction. It can be represented geometrically using a line segment with an arrow. The length of the vector represents the magnitude drawn to scale and the arrow indicates the direction. Velocity is an example of a vector quantity.

Air Speed / Ground Speed / Wind Speed

An aircraft's speed can be greatly enhanced or diminished by the wind. This is the reason for the consideration of two speeds: **ground speed** and **air speed**. Ground speed is the speed at which an aircraft is moving with respect to the ground. Air speed is the speed of an aircraft in relation to the surrounding air. Wind speed is the physical speed of the air relative to the ground. Air speed, ground speed and wind speed are all vector quantities. The relationship between the ground speed V_g, wind speed V_w and air speed V_a is given by

$$V_g = V_a + V_w.$$

Scenario

Knowledge of aircraft navigation is vital for safety. One important application is the search and rescue operation. Imagine that some people are stuck on a mountain in bad weather. Fortunately, with a mobile phone, they managed to contact the nearest Mountain Rescue base for help. The Mountain Rescue team needs to send a helicopter to save these people. With the signal received from the people on the mountain, they determine that the bearing from the helipad (point A) to the mountain (point C) is 054° (i.e. approx. north-east). Also the approximate distance is calculated to be 50 km.

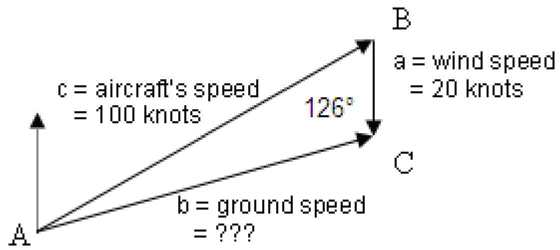
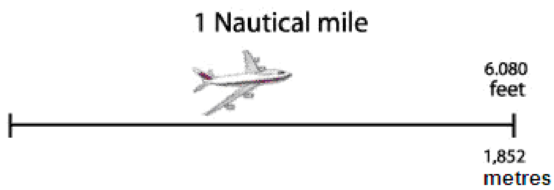


Figure-1

Assume that the rescue helicopter can travel at a speed of 100 knots and a 20-knot wind is blowing on a bearing of 180°. The knot is the standard unit for measuring the speed of an aircraft and it is equal to one nautical mile per hour.



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It is defined as follows in SI:

1 international knot = 1 nautical mile per hour
= 1.852 km/hr exactly
= 1.151 miles/hr approx.
= 0.514 m/sec approx.

Ground Speed and Course

We can quickly and accurately determine the ground speed and course using a sketch to visualise the problem as shown in Figure-1 above. Anticipating the effect of wind, the helicopter needs to fly on a heading aiming at a point B that is slightly more northerly than the mountain (point C).

Using the *cosine rule,* we can find the ground speed b as follows:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$100^2 = 20^2 + b^2 - 2 \times 20 \times b\cos 126$$

$$b^2 - 23.5b - 9600 = 0$$

Using the standard formula for the solution of quadratic equations and taking the positive root:

$$b = 86.7\,knots \approx 45\,m/\text{sec}.$$

Now that we know our ground speed, we can use the *sine rule* to calculate the heading the helicopter should follow. Looking at Figure-1, the heading is equal to the angle *B*. Hence:

$$\sin B = \frac{b\sin C}{c} = \frac{86.7\sin 126}{100}$$

$$B \approx 44.7 \text{ degrees}$$

With this result, we can conclude that if the aircraft takes a heading of 044.7° with a speed of 100 knots under the effect of a wind blowing towards the south with a speed of 20 knots, it actually reduces the ground speed of the aircraft to 86.7 knots and it travels on a bearing of 054°.

Flight Time

The flight time is the actual time an aircraft is in the air flying from its departure point to its destination point. The computed flight time (when the speed is assumed to be constant) is given by the equation:

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

Since the approximate distance between A and C is 50 km and ground speed is approximately 45 m/sec, the helicopter will take approximately 18 minutes flight time as calculated below:

$$\text{Time} = \frac{50 \times 1000}{45} = 1111.11\,\text{sec.} \approx 18\,\text{min}.$$

Fuel Requirements

The pilot uses the following formula to find the fuel consumed during the flight:

$$\text{Fuel Consumed (Litres)} = \text{Fuel Consumption Rate (Litres/hour)} \times \text{Flight Time (Hours)}$$

How much and how fast an aircraft uses fuel is known as the fuel consumption. It varies with engine speed and load (and hence how the aircraft is operated). It can be calculated with the information given in the aircraft operation handbook.

Assume that the fuel consumption rate of this aircraft is 600 litres/hour and it can hold 3200 litres of fuel. Knowing that the helicopter takes 2 minutes to take off vertically to departure height, 2 minutes to land from approach height (both at its base and at the rescue site) and 20 minutes to collect the rescued people at the site with the engine running, if the pilot wishes to have fuel for 30 minutes flying time remaining in the tank when they return to base to allow for emergencies, what is the maximum time they can spend searching for the people?

Time taken from base to point C,

= 2 + 18 + 2 = 22 minutes

Time taken on return journey

= 22 x 2 = 44 minutes

Time taken in rescuing the people = 20 minutes.

Therefore, total time = 20 + 44 = 64 minutes.

Since, the pilot wishes to have fuel for 30 minutes flight time, there must be 300 litres of fuel left in the tank when the helicopter returns to the base. So, the pilot can only use 2900 litres of fuel to complete this task and it will last for 290 minutes, or approximately 4 hours 50 minutes.

$$\text{Flight Time} = \frac{2900}{600} = 4.83\,hrs. \approx 290\,\text{min}$$

$$\approx 4\,hrs. 50\,\text{min}$$

This gives 226 minutes or approximately 3 hours 45 minutes to the rescue team to search for the people on the mountain before they commence the actual lift.

Where to find more

1. *Basic Engineering Mathematics*, John Bird, 2007, published by Elsevier Ltd.
2. www.auf.asn.au/navigation/wind.html
3. www.virtualskies.arc.nasa.gov/navigation/tutorial/tutorial6.html

Dawn Ohlson (M. Eng, C. Eng, FIET) – Director, Educational Affairs, Thales, Surrey

Dawn has spent many years working on Flight Management Systems particularly for Search and Rescue helicopters. This kind of development work uses a lot of maths but is mostly all based on the fundamental principles of trigonometry and vectors. Dawn says that this is engineering at its best as you then have to make it work in the actual helicopter avionics system.

Information for Teachers

The teachers should have some knowledge of

- Vector methods (graphical and component representation of vectors, vector addition and subtraction)
- Manipulating and using trigonometric functions (sine and cosine rules, ratios in right-angled triangle)
- Standard Quadratic Formula to find roots of quadratic equation
- Some terminology related to Aircraft Navigation and Rescue Operation Systems (advisable)

Topics covered from "Mathematics for Engineering"

- Topic 1: Mathematical Models in Engineering
- Topic 4: Functions
- Topic 7: Linear Algebra and Algebraic Processes

Learning Outcomes

- LO 03: Understand the use of trigonometry to model situations involving oscillations
- LO 04: Understand the mathematical structure of a range of functions and be familiar with their graphs
- LO 07: Understand the methods of linear algebra and know how to use algebraic processes
- LO 09: Construct rigorous mathematical arguments and proofs in engineering context
- LO 10: Comprehend translations of common realistic engineering contexts into mathematics

Assessment Criteria

- AC 3.1: Solve problems in engineering requiring knowledge of trigonometric functions
- AC 4.2: Analyse functions represented by polynomial equations
- AC 7.1: Solve engineering problems using vector methods
- AC 9.1: Use precise statements, logical deduction and inference
- AC 9.2: Manipulate mathematical expressions
- AC 9.3: Construct extended arguments to handle substantial problems
- AC 10.1: Read critically and comprehend longer mathematical arguments or examples of applications.

Links to other Units of The Advanced Diploma in Engineering

- Unit-4: Instrumentation and Control Engineering
- Unit-6: Investigating Modern Manufacturing Techniques used in Engineering
- Unit-7: Innovative Design and Enterprise
- Unit-8: Mathematical Techniques and Applications for Engineers
- Unit-9: Principles and Application of Engineering Science